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Determining the Unknown $\Lambda-n$ Interaction by Investigating the $\Lambda n n$ Resonance

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Abstract

The $\Lambda N$ interaction is an essential element in producing a unified baryonic interaction model that describes the strangeness flavor and beyond. However, only limited data exist for $\Lambda p$ scattering, and no $\Lambda n$ scattering data exist at all. In contrast to the normal nuclear situation (in which the binding energy difference between $^3H$ and $^3He$, corrected for the Coulomb energy, is relatively small and charge symmetry is a good approximation for the $NN$ interaction), recent precision experimental results on the mass of the $^4\Lambda H$ ground state and the $\gamma$ transition between the first excited state and the ground state of $^4\Lambda He$ show that charge symmetry breaking (CSB) in the $\Lambda N$ interaction is apparently much more significant. Thus, determining the unknown $\Lambda n$ interaction is critically important to understanding CSB in the strangeness nuclear physics sector.

While a bound 3-body $\Lambda nn$ system is ruled out so far by published theoretical analyses, an actual physical resonance in such a 3-body system has been shown likely to exist with as little as only an ~5% increase in the strength of the $\Lambda n$ interaction over that of the $\Lambda p$ interaction. Even a strong sub-threshold resonance would be evident. By investigating the $\Lambda nn$ resonance, knowledge of the $nn$ interaction would allow one to provide a significant constraint on the low-energy properties of the $\Lambda n$ system; that is, investigation of the 3-body $\Lambda nn$ resonance provides a unique opportunity to determine the unmeasured $\Lambda n$ interaction. Such a $\Lambda n$ interaction determination would quantify the important CSB difference relative to the measured $\Lambda p$ interaction, would provide a realistic interaction basis for understanding the long existing $\Lambda$-hypernuclei data, and would constrain the modeling of neutron stars.

The only way to investigate the $\Lambda nn$ resonance with sufficient precision is to use electroproduction of $K^+$ from a $^3H$ target, i.e. the $^3H(e,e'K^+)(\Lambda nn)$ reaction with a high precision beam at JLab. Currently, there are four approved experiments in Hall A using a common $^3H$ target that will exist until the end of August in 2018. The last one to be
executed in this run is a coincidence experiment (E12-14-011) with the \((e,e'p)\) reaction using a 4.4 GeV beam (we assume 4.524 GeV if the injector energy is included). It is found that with only a kinematic change in momentum and angle of the two HRS spectrometers, a \(^3H(e,e'K^+)(Ann)\) reaction experiment can be carried out with an energy resolution of \(\sim2\)MeV FWHM and an absolute missing mass precision of \(\sim \pm0.20\) MeV. With a similar kinematic condition as in E91-016 (Hall C kaon production from \(H, D, ^3He,\) and \(^4He\) targets) but keeping \(\theta_{\gamma K} = 0^\circ\) (maximizing the \(\Lambda\) photoproduction cross section), a beam time of 5 PAC days would be required to obtain a missing mass spectroscopy with the minimally required statistics in order to meaningfully investigate the \(Ann\) resonance.

This short experiment can generate, for the first time (and possibly for the last time if not forever), \(Ann\) resonance data with high precision that can be used to determine the \(An\) interaction. This can only be done at JLab.

I. Introduction and Physics Motivation

A. Background on the \(AN\) interaction and \(A\)-hypernuclei

A primary goal of nuclear physics is to investigate and understand the behavior of many-body systems bound by the strong interaction as well as the mechanisms that build all forms of nuclear matter observed in the universe today, from few-body nuclei to nuclear matter in the astronomical scale such as neutron stars. Although fundamental particles at the level of mesons and baryons and the level of quarks and gluons are relatively well understood, the actual interaction that gives rise to different properties and formation of matter still remains largely undetermined. Understanding the baryonic interaction that builds the variety of baryonic many-body systems is one of the essential tasks in the overall mission of nuclear physics.

The most current and broadly applied baryonic interaction models (Nijmegen [1–4] or Jülich [5, 6]) are based on the existence of an extensive amount of \(NN\) scattering data but extremely limited or non-existent data beyond \(NN\) scattering. For example, in the case of the \(AN\) interaction, data exist only for \(Ap\) scattering (obtained more than 50 years ago, where statistics are poor with only several hundred events in total and insufficient to provide stringent limits to models). \(An\) scattering data do not exist at all and are impossible to obtain directly. Thus, the two interactions are basically treated as identical in the models. Exploring the formation and properties of \(A\)-hypernuclei, which have one of the nucleons in an ordinary nucleus replaced by an \(A\), becomes the only practical mean to extend the investigation of the baryonic interaction to flavors beyond the nucleons. Although the \(AN\) interaction described by current models appears to be reasonably well understood when experimental and theoretical structure studies of \(A\)-hypernuclei are examined and compared, some obvious puzzles remain, such as the charge symmetry breaking (CSB) in the \(AN\) interaction.

