Calculation of Pion Valence Distribution from Hadronic Lattice Cross Sections

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in Collaboration with

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Why Pion Valence Distribution

★ Pion : lightest bound state and associated with dynamical chiral symmetry breaking

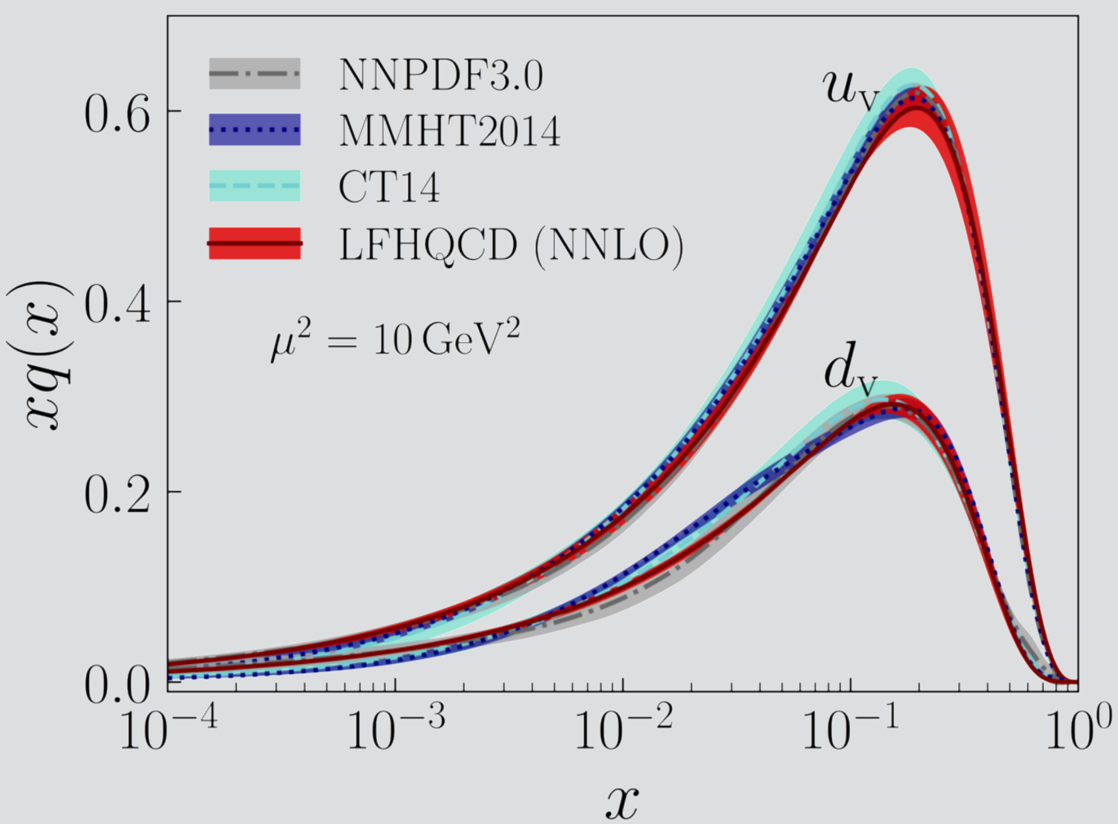
★ Large- x behavior of pion valence distribution is an unresolved problem

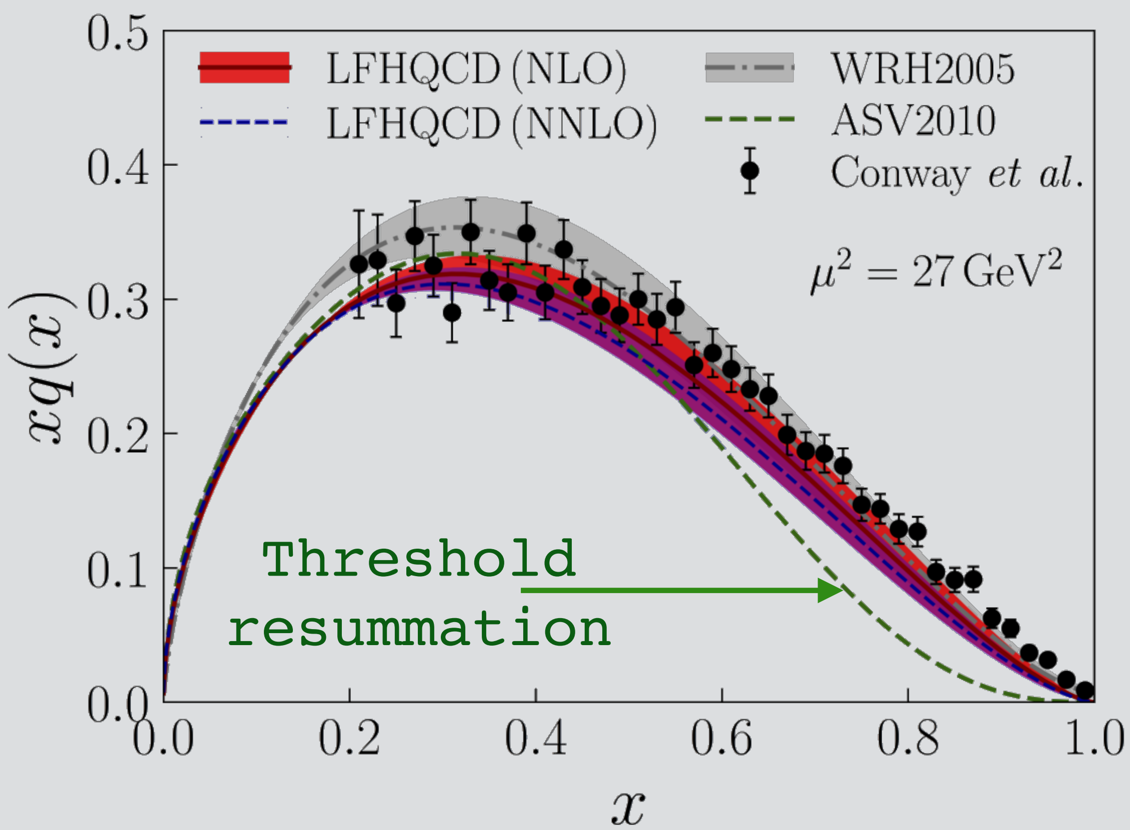
★ From pQCD and different models : $(1-x)^2$ or $(1-x)^1$?

