

# Threshold Contribution to the Deuteron Extended Gerasimov Drell Hearn Sum Rule

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### **Abstract**

We propose to make a high precision measurement of the threshold contribution to the Deuteron extended GDH sum over the momentum transfer range  $0.015 < Q^2 < 0.2 \text{ GeV}^2$ . We plan to use JLab's polarized electron beam with 100–150 nA current and two incident energies of 600 and 1200 MeV. The University of Virginia solid polarized  $\text{ND}_3$  target will be utilized along with the Hall C High Momentum Spectrometer in standard configuration at  $12.5^\circ$  to  $20.5^\circ$ . The HMS excitation energy resolution will be approximately 1 MeV, providing sufficient separation from the elastic scattering process. This experiment will reduce the systematic uncertainty of the Deuteron threshold contribution to the extended GDH sum measurement of PR05-111 [1], and will also provide very precise vector asymmetries and cross sections in the quasielastic region. We request 42 hours to perform this measurement.

# 1 Theoretical Background

## 1.1 The GDH Sum Rule

The Gerasimov-Drell-Hearn (GDH) sum rule for real photon absorption was derived [2] for a nucleon target but is also valid for nuclei. For a target of spin  $S$ , mass  $M$ , and anomalous magnetic moment  $\kappa$ , it reads:

$$\int_{\nu_{th}}^{\infty} \frac{\sigma_P(\nu) - \sigma_A(\nu)}{\nu} d\nu = -4\pi^2 S \alpha \left( \frac{\kappa}{M} \right)^2 \quad (1)$$

Here  $\sigma_P(\sigma_A)$  represents the cross section for absorption of a photon of energy  $\nu$  with spin parallel (anti-parallel) to the target spin and  $\alpha$  is the fine structure constant. The inelastic threshold is signified by  $\nu_{th}$ , which is quasi-free pion production (photodisintegration) for a nucleonic (nuclear) target.

For the nucleons,  $\nu_{thr}$  in Eq. (1) represents the pion production threshold, and we have the following predictions:

$$\begin{aligned} -2\pi^2 \alpha \left( \frac{\kappa}{m} \right)^2 &= -204 \mu b \text{ (Proton; } \kappa = +1.79 \text{)} \\ &= -234 \mu b \text{ (Neutron; } \kappa = -1.91 \text{)} \end{aligned} \quad (2)$$

The Deuteron, on the other hand, has a quite small anomalous magnetic moment, which results in a GDH prediction of:

$$-4\pi^2 \alpha \cdot \left( \frac{\kappa}{m} \right)^2 = -0.65 \mu b \text{ (Deuteron; } \kappa = -0.143 \text{)} \quad (3)$$

To get an idea of the relative size of the ‘‘nuclear’’ contribution to the Deuteron sum rule, we divide the integral for the Deuteron into two regions. Region I extends from breakup to the nucleonic pion production threshold, and region II extends from the pion production threshold to  $\infty$ . The contribution from meson production in region II can be estimated by the incoherent sum of quasifree production from the individual nucleons in the Deuteron. This gives a GDH value of  $-438 \mu b$ , in stark contrast to Eq. (3) which is three orders of magnitude smaller. Clearly, the contribution from region I must play a significant role. Arenhovel [3] points out that the photodisintegration channel, which is the only photoabsorption channel below the pion production threshold, provides a large asymmetry from the M1-transition to the virtual  $^1S_0$  state. This state can only be reached if the spins of photon and Deuteron are antiparallel, and is forbidden for the parallel situation. Fig. (1) displays the intriguing cancellation of high and low energy contributions expected in the Deuteron GDH sum rule.

