Measurement of $^3$He Elastic Electromagnetic Form Factor
Diffractive Minima Using Polarization Observables

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Abstract

We propose to constrain the locations of the first diffractive minima in the electric and magnetic form factors of $^3$He using polarization observables. Significant discrepancies in the locations of the diffractive minima exist between theoretical predictions and experimental data from Rosenbluth separations and global fits of the world data. Thus far, all experimental data on the $^3$He form factor minima come from unpolarized electron scattering results. However, double-polarization experiments have found large disagreement, particularly at high $Q^2$, between the proton form factors extracted via polarization observables and unpolarized Rosenbluth separations.

The double-polarization asymmetry, using a polarized electron beam and a polarized $^3$He target, is proportional to the product of the electric and magnetic form factors. Unlike a Rosenbluth separation, this measurement is sensitive to the signs of the form factors. Thus, the zeros of the asymmetry correspond to the diffractive minima of the form factors. By measuring this asymmetry as a function of $Q^2$ we will constrain the locations of the first diffractive minima for the $^3$He electric and magnetic form factors. Another benefit of using polarization observables is that the ratio in the asymmetry leads to many of the systematic uncertainties cancelling one another. This measurement of $^3$He will determine whether the form factors extracted via Rosenbluth separations and polarization observables differ at high $Q^2$. It will also determine whether theoretical predictions or current experimental results are more accurately predicting the location of the diffractive minima while ruling out numerous models.

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I. INTRODUCTION

The electromagnetic form factors of $^3$He have been determined many times using elastic electron scattering. To date, this has been done by using either the Rosenbluth separation technique or by fitting the cross section world data with some parametrization of the form factors. Each of these methods introduces challenges to extracting form factors, particularly in the region of the form factors’ diffractive minima.

The Rosenbluth formula, given in Equation 1, can be used to extract the electric ($G_E$) and magnetic ($G_M$) form factors of target nuclei.

\[
\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1 + \tau} \left[ G_E^2 (Q^2) + \frac{\tau}{\epsilon} G_M^2 (Q^2) \right]
\]  

(1)

The kinematic factors are defined as $\epsilon = (1 + 2(1 + \tau) \tan^2 \left(\frac{\theta}{2}\right))^{-1}$ and $\tau = \frac{Q^2}{4M^2}$, where $\theta$ is the scattering angle of the electron. To separate the form factors a reduced cross section can be defined as in Equation 2.

\[
\left(\frac{d\sigma}{d\Omega}\right)_r = \left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} \epsilon (1 + \tau) = \left[ \epsilon G_E^2 (Q^2) + \tau G_M^2 (Q^2) \right]
\]  

(2)

By plotting $(\frac{d\sigma}{d\Omega})_r$ against $\epsilon$ the slope of the line gives $G_E^2$ and the $y$-intercept gives $\tau G_M^2$.

Another method to extract form factors is to use a parametrization like the sum of Gaussians (SOG) and fit the world data as done in References [1, 2]. Using both of these elastic electron scattering methods the form factor world data for $^3$He reaches up to a $Q^2$ of 65 fm$^{-2}$. However, the majority of this data is taken at lower $Q^2$ and as such $|G_E|$’s first diffractive minimum is well constrained, but $|G_M|$’s first minimum is much less well known. Figures 1 and 2 show these form factors using the SOG fits of References [1, 2] where an uncertainty band is given for each parametrization around a ‘representative’ fit near the centers of the uncertainty distributions. These figures also show four modern theoretical predictions from Reference [3] using a ‘conventional’ theoretical approach, two $\chi$EFT predictions with different momentum space cutoffs, and a covariant spectator theorem model.

Figures 1 and 2 show that modern theoretical predictions differ from one another significantly. Current experimental form factors are in reasonable agreement with several theory predictions of $G_E$. However, experimental form factors are in strong disagreement with
FIG. 1: $^3\text{He}$ $F_{ch}$ SOG fits and uncertainty bands from References [1, 2] along with four theoretical predictions from Reference [3]. Note that $F_{ch}$ is plotted here and $F_{ch} = G_E$.

theoretical predictions of $G_M$. Previous measurements of the proton elastic form factor have shown significant differences in the EM form factors at high $Q^2$ when determined by Rosenbluth separation versus by polarization observables [4]. This experiment would be the first $^3\text{He}$ form factor measurement using polarization observables and would determine if an effect similar to the proton elastic form factor’s is present in $^3\text{He}$ at higher $Q^2$.