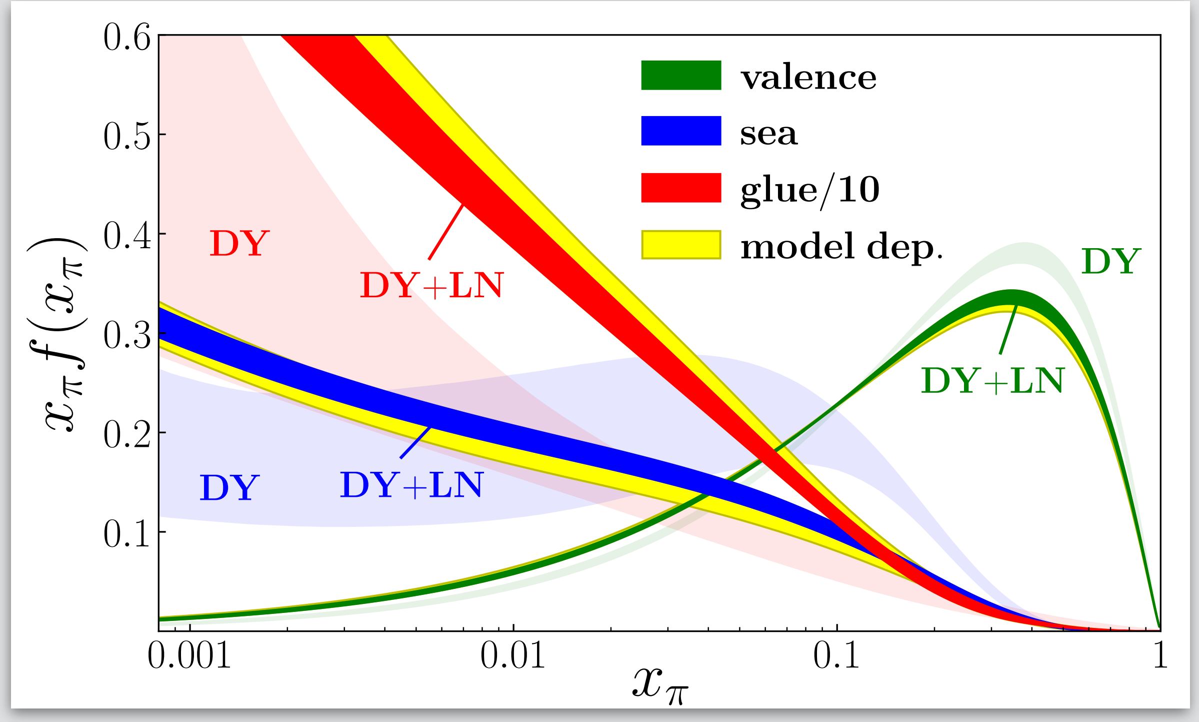
★ C12-15-006 experiment at JLab to explore large-x behavior

With controllable systematics, Lattice QCD can help understanding large-x behavior and test different models

de Téramond, Liu, **RSS**, Dosch, Brodsky, Deur (PRL 2018)







Barry, Sato, Melnitchouk, Ji
To appear in PRL

Calculations of Parton Distributions on the Lattice

★ Hadronic tensor (K. F. Liu, PRL 1994, PRD 200)

★ Position-space correlators (V. M. Braun & D. Müller, EPJ 2008)

★ Inversion Method (A. Chambers, et al PRL 2017)

★ Quasi PDFs (X. Ji, PRL 2013)

★ Pseudo-PDFs (A. Radyushkin, PLB 2017)

Extensive efforts and significant achievements in recent years

★ **Hadronic Lattice Cross Sections (LCSs)**
(Y. Q. Ma, J.-W. Qiu, arXiv 2014, PRL 2018)

What are Good Lattice "Cross Sections" (LCSs)

Single hadron matrix elements:

1. Calculable using lattice QCD with Euclidean time

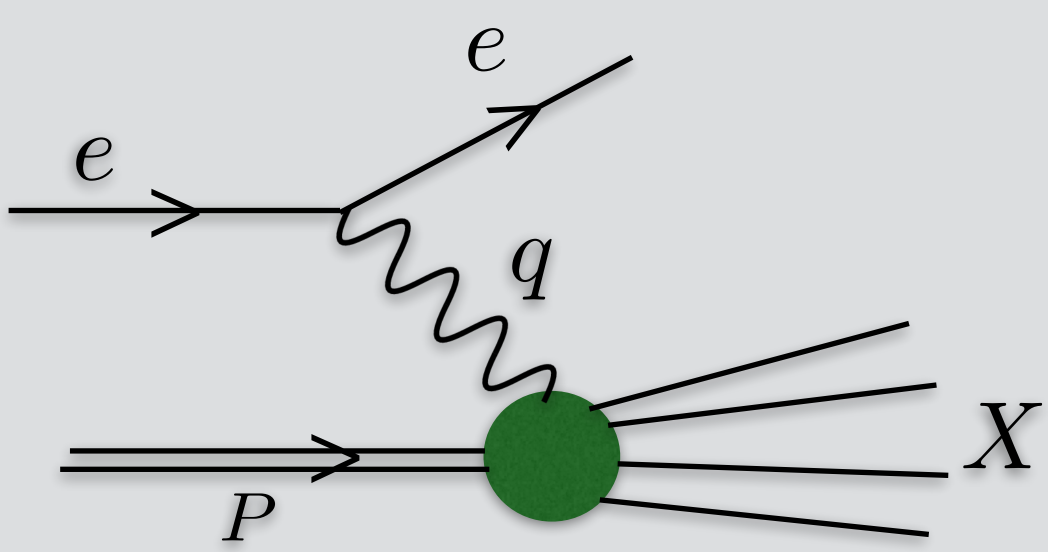
2. Well defined continuum limit ($a \to 0$), UV finite
i.e. no power law divergence from Wilson line in non-local operator

3. Share the same perturbative collinear divergences with PDFs

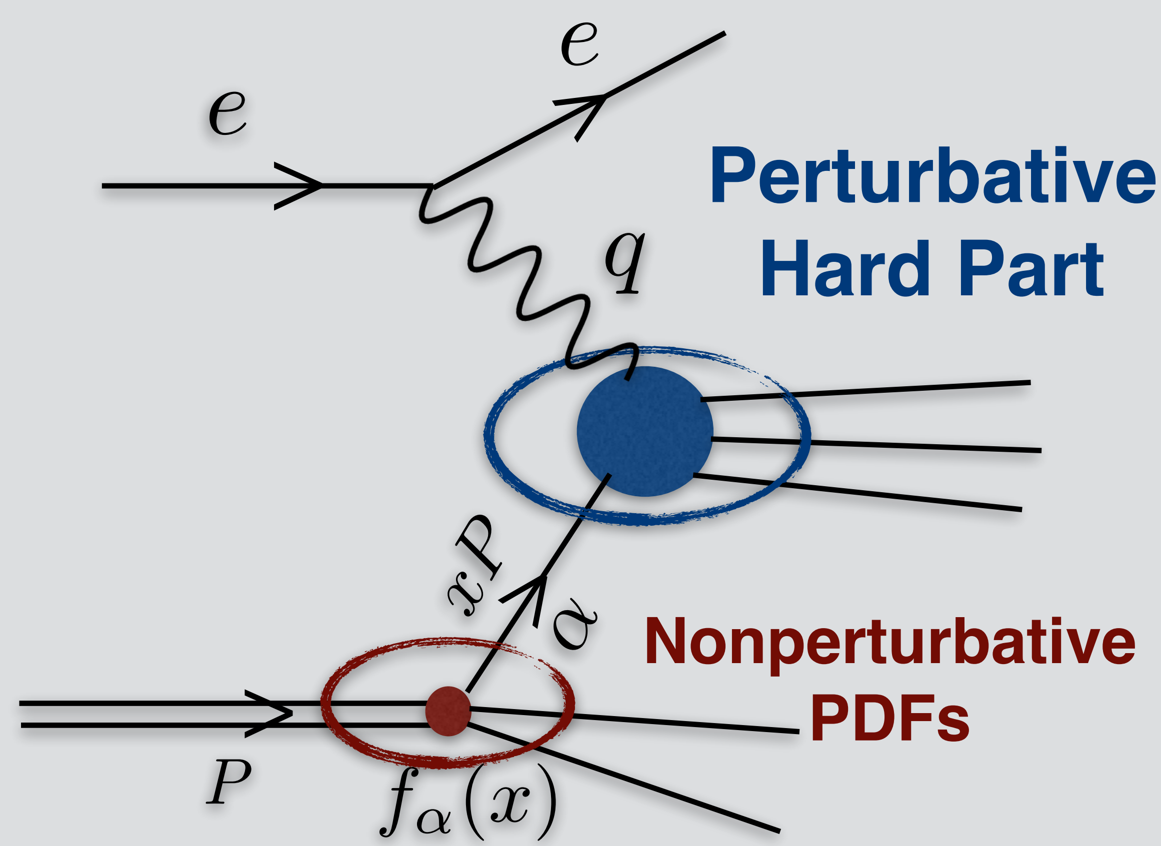
4. Factorizable to PDFs with IR-safe hard coefficients
with controllable power corrections

