TMD FFs from SIDIS

Andrea Signori

Fragmentation Functions
2018

Stresa, Feb. 21st 2018
Outline

1) TMD FFs

2) extraction from SIDIS data

3) what’s next
TMD FFs
**Definition**

Parton Fragmentation Functions (Metz-Vossen) - DOI: 10.1016/j.ppnp.2016.08.003

\[ \Delta_{ij}(z, P_{h \perp}) = \frac{1}{2z} \sum_{\mathcal{X}} \int \frac{d\xi + \xi^2_T}{(2\pi)^3} e^{ik\xi} \langle 0|\psi_i(\xi)|h\mathcal{X}\rangle\langle \mathcal{X}h|\bar{\psi}_j(0)|0 \rangle \]

**Hadronic variables**

(probabilistic interpretation)

---

Although the parton frame is a natural one for defining fragmentation functions as number densities, it is inconvenient for derivations of factorization. The problem is that, in a physical process, there is an integral over parton momentum, and so the parton-frame axes are not fixed. Neither parton momenta nor the resulting parton-frame axes can be determined from experimentally measured quantities. Therefore we will express the definitions of fragmentation functions in hadron-frame coordinates. In the derivation of factorization, we will use a hadron frame defined in terms of measured quantities.

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**Parton frame**

**Hadron frame**

Different frames for different purposes
**TMD FFs**

A similar table exists for gluon TMD FFs

<table>
<thead>
<tr>
<th>hadron pol.</th>
<th>U</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$D_1$</td>
<td></td>
<td>$H_1^\perp$</td>
</tr>
<tr>
<td>L</td>
<td>$G_{1L}$</td>
<td>$H_{1L}^\perp$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>$D_{1T}^\perp$</td>
<td>$G_{1T}$</td>
<td>$H_1$, $H_{1T}^\perp$</td>
</tr>
</tbody>
</table>

**Correlator for spin 1/2 hadron:**

Dirac matrix parametrized by **quark TMD FFs**

$$D_1^{a\rightarrow h}(z, P_{\perp}^2; \mu, \zeta)$$

**Evolution equations with respect to two scales:**
- UV renormalization
- Rapidity renormalization

Twist-2 table

Diagonal: also collinear
- **Red**: T-odd (but universal, unlike TMD PDFs)
- **Blue**: T-even

A similar table exists for gluon TMD FFs
TMDs and their evolution

FT of TMDs:

\[ \tilde{F}_i(x, b_T; Q, Q^2) = \tilde{F}_i(x, b_T; \mu_b, \mu_b^2) \times \exp \left\{ \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \gamma_F[\alpha_s(\mu), Q^2/\mu^2] \right\} \left( \frac{Q^2}{\mu_b^2} \right)^{-K(\mu_b \tau)} \cdot g_K(b_T, \{\lambda\}) \]

Sudakov form factor: perturbative and nonperturbative contributions

Collinear distribution!

[input] TMD distribution: Wilson coefficients and intrinsic part

\[ \tilde{F}_i(x, b_T; \mu_b, \mu_b^2) = \sum_{j=q, \bar{q}, g} C_{i/j}(x, b_T; \mu_b, \mu_b^2) \otimes f_j(x; \mu_b) \tilde{F}_{i, NP}(x, b_T; \{\lambda\}) \]

Nonperturbative parts: power corrections to perturbative calculations
TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

\[ \tilde{F}_{i, NP}(x, \bar{b}_T; \{\lambda}\} \]

Which form?

Soft gluon emission

\[ g_K(\bar{b}_T; \{\lambda}\} \]
TMDs and their evolution

Distribution for intrinsic transverse momentum (and its FT):

\[ \tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\}) \]

Which form?

Soft gluon emission

\[ g_K(\bar{b}_T; \{\lambda\}) \]

Separation of \( b_T \) regions

\[ \hat{b}_T(b_T; b_{\text{min}}, b_{\text{max}}) \]

\[ \sim b_T, \quad b_{\text{min}} \ll b_T \ll b_{\text{max}} \]

\[ \sim b_{\text{max}}, \quad b_T \to +\infty \]

\[ \sim b_{\text{min}}, \quad b_T \to 0 \]