New high precision mass spectroscopy on the binding energy of the \(0^+\) ground state of \(^4AH\) [7] and the energy of the \(\gamma\) transition between the \(1^+\) first excited state and \(0^+\) ground state of \(^4AHe\) [8] now determine a \(A\) separation energy difference between the ground states of \(^4AH\) and \(^4AHe\) to be \(0.27 \pm 0.06\) MeV. With the Coulomb effect expected to be at level of ~0.05 MeV, CSB in \(AN\) interaction suggested by the pair of \(A = 4\) hypernuclei is
surprisingly several times that of the smaller CSB in the $^2H$ and $^3He$ pair due to the $NN$ interaction. Its origin is unknown. Obviously, any opportunity for an experimental determination of the low-energy properties of $AN$ interaction will be extremely valuable, especially in the case of the unmeasured $An$ interaction.

B. A neutral three-body $Ann$ system - $^3An$

A recent experiment (HypHI at GSI) studied the data collected from the reaction of $^6Li$ projectiles at 2A GeV on a fixed graphite($^{12}C$) target for the invariant mass distributions of $d + \pi^-$ and $t + \pi^-$ [9], considered to be from weak decays of few-body ($A = 2$ and 3) hypernuclei produced by heavy ion collisions. The estimated mean values of the invariant mass of $d + \pi^-$ and $t + \pi^-$ systems were reported to be $2059.3 \pm 1.3 \pm 1.7$ MeV/c$^2$ and $2993.7 \pm 1.3 \pm 0.6$ MeV/c$^2$, respectively. Their lifetimes were estimated to be $181^{+30}_{-24}$ 25 ps and $190^{+47}_{-35}$ 36 ps, respectively, significantly shorter than the lifetime of a free $\Lambda$ (\sim 260 ps). These final states were interpreted as the two-body and three-body decay modes of a bound 3-body hypernucleus, thus suggesting a possible observation of a bound neutral $^3An$ system. Note that the identical method of analysis was successfully applied in the case of the $A$, $^3_AH$, and $^4_AH$ [10].

Subsequently, several theoretical analyses concluded that such a bound state cannot exist [11-13] based upon our current understanding of the $AN$ interaction. This is because of the fact that the hypertriton ($^3_AH$) is a barely bound $T = 0$ state [$B_A(^3_AH) = 0.13 \pm 0.05$ MeV], while a $Ann$ state should be a $T = 1$ system. Changing from the $T = 0$ state to the $T = 1$ state, the $(^3S_1 - ^3D_1)$ $np$ interaction, that supports a bound deuteron state, would be replaced by the $^1S_0$ $nn$ interaction that yields an unbound $di-neutron$ state.

The contradiction between the experimental suggestion and the theoretical analyses raises the question of to what extent do we understand the $AN$ interaction. Although current theoretical models can reasonably well interpret the basic nature of the so far experimentally observed spectroscopy of $A$ hypernuclei (except this $^3An$ system), the properties of the $Ap$ interaction rely on very sparse $Ap$ scattering data from early bubble chamber experiments with no $An$ scattering data existing at all. The $An$ interaction is assumed to be identical to the $Ap$ interaction with no CSB as concluded from the $NN$ interaction (i.e., the nucleon separation energy between the $^3H$ and $^3He$ systems with Coulomb correction). However, new high precision data on $^4_AH$ and $^4_AHe$ [7, 8] have established that CSB in the case of the $AN$ interaction is indeed quite significant, at a level of $\sim$270 keV. Although CSB within the $A = 4$ system was recently revisited in a calculation including $A-\Sigma$ coupling [14] that claims good agreement with the experimental results, the contradiction with regard to a bound $^3An$ system remains, i.e. no theory supports a bound $^3An$.

C. The $Ann$ resonance and the $An$ interaction

The questions are then “can a physical resonance exist” and “can such a $Ann$ resonance provide clear clues about the $An$ interaction” for which we do have no experimental data? Recently, a theoretical investigation took an alternative approach, aiming to answer these questions. It looked into the possibility of the existence of a $Ann$ three-body resonance and how it relates specifically to the $An$ interaction [15]. The conclusion is that an
experimental observation of this resonance with good precision can provide a valuable resource to determine properties of the $An$ interaction with rigorous constraints.