Parton Distribution Functions (PDFs) & Factorization

$$\sigma^{DIS}(x, Q^2, \sqrt{s}) = \sum_{\alpha=q,\bar{q},g} \boxed{C_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right)} \otimes \boxed{f_\alpha(x, \mu^2)} + \boxed{\text{Power Corrections}}$$



DIS

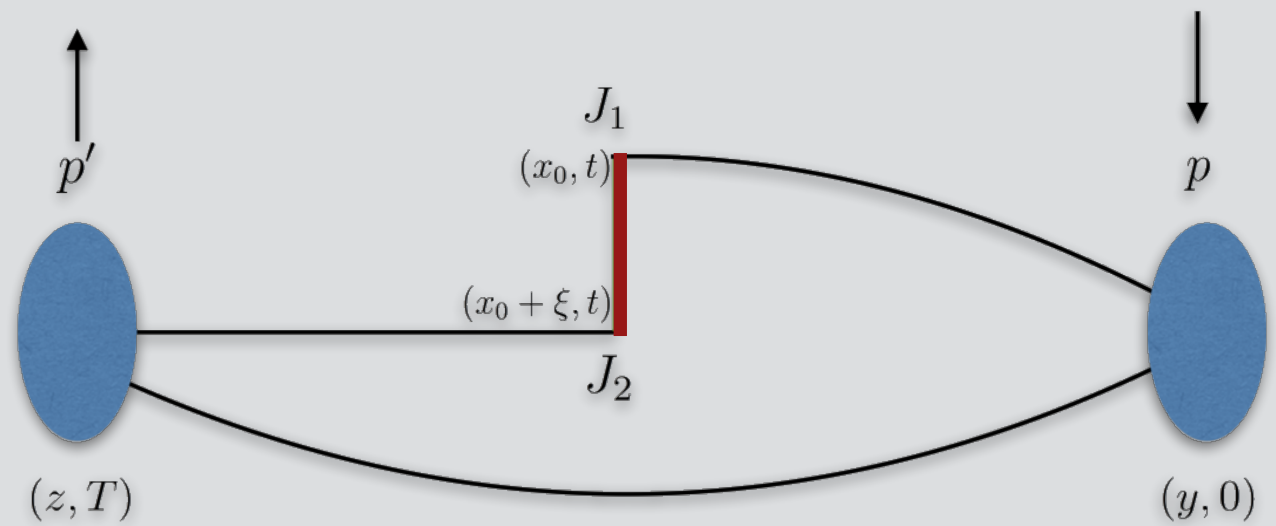


Parton Picture

Factorization scale μ describes which fluctuations should be included in the PDFs and which can be included in the hard scattering part

A good theory can identify its limitations

★ 4-point correlation function is numerically expensive



★ Equal time current insertion : sum over all energy modes can saturate phase space

Use heavy-light flavor changing current to suppress noise from spectator propagator in a systematic way

Good Lattice Cross Sections (LCSs)

★ **Hadron matrix elements:** $\sigma_n(\omega,\xi^2,P^2)=\langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$

Lorentz scalar $\omega\equiv P\cdot\xi$

★ **Current-current correlators**

$$\mathcal{O}_{j_1 j_2}(\xi)\equiv\xi^{d_{j_1}+d_{j_2}-2}\,Z_{j_1}^{-1}\,Z_{j_2}^{-1}j_1(\xi)\,j_2(0)$$

d_j : Dimension of the current

Z_j : Renormalization constant of the current

★ **Different choices of currents**

$$j_S(\xi)=\xi^2 Z_S^{-1}[\overline{\psi}_q\psi_q](\xi),\qquad j_V(\xi)=\xi Z_V^{-1}[\overline{\psi}_q\gamma\cdot\xi\psi_q](\xi),$$

$$j_{V'}(\xi)=\xi Z_{V'}^{-1}[\overline{\psi}_q\gamma\cdot\xi\psi_{q'}](\xi),\qquad j_G(\xi)=\xi^3 Z_G^{-1}[-\frac{1}{4}F^c_{\mu\nu}F^c_{\mu\nu}](\xi),\ldots$$

flavor changing current

gluon distribution

LCSs: Lattice Calculable + Renormalizable + Factorizable

$$\sigma_n(\omega,\xi^2,P^2)=\sum_a\int_{-1}^{1}\frac{dx}{x}f_a(x,\mu^2)\times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2)+\mathcal{O}(\xi^2\Lambda_{QCD}^2)$$

Nonperturbative PDFs of flavor $a=q,g$

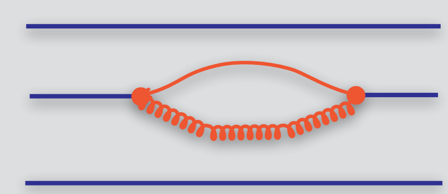
Perturbatively calculable hard hoefficients

P and ξ →

Collision Kinematics

$P\to\sqrt{s}$ Collision energy

$\xi\to\frac{1}{Q}$ Hard Probe



$\mathcal{O}_n$ → Dynamical Features of LCSs

In coordinate space

Factorization holds for any finite ω and $P^2\xi^2$ if ξ is short distance

Lattice Calculation

$$32^3 \times 96, \quad m_\pi \approx 430 \quad \text{MeV}$$
$$a \approx 0.127 \quad \text{fm}$$

Production Recently Finished

★ Projected calculations with

$$24^3 \times 64, \quad m_\pi \approx 430 \quad \text{MeV}$$
$$a \approx 0.127 \quad \text{fm}$$