Each $^3\text{He}$ form factor measurement or fit in the current world data is dependent upon either performing Rosenbluth separations in the area of the diffractive minima, which is extremely challenging, or fitting cross section data using a form factor parametrization with sharp diffractive minima as an assumption. Figure 3 shows the $^3\text{He}$ cross section at 1 GeV and 3 GeV using the form factor parametrizations in Reference [2]. These cross sections are typical in having only a relatively shallow minimum from which we try to extract sharp form factor minima using the Rosenbluth separation technique.

This begs the question of how well parametrizations assuming sharp form factor minima will perform when fitting cross sections, as the shallow cross section minima increase the difficulty of determining the precise locations of the form factors’ diffractive minima. A double-polarization measurement would constrain the the locations of the EM form factors’ first
FIG. 2: $^3\text{He}$ $F_m$ SOG fits and uncertainty bands from References [1, 2] along with four theoretical predictions from Reference [3]. Note that $F_m$ is plotted here and $F_m = G_M/\mu$, where $\mu$ is the $^3\text{He}$ magnetic moment.

FIG. 3: Plots of the $^3\text{He}$ cross section at two different energies. When performing a Rosenbluth separation we are trying to extract form factors with sharp minima from cross sections that display only a single shallow minimum increasing the difficulty of determining the precise locations of the form factors’ diffractive minima. Form factor parametrizations from Reference [2].

diffractive minima without making these assumptions or trying to perform Rosenbluth sep-
arations in the diffractive minima. Further, double-polarization measurements have shown divergent results from theoretical predictions in past $^3$He experiments [5, 6].

The physical asymmetry is given by Equation 3.

$$A_{\text{phys}} = \frac{-2\sqrt{\tau(1 + \tau)} \tan \left( \frac{\theta}{2} \right)}{G_E^2 + \frac{\tau}{2} G_M^2} \left[ \sin (\theta^*) \cos (\phi^*) G_E G_M + \sqrt{\tau \left[ 1 + (1 + \tau) \tan^2 \left( \frac{\theta}{2} \right) \right]} \cos (\theta^*) G_M^2 \right]$$

(3)

$\theta^*$ and $\phi^*$ are the polar and azimuthal angles of the polarization vector of the target (in the lab frame with $\hat{z}$ parallel with the virtual photon momentum, $\hat{q}$, and $\hat{x}$ in the scattering plane). The relative contributions of the cross term, $G_E G_M$, and the $G_M^2$ term can be experimentally controlled through the target polarization direction. The observable to be measured, $A_{\text{meas}}$, is given in Equation 4:

$$A_{\text{meas}} = \frac{N^+ - N^-}{N^+ + N^-},$$

(4)

where $N^+$ ($N^-$) is the normalized counting rate for positive (negative) beam helicity. $A_{\text{meas}}$ is related to the true asymmetry via Equation 5:

$$A_{\text{meas}} = P_t P_l A_{\text{phys}},$$

(5)

where $P_t$ is the degree of polarization of the target and $P_l$ is the degree of polarization of the electron beam.

Currently, knowledge of the diffractive minima for elastic scattering off of light nuclei is constrained only by unpolarized experiments, which use the Rosenbluth formula to extract $G_E^2$ and $G_M^2$. By contrast, the double-polarization asymmetry is sensitive to the signs of $G_E$ and $G_M$ through the cross term in Equation 3. Since the diffractive minima of $G_E^2(Q^2)$ and $G_M^2(Q^2)$ correspond to the zeros of $G_E(Q^2)$ and $G_M(Q^2)$, a measurement of the zeros of the asymmetry cross term immediately determines the locations of the diffractive minima. Figure 4 shows a simple example of the double-polarization asymmetry versus $Q^2$. The exploitation of polarization observables should enable a more precise determination of the location of the first diffractive minima than is possible through Rosenbluth-style measurements of $G_E^2$ and $G_M^2$. 


FIG. 4: Double-polarization asymmetry at 2.216 GeV using the SOG fits in Reference [2]. The zero-crossings in the asymmetry correspond to the diffractive minima in $G_E$ and $G_M$. The points show the statistical uncertainty of the mean of each kinematic setting.

A measurement of the double-polarization asymmetry will also allow us to hypothesis test the different theoretical models. By taking $G_E$ and $G_M$ from the theoretical predictions the double-polarization asymmetry can be plotted for each model. These theory asymmetries can then be compared against the asymmetry measurements of this experiment. (More simply one can just compare the asymmetry zero-crossings to the locations of the theoretical diffractive minima.) This provides a never before used tool to rule out theoretical $^3$He form factor models that are inconsistent with the form factor results from polarization observables.

