PROPOSAL TO JLAB PAC47

High precision measurement of Λ hyperhydrogens

T. Gogami,[1,*] S.N. Nakamura,[2] F. Garibaldi,[3,4] P. Markowitz,[5] J. Reinhold,[5] L. Tang,[6,7]
G.M. Urciuoli,[3] for the JLab hypernuclear collaboration, and the JLab Hall A collaboration

[1]*Division of Physics and Astronomy, Graduate School of Science, Kyoto University, Kyoto, Kyoto 606-8502, Japan*

[2]*Department of Physics, Graduate School of Science, Tohoku University, Sendai, Miyagi 980-8578, Japan*

[3]*INFN, Sezione di Roma, 00185 Rome, Italy*

[4]*Istituto Superiore di Sanità, 00161 Rome, Italy*

[5]*Department of Physics, Florida International University, Miami, FL 33199, USA*

[6]*Department of Physics, Hampton University, Hampton, VA 23668, USA*

[7]*Thomas Jefferson National Accelerator Facility (JLab), Newport News, VA 23606, USA*

(Dated: June 10, 2019)

The Λ binding energies of few-body systems are basic information for constructing ΛN interaction models. Recent experimental new findings about light hypernuclei deepened our understanding of the baryonic interaction and at the same time raised new puzzles. In order to settle confusion, we are proposing high precision measurements on binding energies of ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H with an accuracy of $\Delta B_{\Lambda} \leq \pm 100$ keV using the HKS-HRS spectrometer system at Hall A. The expected results provides new constraints to discussions of (1) a conflict between short lifetime and small binding energy of hypertriton, and (2) the ΛN charge symmetry breaking. Basic understanding of simple three or four body systems is essential to understand heavier and more dense nuclear objects such as neutron stars. An additional twelve days of beamtime with a 50-μA beam (20 μA for calibration data) and cryogenic targets in Hall A to the approved experiment E12-15-008 will enable us to measure ${}^{3}_{\Lambda}$H ($1/2^{+}$, $3/2^{+}$) and ${}^{4}_{\Lambda}$H (1^{+}) with an accuracy of $\Delta B_{\Lambda}^{\text{stat.}} \simeq \pm 70$ and ± 20 keV, respectively.

* gogami.toshiyuki.4a@kyoto-u.ac.jp

I. WHY IS THE STUDY OF SIMPLEST HYPERNUCLEI NECESSARY, NOW?

The precise spectroscopy of Λ hypernuclei, which was established at JLab after a long effort which started in 2000 [1], is now attempting to solve the puzzle of heavy neutron stars (NS). NS can be considered as a single nucleus with a radius of about 10 km and are the most dense object in the Universe. It has special features: 1) isospin asymmetry is almost 1 (number of neutrons $\gg$ number of protons) and its mass number is enormously large. So far, theoretical calculations with the established baryonic force potential models set an upper limit of NS mass as $\sim$1.6 solar mass and thus recent observation of two solar mass NSs triggered hot discussion (hyperon puzzle). Recent observation of gravitational waves from NS merger (GW170817) set a new limit for the maximum mass of NS [2]. Detailed information about baryonic force under high-density and neutron rich environment becomes now more important than before.

It is common understanding that an additional repulsive force other than the established baryonic force is necessary to make the too soft Equation of State hard enough to support two solar mass NSs. The inclusion of a three-body repulsive force with hyperons is the most promising way to explain it. The isospin dependence of light hypernuclei has been studied by detecting the charge symmetry breaking (CSB) effect of light Λ hypernuclei. Originally CSB of the ΛN interaction was discussed in the 1960s with emulsion and NaI γ-ray measurements but the limited precision of data prevented further research on it. Recent precise spectroscopy of ${}^{7}_{\Lambda}$He at JLab [3, 4] reignited the CSB study with state-of-the-art experimental techniques in the 21st century. The mass number 4 hypernuclear system, the ground state of ${}^{4}_{\Lambda}$H [5, 6] and the excitation energy of ${}^{4}_{\Lambda}$He [7] were recently measured very precisely. Though new experimental measurements of excitation energy of ${}^{4}_{\Lambda}$H and ground state energy of ${}^{4}_{\Lambda}$He have not yet been measured, it is recognized that systematic study of CSB or the isospin dependence of not only s-shell hypernuclei but also p-shell and heavier hypernuclei is important [8]. There has been no experimental study of isospin dependence for heavier hypernuclei though such information is essential to understand the structure of neutron rich, high density objects like NS. The first attempt to measure binding energies of ${}^{40}_{\Lambda}$K and ${}^{48}_{\Lambda}$K hypernuclei to study isospin dependence of the ΛNN force (E12-15-008) was already approved by JLab PAC44. Systematic spectroscopy of medium to heavy Λ hypernuclei is also important to extend our knowledge at normal nuclear density (ρ_0) to density of NS core, $3-5\rho_0$. Such study with various targets were now planned with high-intensity, high-resolution π beam at J-PARC [9] and a new proposal of the $(e, e'K^+)$ spectroscopy with a ^{208}Pb target is planned to be submitted to JLab.