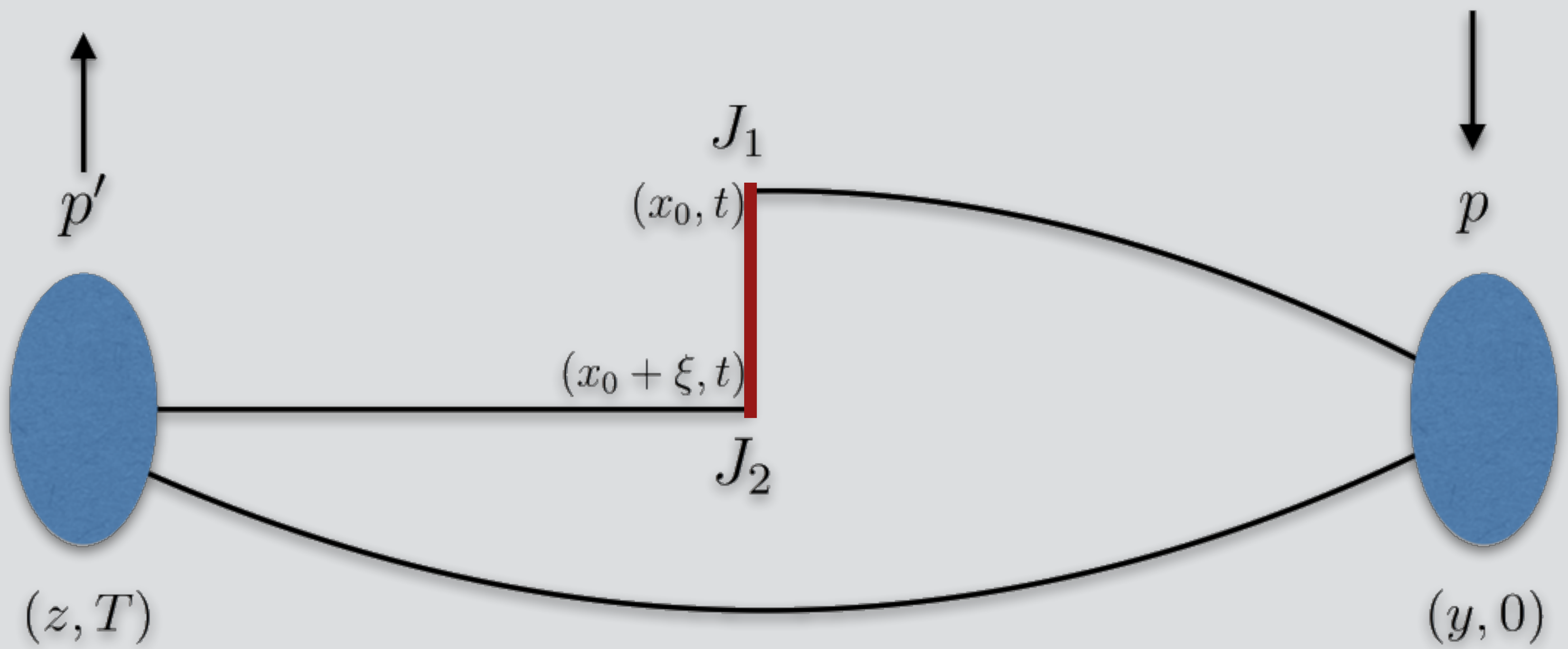
Finite volume effect
Briceño, et al
PRD 2018

$$32^3 \times 64, \quad m_\pi \approx 280 \quad \text{MeV}$$
$$a \approx 0.09 \quad \text{fm}$$

Lattice spacing and pion mass effects

$$64^3 \times 128, \quad m_\pi \approx 170 \quad \text{MeV}$$
$$a \approx 0.09 \quad \text{fm}$$

Lattice Calculation Setup



$\boxed{\text{possible } \xi \text{ on/off axis}}$

★ Analysis shown here on isoClover with 490 Configurations

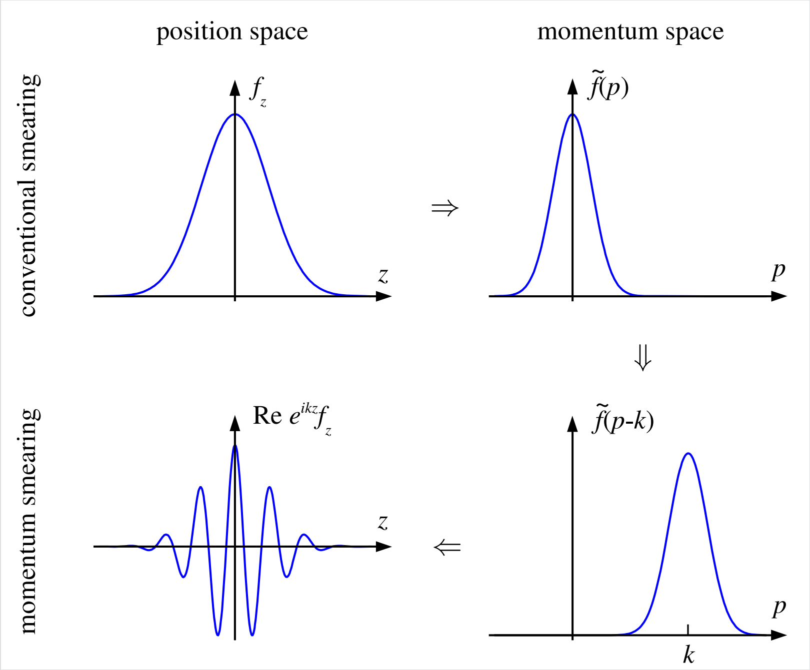
★ Lattice spacing ~ 0.127 fm, $m_\pi \approx 430$ MeV $(32^3 \times 96)$