High \( b_T \) limit: avoid Landau pole

Low \( b_T \) limit: recover fixed order expression
Extraction from SIDIS
<table>
<thead>
<tr>
<th>Framework</th>
<th>HERMES</th>
<th>COMPASS</th>
<th>DY production</th>
<th>Z production</th>
<th>N of points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507</td>
<td>No evo (QPM)</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>1538</td>
</tr>
<tr>
<td>Torino 2014 (+JLab) arXiv:1312.6261</td>
<td>No evo (QPM) (separately)</td>
<td>✓</td>
<td>✓ (separately)</td>
<td>✗</td>
<td>576 (H) 6284 (C)</td>
</tr>
<tr>
<td>ElKV 2014 arXiv:1401.5078</td>
<td>LO-NLL</td>
<td>✓ (x,Q^2) bin</td>
<td>✓ (x,Q^2) bin</td>
<td>✓</td>
<td>500 (?)</td>
</tr>
<tr>
<td>Pavia 2017 arXiv:1703.10157</td>
<td>LO-NLL</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>8059</td>
</tr>
</tbody>
</table>

(only a selection of results!)
Data sets and kinematic coverage

Electron-positron annihilation data are still missing (only some azimuthal asymmetries are available) crucial for analyses of TMD FFs!!
Comparison with collinear PDF fits

- NMC
- SLAC
- BCDMS
- CHORUS
- NTVDMN
- EMCF2C
- HERACOMB
- HERAF2CHARM
- F2BOTTOM
- DYE886
- DYE605
- CDF
- D0
- ATLAS
- CMS
- LHCb

data sets available:

collinear PDFs vs TMD PDFs

see talk by E. Nocera at POETIC2016
Transverse momenta in SIDIS

SIDIS

Photon-proton frame

hadron

Photon-proton frame

Proton

Longitudinal scaling variable

\[ z = \frac{P \cdot P_h}{P \cdot q} \]

Observed transverse momentum

\[ P_{hT} \approx zk_{\perp} + P_{\perp} \]
Data sets and selections - SIDIS

<table>
<thead>
<tr>
<th>Reference</th>
<th>HERMES $p \rightarrow \pi^+$</th>
<th>HERMES $p \rightarrow \pi^-$</th>
<th>HERMES $p \rightarrow K^+$</th>
<th>HERMES $p \rightarrow K^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cuts</td>
<td>$Q^2 &gt; 1.4$ GeV$^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.2 &lt; z &lt; 0.7$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_{hT} &lt; \text{Min}[0.2 , Q, 0.7 , Qz] + 0.5$ GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Points</td>
<td>190</td>
<td>190</td>
<td>189</td>
<td>187</td>
</tr>
<tr>
<td>Max. $Q^2$</td>
<td>9.2 GeV$^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x$ range</td>
<td>$0.06 &lt; x &lt; 0.4$</td>
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</tr>
</tbody>
</table>

TMD factorization 
$[P_{hT}^2/z^2 \ll Q^2]$ 
avoid target fragmentation [?] 
and exclusive contributions [?] 
[low $z$] 
[high $z$]

Problem with normalization in the previous release

<table>
<thead>
<tr>
<th>Reference</th>
<th>HERMES $D \rightarrow \pi^+$</th>
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<th>HERMES $D \rightarrow K^-$</th>
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<th>COMPASS $D \rightarrow h^-$</th>
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<td></td>
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<tr>
<td></td>
<td>$0.20 &lt; z &lt; 0.74$</td>
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<tr>
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<td>Points</td>
<td>190</td>
<td>190</td>
<td>189</td>
<td>189</td>
<td>3125</td>
<td>3127</td>
</tr>
<tr>
<td>Max. $Q^2$</td>
<td>9.2 GeV$^2$</td>
<td></td>
<td></td>
<td></td>
<td>10 GeV$^2$</td>
<td></td>
</tr>
<tr>
<td>$x$ range</td>
<td>$0.04 &lt; x &lt; 0.4$</td>
<td></td>
<td></td>
<td></td>
<td>0.005 &lt; $x &lt; 0.12$</td>
<td></td>
</tr>
<tr>
<td>Notes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>Observable:</strong> $m_{\text{norm}}(x, z, P_{hT}^2, Q^2)$, eq. (3.1)</td>
</tr>
</tbody>
</table>
### Features

<table>
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<td>Pavia 2017 arXiv:1703.10157</td>
<td>LO-NLL</td>
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#### PROs