In this theoretical investigation, the authors used pairwise interactions of rank one, separable form that fit effective range parameters of the $nn$ system (for which experimental data exist) and those predicted for the yet-to-be-observed $An$ system by two different Nijmegen one-boson exchange potentials [16, 17], the Jülich one-boson exchange potential [18], and a chiral $AN$ potential [19] based upon the currently existing $Ap$ scattering data. The use of rank-one separable potentials allowed the authors to analytically continue the $Ann$ Faddeev equations into the second complex energy ($E$) plane in search of resonance poles, by examining the eigenvalue spectrum of the kernel of the Faddeev equations (see detailed discussion on the rank one $S$-wave separable potentials in Ref. 15). Consistency was checked by comparing the result of the code with those of the authors’ previous investigation of the hypertriton ($^3ΛH$) binding energy. In both cases, no $Λ-Σ$ coupling and $ΛΛN$ three-body interactions were included.

With the above four $AN$ potentials that assumed $An = Ap$, the results showed consistently no physical resonance; instead each produced a sub-threshold resonance with $R(E) < 0$, which lies just below the $Ann$ breakup threshold. Since this sub-threshold resonance appears so close to the threshold, the interesting question became how easy would it be to convert this pole into a physical resonance. To explore this question a scaling factor “$s$” ($s = 1.0 + Δs$) was applied to the strength of the $An$ potential in both the $^1S_0$ and $^3S_1$ $An$ channels with the increment Δs starting from 0 and increasing in steps of 0.025 (2.5%). The continuous incrementing of the scaling factor forms trajectories of resonance pole in the complex energy $E$ plane using the four different $AN$ potentials (with the $Ap$ and $nn$ potentials remaining fixed as they were fitted to the scattering data), as shown in Fig. 1 (from Fig.2 in Ref 15).

![Figure 1](image)

**Figure 1.** Trajectory of the resonance pole as one varies only the strength of the $An$ interaction by a scaling factor “$s$” with a Δs = 0.025 (2.5%) increment for the four different $AN$ potentials which fit the $Ap$ scattering data and with the same fitted $nn$ potential [Fig. 2 in Ref. 15]. The energy scale is in MeV.
The path of these trajectories start from the lower left with $\Delta s = 0.0$. All four current $AN$ potentials indicate a sub-threshold resonance with $R(E) < 0$ and no bound state with $Im(E) < 0$ for the $Ann$ system. Following the path, however, it was found that the sub-threshold pole turns first into a real physical unbound $Ann$ resonance with as little as a 5% increase in the $An$ potential for all four models, and then into a bound state that requires a change of at least 25%.

This means that a resonance possibility for the $Ann$ system certainly lies well within the uncertainties of the observed $Ap$ scattering data. In contrast the existence of a bound state would seem to be ruled out by $Ap$ scattering data unless there exists a sizable charge symmetry breaking well beyond the uncertainties in the $Ap$ scattering length and effective range. On the other hand, this resonance pole investigation suggests that one may use the $Ann$ system as a tool to examine the strength of the $An$ interaction while constraining the $Ap$ interaction within the uncertainties of the $Ap$ scattering data.

D. **Uniqueness for a JLab experiment in determining the $An$ interaction**

The low energy properties of the $An$ interaction cannot be determined without $An$ scattering data or experimental data that can at least provide a rigorous constraint on its properties. This unfortunate situation will last forever without an $A$ beam and a pure neutron target.

There exist other possible experiments but each has its difficulty. One example is the consideration of producing a $An$ system from a $D$ target using either ($K^-, \pi^0$), ($e,e'K^+$), or ($\gamma,K^+$) reactions. A $An$ system is clearly unbound and the hope would be to model the final-state interaction (FSI) to extract the properties of the $An$ interaction. Actually, in this type of analysis it is very difficult to obtain unambiguous results because of the strong model dependence due to there being three strongly interacting particles in the final state. Another suggestion is an experiment using the ($K^-\text{stop}, \gamma$) reaction to obtain a FSI for only the $An$ system. Model dependence will be much less severe. However, the difficulty lies in detecting a $\gamma$ with energy of ~300 MeV and needing a resolution of better than 1%. Such a detector system requires funds in the millions of dollars. It appears unrealistic.

Because the HypHI experiment suggested a state of the $Ann$ system was observed, regardless of whether it is a bound state or an unbound resonance state, its binding energy should likely be within ~±1 MeV around the three-body threshold. The above-presented theoretical study shows that it is highly likely to be an unbound resonance. It can be easily produced by the $(e,e'K^+)$ reaction with a tritium ($^3H$) target and with good precision utilizing the JLab beam and experimental conditions. It has clear and clean advantages over the other possibilities to obtain the $An$ interaction.