II. PROPOSED PROCEDURE

A. Measurement

We propose to precisely determine the double-polarization asymmetry in elastic electron scattering off a polarized $^3$He target in Hall C. If the target polarization is chosen such that $\cos(\phi^*) \approx 1$ and $\theta^* \approx \frac{\pi}{2}$, the asymmetry becomes proportional to $G_E G_M$ (see Equation 3).
TABLE I: Expected $^3$He Target Characteristics

<table>
<thead>
<tr>
<th>Length [cm]</th>
<th>Max Rate [$\mu$A]</th>
<th>Target Polarization</th>
<th>Beam Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>30</td>
<td>55%</td>
<td>85%</td>
</tr>
</tbody>
</table>

B. Apparatus

This measurement will require the standard Hall C equipment. In particular, we propose to employ both the HMS and SHMS in their standard configurations to make these measurements. We plan to position the HMS at a single angle for the entirety of the run. The HMS will be taking data centered on the anticipated magnetic form factor diffractive minima such that the maximum amount of data may be collected in this region. The SHMS will begin at small angles and step up incrementally allowing it to scan the $Q^2$ region passing the first diffractive minima in the electric form factor up to the anticipated first minima of the magnetic form factor. This will verify the position for $G_E$’s first diffractive minimum and improve knowledge of the location of $G_M$’s first diffractive minimum. The target will be the $^3$He target developed for E12-06-110 and E12-06-121. The expected target characteristics are shown in Table I along with beam polarization.

C. Beam Requirements

We propose to parasitically make these measurements at 2.216 GeV when experiment E12-06-121 [7] takes data on the product of the target polarization and the beam polarization. (Note: that we are not dependent upon using a highly precise beam energy, but merely chose the energy already planned for polarization tests.) As such we request no beam time for this experiment and instead propose to make these measurements during the previously allocated one PAC day of beam time E12-06-121 plans at this kinematic. Table II shows the planned kinematic settings for the spectrometers along with Monte Carlo rate estimates for the proposed experiment. Figure 4 shows the expected asymmetry with each kinematic setting and its projected statistical uncertainty.
TABLE II: Spectrometer Central Kinematics

<table>
<thead>
<tr>
<th>Spectrometer</th>
<th>$E_{beam}$</th>
<th>Label</th>
<th>$\theta$ [°]</th>
<th>$Q^2$ [fm$^{-2}$]</th>
<th>Estimated Cross Section [mb/sr]</th>
<th>Rate [Events/hr]</th>
<th>Time [hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHMS</td>
<td>2.216</td>
<td>k1</td>
<td>11</td>
<td>4.57</td>
<td>4.39×10$^{-4}$</td>
<td>2,605,270</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k2</td>
<td>13</td>
<td>6.34</td>
<td>5.14×10$^{-5}$</td>
<td>305,609</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k3</td>
<td>15</td>
<td>8.38</td>
<td>4.37×10$^{-6}$</td>
<td>25,946</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k4</td>
<td>17</td>
<td>10.66</td>
<td>2.22×10$^{-7}$</td>
<td>1,319</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>k5</td>
<td>19</td>
<td>13.18</td>
<td>5.97×10$^{-8}$</td>
<td>355</td>
<td>11</td>
</tr>
<tr>
<td>HMS</td>
<td>2.216</td>
<td>k6</td>
<td>21</td>
<td>15.93</td>
<td>3.99×10$^{-8}$</td>
<td>427</td>
<td>24</td>
</tr>
</tbody>
</table>

For the high rate kinematics, these measurements will not be statistics limited and those points will be used to check systematics. For the lowest $Q^2$, this measurement can be used to precisely determine the product of beam-target polarization.

III. CONCLUSION

In collaboration with the $d_2^n$ experiment, we will take and analyze elastic scattering double-polarization asymmetry data over a range of $Q^2$. This data will verify the location of the first diffractive minima of the $^3$He electric form factor and will significantly constrain the location of the magnetic form factor’s first minima. This will be the first time a double-polarization asymmetry technique has been used to determine the location of a diffractive minima and, while it will likely provide expected results, history has shown that asymmetry measures can reveal problems with cross section extracted form factors [8].


[7] B. Sawatzky et al., “A Path to ‘Color Polarizabilities’ in the Neutron: A Precision Measurement of the Neutron $g_2$ and $d_2$ at High $Q^2$ in Hall C.”.