- almost a **global fit** of quark unpolarized TMDs
- includes **TMD evolution**
- **replica (bootstrap)** fitting methodology
- **kinematic dependence** in intrinsic part of TMDs
- intrinsic momentum: **beyond the Gaussian** assumption

#### CONs

- no “pure” info on TMD FFs
- accuracy of TMD evolution: not the state of the art
- only “low” transverse momentum (no fixed order and Y-term)
- flavor separation in the transverse plane: problematic
Intrinsic transverse momentum

\[ f^{a}_{1NP}(x, k_{\perp}^{2}) = \frac{1}{\pi} \frac{(1 + \lambda k_{\perp}^{2})}{g_{1a} + \lambda g_{1a}^{2}} e^{-\frac{k_{\perp}^{2}}{g_{1a}}} \]

weighted sum of two Gaussian distributions:
- same widths for TMD PDFs
- different widths for TMD FFs

\[ \hat{x} = 0.1 \]
\[ g_{1}(x) = \frac{N_{1} (1 - x)^{\alpha} x^{\sigma}}{(1 - \hat{x})^{\alpha} \hat{x}^{\sigma}} \]

\[ \hat{z} = 0.5 \]
\[ g_{3,4}(z) = \frac{N_{3,4} (z^{\beta} + \delta)(1 - z)^{\gamma}}{(\hat{z}^{\beta} + \delta)(1 - \hat{z})^{\gamma}} \]

\[ D^{a\rightarrow h}_{1NP}(z, P_{\perp}^{2}) = \frac{1}{\pi} \frac{1}{g_{3a\rightarrow h} + \left( \lambda_{F} / z^{2} \right) g_{4a\rightarrow h}^{2}} \left( e^{-\frac{P_{\perp}^{2}}{g_{3a\rightarrow h}}} + \lambda_{F} \frac{P_{\perp}^{2}}{z^{2}} e^{-\frac{P_{\perp}^{2}}{g_{4a\rightarrow h}}} \right) \]

Inspired by model calculations:
- Matevosyan et al.
  Phys. Rev. D85, 014021 (2012), 1111.1740
- Bacchetta et al.
- Bacchetta et al.

There are 11 free parameters in a flavor independent scenario [one for evolution]
Models - evolution and $b_T$ regions

$$g_K(b_T; g_2) = -g_2 \frac{b_T^2}{2}$$

$$\hat{b}(b_T; b_{\min}, b_{\max}) = b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)$$

These choices guarantee that for $Q=1$ GeV the TMD coincides with the NP model.
The phenomenological importance of $b_{\text{min}}$ is a signal that -especially in SIDIS data at low $Q$- we are exiting the TMD region, entering the collinear factorization region.
Agreement data-theory

Flavor independent configuration | 11 parameters

\[
\begin{array}{|c|c|c|c|}
\hline
\text{HERMES} & \text{HERMES} & \text{HERMES} & \text{HERMES} \\
\text{\( p \to \pi^+ \)}} & \text{\( p \to \pi^- \)}} & \text{\( p \to K^+ \)}} & \text{\( p \to K^- \)}} \\
\hline
| Points | 190 | 190 | 189 | 187 \\
| \( \chi^2/\text{points} \) | 4.83 | 2.47 | 0.91 | 0.82 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{HERMES} & \text{HERMES} & \text{HERMES} & \text{HERMES} & \text{COMPASS} & \text{COMPASS} \\
\text{\( D \to \pi^+ \)}} & \text{\( D \to \pi^- \)}} & \text{\( D \to K^+ \)}} & \text{\( D \to K^- \)}} & \text{\( D \to h^+ \)}} & \text{\( D \to h^- \)}} \\
\hline
| Points | 190 | 190 | 189 | 189 | 3125 | 3127 \\
| \( \chi^2/\text{points} \) | 3.46 | 2.00 | 1.31 | 2.54 | 1.11 | 1.61 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{E288 [200]} & \text{E288 [300]} & \text{E288 [400]} & \text{E605} \\
\hline
| Points | 45 | 45 | 78 | 35 \\
| \( \chi^2/\text{points} \) | 0.99 | 0.84 | 0.32 | 1.12 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{CDF Run I} & \text{D0 Run I} & \text{CDF Run II} & \text{D0 Run II} \\
\hline
| Points | 31 | 14 | 37 | 8 \\
| \( \chi^2/\text{points} \) | 1.36 | 1.11 | 2.00 | 1.73 \\
\hline
\end{array}
\]