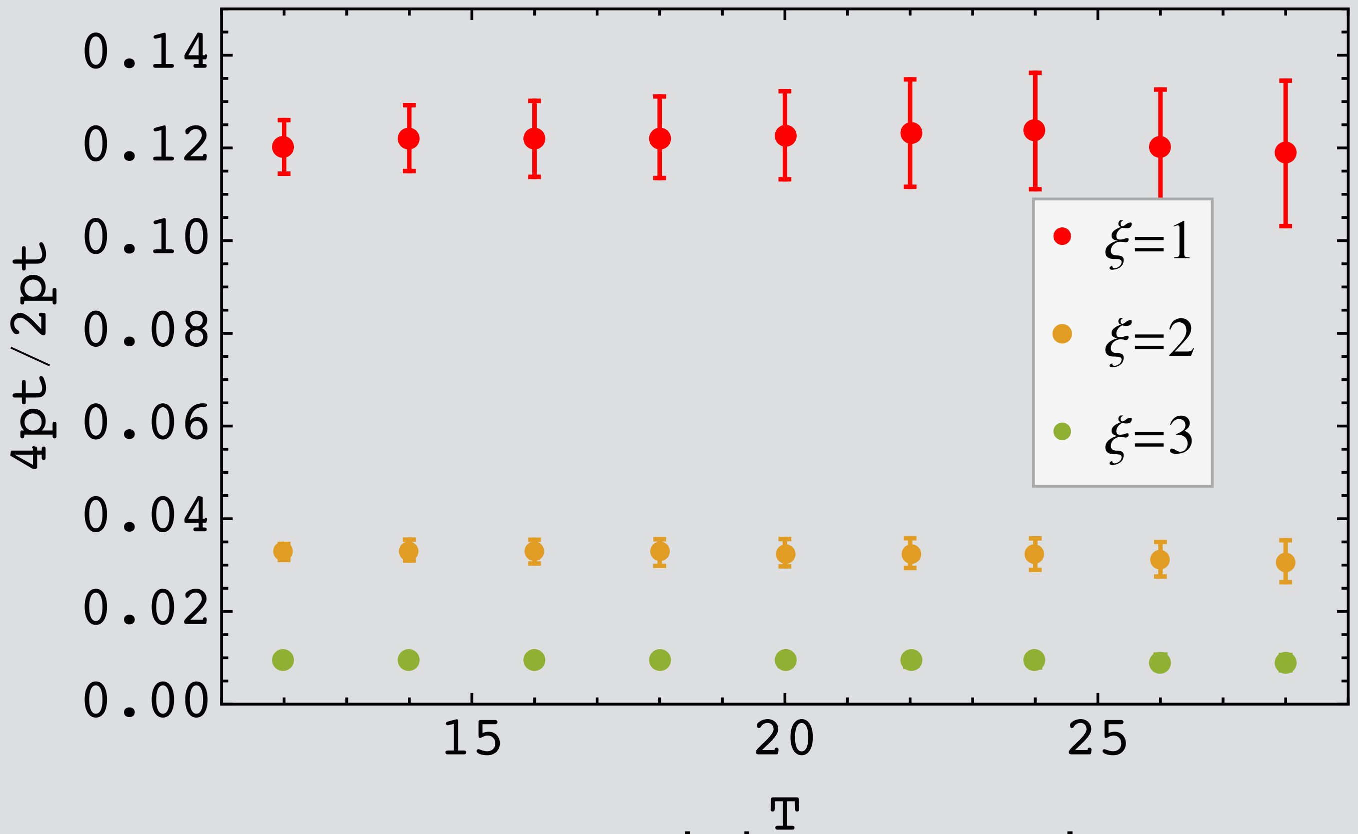
Example Lattice Matrix Elements

- About 10 different current-current correlations are being analyzed

- Momentum smearing for higher momentum

Gunnar S. Bali, et al
(PRD 2016)





