Strange Baryon Physics in Full Lattice QCD

Huey-Wen Lin

Theoretical Physics Seminar
University of Kentucky
2007 Nov. 5
Outline

**Lattice QCD**
- Background, actions, observables, …

**Spectroscopy**
- Group theory, operator design, first results

**Coupling constants and form factors**
- Hyperon axial coupling constants
- Strangeness in nucleon magnetic and electric moments
- Hyperon semi-leptonic decays
Lattice QCD is a discrete version of continuum QCD theory.

\[ \psi(x + \mu) \]

\[ U_\mu(x) \]

\[ \psi(x) \]
Lattice QCD is a discrete version of continuum QCD theory.

Physical observables are calculated from the path integral

\[ \langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int [dA][d\bar{\psi}][d\psi] O(\bar{\psi}, \psi, A) e^{i \int d^4 x L^{QCD}(\bar{\psi}, \psi, A)} \]

Use Monte Carlo integration combined with the “importance sampling” technique to calculate the path integral.

Take \( a \to 0 \) and \( V \to \infty \) in the continuum limit.
Lattice QCD

- A wide variety of first-principles QCD calculations can be done:
  - Since 1970, Wilson started to write down the actions
- Progress is limited by computational resources
  - But assisted by advances in algorithms
T.D. Lee uses an “analog computer” to calculate stellar radiative transfer equations
Lattice QCD

2007: The 13 Tflops cluster at Jefferson Lab

Other joint lattice resources within the US: Fermilab, BNL. Non-lattice resources open to USQCD: ORNL, LLNL, ANL.
Lattice QCD

Lattice QCD is computationally intensive

Cost \approx \left( \frac{L}{\text{fm}} \right)^5 L_s \left( \frac{\text{MeV}}{M_\pi} \right) \left( \frac{\text{fm}}{a} \right)^6 \left( C_0 + C_1 \left( \frac{\text{fm}}{a} \right) \left( \frac{\text{MeV}}{M_K} \right)^2 + C_2 \left( \frac{a}{\text{fm}} \right)^2 \left( \frac{\text{MeV}}{M_\pi} \right)^2 \right)

Norman Christ, LAT2007

Current major US 2+1-flavor gauge ensemble generation:
- MILC: staggered, \( a \sim 0.06 \text{ fm} \), \( L \sim 3 \text{ fm} \), \( M_\pi \sim 250 \text{ MeV} \)
- RBC+UKQCD: DWF, \( a \sim 0.09 \text{ fm} \), \( L \sim 3 \text{ fm} \), \( M_\pi \sim 330 \text{ MeV} \)
- Chiral domain-wall fermions (DWF) at large volume (6 fm) at physical pion mass may be expected in 2011

But for now….

need a pion mass extrapolation \( M_\pi \rightarrow (M_\pi)_{\text{phys}} \)
(use chiral perturbation theory, if available)
Lattice Fermion Actions

**Chiral fermions (e.g., Domain-Wall/Overlap):**
- Automatically $O(a)$ improved,
suitable for spin physics and weak matrix elements
- Expensive

$$D_{x,s;x',s'} = \delta_{x,x'} D_{s,s'}^{\perp} + \delta_{s,s'} D_{x,x'}^{\parallel}$$

$$D_{s,s'}^{\perp} = \frac{1}{2} [(1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2\delta_{s,s'}]$$

- (Improved) Staggered fermion
  - Relatively cheap for dynamical fermions (good)
  - Mixing among parities and flavors or “tastes”
  - Baryonic operators a nightmare — not suitable

**Wilson/Clover action:**
- Moderate cost; explicit chiral symmetry breaking

**Twisted Wilson action:**
- Moderate cost; isospin mixing
**Mixed Action Parameters**

**Mixed action:**
- Staggered sea (cheap) with domain-wall valence (chiral)
- Match the sea Goldstone pion mass to the DWF pion
- Only mixes with the “scalar” taste of sea pion
- Free light quark propagators (LHPC & NPLQCD)

