Duality in Meson Electroproduction

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Duality in Meson Electroproduction

Abstract

We propose to study quark-hadron duality in meson electroproduction. While duality between inclusive electron-hadron scattering in the resonance and deep inelastic regimes (Bloom-Gilman or quark-baryon duality) is well established, the existence of a similar duality in meson electroproduction has not yet been tested. In the inclusive case, duality implies that single-quark scattering governs the scale of the reaction: the nucleon resonances act at low $Q^2$ as one would expect from a scaling behavior. In the semi-inclusive case, the nucleon resonances may similarly affect a scaling behavior in the final meson channel. On the other hand, it could be that (for example) the $\Delta$ resonance does not fall anomalously fast in semi-inclusive scattering, which would provide us with important clues about that phenomenon.

New evidence for factorization of the cross section at low energy loss into two pieces – quark scattering and quark fragmentation – guides our approach to this measurement. We discuss an appropriate framework in which to investigate semi-inclusive duality, and propose a measurement designed to look for the existence of pion- and, with lower statistics, kaon-tagged duality, which could be at the root of the unexpected low energy cross section factorization.

This proposal addresses two issues, which are possibly related. Firstly, does the cross section still factorize at low energy loss, and can one reproduce the fragmentation functions determined from high energy scattering? Secondly, do the nucleon resonances average around these high energy fragmentation functions?

1 Motivation

Introduction

In the early 1970s Bloom and Gilman made the phenomenological observation that there exists a duality between electron-hadron scattering in the resonance region and electron-nucleon scattering in the deep-inelastic regime [Blo70, Blo71]. In particular, they made two observations: 1) the deep-inelastic scaling curve ($F_2(x)$ at large $Q^2$) is a good approximation to the average strength of the resonances ($\nu W_2(\nu, Q^2)$ at smaller $Q^2$), and 2) the ratio of resonant to non-resonant background strength is roughly constant (i.e., they both decrease at about the same rate with increasing $Q^2$). Both of these observations are evident in Fig. 1.

This duality of behavior between resonance region scattering and deep inelastic scattering (DIS) was unexpected. It was not obvious that there should be a strong relationship between the resonance region, in which the lepton is scattering from a target hadron traditionally treated as a bound system of massive constituent quarks, and the deep-inelastic region, where the lepton is essentially scattering from a single free quark. In 1977, De Rújula, Georgi, and Politzer offered an explanation of the first Bloom-Gilman observation in terms of Quantum Chromodynamics (QCD) [DeR77a, DeR77b]. In 1990, Carlson and Mikhapadhyay showed that QCD can also account for the second observation of Bloom and Gilman [Car90].

More recently, studies have shown that quark-baryon duality is exhibited over a broader kinematic range, and with greater precision, than was previously known [Nic99, Nic00a]. In addition, local duality (individual resonant peaks averaging to the deep inelastic scattering, or DIS, scaling curve) was shown to hold quite well even down to $Q^2 \approx 0.5$ (GeV/c)$^2$. Alternatively, the individual resonance scans average to some global curve even down to $Q^2 \approx 0.1$ (GeV/c)$^2$ [Nic99, Nic00b]. This global curve coincides with the DIS scaling curve at larger $x$ (or $Q^2$). This is illustrated further in Fig. 2. The observation of duality tells us that higher twist terms mostly cancel, or are small, even at these low values of $Q^2$, when averaging over resonances. This means that single-quark scattering is the dominant process.

While the phenomenon of duality in inclusive electron scattering is now well-established, duality in the related case of semi-inclusive meson electroproduction has not yet been experimentally tested. The
Figure 1: $W_2$ versus $\xi$ for Jefferson Lab resonance data [Nic99, Nic00a] at three different values of $Q^2$ (the variable $\xi$ may be treated as Bjorken $x$ modified to account for target mass effects). The data are $(e,e')$ scans in the resonance region, corrected to the central value of $Q^2$ indicated (arrows indicate the position of the elastic peak). The lines show the NMC parameterization [Arn95] at $Q^2=5$ (GeV/c)$^2$. 
Figure 2: $F_2$ versus $x$ for Jefferson Lab and SLAC resonance data [Nic99, Nic00b], and SLAC and NMC deep-inelastic data, at four different values of $Q^2$. The solid curves show the GRV parameterization [Glu98] at $Q^2=3$ (GeV/c)^2 and $Q^2=1$ (GeV/c)^2. The dashed curves at the lower $Q^2$ panels show, for comparison only, the GRV parameterization at $Q^2=1$ (GeV/c)^2.
Figure 3: A diagram of meson electroproduction. In the proposed experiment, we detect the outgoing electron \( e \) and the outgoing meson \( m^\pm \) (a negative or positive charged pion or kaon).