For a resonance, the $Ann$ system is a pure $T = 1$ three-body system which is dominated by the $An$ and $nn$ interactions, while the $nn$ interaction can be modeled in terms of existing experimental data. The influence from the FSI involving the $K^-$ will be rather minor but calculable even though some level of model dependence exists. Therefore, with this data available, investigating the $Ann$ final state (resonance or sub-threshold resonance) is feasible; modeling the position and shape of the spectrum, taking into account the possible CSB in the $AN$ interaction, would provide significant constraints on the scattering length and effective range of the heretofore un-measured $An$ interaction.
The opportunity is thus so unique, only for an experiment at JLab, because of the achievable energy resolution (a few MeV FWHM) and precision (~±100-200keV) in determination of the binding energy. There already exist four approved Hall A experiments (E12-10-103, E12-11-112, E12-14-109, and E12-14-011) which will share a common tritium target, as well as other gas targets such as $H_2$, $D_2$, and $^3He$ for different physics. The tritium target will be available just at the beginning of the first tritium target experiment then remain in the target chamber until completion of all four experiments within a one-year limit. The four experiments will all use the same Hall A HRS spectrometers but with different kinematic configurations and beam energy. Under the current scheduling consideration, the three single arm experiments will run in the beginning of the schedule, while the $(e,e'p)$ coincidence experiment will run last.

By carefully examining the running conditions of this $(e,e'p)$ coincidence experiment, we have found that using the same beam energy as this last experiment, simply reconfiguring the kinematics by setting the two HRS spectrometers at different angles and momenta, we can reconfigure this experiment into a $(e,e'K^+)$ experiment without any additional investment. The switch can be done within a few hours. Although using HRS as a $K^+$ spectrometer is not ideal, because of the small kaon survival rate that limits the production yield, but with the exceptionally precise characteristics of the entire system as well as the level of cleanliness of the resonance relative to the quasi-free distribution, even a limited number of events obtained in the resonance region can offer a very productive outcome and certainly will be the only experimental data for determination of the $\Lambda n$ interaction for decades to come.

II. Experiment and Measurement

E. Experimental configuration

The basic concepts in designing this experiment are (1) utilizing the existing tritium target as well as the other targets for calibration of $K^+$ production (such as the $LH_2$ target) and (2) using only the standard Hall A equipment (the HRS system). Only kinematic change is needed and most of the operational and calibration studies should already exist and be helpful through the front running experiments, especially the coincidence experiment E12-14-011. An additional short run on $K^+$ production with the $LH_2$ target is to establish $K^+$ particle identification, calibrate the absolute missing mass, and to obtain the energy resolution (and peak shaping).

The experiment will use the same beam energy, 4.4 GeV (4.524 GeV assumed to include the injector energy), as that used by E12-14-011. Thus, no beam energy change is needed. The HRS(left) used for scattered electrons ($e'$) will be relocated at 12.5° (assumed minimum angle of the spectrometer) and its central momentum will be set for 2.725 GeV/c. This is to maximize the virtual photon flux with the electron arm set at the smallest angle ($|Q^2|$ minimized to ~0.58 GeV$^2$). The virtual photons ($W = E - E' = 1.8$ GeV) will be emitted to the right at a central angle of 17.5° with respect to the beam axis. The kinematics (experimental configuration) is summarized in Table 1.
Table 1. Kinematic configuration of the proposed experiment

| Electron beam energy (2-passes, 2.2 GeV per pass) | 4.524 GeV |
| e’ HRS central momentum (acceptance) | 2.725 GeV/c (±4.5%) |
| e’ HRS central angle (acceptance) | 12.5° (6 msr) |
| K⁺ HRS central momentum (acceptance) | 1.5 GeV/c (±4.5%) |
| K⁺ HRS central angle (acceptance) | 17.5° (6 msr) |

The HRS(right) used for reaction kaons (K⁺) will be set at the virtual photon emission angle, 17.5°, so that θₚK = 0°. At this angle, the momentum transfer to the Λ is minimized (~450 MeV/c) to maximize the Λ “sticking” probability to the nn system, i.e. maximizing the production cross section (or yield) of the Ann system.

This experimental configuration is the same as that of the previously completed Hall C experiment E91-016, when the Λ production from a 3He target was measured at θₚK = 0°, except a 3.245 GeV beam was used by E91-016 with HMS for e’ at 15° and SOS for K⁺ at 13.33°. The physics outcome will be different. What E91-016 produced in the Λ production region at this kinematics point was a mixture of T = 0 Λd two-body plus Apn three-body and T = 1 Apn three-body systems. The measured production was mainly the quasi-free production, since the bound 3H state is too close to the Λ break-up threshold (Bₜ was measured to be only ~0.13 MeV); thus, its cross section is too small (due to large momentum transfer to Λ) for E91-016 to see. Furthermore, without a real resonance, the overall strength of the FSI for the T = 0 Λd two-body plus Apn three-body and T = 1 Apn three-body systems is weak (the thresholds are slightly different for the two-body and three-body systems). In addition, the energy resolution was only of the order of ~4-5 MeV; thus, the limited number of events showing FSI effects were simply smeared near and above the threshold region.