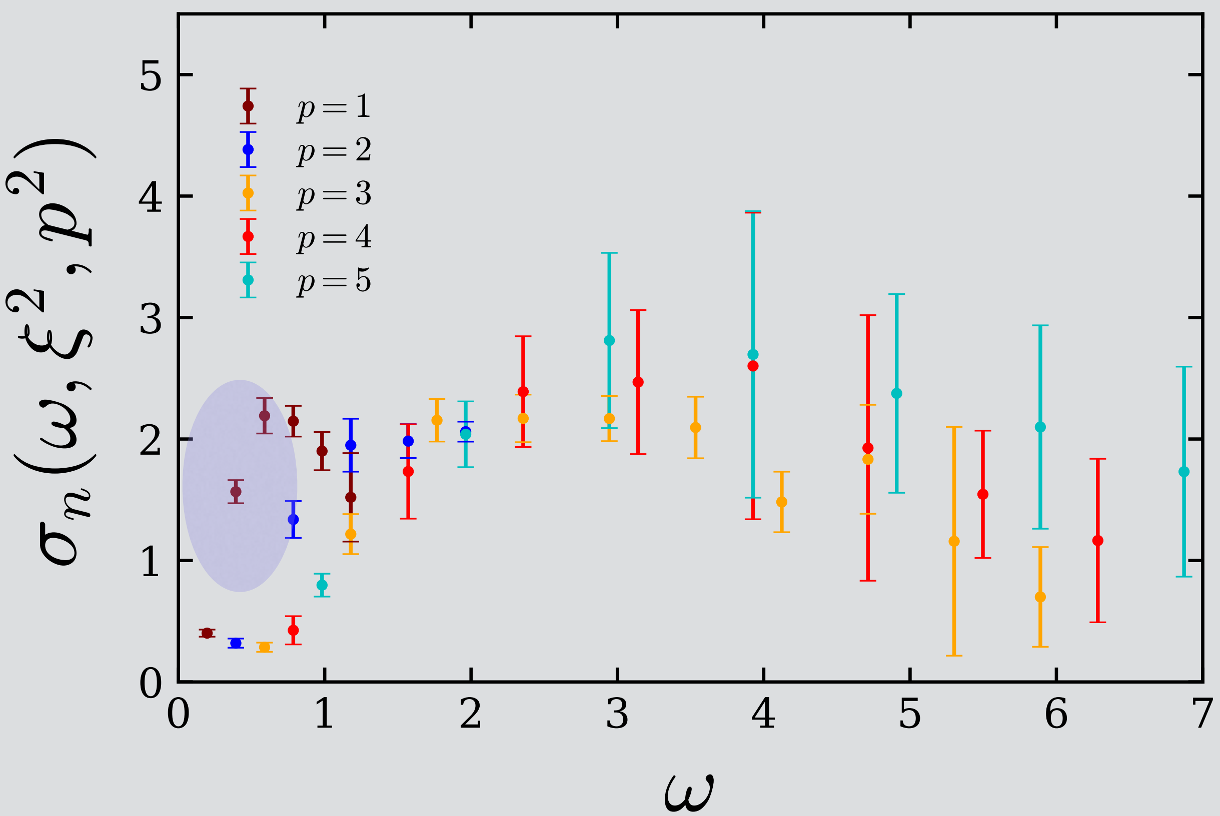
source-sink separation

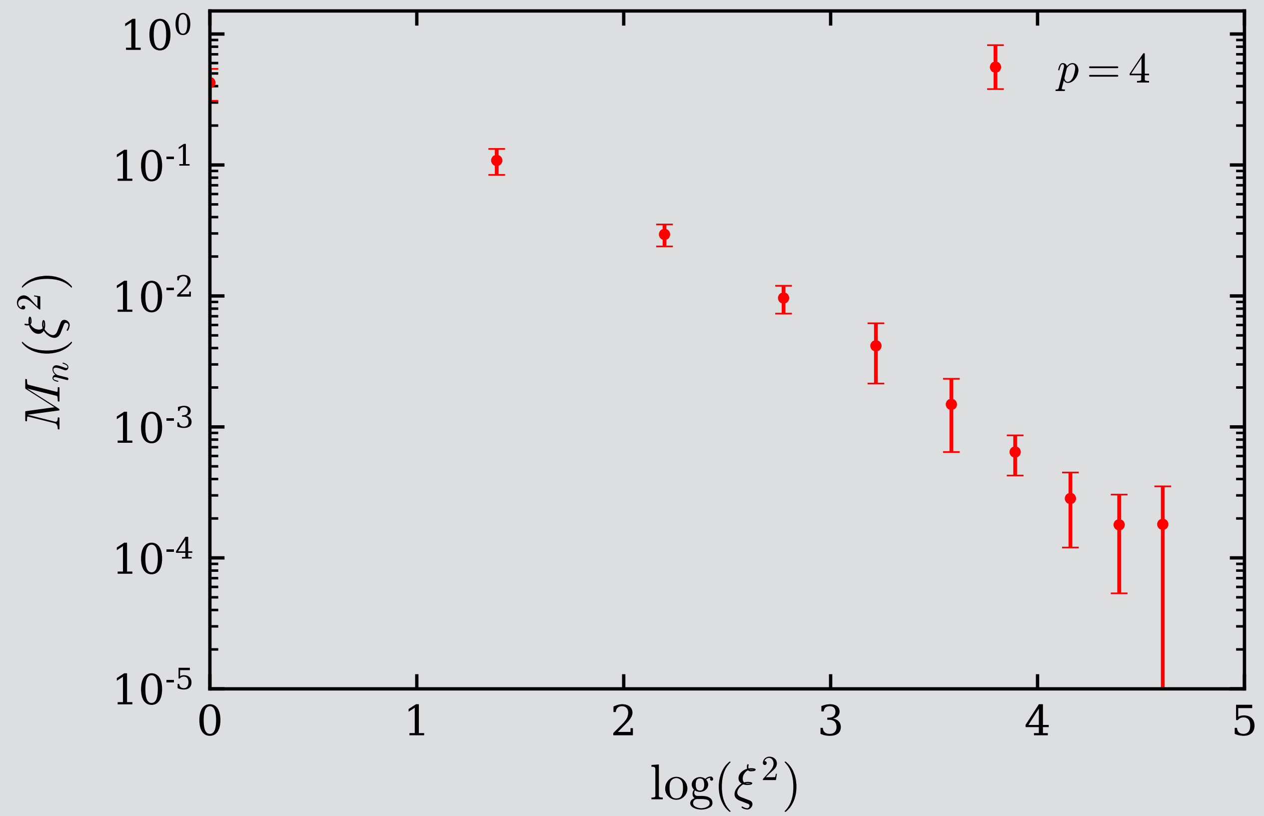
V-A matrix element

Reliable extraction of matrix elements

★ Only about 1/4 statistics of p=3,4,5 data analyzed

V-V current correlation





★ p=1 (0.3 GeV) data deviates

Does the calculated correlation matrix lead to consistent description of pion PDF ?

$$f(x) \approx Ax^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\delta x)$$

Preliminary Lattice Results

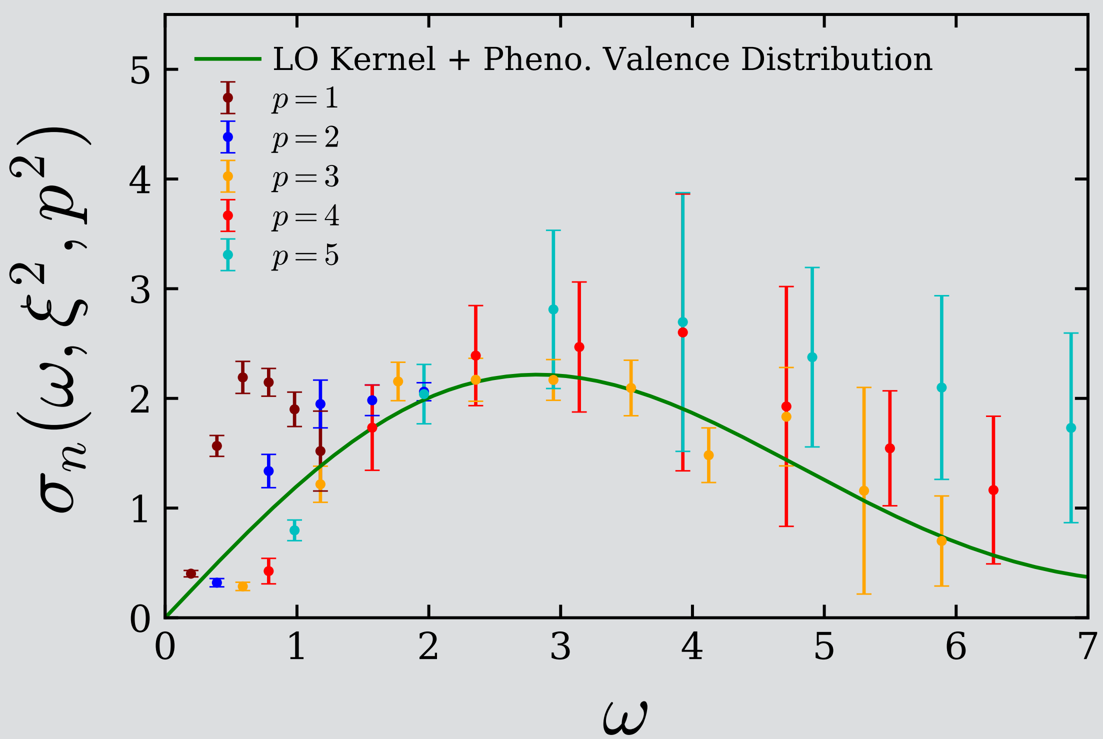
$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) \times K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda^2_{QCD})$$

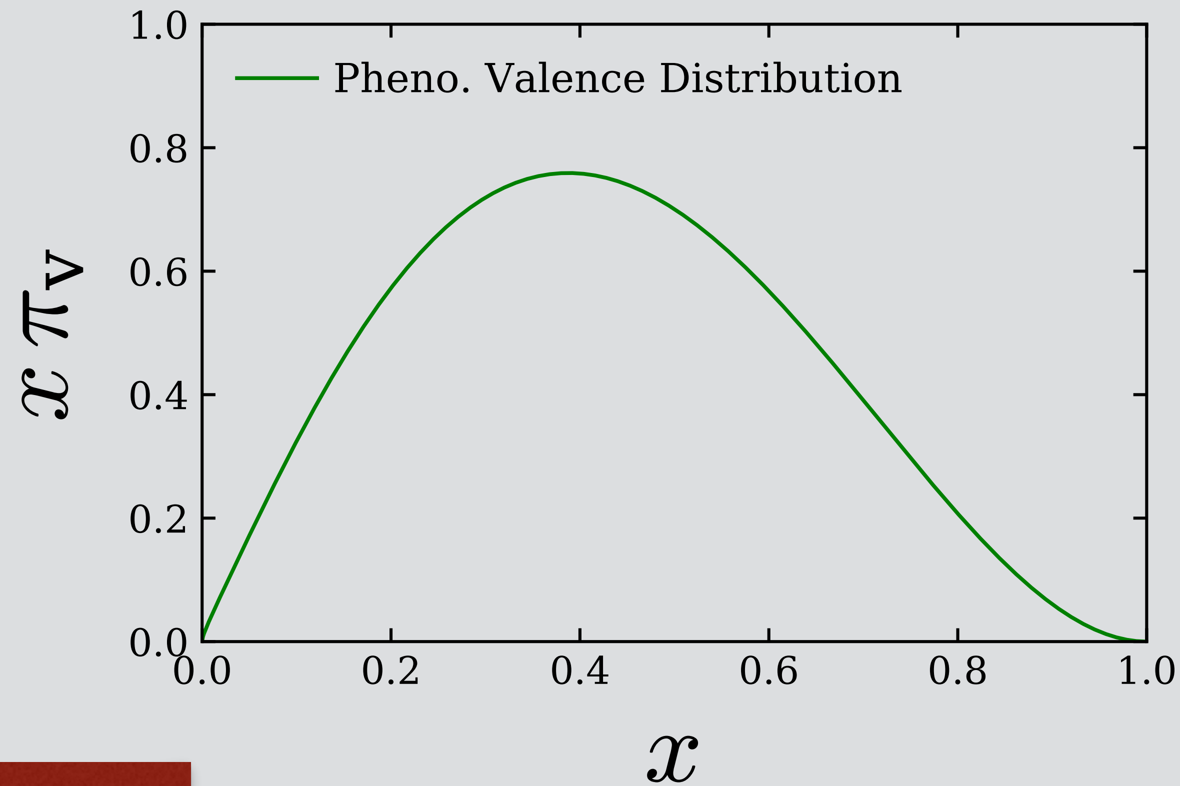
calculate on lattice

extract PDF

PQCD

NLO perturbative kernel will give ξ^2 correction





NOT a fit yet

e.g. like global fits to data from different experiments !