The goal of this proposal is to explore the extent to which the electroproduction of mesons exhibits this same dual behavior between resonance region scattering and the scaling region. In analogy with the inclusive case, Carlson suggests several phenomena one could look for by ‘tagging’ a meson in the final state [Car98]:

- Do we observe scaling behavior as \( Q^2 \) increases?
- Do the resonances tend to fall along the DIS scaling curve?
- Does the ratio of resonant to non-resonant strength remain constant as we change \( Q^2 \)? It could be that the anomalously rapid fall of the \( \Delta \) resonance with momentum transfer observed in some channels is ‘accidental’ and that we might not see the same behavior in meson-tagged measurements.\(^1\)

Fig. 3 shows a diagrammatic figure of meson electroproduction. For reasons that will be explained below, we treat the interaction at lowest order as knockout of a quark and subsequent (independent) hadronization. The struck quark carries momentum \( xp \), where \( x \) is the fraction of the light-cone momentum of the target nucleon carried away by the struck quark.\(^2\) The outgoing meson is detected (as is the outgoing electron), and we define \( z \) in terms of target, photon, and meson four vectors: \( z = (p \cdot m)/(p \cdot \gamma) \).

In the target rest (lab) frame, this becomes \( z = E_m/\nu \), the fraction of the virtual photon energy taken away by the meson. In the elastic limit, \( z = 1 \), and the meson carries away all of the photon’s energy. We define \( p_\perp \) to be the transverse momentum of the meson in the virtual photon-nucleon system.

We reconstruct the invariant mass \( m_x \) that goes undetected. Here we refer to this quantity as \( W' \) to highlight the fact that it could play a role analogous to \( W \) for duality in the inclusive case [Afa00]. If we neglect the meson mass, \( W'^2 \) is given by

\[
W'^2 = W^2 - 2z\nu (m_p + \nu - |\mathbf{\gamma}| \cos\theta_{\gamma m})
\]

where \( \nu = E - E' \) and \( \theta_{\gamma m} \) is the relative angle between the virtual photon momentum \( |\mathbf{\gamma}| \) and the outgoing meson momentum \( |\mathbf{\gamma}| \). As in the usual inclusive scattering case, the square of the (inclusive) invariant mass \( W \) is given by

\[
W^2 = m_p^2 + Q^2 \left( \frac{1}{x} - 1 \right)
\]

If we further limit the outgoing meson to be collinear with the virtual photon momentum and require that \( Q^2/\nu^2 \ll 1 \), we can express \( W'^2 \) in terms of \( z, x \), and \( Q^2 \) as

\[
W'^2 = m_p^2 + Q^2 (1 - z) \left( \frac{1}{x} - 1 \right)
\]

\(^1\) In particular, the fall of the \( \Delta \) resonance may be the result of accidental cancellation of terms that contribute to \( G_+(Q^2) \). It may be that measurements in different channels are not subject to the same cancellation and do not observe this rapid fall.

\(^2\) For our purposes we take \( x \) to be Bjorken \( x \), but ultimately it should contain appropriate modifications.
At high energies, we expect from perturbative QCD (PQCD) that (as implied by Fig. 3) there will be factorization between the virtual photon-quark interaction and the subsequent quark hadronization. By this we mean that the cross section can be decomposed into one part dependent only on the quark hadronization, \( f(z) \), and another dependent only on the hard quark-photon interaction, \( g(x, Q^2) \):

\[
\sigma \propto f(z) g(x, Q^2).
\]

At lower energies, it does not seem obvious that we can make such a simplifying assumption. In this regime we would normally view the nucleon as a collection of constituent quarks, in which case it is not clear that the cross section would factorize. Indeed, in the proceedings of the 1994 Workshop on CEBAF at Higher Energies, B. Frois and P. J. Mulders wrote about the potential for semi-inclusive experiments at beam energies accessible to CEBAF in the near future:

In order for factorization to be valid and to have a sufficiently clear separation of the target and current fragmentation region, an electron beam energy of 10 GeV is too low [Fro94].