The quasi-free T = 1 Ann three-body system produced by the 3H(e,e’K⁺) reaction from the proposed experiment is basically the same as the T = 1 Apn three-body system by the 1He(e,e’K⁺) reaction from E91-016. However, a Ann system is pure and has no mixing of the T = 0 channels, so that calculation of the FSI is significantly easier. On the other hand, for a real resonance close to the threshold, its strength will be significantly stronger than that of final states without a resonance. It will be much easier to observe a peak-like structure within the ~ ±2.0 MeV region around the three-body break-up threshold. The resonance will be dominated by An and nn interactions while the FSI involving K⁺ will not have as strong an influence as in the non-resonance cases. Overall, the spectroscopy of Ann around a narrow threshold region should be much cleaner and simpler.

The only extra equipment requirement for this experiment is installation of the existing aerogel Čerenkov detector into the HRS spectrometer that will be used for hadrons at least before the (e, e’p) tritium experiment. This is to add π⁺ online rejection capability. In offline analyses, K⁺ is relatively easy to be separated from π⁺ and p using our standard technique (combining TOF, coincidence time, Čerenkov and dE/dx).
F. Yield estimation

F.1 Quasi-free $\Lambda$ production from E91-016 with $^3$He(e,e'K) reaction

Figure 2 shows the measured missing mass spectrum obtained by E91-016 at the kinematic point with $\theta_{\gamma K} = 0^\circ$ [20]. With the measured quantities and known parameters from E91-016, the yield-estimate using $\Lambda$ photoproduction cross section can be verified for consistency. Details are presented in the next sub-section.

The yield of quasi-free $\Lambda$ is estimated to be 0.7 counts/sec or $2.5 \times 10^3$ /hour. The total number of quasi-free $\Lambda$ was extracted to be $\sim 86,000$ in the spectrum after excluding $\Sigma^0$ and $\Sigma$, as well as the accidental background [20]. It means this spectrum was obtained in $\sim 34$ hours of running time with average beam current of 25 $\mu$A. The yield rate estimate made by using the $\Lambda$ photoproduction cross section is only slightly under estimated ($<10\%$). It is sufficient for the purpose of this proposal.

Figure 2. Missing mass spectrum obtained by E91-016 from the $^3$He target at the kinematic point with $\theta_{\gamma K} = 0^\circ$. The $\Lambda$, $\Sigma^0$, and $\Sigma$ quasi-free distributions were simulated by SIMC using an Atti spectral function. The solid lines are scaled simulation and sum [20].

In addition, the number of events that contain the enhancement over the quasi-free distribution at the threshold region due to the FSI can also be estimated from this measured spectrum. In case of the $^3$He target, there are two $\Lambda$ thresholds, at 2.9913 GeV for the $\Lambda d$
two-body system and 2.9935 GeV for the \( \Lambda pn \) three-body system and their average is at 2.9925 GeV. The energy resolution for E91-016 is \( \sim 5 \) MeV FWHM. Therefore, the FSI enhancement can be smeared to about 30 MeV wide around the averaged threshold. The cross section of \( ^3A\!H \) ground state is very small and locates almost right at the threshold. A \( \sim 30 \) MeV shaded area in Fig. 2 illustrates the considered region for FSI enhancement plus small contribution of \( ^3A\!H \) ground state. The SIMC simulation made by using an Atti spectral function estimates the missing mass distribution of the \( A \) quasi-free. The extra events distributed above this quasi-free distribution were considered to be from FSI enhancement. Counting only within the shaded region, the number of events is \( \sim 2600 \), which is \( \sim 3.0\% \) of the overall \( \Lambda \) production. In case of the \( ^3H \) target, both FSI and \( \Lambda \) quasi-free contain only \( T = 1 \) channel so that the ratio (3%) remains the same.

**F.2 Quasi-free \( \Lambda \) production estimation with \( ^3H(e,e'K^+) \) reaction for the proposed experiment**

The following shows the yield estimation using the same method and the known \( \Lambda \) photoproduction cross section.