In this calculation:
- Pion mass ranges 300–750 MeV
- Volume fixed at 2.6 fm, box size of $20^3 \times 32$
- $a \approx 0.125$ fm, $L_s = 16$, $M_5 = 1.7$
- HYP-smeared gauge fields
Lattice QCD: Observables

Two-point Green function

- e.g. spectroscopy

Three-point Green function

- e.g. form factors, structure functions, …

\[
\sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \left\langle J(X_{\text{snk}}) J(X_{\text{src}}) \right\rangle_{\alpha, \beta}
\]

\[
\sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \left\langle J(X_{\text{snk}}) O(X_{\text{int}}) J(X_{\text{src}}) \right\rangle_{\alpha, \beta}
\]
**Lattice QCD: Observables**

- **Two-point Green function**
  
  e.g. spectroscopy

  \[ \sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}), J(X_{\text{src}}) \rangle_{\alpha,\beta} \]

  After taking spin and momentum projection
  (ignore the variety of boundary condition choices)

- **Three-point Green function**
  
  e.g. form factors, structure functions, …

  \[ \sum_{\alpha,\beta} \Gamma^{\alpha,\beta} \langle J(X_{\text{snk}}) O(X_{\text{int}}) J(X_{\text{src}}) \rangle_{\alpha,\beta} \]

  Two-point correlator

  \[ \sum_n Z_{n,B} e^{-E_n(\vec{P})t} \]

  At large enough \( t \), the ground-state signal dominates

  Three-point correlator

  \[ \sum_n \sum_{n'} Z_{n',B}(p_f) Z_{n,A}(p_i) \times \text{FF's} \times e^{-(t_f-t)E_{n'}(\vec{p}_f)} e^{-(t-t_i)E_n(\vec{p}_i)} \]
Two-Point Green Functions

work with

Lattice Hadron Physics Collaboration (LHPC)
Why Baryons?

Lattice QCD spectrum

- Successfully calculates many ground states (Nature, …)
- Nucleon spectrum, on the other hand… not quite

Example: \( N, P_{11}, S_{11} \) spectrum
Strange baryons are of special interests; challenging even to experiment

Example from PDG Live:

<table>
<thead>
<tr>
<th>$\Xi$ BARYONS ($S = -2, I = 1/2$)</th>
<th>$\Xi^0 (u \bar{s} \bar{n})$</th>
<th>$\Xi^-(d \bar{s} \bar{n})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi^-$</td>
<td>$1/2(1/2^-)$</td>
<td>$\Xi(1820) D_{13}$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$1/2(1/2^-)$</td>
<td>$\Xi(1950)$</td>
</tr>
<tr>
<td>$\Xi(1530) P_{13}$</td>
<td>$1/2(3/2^-)$</td>
<td>$\Xi(2030)$</td>
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<tr>
<td>$\Xi(1620)$</td>
<td>$1/2(3/2^-)$</td>
<td>$\Xi(2120)$</td>
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<tr>
<td>$\Xi(1690)$</td>
<td>$1/2(3/2^-)$</td>
<td>$\Xi(2250)$</td>
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</table>

<table>
<thead>
<tr>
<th>$\Omega$ BARYONS ($S = -3, I = 0$)</th>
<th>$\Omega^-$</th>
<th>$\Omega(2250)$</th>
<th>$\Omega(2380)$</th>
<th>$\Omega(2470)$</th>
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<tbody>
<tr>
<td>$\Omega^-$</td>
<td>$0(3/2^-)$</td>
<td>$0(3/2^-)$</td>
<td>$**$</td>
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</tbody>
</table>
# Operator Design

- All baryon spin states wanted: $j = 1/2, 3/2, 5/2, \ldots$
- Rotation symmetry is reduced due to discretization rotation $O(3) \Rightarrow$ octahedral $O_h$ group