This is in keeping with the high energy picture in which factorization comes about as a result of quark fragmentation after it has left the vicinity of the target. We point out, however, that while at low energies there may not be clear separation of target and current fragmentation regions, \textit{if duality holds we might see behavior consistent with factorization as well as scattering whose scale is consistent with scattering in the current-dominated regime}. Consider the analogous situation with \((e,e')\) scattering, in which resonance scattering appears to be connected on a fundamental level with deep inelastic scattering.

In this case, the overall scale of the interaction in the low-\( W \) regime mirrors that at high \( W \). Similarly, if duality holds for semi-inclusive scattering, the overall scale of scattering in the low-\( W' \) region must mirror that at high \( W' \). This may come about if the various decay channels from resonances with varying \( W' \) interfere such that factorization holds. Several recent results indicate that factorization does indeed seem to hold, even down to very low energy losses. We will illustrate this following the next section.

**Duality in Meson Electroproduction**

The usual Bloom-Gilman duality involves comparison of a structure function over some range in Bjorken \( x \) at low \( W^2 \) (and hence low \( Q^2 \)) with that structure function over the same range in \( x \) but at high \( W^2 \) (and \( Q^2 \)) (as in Fig. 1). In the case of semi-inclusive meson electroproduction, if we find that the cross section factorizes, we want to extract information about the \( f(z) \) of Eq. 1. In other words, given the \( x \)- and \( Q^2 \)-dependent part of Eq. 2, we can see if the \( z \)-dependent part exhibits dual behavior. In order to warrant the single-quark scattering assumption, \( W^2 \) will be kept above 4 GeV^2.

In practice, we will extract the meson yield \( \frac{\Delta N^m}{\Delta z} \) over a range of \( z \) at several values of \( x \) and \( Q^2 \). This allows the comparison of \( \frac{\Delta N^m}{\Delta z} \) in the resonance region to that in the deep inelastic regime, which we obtain from the quark model or from parameterizations of data.

Fig. 4 shows what extracted fragmentation functions might look like as a function of \( z \) if meson duality holds. As in the case of Bloom-Gilman duality, we would like to know, firstly, whether spectra in \( z \) at low \( Q^2 \) and \( W^2 \) tend to average to the scaling curve obtained at high \( Q^2 \) and \( W^2 \). Secondly, is the relative strength of resonant bumps to background fairly constant for spectra at different \( Q^2 \), analogous to the case with inclusive scattering? Finally, what is the \( Q^2 \) behavior of the resonant bumps?

In addition to using a pion tag to study duality in electroproduction, we are considering the possibility of tagging kaons. We are presently studying whether the installation of an aerogel detector in the HMS for kaon-proton separation is feasible. If so, we plan to perform measurements of duality (requiring no additional beam time) in semi-inclusive kaon electroproduction.

\[\text{The reduction of the experimentally measured cross section to } \frac{\Delta N^m}{\Delta z} \text{ is discussed in a later section (Cross Section Reduction). In particular, we discuss how we can use the (simultaneously measured) inclusive cross section as well as the relatively small acceptance of the meson spectrometer to facilitate this reduction.}\]
Evidence of Factorization

We now review several pieces of evidence that factorization does indeed hold at low energies in meson-tagged reactions. Comparison between a recent HERMES result [Ack98] at moderate energies and a Drell-Yan experiment at high energy [Haw98] shows very interesting evidence that factorization is valid at energies where it is not necessarily expected to work (because of overlapping target and fragmentation regions) [Slo88]. The HERMES experiment measured semi-inclusive pion electroproduction ($\gamma^* N \rightarrow \pi^\pm X$) in the DIS regime, over the ranges $13 < \nu < 19$ GeV and $21 < W^2 < 35$ GeV$^2$, with an average four-momentum transfer $\langle Q^2 \rangle = 2.3$ (GeV/c)$^2$. The HERMES analysis explicitly assumed factorization in order to extract the sea asymmetry $\bar{d} - \bar{u}$. In particular, it was assumed that the charged pion yield $N^{\pi^\pm}$ factorized into quark density distributions $q_i(x)$ and fragmentation functions $D_i^{\pi^\pm}(z)$:

$$
N^{\pi^\pm}(x, z) \propto \sum_i e_i^2 \left[ q_i(x) D_i^{\pi^+}(z) + \bar{q}_i(x) D_i^{\pi^-}(z) \right],
$$

where $i$ denotes quark flavor and $e_i$ is the quark charge. The results of the HERMES analysis and comparison with Drell-Yan experiment E866 (which is at dramatically higher energies) are shown in Fig. 5. While the new data are of poor statistics, there is a clear agreement between the two data sets, indicating that the factorization assumption used in the HERMES analysis appears to be valid for the nucleon sea even at energy losses as low as 13 GeV.
Table 1: Central kinematics for the 1999 Jefferson Lab test runs.