## 1.2 Generalization to finite $Q^2$

The GDH sum can be generalized to include an arbitrary  $Q^2$  dependence, i.e. to extend the integral away from the real photon point to treat virtual photons in electron scattering. Many different approaches have been proposed but we focus here on the approach of Ref. [7] which has the clearest theoretical explanation. Ji and Osborne [7] suggest

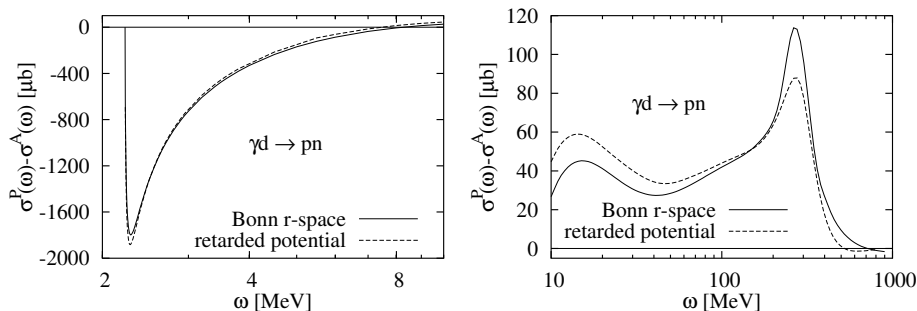


Figure 1: Spin asymmetry of Deuteron photodisintegration using (a) Bonn r-space potential [4]+MEC+IC+RC and (b) retarded potential + retarded  $\pi$ -MEC,  $\Delta$ -degrees in coupled channel,  $\pi d$ -channel + RC [5]. Left panel: low energy region; right panel: high energy region. Plot from [6].

a generalization of the GDH sum rule that takes advantage of the relationship between the forward virtual Compton amplitudes and the spin dependent structure functions. They point out that since the GDH sum rule is derived from the dispersion relation for the invariant Compton amplitude  $S_1(\nu, Q^2 = 0)$  at the real photon point, a generalized sum rule can also be constructed from the same dispersion relation at nonzero  $Q^2$ :

$$S_1(0, Q^2) = \frac{8}{Q^2} \int_0^1 g_1(x, Q^2) dx \quad (4)$$

This serves as a natural extension of the GDH sum rule to virtual-photon scattering, and represents a  $Q^2$ -dependent sum rule, provided theoretical predictions for the  $S_1$  Compton amplitude can be extended beyond the low energy theorem results at  $Q^2 = 0$ .

As in the real photon case, it is expected that the integral will be dominated by the lowest mass states. For a nuclear target, quasielastic scattering and electro-disintegration therefore play an important role.

## 2 Motivation

In this brief proposal, we describe how a short measurement in Hall C would reduce the systematic uncertainties of a measurement proposed for Hall B (PR05-111 [1]). One of the goals of PR05-111 is to determine the extended GDH sum on Deuteron at low  $Q^2$  in Hall B. A difficulty for that measurement will be to separate the elastic reaction from electrodisintegration, since it is only separated by a few MeV from the two-body breakup threshold, while the CLAS resolution is about 10 MeV. We describe in the updated version of PR05-111 the technique we plan to use to disentangle the sub-threshold contributions, which will have an associated uncertainty of about 10%. Hence, it will be the dominant error associated with the generalized GDH Deuteron

measurement\*, with the next uncertainties entering at the 5% level. **We stress that this issue has no impact on the neutron results of PR05-111, which do not require any data below  $W = 1073$  MeV.**

Establishing the  $Q^2$  dependence of the Deuteron sum rule at low  $Q^2$  is important to test the first chiral perturbation theory and lattice QCD calculations [8, 9, 10, 11] that are becoming available for nuclei. Unfortunately, an optimal measurement of the generalized GDH sum on the Deuteron at very low  $Q^2$  cannot be performed in a single JLab experimental hall. Such a measurement requires:

- a polarized Deuteron target,
- a high energy resolution detector,
- the possibility to measure very forward angles.

None of the JLab halls combines these three conditions, as shown in Table 1.