**Beam energy \((E)\):** 4.5238 GeV;
**Scattered \( e' \) \((E')\):** 2.725 GeV (\( \pm 4.5\% \) with HRS)
**Scattering angle \((\theta_{e'}\):** 12.5° (acceptance: \( \Delta \Omega_{e'} = 6 \) msr with HRS)
**Beam current:** 20 \( \mu \)A (i.e. \( 20 \times 6.24 \times 10^{12} = 1.25 \times 10^{14} \) e/s)
**Integrated virtual photon flux:** \( N_\gamma = 9.47 \times 10^{-6} \frac{\gamma}{e} \times 1.25 \times 10^{14} \frac{e}{s} = 1.18 \times 10^9 \frac{\gamma}{s} \)
**Target:** \( ^3H(T_2 \text{ gas}) \) \((A = 6 \text{ and } N_p = 2 \) per molecule); \**Target length \((d)\):** 25 cm
**Target density \((p)\):** 0.0033 g/cm\(^3\); \**Mass thickness \((t)\):** 0.083 g/cm\(^2\)

Number of scattering centers (protons) per cm\(^2\):

\[
N_{\text{target}} = \frac{t}{A} \frac{N_A \cdot N_p}{6.0 \text{ [g/mol]}} \times 6.022 \times 10^{23} \text{ [mol]} \times 2 = 1.67 \times 10^{22} \text{ [/cm}^2]\]

**Assumed photoproduction cross section:** \( \frac{d\sigma_\gamma}{d\Omega_{K^+}} \approx 0.390 \text{ [\mu b/sr] = 3.9 \times 10^{-31} \text{ cm}^2/sr} \)

**K\(^+\) momentum \((P_{K^+})\):** 1.50 GeV/c (\( \pm 4.5\% \) with HRS)
**K\(^+\) angle \((\theta_{K^+} = 0^\circ)\):** 17.5° (acceptance: \( \Delta \Omega_{K^+} = 6 \) msr with HRS)
**K\(^+\) survival rate (path length of 24 m):** \( \sim 12\% \)
**K\(^+\) detection efficiency:** \( \sim 80\% \)
**K\(^+\) combined efficiency \( \epsilon_{K^+} \):** \( \sim 0.10 \) (10%)\)

Thus, the \( \Lambda \) production rate is:

\[
N_A = N_\gamma \times N_{\text{target}} \times \frac{d\sigma_\gamma}{d\Omega_{K^+}} \times \Delta \Omega_{K^+} \times \epsilon_{K^+}
\]

\[
= 1.18 \times 10^9 \frac{\gamma}{s} \times 1.67 \times 10^{22} \text{ [cm}^2]\times 3.9 \times 10^{-31} \text{ [cm}^2]\times 0.006 \text{ [Sr]} \times 0.10
\]

\[
\approx 0.00461 \text{ /sec or } \approx 16.6 \text{ /hour}
\]
Under the assumption of a minimum data collection time of 5 PAC days, the total number of \( \Lambda \) in the quasi-free production will be \( \sim 2000 \) counts. The shape of the \( \Lambda \) part (as well as the \( \Sigma^0 \) part) of quasi-free distribution will be similar to that shown in Fig. 2, except the \( \Sigma \) quasi-free production from neutrons is expected to be enhanced by a factor of 2 relatively. Statistically, it is like the vertical scale of the distribution in Fig. 2 being reduced by a factor of \( \sim 44 \), while the \( \Sigma \) quasi-free is twice higher.

**F.3 Yield of the \( \Lambda \)n resonance**

Since \( \Lambda nn \) system is a pure \( T = 1 \) three-body system without additional structures, significant strength should be pulled into this single resonance from the entire effective region for FSI. It is thus expected that the number of events in the resonance should be \( \sim 3\% \) (or more) of the quasi-free \( \Lambda \) production estimated previously in section **F.1**. If one assumes 3\% to be the minimum, the minimum number of events in the resonance will be \( \sim 60 \) counts for the data collection time of 5 PAC days. This minimum is certainly a small number. However, energy resolution, peak location in the spectrum, and the accidental background will play key roles in extracting important information from analysis of this resonance when the obtained statistics is limited.

**III. Expected Result, Uncertainties, and Beam Time Request**

**G. Simulation for expected result**

Since the missing mass distribution of quasi-free \( \Lambda \) production is basically known and the resonance is expected to locate very close to the \( \Lambda \) separation threshold, the expected result can be simulated to study the significance, uncertainty, and thus the needed beam time.