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Figure 6: A coincidence time spectrum from one of the positive polarity Jefferson Lab test runs at a hadron momentum $p_h = 3.0$ GeV/c, showing clear peaks at 0 ns ($\pi^+$) and at -4 ns (protons). There is a small kaon peak (just visible) at -1 ns. The 2 ns RF structure of the electron beam is visible in the multiple accidental peaks.

Evidence of factorization in meson electroproduction at even lower energies is apparent in the results of several test runs made in Hall C of Jefferson Lab in August 1999. The data included measurements of semi-inclusive pion electroproduction, $(e, e'\pi^-)X$, on $^1$H, $^2$H, and Al at low energy [$\nu = 3.9$ GeV, $W'^2 = 5.9$ GeV$^2$, $Q^2 = 2.4$ (GeV/c)$^2$]. The kinematics, which are similar to those proposed for this experiment, are given in Table 1. A coincidence time spectrum for one of the runs, showing the easily separable pion peak, is given in Fig. 6. We assume that factorization holds, as in Eq. 2, which allows us to extract the fragmentation functions $D_{\pi^-}^{e^+} (z)$ from the data. Fig. 7 shows the results, plotted as a function of $z$. The solid line is a next-to-leading order (NLO) fragmentation function fit to $e^+e^-$ data at high energy ($s = W'^2 = 840$ GeV$^2$) [Bin95] and evaluated at the momentum transfer of our data. The similarity between the data and the NLO curve suggests that factorization is working quite well in this kinematic regime.

Using the Jefferson Lab test data on both hydrogen and deuterium, we can extract the ratio of probability distributions for finding down to up valence quarks in the nucleon, $d_v/u_v$. We assume charge conjugation invariance, isospin symmetry and a charge ratio of valence quarks $q_v^d/q_v^u = 4$, which allows us to use charged pion yields on both targets to extract $d_v/u_v$. The procedure, which follows that found in Ref. [Mar77], assumes factorization. The single Jefferson Lab point is plotted in Fig. 8 together with a collection of data from neutrino measurements at hundreds of GeV (CDHS, WA 21/25) [Glu98, and references therein]. Here, again, the excellent agreement between experiments from very low to very high energy loss strongly suggests that factorization holds at low energy losses.

The behavior witnessed in the above examples is quite remarkable, although all of it would be consistent with dual behavior in meson electroproduction between the resonance and DIS regions. Regardless, in this experiment we will investigate whether the factorization approach is valid at relatively low energy loss $\nu$ but large $W'^2$ ($> 3$ GeV$^2$) by trying to extract the fragmentation functions $D_{\pi^\pm}^{e^\pm} (z)$ from this data and verifying the $p_{\perp}$ dependence. In particular, are the extracted fragmentation functions and
The average $\pi^\pm$ Fragmentation Functions

Figure 7: A comparison between $\pi^\pm$ electroproduction data at low energy and a next-to-leading order (NLO) fragmentation function fit to high energy data (solid line). The curve corresponds to the average of positive and negative fragmentation functions, which is compared to the average of $\pi^+$ and $\pi^-$ data points. The data shown here were limited to pion momenta above 2.4 GeV/c in order to minimize $\pi^\pm N$ interactions in the final state.

Figure 8: The ratio $d_v/u_v$ from several high energy experiments (plot taken from Ref. [Glu98]), together with a single point extracted from the Jefferson Lab test runs (very low energy).
associated $p_{\perp}$ dependence similar to those extracted from high energy data? Furthermore, does one see a dual behavior for the kinematics with small $W^2$ ($< 3 \text{ GeV}^2$)?

We note that, in case factorization proves to be valid at these low energies, it could prove to be a powerful tool for determining ratios of probability distributions like $d_{e^+}/u_{e^-}$. The technique would be very well suited to a higher energy Jefferson Lab upgrade.