Hall	Energy resolution	Pol. D target	Forward angles
A	✓		✓
B		✓	✓
C	✓	✓	

Table 1:

However, most of the measurement of the GDH integral can be done in Hall B, since high resolution is critical only for a very small region just above Deuteron two-body breakup. This region can be measured well in Hall C, by using a low incident energy beam. Only six  $Q^2$  points are needed to complement PR05-111, and this quick measurement can be done opportunistically after one of the Hall C experiments [12, 13, 14] that require the polarized target.

### 3 Experimental Setup

We will require the High Momentum Spectrometer (HMS) in standard configuration at forward angle for an inclusive measurement of the Deuteron polarized cross sections in the threshold region. The kinematic coverage, which is shown in Fig. 2, will match the lowest momentum transfer reached by Hall B PR05-111. Comparison to Fig. 3 shows that we will cover well the region of interest. The incident energies will be 600 and 1200 MeV, and the HMS scattering angle ranges from  $12.5^\circ$  to  $20.5^\circ$ . Polarized beam with 80% longitudinal polarization, and 100 to 150 nA beam current will be required.

We will require the Hall C HMS spectrometer in the HMS-100 point to point tune, and approximately 0.1 - 0.2%  $\delta p/p$  resolution with an 18% momentum bite. The standard detector package will be used to identify electrons. We assume that the data

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\*The total systematic of the Hall B determination of the  $W < 1073$  MeV contribution to the Deuteron extended sum will be 13.5%.

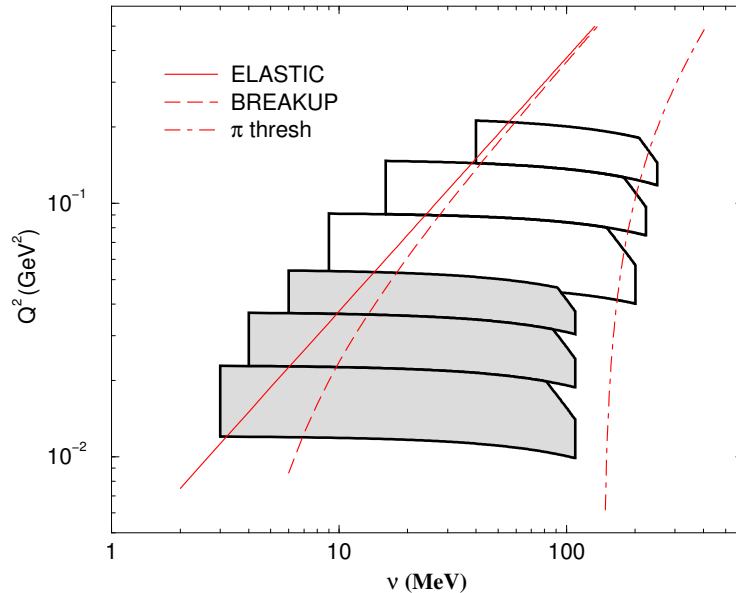


Figure 2: Kinematics coverage.  $E_0 = 600$  MeV is represented by the filled regions.  $E_0 = 1200$  MeV is represented by the open regions.

acquisition system is capable of 3 kHz event rate, although it is possible to increase this [15] with a hardware upgrade. This would be beneficial to us, and should be quite useful to the hall C program in general. The Hall C Moller polarimeter will be utilized once during this short run to evaluate the beam polarization.

The target field will be held parallel to the beam direction, and the scattering is all at small angle, so the effect of the 5T target field on the beam will be minimized. For the downstream section of beamline a helium bag will be used to transport the beam to the beam dump.

We will require the existing Hall C slow rastering system, used for previous polarized target experiments, which produces a 2 cm diameter beam spot at the target. The existing Secondary Emission Beam Position Monitor (SEM) will also be needed.