### Points | Parameters | \( \chi^2 \) | \( \chi^2/\text{d.o.f.} \)
--- | --- | --- | ---
8059 | 11 | 12629 ± 363 | 1.55 ± 0.05

**Hermes** P/D into \( \pi^+ \): problems at low \( z \)

**Hermes** kaons better than pions: larger uncertainties from FFs

**Compass**: better agreement due to points and normalization

Let’s see what happens with the new data
Average transverse momenta

Red/orange regions: 68% CL from replica method

Inclusion of *Compass* increases the $\langle p_{\perp}^2 \rangle$ and reduces its spread

Inclusion of *DY/Z* diminishes the correlation

*e+e- data would further reduce the correlation*

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Bacchetta, Delcarro, Pisano, Radici, Signori (JHEP 2017)
Signori, Bacchetta, Radici, Schnell arXiv:1309.3507
Schweitzer, Teckentrup, Metz, arXiv:1003.2190
Anselmino et al. arXiv:1312.6261 [HERMES]
Anselmino et al. arXiv:1312.6261 [HERMES, high z]
Anselmino et al. arXiv:1312.6261 [COMPASS, norm.]
Anselmino et al. arXiv:1312.6261 [COMPASS, high z, norm.]
Echevarria, Idilbi, Kang, Vitev arXiv:1401.5078 (Q = 1.5 GeV)
Kinematic dependence

\[ \langle P_{\perp}^2 \rangle(z) = \frac{\int d^2 P_{\perp} P_{\perp}^2 D_{1}^{a \rightarrow h}(z, P_{\perp}^2, Q = 1 \text{ GeV})}{\int d^2 P_{\perp} D_{1}^{a \rightarrow h}(z, P_{\perp}^2, Q = 1 \text{ GeV})} \]

Average square transverse momentum in TMD FF

Color code: same as previous slide

Flavor-independent scenario: no differences in quark/hadron flavor

z-dependence: important to fit the data

GMC trans

Anselmino et al.
hep-ph/9901442
What’s next

Just a selection of topics to feed the discussion
Target vs current vs central regions

current fragmentation

target fragmentation

“soft” fragmentation
Target vs current vs central regions

See O. Gonzalez’s talk

“collinearity” criterion
Target vs current vs central regions

Description of Hermes data within the quark parton model

Widths described as a function of the rapidity difference between the incoming proton and outgoing hadron

M. Albright et al. 2018
Completing the formalism to study TMD FFs
fixed Q, variable $q_T$

$e^+e^- \rightarrow h_1 h_2 X$

$d\sigma/dq_T$
$q_T \ll Q$

TMD factorization

$\Lambda_{QCD}$

Where does the matching occur in $q_T$?
At which value of $q_T/Q$?

Collins 2011 (TMD factorization)

Where does the matching occur in $q_T$?
At which value of $q_T/Q$?

Several phenomenological works, e.g.:
* Bacchetta, Echevarria, Mulders, Radici, AS - JHEP 2015

fixed-order term

Coll. FFs

New calculation in progress!
Moffat, Rogers, AS

Jefferson Lab
Matching

development of a new scheme (InEW - inverse error weighting)
and comparison to improved CSS subtraction

\[ d\sigma(q_T, Q) = \omega_1 \mathcal{W}(q_T, Q) + \omega_2 \mathcal{Z}(q_T, Q) \]

\[ \omega_1 \sim \Delta \mathcal{W}^{-2}, \quad \Delta \mathcal{W} \sim \mathcal{O}\left(\frac{q_T}{Q}\right)^a + \mathcal{O}\left(\frac{m}{Q}\right)^{a'} \]

\[ \omega_2 \sim \Delta \mathcal{Z}^{-2}, \quad \Delta \mathcal{Z} \sim \mathcal{O}\left(\frac{m}{q_T}\right)^b \]

Drell-Yan at LHC
(Q=12 GeV)
(data available)
development of a new scheme ([InEW - inverse error weighting])
and comparison to improved CSS subtraction
Inputs for discussion

FORMALISM:
* definition of the fragmentation regions
* which variables shall we use to describe the momentum fractions? (see Gunar’s talk)
* matching schemes: how to estimate uncertainties associated to the matching prescription
* jet fragmentation functions

PERTURBATIVE ASPECTS:
* implementation of evolution (transverse momentum, threshold resummation, zeta prescription, …?)
* fixed-order calculations in SIDIS: is it sufficient to describe data at higher qT? Or do we need power corrections/higher twist, contributions from soft region, etc?