With these encouraging results, we are very excited !!!

- ★ A combined fit to many LCSs on an ensemble will lead to precise determination of PDFs
- ★ LCSs can be a tool to test different model calculations
- ★ K_n^a at LO and NLO for different currents being calculated
- ★ Extensions such as kaon, nucleon PDFs on their way....

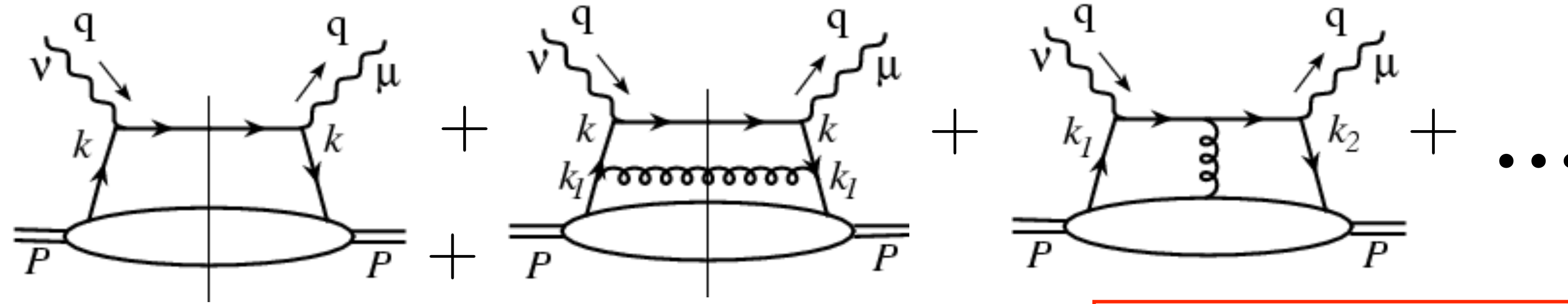
Thank You

Backup

$$
\boxed{
\begin{aligned}
&\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle = \\
&= \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\,\bar{q}\,J_2\, q(x_0+\xi,t)\,\bar{q}\,J_1\, q(x_0,t)\,\bar{q}\,\Gamma_\Pi\, q(y,0)\rangle \\
&= \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{tr}[J_2\, D^{-1}(x_0+\xi,t;x_0,t)\, J_1\, D^{-1}(x_0,t;y,0)\,\Gamma_\Pi \\
&\qquad \times D^{-1}(y,0;z,T)\Gamma_\Pi\, D^{-1}(z,T;x_0+\xi,t)],
\end{aligned}
}
$$

❑ DIS cross section is infrared divergent, and nonperturbative!

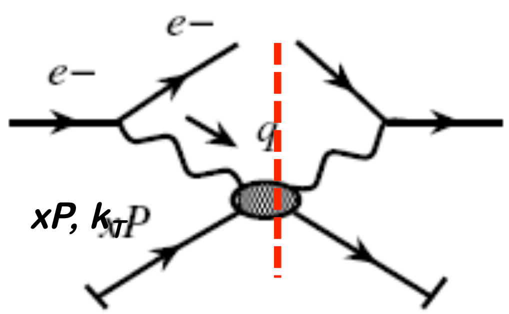
$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})} \propto$$



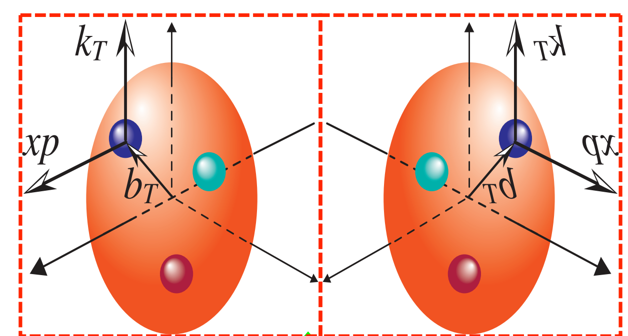
❑ QCD factorization (approximation!)