This experiment will utilize the UVA polarized target, which has been successfully used in E143/E155/E155x at SLAC, and E93-026 and E01-006 at JLab. This target operates on the principle of Dynamic Nuclear Polarization, to enhance the low temperature (1K), high magnetic field (5T) polarization of solid materials (ammonia, lithium hydrides) by microwave pumping. The polarized target assembly contains two 3 cm long target cells that can each be located in the uniform field region of a superconducting Helmholtz pair. The permeable target cells are immersed in a vessel filled with liquid He and maintained at 1 K by use of a high power evaporation refrigerator.

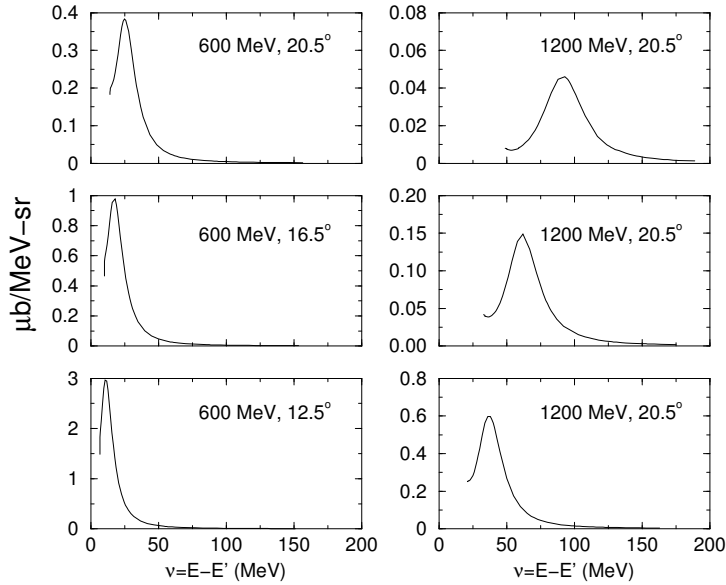


Figure 3: Deuteron unpolarized cross sections [16, 17, 18] at proposed kinematics.

## 4 Experimental Method

We will perform a polarized cross section measurement in order to determine the spin structure function  $g_1^d$  and ultimately the threshold contribution to the GDH sum, which is defined [7] as:

$$I(Q^2) = \frac{8}{Q^2} \int_0^1 g_1(x, Q^2) dx \quad (5)$$

The spin structure function  $g_1$  is related to the spin-dependent cross sections via:

$$g_1 = \frac{MQ^2}{4\alpha^2} \frac{y}{(1-y)(2-y)} \left( \Delta\sigma_{\parallel} + \tan \frac{\theta}{2} \Delta\sigma_{\perp} \right) \quad (6)$$

Here, the polarized cross section differences are represented by  $\Delta\sigma_{\parallel}$  and  $\Delta\sigma_{\perp}$ . We note that we will be measuring only  $\Delta\sigma_{\parallel}$ . The effect of this on the uncertainty is studied in detail in Ref. [1]. We state here only the conclusion that the effect arising from lack of  $\Delta\sigma_{\perp}$  ranges from 0.3 to 2.2% at the proposed kinematics. For full details, see [1].

Measuring polarized cross section differences results in the cancellation of the contribution from any unpolarized target material, so the dilution factor does not enter the systematic uncertainty. This also eliminates the need for external input from  $R$  or  $F_2$ .

$P_0$ (MeV)	$\Delta p$ (MeV)
550	0.55-0.86
1100	0.88-1.32

Table 2: HMS momentum resolution.

Source	(%)
Target Polarization	4
Beam Polarization	2
Radiative Corrections	4
Cross section	3
Transverse contribution	<2
Total	6.9

Table 3: Systematic Uncertainties

## 4.1 Resolution

In order to separate the elastic contribution from electrodisintegration, the high resolution of the HMS will be needed. The design standards [19] call for  $\delta p/p$  to be better than 0.1%. This has been confirmed [20] at the proposed kinematics, via simulation using the Hall C montecarlo. The results of the study are summarized in Table 2. The range reflects the difference between the RMS and Gaussian width of the resolution. From previous experience [20], the Monte Carlo usually predicts 20% better resolution than the data.