**Cornell Data**

There is one additional existing series of measurements at kinematics close to what we propose here: at Cornell a series of measurements of semi-inclusive pion electroproduction was carried out with both hydrogen and deuterium targets [Beb75, Beb76, Beb77]. This series of measurements covered a region in $Q^2$ ($1 < Q^2 < 4 \text{ GeV}^2$) and $\nu$ ($2.5 < \nu < 6 \text{ GeV}$) close to the one proposed here. This series of measurements was analyzed in terms of an invariant structure function (comparable to $N^{x^+}(x, z)$ of Eqn. 2), written in terms of the sum of products of parton distribution functions and parton fragmentation functions. The authors conclude that this invariant structure function shows no $Q^2$ dependence, and a weak $W^2$, within their region of kinematics.

This is particularly striking if one realizes that their kinematics cover a region in $W^2$ between 4 and 10 GeV$^2$, and in $z$ between 0.1 and 1. Furthermore, the final pion momentum is often only 1 GeV/c, while $p_{\perp}$, the average transverse momentum of the meson, is typically less than 0.1. In fact, for a fraction of their kinematics they are in the region $1 < W^2 < 4$, that we would associate with the nucleon resonance region. To illustrate, Fig. 9 shows the extracted invariant structure function of Ref. [Beb77] solely in the nucleon resonance region ($1 < W^2 < 4$). Circles, triangles, and squares correspond to various $W^2$ regions. Open (closed) symbols correspond to $Q^2 > (<) 2 \text{ GeV}^2$. We have not attempted to perform any correction for the different $Q^2$ of these data using e.g. the NLO parameterizations of the fragmentation functions by Ref. [Bin98], as we consider this to be beyond the region of validity of these parameterizations.

Unfortunately, not enough statistics is available, and, similarly, not enough information is available anymore, to warrant a careful check of duality or factorization in the Cornell data. However, the data seem to inform us that meson duality may start to work in these kinematics.
2 The Experiment

This experiment will use the SOS for electron detection and the HMS for detection of the (high momentum) mesons, together with the standard Hall C cryogenic target system. For reasons explained below, we require a minimum beam energy of 6 GeV in order to access kinematics with adequate meson momentum in the final state. Reinstallation of a small-profile beam pipe downstream of the target may be required in order to allow HMS operation at 10.5 degrees.

The main focus of the experiment will be on pion production. Separation of pions from protons and kaons in the 2.0 to 4.0 GeV/c range can be accomplished with the HMS Čerenkov operating with about 1.4 atmospheres of C$_4$F$_{10}$ (at room temperature). The HMS Čerenkov was designed for high-pressure operation and has been hydraulically pressure tested (with thick Al windows) up to 2.4 atmospheres. In order to extract kaon electroproduction cross sections one additionally needs to separate kaons from protons in this momentum range. The difference in coincidence timing between electron-proton and electron-kaon coincidences is of order 2 ns. However, as many more protons than kaons will be detected it may be prudent to utilize an additional particle identification technique for kaon-proton separation. For this purpose the collaboration considers building and instrumenting a suitable aerogel detector (similar to that used previously in the (SOS) and, with the assistance of Hall C staff, performing modifications to the HMS detector stack to allow its installation. In principle nine inches of real estate is available between the HMS wire chambers and the first hodoscope plane, to be compared to the close to ten inches total thickness of the SOS Aerogel detector. The experiment will take $(e,e')$ data concurrently with semi-inclusive data in order to check single arm cross sections against existing data, as well as to facilitate the reduction of cross section to $\frac{d^2 \sigma}{d^2 \omega}$ (which is discussed below).

Proposed Kinematics

In order to avoid complications from $\pi N$ final state interactions, we limit our kinematics such that the momenta of the outgoing pions is greater than 2 GeV/c. See, for example, the plots of total $\pi^\pm p$ cross sections given in Fig. 10, which are taken from the Particle Data Group [Cas08]. It is clear from these plots that, below pion momenta of a few GeV/c, there will be interactions between the final state $\pi$ and $p$ that complicate interpretation of the pion yields. Restricting kinematics to larger pion momenta permits the use of Glauber calculations to address differences in $\pi^+$ and $\pi^-$ rescattering, if necessary. In a simple Glauber calculation we estimated the total absorption correction to be 5% for a deuterium target, and the difference between the absorptions of $\pi^+$ and $\pi^-$ to be less than 1%.

Similarly, radiative correction factors seem well under control. In the soft-photon approximation a correction factor of 10% was obtained for the $(e,e'\pi)$ reaction. This number was confirmed with an independent calculation with the POLRAD code [Aku00].