<table>
<thead>
<tr>
<th>I</th>
<th>J</th>
<th>6 $C_4$</th>
<th>8 $C_6$</th>
<th>8 $C_3$</th>
<th>6 $C_9$</th>
<th>6 $C'_9$</th>
<th>12 $C_4'$</th>
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<td>A₂</td>
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<td>-2 1</td>
<td>0 -1</td>
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Diagram: 6 C_4(1)
Operators Design

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Huey-Wen Lin — Univ. of Kentucky
Operators Design

- All baryon spin states wanted: \( j = 1/2, 3/2, 5/2, \ldots \)
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  \( \text{rotation } O(3) \Rightarrow \text{octahedral } O_h \)

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<table>
<thead>
<tr>
<th>j</th>
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<tr>
<td>1/2</td>
<td>( G_1 )</td>
</tr>
<tr>
<td>3/2</td>
<td>( H )</td>
</tr>
<tr>
<td>5/2</td>
<td>( G_2 \oplus H )</td>
</tr>
<tr>
<td>7/2</td>
<td>( G_1 \oplus G_2 \oplus H )</td>
</tr>
<tr>
<td>9/2</td>
<td>( G_1 \oplus 2H )</td>
</tr>
<tr>
<td>(11/2)</td>
<td>( G_1 \oplus G_2 \oplus 2H )</td>
</tr>
<tr>
<td>(13/2)</td>
<td>( G_1 \oplus 2G_2 \oplus 2H )</td>
</tr>
<tr>
<td>(15/2)</td>
<td>( G_1 \oplus G_2 \oplus 3H )</td>
</tr>
<tr>
<td>(17/2)</td>
<td>( 2G_1 \oplus G_2 \oplus 3H )</td>
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<tr>
<td>(19/2)</td>
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<td>(21/2)</td>
<td>( G_1 \oplus 2G_2 \oplus 4H )</td>
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<tr>
<td>(23/2)</td>
<td>( 2G_1 \oplus 2G_2 \oplus 4H )</td>
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Baryons
Operators Design

The basic building blocks

\[ \bar{B}^{ABC}_{\alpha\beta\gamma}(x) = \bar{\psi}^A_i \bar{\psi}^B_j \bar{\psi}^C_k \epsilon^{ijk} \]

- \( A, B, C \): quark flavor
- \( i, j, k \): color
- \( \alpha, \beta, \gamma \): Dirac indices

Project onto irreducible representations (irreps)

\[ \bar{B}^{\Lambda,n}_\lambda(x) = \Gamma^{\Lambda,n}_\lambda(\alpha, \beta, \gamma) \bar{B}_{\alpha,\beta,\gamma}(x) \]

- \( \Lambda \): irrep
- \( \lambda \in [1, \text{dim}(\Lambda)] \)
- \( n \): element of interoperating op

Correlator matrix

\[ C^{m,n}_\Lambda(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 | \bar{B}^{\Lambda,m}_\lambda(\vec{x}, t) \bar{B}^{\Lambda,n}_\lambda(0) | 0 \rangle \]

For more details and extended-link operator

### Variational Method

Construct the matrix

\[ C_{i,j}(t) = \langle 0 | \mathcal{O}_i(t)\dagger \mathcal{O}_j(0) | 0 \rangle \]

Solve for the generalized eigensystem of

\[ C(t_0)^{-1/2}C(t)C(t_0)^{-1/2}v = \lambda(t, t_0)v \]

with eigenvalues

\[ \lambda_n(t, t_0) = e^{-(t-t_0)E_n}(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})) \]

At large \( t \), the signal of wanted state dominates.