## 4.2 Systematic Uncertainties

Several JLab experiments (for ex., see [21, 22, 23]) have performed measurements similar to what we propose here. From these previous endeavors, we can make an estimate of the dominant<sup>†</sup> contributions to the systematic uncertainty. Table 3 gives conservative estimates of the most significant sources of error.

We expect 6.9% systematic accuracy on  $I(Q^2)$  in the threshold region, which represents a significant reduction from what is possible with a Hall B run alone.

## 5 Rates and Beam Time Request

The count rate of scattered electrons from the polarized target is given by:

$$\dot{N} = \frac{\mathcal{L}\Delta\Omega\Delta E'\sigma}{f} \quad (7)$$

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<sup>†</sup>The presence of  $^{15}\text{N}$  in the ammonia used in the polarized target introduces a small asymmetry [24] which will be negligible.



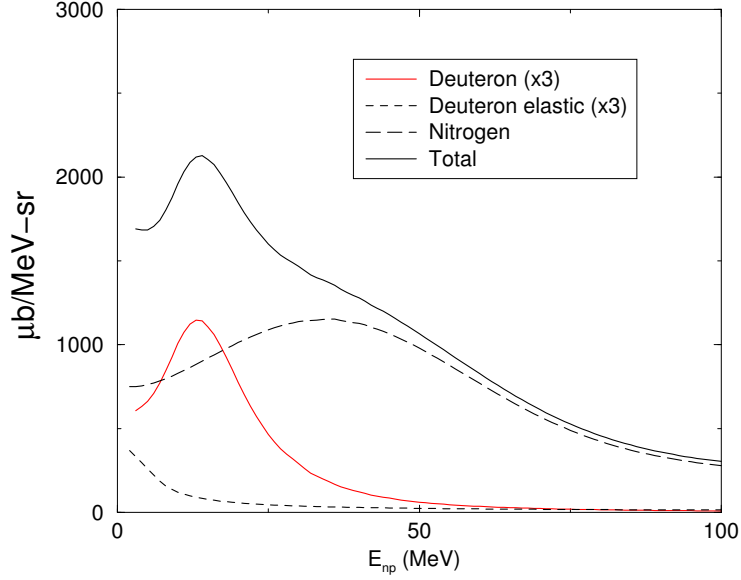


Figure 4:  $\text{ND}_3$  cross section at  $E_0 = 600 \text{ MeV}$ ,  $20.5^\circ$ .

where  $\mathcal{L}$  is the luminosity,  $\Delta\Omega$  is the angular acceptance,  $\Delta E'$  is the momentum bite,  $\sigma$  represents the  $\text{ND}_3$  cross section, and  $f$  is the dilution factor which accounts for scattering from unpolarized nucleons in the target. The value of these quantities can be estimated from previous Hall C polarized target experiments [21]:

- $\mathcal{L} = 0.85 \cdot 10^{35} \text{ cm}^{-2} \cdot \text{Hz}$
- $\Delta\Omega = 6.4 \text{ msr}$
- $\Delta E' = 1 \text{ MeV}$
- $f \approx 0.35$

We construct the ammonia cross section by combining Deuteron cross sections from Arenhovel [16, 17, 18], with Nitrogen spectra from the quasifree scattering model QFS [25, 26]. Radiated Deuteron and Nitrogen elastic tails are also included in the model. A representative spectrum is shown in Fig. 4. We note that our model predicts dilution factors of approximately 0.5, but we take the more conservative value of 0.35 in our rate calculations.

We estimate the uncertainty on the measured asymmetry  $\delta A$  by:

$$\delta A = \frac{1}{f P_b P_T \dot{N} t} \quad (8)$$

where we have assumed 80 and 25 percent for the beam and target polarization respectively. The running time and spectrometer configurations are summarized in Table 4.