Table 2 gives the central kinematics we propose for this experiment. We will take data on three targets: $^1$H, $^2$H, and Al (dummy target). The quantities $Q^2$, $x$, $W^2$, $\nu$, $E'$, $\theta_L$, $\phi_L$, and $\epsilon$ are defined according to the usual electron scattering conventions. $\theta_m$ and $p_m$ are the lab angle and central momentum of the meson spectrometer (HMS), and $z = \frac{E_m}{E'}$, the fraction of the virtual photon energy taken away by the outgoing meson. We will separately map the $x$-dependence at fixed $z$, and the $z$-dependence at fixed $x$. The kinematics have been selected to have uniform $p_{\perp}$ (up to $p_{\perp} = 0.35$ GeV/c) and $\phi$ coverage $2\pi$ at all $t$. Additional kinematics have been selected adding $5^\circ$ to the pion angle to enhance the coverage in $p_{\perp}$ up to 0.6 GeV/c. Similarly, the Mandelstam $t$ coverage will be up to 0.6 GeV/c. As an example, we show the $p_{\perp}$ coverage for $x = 0.3$ in Fig. 11. Please note that our kinematics coverage is very similar to the kinematics of the Cornell experiments [Beb75, Beb76, Beb77], apart from our far larger coverage in $p_{\perp}$.

4Note that such a detector, capable of $e-p$ separation at momenta approaching 5 GeV/c, would be a useful addition to the HMS if Hall C goes to an HMS-SHMS configuration at higher machine energies.
Figure 38.20: Total and elastic cross sections for $\pi^\pm p$ and $\pi^\pm d$ (total only) collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at http://pdg.lbl.gov/sect/contents.html. (Courtesy of the COMPAS Group, IHEP, Protvino, Russia, April 1998.)

Figure 10: A plot from the Particle Data Group showing $\pi^\pm p$ and $\pi^\pm d$ cross sections.
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Table 2: Proposed $(e,e'\pi^0)$ kinematics for running on $^1H$, $^2H$, and Al targets at $E_h = 6$ GeV. Data on $(e,e')$ and possibly $(e,e'k^\pm)$ will be taken concurrently.

![Graphs showing coverage in $p_T$ for proposed kinematics at $x = 0.30$, for $z = 0.55, 0.65, 0.75$, and 0.85, respectively. The dashed curves indicate a coverage obtained by rotating the H(MS) spectrometer by another 5 degrees.](image)

Figure 11: Coverage in $p_T$ for the proposed kinematics at $x = 0.30$, for $z = 0.55, 0.65, 0.75$, and 0.85, respectively. The dashed curves indicate a coverage obtained by rotating the H(MS) spectrometer by another 5 degrees.
Cross Section Reduction

We measure the coincidence cross section $\frac{d\sigma}{dx dp_{\perp} d\phi}$ and wish to reduce this to the differential meson yield $\frac{dN}{dz}$. This reduction is often accomplished using a Monte Carlo simulation. We now discuss two techniques we can use to reduce our dependence on Monte Carlo simulation.\(^5\) The first technique uses the simultaneously measured inclusive cross section $\frac{d\sigma}{dx dE_x}$ to eliminate the need for a Monte Carlo simulation of the electron spectrometer, and the second uses the small acceptance of the meson spectrometer to allow an analytical integration over $p_{\perp}$, eliminating the need for a full simulation of the meson spectrometer.

The meson yield $\frac{dN}{dz}$ conventionally arises in terms of a fit (with parameters $b$, $A$, and $B$) to the cross section [Dak73, Eq. 3], with normalization provided by the corresponding inclusive cross section:

$$\frac{d\sigma}{dx dE_x} = \frac{dN}{dz} b \exp(-bp_{\perp}^2) \frac{1 + A \cos(\phi) + B \cos(2\phi)}{2\pi}.$$  \hspace{1cm} (3)

We use the relationships $d\Omega_m dp_m = \frac{1}{2p_m} dp_{\perp} dp_\parallel d\phi$ and $dp_\parallel = \nu dx$ to rewrite Eq 3 as

$$\frac{d\sigma}{dx dE_x} = \frac{2p_m^2}{\nu} \frac{dN}{dz} b \exp(-bp_{\perp}^2) \frac{1 + A \cos(\phi) + B \cos(2\phi)}{2\pi}.$$  \hspace{1cm} (4)