Unfortunately, we cannot see a clear radial excited state with the smeared propagators we got for free.
Spectroscopy Results

The non-strange baryons ($N$)

Symbols: $J^P = 1/2^+ \triangle$, $1/2^- \triangledown$, $3/2^+ \Diamond$, $3/2^- \Box$

$N$  $N$  $N(1535)$  $N(1720)$  $N(1520)$
Spectroscopy Results

The non-strange baryons ($N$ and $\Delta$)

Symbols: $J^P = \frac{1}{2}^+, \triangle, \frac{1}{2}^-, \triangledown, \frac{3}{2}^+, \diamondsuit, \frac{3}{2}^-$

- $N(1535)$
- $N(1720)$
- $N(1520)$
- $\Delta(1620)$
- $\Delta(1700)$
Spectroscopy Results

The singly strange baryons: (Σ and Λ)

Symbols: $J^P = 1/2^+, \triangle$, $1/2^-, \blacktriangle$, $3/2^+, \blacklozenge$, $3/2^-, \square$

- $\Sigma$: $\Sigma(1620)$, $\Sigma(1580)$
- $\Lambda$: $\Lambda(1405)$, $\Lambda(1890)$, $\Lambda(1520)$

![Graphs showing $M_\Sigma$ and $M_\Lambda$ vs. $M_{\pi}^2 / f_\pi^2$](image)
Spectroscopy Results

The less known baryons ($\Xi$)

Symbols: $J^P = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^+ , \frac{3}{2}^-$

$\Xi$, $\Xi(1690)$?, $\Xi(1530)$, $\Xi(1820)$
The less known baryons (Ξ )
Symbols: $J^P = 1/2^+ \triangle$, $1/2^- \blacklozenge$, $3/2^+ \blacklozenge$, $3/2^- \square$
Ξ (1690) Ξ (1530) Ξ (1820)

Babar at MENU 2007: Ξ (1690)$^0$ negative parity $-1/2$
Spectroscopy Results

The less known baryons ($\Xi$ and $\Omega$)

Symbols: $J^P = \frac{3}{2}^+ \bigtriangleup, \frac{1}{2}^- \blacktriangledown, \frac{1}{2}^- \blacktriangle, \frac{3}{2}^- \blacklozenge$

$\Xi(1690)\ ? \ \Xi(1530) \ \Xi(1820)$

Could they be $\Omega(2250)$, $\Omega(2380)$, $\Omega(2470)$?
Summary/Outlook — I

What we have done:
- 2+1-flavor calculations with volume around 2.6 fm
- Ground states of $G_{1g/u}$ and $H_{g/u}$ for each flavor
- Preliminary study with lightest pion mass 300 MeV
- Correct mass ordering pattern is seen

Currently in progress:
- Increase the statistics on the lightest two pion-mass points
- Mixed action chiral extrapolation for octet and decuplet
- Open-minded for extrapolation to physical pion mass for other states

In the future:
- Lower pion masses to confirm chiral logarithm drops
Three-Point Green Functions

in collaboration with

Kostas Orginos

- Hyperon axial coupling constants
- Strangeness in nucleon magnetic and electric moments
- Semi-leptonic decays
Green Functions

Three-point function with connected piece only

\[ C_{3pt}^{\Gamma, \mathcal{O}} (\vec{p}, t, \tau) = \sum_{\alpha, \beta} \Gamma^{\alpha, \beta} \langle J_\beta (\vec{p}, t) \mathcal{O}(\tau) J_\alpha (\vec{p}, 0) \rangle \]

Two constructions:

- Iso-vector quantities
- Use ratios to cancel out the unwanted factors

\[ \frac{\Gamma_{\mu, CG}^{BB}(t_i, t_f, \vec{p}_i, \vec{p}_f; T)}{\sqrt{\Gamma_{PG}^{BB}(t_i, t_f, \vec{p}_i; T) \Gamma_{PG}^{BB}(t_i, t_f, \vec{p}_f; T)}} \cdot \frac{\Gamma_{GG}^{BB}(t_i, t_f, \vec{p}_i; T)}{\sqrt{\Gamma_{PG}^{BB}(t_i, t_f, \vec{p}_i; T) \Gamma_{PG}^{BB}(t_i, t_f, \vec{p}_f; T)}} \]
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

Definition:

- Has applications such as hyperon scattering, non-leptonic decays, ...
- Cannot be determined by experiment
- Existing theoretical predictions:
  - Chiral perturbation theory
    