NONPERTURBATIVE ASPECTS:
* functional form at low transverse momentum
* its kinematic dependence
* its flavor dependence
* nonperturbative contribution to TMD evolution

DATA:
* impact of the new release of Compass data
* A Fixed Target Experiment at the LHC?
* what can be done with the forthcoming e+e- data concerning TMD FFs (also including matching to high qT)
* how well does the fixed order describes data at large transverse momentum
* …
Backup
quark TMD PDFs

\[ \Phi_{ij}(k, P; S) \sim \text{F.T.} \langle PS' | \bar{\psi}_j(0) U_{[0,\xi]} \psi_i(\xi) | PS' \rangle_{LF} \]

extraction of a quark not collinear with the proton
Status of TMD phenomenology

Theory, data, fits: we are in a position to start validating the formalism

<table>
<thead>
<tr>
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<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1$</td>
<td></td>
<td>$h_1^T$</td>
</tr>
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<td>L</td>
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<td>$h_{1L}^T$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^T$</td>
<td>$g_{1T}$</td>
<td>$h_1, h_{1T}^T$</td>
</tr>
</tbody>
</table>

Twist-2 TMDs

Limited data, theory, fits

see, e.g., Bacchetta, Radici, arXiv:1107.5755
Anselmino, Boglione, Melis, PRD86 (12)
Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)
Anselmino, Boglione, D’Alesio, Murgia, Prokudin, arXiv: 1612.06413
Anselmino et al., PRD87 (13)
Kang et al. arXiv:1505.05589

Only first attempts

Lu, Ma, Schmidt, arXiv:0912.2031
Lefky, Prokudin arXiv:1411.0580
## Beware of different notations...

<table>
<thead>
<tr>
<th>Amsterdam</th>
<th>Seattle (arXiv:1108.1713)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>( k )</td>
</tr>
<tr>
<td>( p_T )</td>
<td>( k_\perp )</td>
</tr>
<tr>
<td>( k )</td>
<td>( p )</td>
</tr>
<tr>
<td>( k_T )</td>
<td>( p_\perp )</td>
</tr>
<tr>
<td>( K_T )</td>
<td>( P_\perp )</td>
</tr>
<tr>
<td>( P_{h_\perp} )</td>
<td>( P_{hT} )</td>
</tr>
</tbody>
</table>

- \( p \): momentum of parton in distribution function
- \( k \): momentum of fragmenting parton
- \( k_\perp \): parton transverse momentum in distribution function
- \( p_\perp \): trans. momentum of fragmenting parton w.r.t. final hadron
- \( P_\perp \): trans. momentum of final hadron w.r.t. fragmenting parton
- \( P_{hT} \): transverse momentum of final hadron w.r.t. virtual photon

Let’s agree on the notation!
Collinear and TMD factorization

Let’s consider a process with three separate scales:

- hadronic mass scale
- hard scale

(related to the) transverse momentum of the observed particle

The ratios

\[ \frac{\Lambda_{QCD}}{Q} \quad \frac{\Lambda_{QCD}}{q_T} \quad \frac{q_T}{Q} \]

select the factorization theorem that we rely on.

According to their values we can access different “projections” of hadron structure

(SIDIS, Drell-Yan, e+e- to hadrons, pp to quarkonium, …)
Collinear and TMD factorization

The key of phenomenology: emergence of TMD and collinear distributions from factorization theorems

fixed $Q$, variable $q_T$

$$\frac{d\sigma}{dq_T}$$

$q_T \geq Q$

fixed-order term

$$\sim \Phi_A(x_a) \Phi_B(x_b)$$
collinear PDFs

relative error = $O(\frac{\Lambda_{QCD}}{q_T})$

degraded description!
Collinear and TMD factorization

The key of phenomenology: emergence of TMD and collinear distributions from factorization theorems

fixed $Q$, variable $q_T$

\[ \frac{d\sigma}{dq_T} \]

$q_T \ll Q$

resummed term $[W]$

collinear factorization

fixed-order term

$\sim \Phi_A(x_a) \Phi_B(x_b)$

collinear PDFs

relative error $= O(q_T/Q)$

degraded description!