There is no evidence of a $\phi$ dependence [Dak73], and where possible we position the meson spectrometer along the direction of momentum transfer, effectively integrating over $\phi$:

$$\frac{d\sigma}{dx dE_x} = \frac{2p_m^2}{\nu} \frac{dN}{dz} b \exp(-bp_{\perp}^2).$$  \hspace{1cm} (5)

We now take advantage of the small $p_{\perp}$ acceptance of the apparatus by expanding $\exp(-bp_{\perp}^2)$ and performing an analytical integration from 0 to $p_{\perp}\max$ (the latter can be obtained to good accuracy from geometrical arguments or from a very simple Monte Carlo simulation). The parameter $b$, which we will extract from our data, is also available from fits to other data [Dak73]. The result is an expression of the meson yield in terms of the measured inclusive and coincidence cross sections:

$$\frac{dN}{dz} = \frac{\nu}{2p_m^2 b_{p_{\perp}\max}^2} \frac{d\sigma}{dx dE_x}.$$  \hspace{1cm} (6)

Expected Rates and Background Estimates

Pion coincidence event rates for this experiment (assuming 50 $\mu$A beam current) are of order 0.3 Hz (estimate consistent with the Hall C test runs), with kaon rates down from that by factors of roughly twenty (a factor of ten due to the difference in fragmentation functions and an additional factor of two due to the kaon survival fraction being 20-40% in this momentum range). Note that these rates can be easily estimated from the procedure described in the Section on “Cross Section Reduction”, from existing parton distribution functions and fragmentation functions. The rates are hardly dependent on $z$, for fixed $x$, as the drop in fragmentation function gets compensated by the Jacobian transforming from $dp_m$ to $dp_{\perp} d\phi$ as indicated in Eqn. 4. Our goal is to collect statistics that will result in 1–2% (5–10%) uncertainties in the pion (kaon) yield per hadron setting, which corresponds to typically three hours per setting (or one hour per setting for the aluminum dummy target). The exceptions to this rule are the highest $x$ kinematics, $x = 0.6, 0.7$, where the rates for pions are only 0.05 and 0.02 Hz, respectively. Here we request ten hours per setting (three for the Aluminum dummy targets). Thus, for the highest $x$ kinematics we will only obtain a 5% uncertainty in the pion yield. The systematic uncertainty will in

\(^5\) We plan to perform the final analysis using full Monte Carlo simulations and the measured $p_{\perp}$ and $\phi$ dependences; the techniques discussed here serve as a means to quickly extract preliminary results and as a check on the Monte Carlo simulations.
<table>
<thead>
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<th>Activity</th>
<th>Time (hours)</th>
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<tr>
<td>Target changes (80)</td>
<td>20</td>
</tr>
<tr>
<td>Momentum changes (20)</td>
<td>10</td>
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<td>Polarity changes (6)</td>
<td>12</td>
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<td>Calibration and checkout</td>
<td>30</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>480 (20 days)</strong></td>
</tr>
</tbody>
</table>

Table 3: Beam time request for the proposed experiment

all cases be as large or larger than these statistical uncertainties. As we do not have complete $p_\perp$ and $\phi$ coverage, we need to assume $p_\perp$ and $\phi$ dependences of the cross section beyond our acceptances (in particular to estimate the effect of radiative processes). However, we believe a systematic uncertainty of 5 to 10% is achievable and will be sufficient for this initial test. Based on the test runs we also estimate that worst-case accidental coincidences will result in a signal-to-noise ratio in the coincidence peak of approximately 4:1.

Table 3 gives the beam time requested for the experiment. When estimating time for configuration changes, we assumed 15 minutes per target change, 30 minutes per HMS or SOS momentum change, and 2 hours for a single HMS polarity change.

3 Summary

We request 20 days of beam in Jefferson Lab Hall C in order to perform the first experimental tests of factorization at low $\nu$ and duality in pion electroproduction, a phenomenon whose analog in the inclusive realm has been shown to hold to high precision over a large range of kinematics. Several facts indicate that we can utilize fragmentation functions to extract information about duality from pion electroproduction data, and recent Jefferson Lab test runs strongly suggest the existence of such dual behavior.

Sparse information from previous Cornell data suggest that we are in the right kinematics region to investigate the onset of such a phenomenon in meson electroproduction. If confirmed, a new window on the investigation of flavor decomposition will be opened.

References