    
    $0.35 \leq g_{\Sigma\Sigma} \leq 0.55 \quad 0.18 \leq -g_{\Xi\Xi} \leq 0.36$

    M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - Large-$N_c$
    
    $0.30 \leq g_{\Sigma\Sigma} \leq 0.36 \quad 0.26 \leq -g_{\Xi\Xi} \leq 0.30$

    R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

- Loose bounds on the values
- Lattice QCD can provide substantial improvement.
Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Pion mass: 350–750 MeV
- First lattice calculation of this quantity

Chiral perturbation theory fails to describe the data

$W. \ Detmold \ and \ C. \ J. \ D. \ Lin, \ Phys. \ Rev. \ D71, \ 054510 \ (2005)$

- Including quadratic and log terms with coefficients consistent with 0
- Systematic errors: finite volume + finite $a$

$g_{\Sigma\Sigma} = 0.437(16)_{\text{stat}}(22)_{\text{syst}} \quad g_{\Xi\Xi} = -0.279(12)_{\text{stat}}(16)_{\text{syst}}$
Strange Magnetic Moment of Nucleon

- Purely sea-quark effect
- First strange magnetic moment was measured by SAMPLE
  \( G_M^s(Q^2 = 0.1 \text{ GeV}^2) = 0.23(37)(25)(29) \)
- More data is being collected today
  \textit{HAPPEX and G0 collaborations at Jefferson Lab, SAMPLE at MIT-BATES,}
  \textit{and A4 at Mainz}
- Lattice calculations

\[
\langle B | V_\mu | B \rangle(q) = \bar{u}_B(p') \left[ \gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_B} \right] u_B(p) e^{-iq\cdot x}
\]

\textit{The disconnected diagram is a must}
- Noisy estimator
  \(-0.28(10)\) to \(+0.05(6)\)
- Help with chiral perturbation theory
  \(-0.046(19)\)
- Quenched approximation

Kentucky Field Theory group (97-01)
Adelaide-JLab group (06)
Quenched Approximation

Full QCD: \[
\langle O \rangle = \frac{1}{Z} \int [dU][d\psi][d\bar{\psi}] e^{-S_F(U,\psi,\bar{\psi}) - S_G(U)} O(U, \psi, \bar{\psi})
\]

\[
= \frac{1}{Z} \int [dU] \text{det} M e^{-S_G(U)} O(U)
\]

Quenched: Take \( \text{det} M = \text{constant} \).

Historically used due to the lack of computation power

Bad: Uncontrollable systematic error
Good? Cheap exploratory studies to develop new methods
Strange Magnetic Moment of Nucleon

Disconnected diagrams are challenging
Much effort has been put into resolving this difficulty
Alternative approach:

Assume charge symmetry:

\[ p = e_u u^p + e_d d^p + O_N; \quad n = e_d u^p + e_u d^p + O_N, \]
\[ \Sigma^+ = e_u u^\Sigma + e_s s^\Sigma + O_\Sigma; \quad \Sigma^- = e_d u^\Sigma + e_s s^\Sigma + O_\Sigma, \]
\[ \Xi^0 = e_s s^\Xi + e_u u^\Xi + O_\Xi; \quad \Xi^- = e_s s^\Xi + e_d u^\Xi + O_\Xi. \]

The disconnected piece for the proton is

\[ O_N = \frac{2}{3} l G_M^u - \frac{1}{3} l G_M^d - \frac{1}{3} l G_M^s. \]

The strangeness contribution is

\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ \frac{2p + n}{u^\Sigma} - \frac{u^p}{u^\Sigma} (\Sigma^+ - \Sigma^-) \right] \]
\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ p + 2n - \frac{u^n}{u^\Xi} (\Xi^0 - \Xi^-) \right] \quad \text{with} \quad l R_d^s \equiv l G_M^s / l G_M^d \]
Strange Magnetic Moment of Nucleon

Disconnected diagrams are challenging
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Alternative approach:

Assume charge symmetry:

\[ p = e_u u^p + e_d d^p + O_N; \quad n = e_d u^p + e_u d^p + O_N, \]
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The disconnected piece for the proton is

\[ O_N = \frac{2}{3} l G^u_M - \frac{1}{3} l G^d_M - \frac{1}{3} l G^s_M. \]

The strangeness contribution is

\[ G^s_M = \left( \frac{l R^s_d}{1 - l R^s_d} \right) \left[ 3.673 - \frac{u^p}{u^\Sigma} (3.618) \right] \mu_N \]

\[ G^s_M = \left( \frac{l R^s_d}{1 - l R^s_d} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N \]

with \( l R^s_d \equiv l G^s_M / l G^d_M \)

Needs better statistics

Strange Magnetic Moment of Nucleon

- Magnetic moment $\mu_B = F_2(q^2=0)$
- Dipole-form extrapolation to $q^2 = 0$

Example: $u$-quark contribution in $\Sigma$ form factor $F_2(q^2)$
Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)


Strange Magnetic Moment of Nucleon

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\[ G_M^s = \left( \frac{l R_d^s}{1 - l R_d^s} \right) \left[ -1.033 - \frac{u^n}{u^\Xi} (-0.599) \right] \mu_N \]
Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$
- Magnetic-moment ratios (linear extrapolation, for now)

\[ R_d^s = 0.139(42) \]


We find

\[ G_M^s = -0.066(12)_{\text{stat}}(23)_{\text{sys}} R_d^s \]

H WL, arXiv:0707.3844[hep-lat]
Strange Electric Moment of Nucleon

- $G_E^s$ is proportional to $Q^2 \ r^2 \ s$


\[
\langle r^2 \rangle^s = \frac{r_d^s}{1 - r_d^s} \left[ 2\langle r^2 \rangle^p + \langle r^2 \rangle^n - \langle r^2 \rangle^u \right] \quad r_d^s = 0.16(4)
\]

- $u$-quark form contribution of vector form factors

We find

\[
G_E^s(Q^2 = 0.1 \text{ GeV}) = 0.022(61)
\]

Preliminary
Strangeness

$G^s_E - G^s_M$ plots

Hyperon Decays

Matrix element of the hyperon $\beta$-decay process $B_1 \rightarrow B_2 e^- \bar{\nu}$

$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O^V_\alpha + O^A_\alpha) u_{B_1} \overline{u}_e \gamma^\alpha (1 + \gamma_5) v_\nu$$

with

$$O^V_\alpha = f_1(q^2) \gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{f_3(q^2)}{M_{B_2}} q_\alpha$$

$$O^A_\alpha = \left( g_1(q^2) \gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}} \sigma_{\alpha\beta} q^\beta + \frac{g_3(q^2)}{M_{B_2}} q_\alpha \right) \gamma_5$$
Hyperon Decay Experiments

Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48

Summary N. Cabibbo et al. 2003 with $f_2/f_1$ and $f_1$ at the SU(3) limit

<table>
<thead>
<tr>
<th>Decay</th>
<th>Rate ($\mu s^{-1}$)</th>
<th>$g_1/f_1$</th>
<th>$V_{us}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda \rightarrow p e^{-}\bar{\nu}$</td>
<td>3.161(58)</td>
<td>0.718(15)</td>
<td>0.2224 ± 0.0034</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n e^- \bar{\nu}$</td>
<td>6.88(24)</td>
<td>-0.340(17)</td>
<td>0.2282 ± 0.0049</td>
</tr>
<tr>
<td>$\Xi^- \rightarrow \Lambda e^- \bar{\nu}$</td>
<td>3.44(19)</td>
<td>0.25(5)</td>
<td>0.2367 ± 0.0099</td>
</tr>
<tr>
<td>$\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$</td>
<td>0.876(71)</td>
<td>1.32(+.22/-.18)</td>
<td>0.209 ± 0.027</td>
</tr>
<tr>
<td>Combined</td>
<td>—</td>
<td>—</td>
<td>0.2250 ± 0.0027</td>
</tr>
</tbody>
</table>

Better $g_1/f_1$ from lattice calculations?