The simulation assumed: (1) an energy resolution of 2.0 MeV (FWHM) and (2) resonance is simplified with a Gaussian distribution and locates at \( B_\Lambda = 0.5 \) MeV with a natural width of 0.5 MeV (FWHM). SIMC with the Atti spectral function was used to generate the \( \Lambda nn \) three-body quasi-free distribution with desired statistics. Since \( \Sigma \) production \( (B_\Lambda > 76 \text{ MeV}) \) has no influence to the resonance peak but only the \( K^+ \) singles rate, it was ignored in the simulation. Using the E91-016 accidental rate, taking into account the new 4ns beam pulse separation and singles rate reduction due to smaller acceptance of the HRS spectrometers and lower \( K^+ \) survival rate, the maximum accidental background rate was entered in the simulation. Due to much thinner target thickness, the real accidental rate should be actually lower than that was assumed.

Figure 3 shows the expected spectroscopy for two difference statistics: (a) running time of 5-PAC days as previously discussed, i.e. 2000 events for the \( \Lambda \) quasi-free and 60 events (3\%) for the resonance; and (b) running time of 10-PAC days with 4000 events for the \( \Lambda \) quasi-free and 120 events for the resonance. The peak significance for 5-PAC days is \( \sim 6\sigma \) while for 10-PAC days it is \( > 8\sigma \). A data collection with 5-PAC days appears to be sufficient to at least identify the resonance clearly.
Figure 3. Simulated spectroscopy that contains the $\Lambda nn$ resonance and the $\Lambda$ quasi-free production only with two different statistics: (a) 2000 events for the $\Lambda$ quasi-free and 60 for the resonance and (b) 4000 events for the $\Lambda$ quasi-free and 120 for the resonance. The resonance is assumed to locate at $B_\Lambda = 0.5$ MeV with a natural width of 0.5 MeV (FWHM) and experimental resolution of 2 MeV (FWHM).

H. Uncertainties

However, the statistical uncertainty for determination of the binding energy (or excitation energy) of the $\Lambda nn$ resonance ($B_\Lambda$) and its natural width should be the main concern. Thus, a few sets of independent dummy data were generated and were fitted with a Gaussian function for the resonance peak, simply with a polynomial function for the $\Lambda$ quasi-free, and the known accidental background function (which can be typically obtained precisely with high statistics using mixing-events analysis technique). The uncertainty was then examined by comparison between the fitted and given values.

Figure 4. Example of fitting the dummy data with two different statistics: (a) 2000 events for the $\Lambda$ quasi-free and 60 for the resonance and (b) 4000 events for the $\Lambda$ quasi-free and 120 for the resonance. The resonance is assumed to locate at $B_\Lambda = 0.5$ MeV with a natural width of 0.5 MeV (FWHM) and experimental resolution of 2 MeV (FWHM).
Figure 4 shows examples of the spectroscopy fitting with simplified functions and two different statistics (5-PAC days and 10-PAC days). Result of these two fitting studies is shown in Table 2 for comparison. Uncertainties are improved significantly for number of events in the resonance increased from 60 to 120.

Table 2. Statistical uncertainties for the fitted parameters, binding energy $B_\Lambda$, peak width $\sigma$, and number of events $N$, in the resonance with two different statistics.

<table>
<thead>
<tr>
<th>Statistics vs parameters</th>
<th>$-B_\Lambda$ (MeV)</th>
<th>Width $\sigma$ (MeV)</th>
<th>Number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-PAC days (Given value)</td>
<td>0.71 ± 0.18 (0.5)</td>
<td>0.95 ± 0.20 (0.875)</td>
<td>53 ± 16 (56)</td>
</tr>
<tr>
<td>10-PAC days (Given value)</td>
<td>0.43 ± 0.09 (0.5)</td>
<td>0.81 ± 0.07 (0.875)</td>
<td>118 ± 18 (120)</td>
</tr>
</tbody>
</table>

Studies were also done for further increased beam time, up to 16-PAC days. It showed that further improvement of the uncertainty for the fitted binding energy $B_\Lambda$ and width becomes small ($\delta B_\Lambda$ from ±0.090 MeV to ±0.073 MeV and $\delta \sigma$ from ±0.07 MeV to ±0.06 MeV for beam time from 10-PAC days to 16-PAC days). The turning point for the significance of uncertainty improvement appears around 10-PAC days.

The systematic uncertainty for binding energy $B_\Lambda$ and width of resonance depends on the calibration using $H_2$ target and production of $\Lambda$ and $\Sigma^0$ whose masses are known. This calibration is critical and is the same technique used by the HKS experiment to obtain ≤ ±0.07 MeV systematic uncertainty for binding energy $B_\Lambda$ [21]. The peak shape and width of $\Lambda$ and $\Sigma^0$ will be used to obtain the energy resolution and radiative tail effect to the Gaussian shape in order to extract the natural width of the resonance with a systematic uncertainty ≤ ±0.06 MeV. This systematic uncertainty depends on the statistics of $\Lambda$ and $\Sigma^0$.