Color entanglement
Approximation

$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})} =$$



$\otimes$



$$+\ O\left(\frac{1}{QR}\right)$$

Physical
Observable

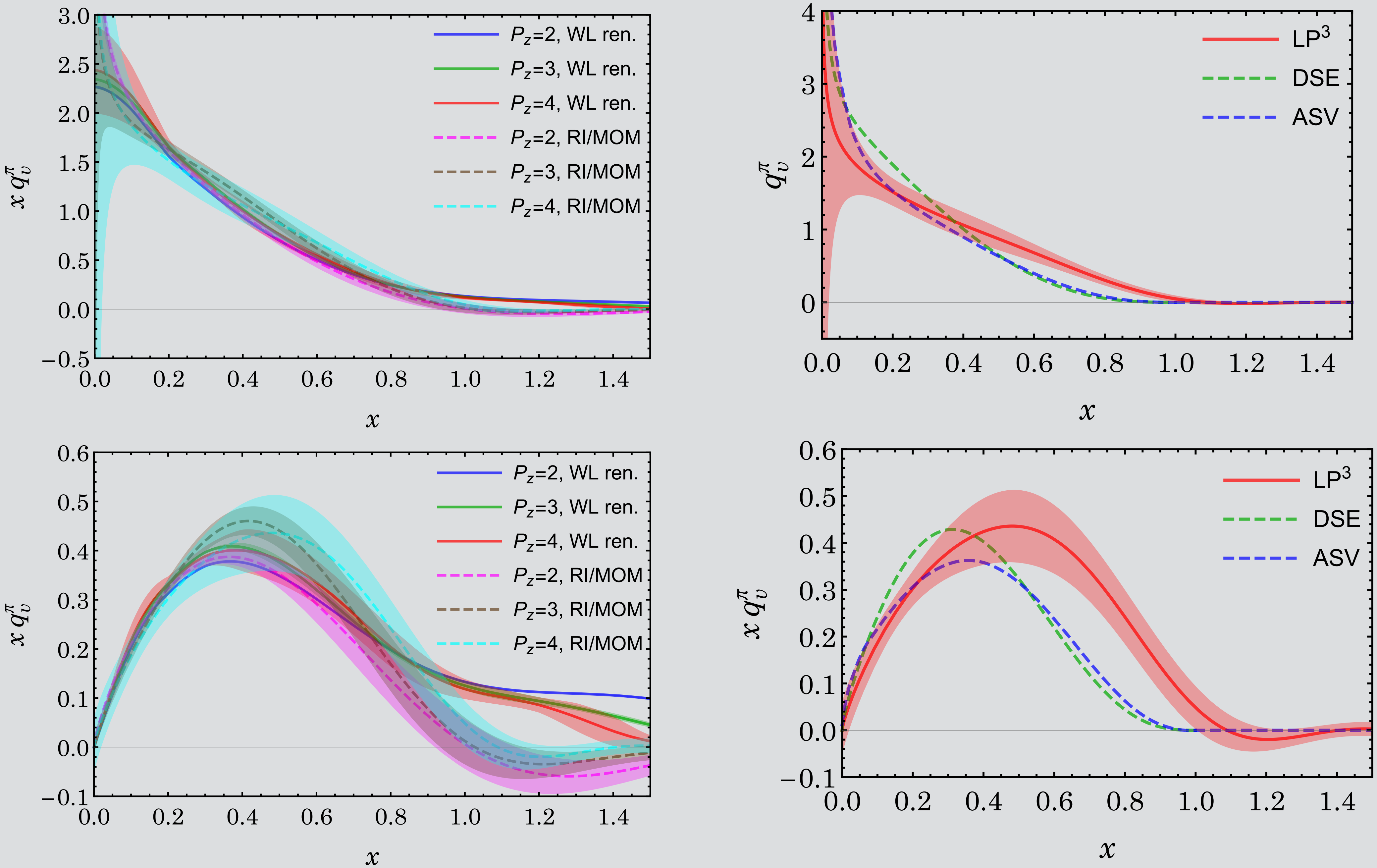
Controllable
Probe

Quantum Probabilities
Structure

Quasi-Distribution of Pion

$m_\pi \simeq 300\,\mathrm{MeV}$

LP3, arXiv:1804.01483



where

$$\tilde{f}_\alpha(x,\rho) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{x-\rho}{(1-x)(1-\rho)} + \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-3x+2x^2+\rho}{(1-x)(1-\rho)} + \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{x-\rho}{(1-x)(1-\rho)} - \frac{2x(2-x)-\rho(1+x)}{2(1-x)(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}$$
$$+ \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{\rho(-3x+2x^2+\rho)}{2(1-x)(1-\rho)(4x-4x^2-\rho)} + \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-x+\rho}{2(1-x)(1-\rho)} + \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{\rho(-3x+2x^2+\rho)}{2(1-x)(1-\rho)(4x-4x^2-\rho)} - \frac{-\rho}{4(1-\rho)^{3/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}, \tag{44}$$

$$\tilde{f}_z(x,\rho) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{-2\rho(1-7x+6x^2)-\rho^2(1+2x)}{(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} + \frac{4x(1-3x+2x^2)-\rho(2-11x+12x^2-4x^3)-\rho^2}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} \\ \quad + \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-2+2x-\rho(1-4x)}{(1-\rho)^2} g_{z\alpha} + \frac{(-1+2x)(2-3x+\rho)}{(1-x)(1-\rho)^2} \\ \quad + \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{-2\rho(1-7x+6x^2)-\rho^2(1+2x)}{(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} - \frac{4x(1-3x+2x^2)-\rho(2-11x+12x^2-4x^3)-\rho^2}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} \\ \quad - \left[\frac{\rho(4-6x-\rho)}{2(1-\rho)^{5/2}} g_{z\alpha} + \frac{2-4x+4x^2-5x\rho+2x^2\rho+\rho^2}{2(1-x)(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}$$
$$+ \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{\rho(1-2x)[-4x(1-x)(2+\rho)+3\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} + \frac{\rho[-4x(2-9x+6x^2)+\rho(1-10x+2\rho)]}{2(1-\rho)^2(4x-4x^2-\rho)^2} \\ \quad + \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{-3\rho g_{z\alpha}-1-2\rho}{2(1-\rho)^2} + \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{\rho(1-2x)[-4x(1-x)(2+\rho)+3\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} - \frac{\rho[-4x(2-9x+6x^2)+\rho(1-10x+2\rho)]}{2(1-\rho)^2(4x-4x^2-\rho)^2} \\ \quad - \frac{\rho[(2+\rho)g_{z\alpha}+3)]}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}, \tag{45}$$

$$\tilde{f}_p(x,\rho) = \frac{\alpha_s C_F}{2\pi}\begin{cases} \frac{-4x\rho(3-5x+2x^2)+\rho^2(4-3x+4x^2-4x^3)-\rho^3}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} + \frac{-2x\rho(5-6x)+\rho^2(3-2x)}{(1-\rho)^2(4x-4x^2-\rho)} \\ \quad + \left[\frac{-2\rho(1-4x+2x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{\rho(1-2x)(4-3x-\rho)}{(1-x)(1-\rho)^2} g_{z\alpha} + \frac{-2x+3\rho-4x\rho}{(1-\rho)^2} \\ \quad + \left[\frac{-\rho(2-8x+4x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{-4x\rho(3-5x+2x^2)+\rho^2(4-3x+4x^2-4x^3)-\rho^3}{(1-x)(1-\rho)^2(4x-4x^2-\rho)} g_{z\alpha} - \frac{-2x\rho(5-6x)+\rho^2(3-2x)}{(1-\rho)^2(4x-4x^2-\rho)} \\ \quad - \left[\frac{-2\rho(1-4x+2x^2)-\rho^2(2-x+2x^2)+\rho^3}{2(1-x)(1-\rho)^{5/2}} g_{z\alpha} + \frac{-\rho(2-6x+\rho)}{2(1-\rho)^{5/2}}\right] \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}$$
$$+ \frac{\alpha_s C_F}{2\pi}(1-\tau)\begin{cases} \frac{16x\rho(1-3x+2x^2)+4x^2\rho^2(3-2x)-\rho^3(5-2x)+2\rho^4}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} \\ \quad + \frac{\rho(1-2x)[16x(1-x)-2\rho(1+2x-2x^2)-\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} + \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x>1 \\ \frac{\rho(5-2\rho)g_{z\alpha}+2+\rho}{2(1-\rho)^2} + \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{1+\sqrt{1-\rho}}{1-\sqrt{1-\rho}} & 0<x<1 \\ -\frac{16x\rho(1-3x+2x^2)+4x^2\rho^2(3-2x)-\rho^3(5-2x)+2\rho^4}{2(1-\rho)^2(4x-4x^2-\rho)^2} g_{z\alpha} \\ \quad - \frac{\rho(1-2x)[16x(1-x)-2\rho(1+2x-2x^2)-\rho^2]}{2(1-\rho)^2(4x-4x^2-\rho)^2} - \frac{-\rho(4-\rho)(g_{z\alpha}+1)}{4(1-\rho)^{5/2}} \ln\frac{2x-1+\sqrt{1-\rho}}{2x-1-\sqrt{1-\rho}} & x<0 \end{cases}. \tag{46}$$

ξ^2 be small but not vanishing

Apply OPE to non-local op $\mathcal{O}_n(\xi)$

$$\sigma_n(\omega, \xi^2, P^2) = \sum_{J=0} \sum_a W_n^{(J,a)}(\xi^2, \mu^2)\, \xi^{\nu_1} \cdots \xi^{\nu_J} \\ \times \langle P | \mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2) | P \rangle \,,$$

$\mathcal{O}^{(J,a)}_{\nu_1 \cdots \nu_J}(\mu^2)$ Local, symmetric , traceless op