PDG 2006 number
Two quenched calculations, different channels

- Pion mass > 700 MeV
- $f_1(0) = -0.988(29)_{\text{stat}}$
- $|V_{us}| = 0.230(5)_{\exp(7)}_{\text{lat}}$

Guadagnoli et al.

- Pion mass ≈ 530–650 MeV
- $f_1(0) = 0.953(24)_{\text{stat}}$
- $|V_{us}| = 0.219(27)_{\exp(5)}_{\text{lat}}$

Sasaki et al.

No systematic error estimate from quenching effects!
Ademollo-Gatto Theorem

Chiral extrapolation:

SU(3) symmetry-breaking Hamiltonian

\[ H' = \frac{1}{\sqrt{3}} \left( m_s - \frac{m_d + m_u}{2} \right) q \lambda \tilde{q} \]

There is no first-order correction \( O(H') \) to \( f_1(0) \); thus

\[ f_1(0) = f_1^{SU(3)}(0) + O(H'^2) \]

Common choice of observable for \( H' : M_K^2 - M_\pi^2 \)

Step I: \[ R(M_K, M_\pi) = \frac{1 - |f'(0)|}{a^4(M_K^2 - M_\pi^2)^2} \]

Step II: \[ R(M_K, M_\pi) = b_0 + b_1 a^2(M_K^2 + M_\pi^2) \]

Obtain \( |V_{us}| \) from

\[ \Gamma = G_F^2 |V_{us}|^2 \frac{\Delta m^5}{60\pi^3} (1 + \delta_{rad}) \]

\[ \times \left[ \left( 1 - \frac{3}{2} \beta \right) \left( |f_1|^2 + |g_1|^2 \right) + \frac{6}{7} \beta^2 \left( |f_1|^2 + 2 |g_1|^2 + \Re(f_1 f_2^*) + \frac{2}{3} |f_2|^2 \right) \right] + \delta_{q^2} \]

with \( g_1/f_1 \) (exp) and \( f_2/f_1 \) (SU(3) value)
Construct an Ademollo-Gatto ratio

\[ R(M_K, M_\pi) = \frac{1 - f_1(0)}{a^4(M_K^2 - M_\pi^2)^2} \]

and extrapolate mass dependence as

\[ R(M_K, M_\pi) = b + ca^2(M_K^2 + M_\pi^2) \]
Do the mass extrapolation as

\[ f_1(0) = -1 + (b_0 + b_1 a^2 (M_K^2 + M_\pi^2)) \times a^4 (M_K^2 - M_\pi^2)^2 \]
Simultaneous Fit

Combine the momentum and mass extrapolation into one fitting form

\[ f_+(q^2) = \frac{1 + (M_K^2 - M_\pi^2)^2 (A_1 + A_2 (M_K^2 + M_\pi^2))}{\left(1 - \frac{q^2}{M_0 + M_1 (M_K^2 + M_\pi^2)}\right)^2} \]
Simultaneous Fit

Combined momentum and mass dependence

\[ f_1(0) = -0.88(15) \text{ (Preliminary)} \]
Summary/Outlook — II

From hyperon analysis

- Predictions for $g_{\Sigma\Sigma} = 0.437(16)(22)$ and $g_{\Xi\Xi} = -0.279(12)(16)$
- Preliminary proton strange magnetic and electric moments directly from full QCD: $-0.066(12)(23)$ and $-0.022(61)$
- Looking for improvements in $G_E$

More work to be done in hyperon semi-leptonic decay

- First dynamical calculation
- Preliminary result from Lin-Orginos is consistent with the previous calculation
- We need much higher statistics for a lighter-pion mass calculation (compared with the quenched one)
- Higher precision $g_1/f_1$:
  Will make the $|V_{us}|$ equivalent to or better than the one from $K_{l3}$ channel