$\Phi_A(x_a, k_{Ta}) \Phi_B(x_b, k_{Tb}) \sim$ TMD PDFs

Jefferson Lab
Collinear and TMD factorization

The key of phenomenology: emergence of TMD and collinear distributions from factorization theorems

fixed $Q$, variable $q_T$

$\frac{d\sigma}{dq_T}$

$q_T \ll Q$

Matching region

resummed term $W$

TMD factorization

collinear factorization

degraded descriptions

fixed-order term

collinear PDFs

fixed $Q$, variable $q_T$

$\Lambda_{QCD}$

$Q$

$W$, relative error = $O(q_T/Q)$

F.O., relative error = $O(\Lambda_{QCD}/q_T)$

We need a prescription to deal with the region where both descriptions are not good
Collinear and TMD factorization

The key of phenomenology: emergence of TMD and collinear distributions from factorization theorems

The extraction of the nonperturbative part of TMDs is affected by the description of the whole $q_T$ range

Crucial, especially at low $Q$ (e.g. JLab kinematics), where the regions shrink

fixed $Q$, variable $q_T$

$d\sigma/dq_T$

$q_T \ll Q$

Matching region

fixed-order term

collinear PDFs

resummed term (W)

TMD factorization

degraded descriptions

TMD PDFs

Crucial, especially at low $Q$ (e.g. JLab kinematics), where the regions shrink

polarization?
Global fit

SIDIS

Drell-Yan

Z production

Bacchetta et al. JHEP 1706 (2017) 081
COMPASS, selected bins

Deuteron $h^-$, $\chi^2$/dof = 1.58

First points are not fitted, but used as normalization
\[ \chi^2 / \text{dof} = 4.80 \]

The worst of all channels...

However normalizing the theory curves to the first bin, without changing the parameters of the fit, \( \chi^2 / \text{dof} \) becomes good.

Contributions to chi2 mainly from normalization, not shape (also in Z-boson production)
**Z-boson @ Fermilab**

**Narrow bands**, driven mainly by $g_2$ values [reduced sensitivity to intrinsic $k_T$]

Contributions to chi2 mainly from **normalization**, not shape
proton target \hspace{1cm} \text{global} \hspace{1cm} \chi^2 / \text{d.o.f.} = 1.63 \pm 0.12
\hspace{1cm} \text{no flavor dep.} \hspace{1cm} 1.72 \pm 0.11

\[\pi^-\]
\[1.80 \pm 0.27\]
\[1.83 \pm 0.25\]

\[K^-\]
\[0.78 \pm 0.15\]
\[0.87 \pm 0.16\]

\[\pi^+\]
\[2.64 \pm 0.21\]
\[2.89 \pm 0.23\]

\[K^+\]
\[0.46 \pm 0.07\]
\[0.43 \pm 0.07\]
The multiplicities of charged hadrons per FO (differential in $x_B$) in SIDIS are shown in Fig. 9. One can notice, going from left to right, that data with very close value of $z$ dependence. This can hardly be reproduced by Eq. (12), even considering eventual higher order corrections. Similar considerations apply to Fig. 10.

The EMC Collaboration measured the SIDIS multiplicity data used in our present fits result from the most recent analyses of the HERMES multiplicities and its origin, at present, cannot easily be explained and deserves further studies.

The resulting value of $Q_B^2$ corresponds to a 5% variation of the total normalization $T$. Let us consider, for example, the data in the di $x$ and $Q_B^2$ for different nuclear targets, without any identification of the final hadrons (not even their charges), and arranging the data in three different bins of $z$.

The EMC measurements of the azimuthal dependence of the SIDIS cross section, for $0 < Q_B^2 < 200$ (in GeV$^2$) with the available EMC and JLab measurements.

However, it is worth and interesting to check whether or not the parameters extracted here are consistent with the COMPASS measurements for SIDIS production of $h^+$ and $h^-$. Additional measurements are provided by the early EMC results of Ref. [10] or by the more recent MES and COMPASS Collaborations. They represent, so far, the only multivariate analyses available. Further studies will be needed to fully understand the origin of the large values of $P_T$ corrections. Similar considerations apply to Fig. 10.