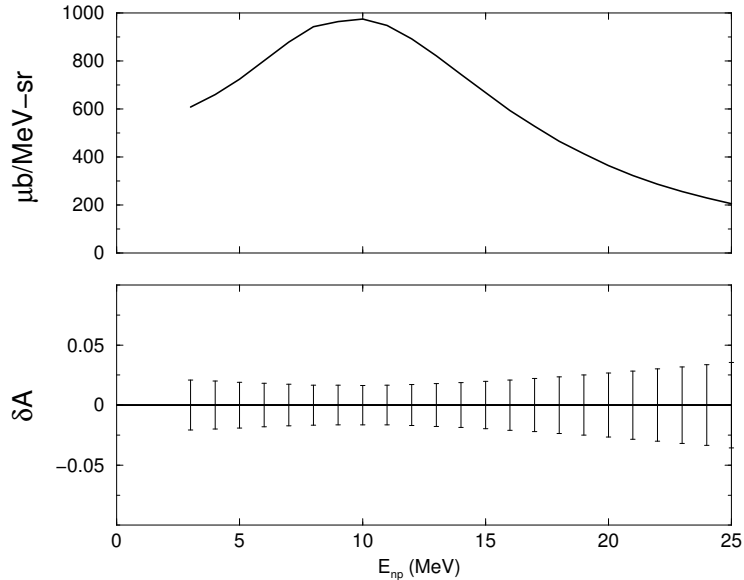


Figure 5: Statistical accuracy on the asymmetry at  $E_0 = 600$  MeV,  $16.5^\circ$ . Top panel shows the unpolarized cross section for reference.

The fifth column represents the rate<sup>‡</sup> from the Deuteron, while the sixth shows the total rate. With this choice of settings, we can achieve  $\delta A \approx 0.025$  for each 1 MeV bin in the region between breakup and the quasielastic peak.

We estimate 2 hour needed for a single Moller measurement. The beam pass change typically requires 4 hours. We include 15 minutes each to accomplish the angle and momentum setting adjustments. Four target anneal cycles, each of an hour will be required. We will opportunistically schedule them to coincide with the Moller measurement, pass change and daily accelerator beam studies, so that they do not impact the overhead. This brings the total overhead to 7.25 hours, and the total requested beamtime to 42.25 hours.

## 6 Summary

We request 42 hours of beam time to perform an opportunistic measurement of the Deuteron extended GDH sum at low  $Q^2$  in the threshold region. This short run will be used to make a precise measurement of the threshold region contribution to the GDH sum rule, and is intended to complement PR05-111. This run can remove the dominant systematic uncertainty of the Hall B run, which arises from the coarse CLAS resolution. The benefit would be a reduction of the systematic error on the threshold contribution to the Deuteron extended GDH Sum from 13.5% to 6.9%. A byproduct

<sup>‡</sup>The rate and time estimates include a prescale correction in order to reduce the total rate to 3kHz.

$E_S$ (MeV)	$\Theta_e$ (deg)	$P_0$ (MeV)	$\langle Q^2 \rangle$ (GeV <sup>2</sup> )	Rate D (Hz)	Tot Rate (Hz)	Time (h)
600.0	12.5	545.0	0.015	249	2999	7.0
600.0	16.5	545.0	0.026	457	2985	5.2
600.0	20.5	545.0	0.039	672	2958	4.3
1200.0	12.5	1109.6	0.063	869	3005	6.0
1200.0	16.5	1084.8	0.109	986	2884	6.0
1200.0	20.5	1054.9	0.165	1084	2835	6.5

Table 4: Beam request summary

of this experiment will be a precise measurement of the Deuteron vector asymmetry at low  $Q^2$  in the quasielastic region.

All equipment needed for this proposal will be already installed in Hall C and utilized by the upcoming polarized target runs [12, 13, 14], dramatically reducing the overhead typically needed for installation and commissioning.

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