The $H_2$ gas target (with identical cell geometry) to be used by the approved tritium experiments has a density of 2.75 $\mu$g/cm$^3$ and 68.8 mg/cm$^2$ thickness. Assuming the same beam current (20 µA), the same calculation as in previous section $F.2$ (with $A = 2$ and $N_p = 2$ per molecule) results 42/hour yield rate for $\Lambda$. 2000 events of $\Lambda$ can be obtained in 2-PAC days while there will be ~300-400 events for $\Sigma^0$. This is the needed statistics to ensure the above systematic uncertainties for the binding energy and width of the $\Lambda nn$ resonance.

J. Beam time request

The required beam time impacts the uncertainty of the scaling factor “$\Delta s$” for the $\Lambda n$ interaction in the resonance pole search from the uncertainty of $B_\Lambda (Re[E])$ and width ($Im[E]$) of the resonance measurement. To illustrate this, uncertainty boxes are drawn onto the resonance pole trajectory figure at arbitrary points with two different statistics assumptions, as shown in Fig. 5.
Figure 5. Illustration of the impact to the pole search scaling factor “\(\Delta s\)” by the uncertainty size with the two different statistics assumptions.

It should be emphasized that control of the uncertainty size is not primarily aiming to distinguish which of the baryonic interaction models (of the four used in this study) is more correct. It actually impacts the scaling factor “\(\Delta s\)” that is the percentage increase of the \(A n\) interaction strength over the \(A p\) interaction. Under the current assumption of \(A p\) equal to \(A n\) (without CSB), all four models consistently predict that no physical resonance or bound state of \(A nn\) system exists (the lower left four points in Fig. 1 and 5). Depending on the point location measured by the proposed experiment, the uncertainty in “\(\Delta s\)” given by the statistics from 5-PAC days is ~6-8% (better controlled by the width \(Im[E]\)), while that given by the statistics from 10-PAC days is ~3-4%.

Given that the \(A n\) interaction will be determined by this resonance measurement together with the \(A p\) interaction fitted to the currently existing \(A p\) scattering data and the more precisely and most recently measured CSB (0.27 ± 0.06 MeV), the uncertainty in “\(\Delta s\)” from the statistics of 5-PAC days is just barely compatible to the other two known pieces of information. On the other hand, the statistics of 10-PAC days will give the uncertainty in “\(\Delta s\)” better than the other two, especially the \(A p\) interaction. Since there are efforts to obtain further improved \(A p\) scattering data, a better “\(\Delta s\)” uncertainty would certainly have more of a key impact on the \(A N\) interaction, especially when there is no other possibility to obtain actual \(A n\) scattering data at all. Therefore, we make a beam time request based on the minimum and maximum impacts in the determination of the \(A N\) interaction, as listed in Table 3. We should point out here that the minimum impact does not mean nothing is determined. The proposed experiment will, for the first time and probably the last time, provide precision experimental data that can be used to determine the \(A n\) interaction. The maximum option is to be preferred in order to minimize the “\(\Delta s\)” uncertainty to almost the lowest level possible from such type of a resonance spectroscopy experiment.
Table 3. Beam time request based on the level of impact on physics

<table>
<thead>
<tr>
<th>Minimum impact</th>
<th>$T_2$ run ($Ann$ spectroscopy)</th>
<th>$H_2$ run (Calibration with $\Lambda$ and $\Sigma^0$)</th>
<th>5 PAC days</th>
<th>2 PAC days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum impact</td>
<td>$T_2$ run ($Ann$ spectroscopy)</td>
<td>$H_2$ run (Calibration with $\Lambda$ and $\Sigma^0$)</td>
<td>10 PAC days</td>
<td>2 PAC days</td>
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</table>

IV. Summary

The availability of a $T_2$ gas target for the four currently approved Hall A experiments provides a unique and the only opportunity for an experiment to measure the $Ann$ three-body resonance. Only at JLab with the $(e,e'K^+)$ reaction, its binding (or excitation) energy and width can be measured with the minimally required precision. With this data, a theoretical resonance pole search technique can be used to determine experimentally for the first time the $Ann$ interaction together with the $Ap$ scattering and CSB data. Different baryonic models can then be used in calculations of hypernuclear spectroscopy (from few-body to heavy nuclei) and further investigation of the isospin dependence, $ANN$ three-body force, and the EoS for neutron stars.

References

[9] “Search for evidence of $^3\Lambda n$ by observing $d + \pi^-$ and $t + \pi^-$ final states in the reaction of $^6\text{Li} + ^{12}\text{C}$ at $2\text{A GeV}$”, C. Rappold et al., Phys. Rev. C 88, 041001(R) (2013).