$\chi^2$/dof = 3.79 with ad-hoc normalization.

see Compass coll. Erratum

Anselmino, Boglione, Gonzalez, Melis, Prokudin, JHEP 1404 (14)
The normalization factor for E288 data is always within their experimental uncertainty, while for R209 it is a bit smaller.

\[
\frac{\sigma}{dq} = \frac{1}{1 + \chi^2/dof} \approx 0.81
\]

D’Alesio, Echevarría, Melis, Scimemi, JHEP 1411 (14)

NLO-NNLL analysis with evaluation of theoretical uncertainties very good
$Q_0 = 3.2 \text{ GeV}$

$b_{\text{max}} = 0.5 \text{ GeV}^{-1}$

$b_{\text{max}} = 1.5 \text{ GeV}^{-1}$
Parametrizations for intrinsic momenta and soft gluon emission:

\[ F_{NP}(b_T, Q)_{pdf} = \exp \left[ -b_T^2 \left( g_1^{pdf} + \frac{g_2}{2} \ln(Q/Q_0) \right) \right] \]

\[ F_{NP}(b_T, Q)_{ff} = \exp \left[ -b_T^2 \left( g_1^{ff} + \frac{g_2}{2} \ln(Q/Q_0) \right) \right] \]

**Pros and Cons:**

1) a global analysis of SIDIS and DY/Z/W data

2) TMD evolution at LO-NLL

3) multidimensionality not exploited

4) chi-square not provided

5) can’t be considered as a “complete” fit
functions from the experimental data. The data points from top to bottom correspond to different invariant mass bins.

In this section we will first extract the quark Sivers functions from the experimental data. The data points from top to bottom correspond to different regions: [0, 2], [2, 4], and [4, 6].

To compare with experimental data, we use the unpolarized parton distribution functions for the collinear nuclear PDFs in the nucleus.

Likewise, we restrict our comparison with the recent COMPASS experimental data for the charged hadron production at both the Tevatron and LHC energies. With QCD evolution formalism, we are implementing the evolution at NLL accuracy along with the LO co-factor term [39–41, 66]. To be consistent with our formalism, we thus rest our QCD formalism is the very first attempt to the fragmentation functions [75].

\[ b_{max} = 1.5 \text{ GeV}^{-1} \]
\[ g_2 = 0.16 \]

Echevarria et al. arXiv:1401.5078
Other studies

CSS formalism on DY/Z/W data:

1) Davies-Webber-Stirling [DOI: 10.1016/0550-3213(85)90402-X]


3) BLNY [DOI: 10.1103/PhysRevD.63.013004]


... combined SIDIS/DY/W/Z:

5) Sun, Yuan [arXiv:1308.5003]

6) Isaacson, Sun, Yuan, Yuan [arXiv:1406.3073]

...
The replica method

$m(x,z,P_{hT},Q^2)$, proton target

$\langle x \rangle \sim 0.15$
$\langle Q^2 \rangle \sim 2.9$ GeV$^2$

$0.27 < z < 0.30$
$0.38 < z < 0.48$

Sample of original data
The replica method

\[ m(x, z, P_{hT}, Q^2), \text{proton target} \]

\[ \langle x \rangle \sim 0.15 \]
\[ \langle Q^2 \rangle \sim 2.9 \text{ GeV}^2 \]

- Green triangles: \( 0.27 < z < 0.30 \)
- Brown squares: \( 0.38 < z < 0.48 \)

Replica of the original data with Gaussian noise
The replica method

\( m(x, z, P_{hT}, Q^2) \), proton target

\( \langle x \rangle \sim 0.15 \)
\( \langle Q^2 \rangle \sim 2.9 \text{ GeV}^2 \)

\( 0.27 < z < 0.30 \)
\( 0.38 < z < 0.48 \)

Fit of the replicated data
The replica method

\[ m(x, z, P_{hT}, Q^2), \text{proton target} \]

- \( \langle x \rangle \approx 0.15 \)
- \( \langle Q^2 \rangle \approx 2.9 \text{ GeV}^2 \)
- \( 0.27 < z < 0.30 \)
- \( 0.38 < z < 0.48 \)

Repeat the generation and the fit N times
The replica method

\[ m(x, z, P_{hT}, Q^2), \text{proton target} \]

\[ \langle x \rangle \sim 0.15 \]

\[ \langle Q^2 \rangle \sim 2.9 \text{ GeV}^2 \]

- \( 0.27 < z < 0.30 \)
- \( 0.38 < z < 0.48 \)

Obtain distributions of best values - calculate 68% CL bands