Such study will reveal the existence of the ΛNN repulsive force and, its isospin and mass dependencies to solve the hyperon puzzle, however, the study of simple systems is more direct for the study of the origin of such forces. The simplest system to study 3-body force is $A = 3$ nuclei and recent experiments reveal that mysteries remain even now for the $A = 3$ hypernuclear system. Though we believed that light hypernuclear systems were reasonably understood based on emulsion data taken in 1960s and various spectroscopic results of hypernuclei, lifetime and binding energy of the simplest hypernucleus, ${}^{3}_{\Lambda}$H contradict each other suggesting we have been missing some important issues. The hypertriton (${}^{3}_{\Lambda}$H) puzzle will be discussed more in the next section. Another mystery on $A = 3$ hypernuclear system is the possibility of the ${}^{3}_{\Lambda}$n, atomic number zero hypernucleus. Its observation as a binding state was reported [10], but its existence cannot be explained with any available baryonic potential models [11, 12]. A conclusion will not be reached until our JLab E12-17-003 experiment reveals its existence or non-existence of binding or resonance state [13]. The data taking with a gaseous tritium target to investigate the Λnn three body system has been completed in 2018, and data are being analyzed.

As we described above, new generation experiments on heavier hypernuclear systems as well as light hypernuclei are on-going or proposed at JLab. Academic base is now ready to connect discussion on hypernuclei and NS systematically, and precise determination of the ΛN interaction is quite important for discussion of heavier hypernuclear systems. Here, we propose to add reasonable amount of beamtime to the approved experiment E12-15-008. It will enable us to perform the precise measurement of the hyperhydrogens ${}^{3,4}_{\Lambda}$H that is a key to resolve the (a) a contradiction between the short lifetime and small binding energy of ${}^{3}_{\Lambda}$H and (b) the CSB effect in the $A = 4$ iso-doublet hypernuclear system.

A. A contradiction between the short lifetime and small Λ binding energy of ${}^{3}_{\Lambda}$H

The Λ binding energy of hypertriton ${}^{3}_{\Lambda}$H was measured to be $B_\Lambda = 130 \pm 50$ keV in the emulsion experiment [14]. The Λ hypertriton is considered to be a loosely bound system of a Λ and a deuteron, and the spatial extent can be simply estimated by the root mean square radius of a two body system as follows:

$$\sqrt{< r^2 >} = \frac{\hbar}{\sqrt{4\mu B_\Lambda}} \quad (1)$$

where, μ is the reduced mass $\mu = m_\Lambda m_d/(m_\Lambda + m_d)$ which is about 76% of the nucleon mass. The root mean square radius $\sqrt{< r^2 >}$ of hypertriton is then evaluated to be about 10 fm which is about five times larger than that of deuteron. Figure 1 shows a calculated density distribution of Λ hypertriton for which only s-waves are included [15]. In such a Λ-halo hypernuclear system, a wave

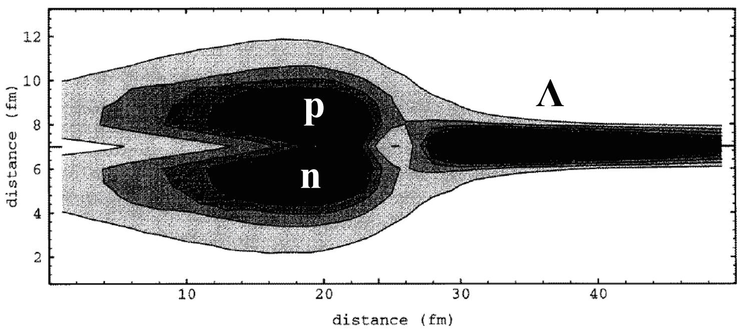

FIG. 1. A theoretical calculated probability distribution of proton, neutron and Λ in the Λ hypertriton [15]. Center of mass is fixed at the center of the figure and only s-waves are included for the calculation. The distance between dense parts of nucleons and Λ is apart by more than 10 fm, and Λ and nucleons have little overlap.

function overlap between the core and Λ is small, and the Λ is almost free from interactions due to the core nucleus. Therefore, the hypertriton lifetime is naively expected to be similar to that of a free Λ hyperon. A theoretical calculation by a three-body Faddeev equations with realistic NN and YN interactions predicts that the lifetime of the hypertriton nearly unchanged from that of a Λ hyperon (shorter by only 3%) [17].

However, recent heavy-ion beam experiments at GSI, LHC and RHIC consistently showed much shorter lifetime of hypertriton than a free Λ by 10–50% that is apart from theoretical predictions. Rappold *et al.* applied statistical analysis for old data including heavy-ion data at GSI, and the hypertriton lifetime was deduced to be 216^{+19}_{-16} ps which is about 18% shorter than that of Λ [18]. Figure 2 shows experimental data and theoretical calculations of Λ hypertriton lifetime [19]. An average of recent five data that were obtained in heavy ion experiments shows the Λ hypertriton lifetime is shorter by $30 \pm 8\%$. Gal and Garcilazo recently took into account the pion final state interaction in the hypertriton decay process [19]. It was found that the pion FSI could enhance the decay rate and the about 20% shortage of hyper tron lifetime from a free Λ is conceivable. However, something is missing to explain the 30% lifetime shortage of hypertriton compared to a free Λ.

To experimentally confirm the hypertriton lifetime, new experiments are now being prepared. At FAIR using a heavy-ion beam, the lifetime of ${}^{3}_{\Lambda}$H will be measured with higher statistics by more than a factor of ten [20] than that obtained in GSI. While the new heavy ion-beam experiment

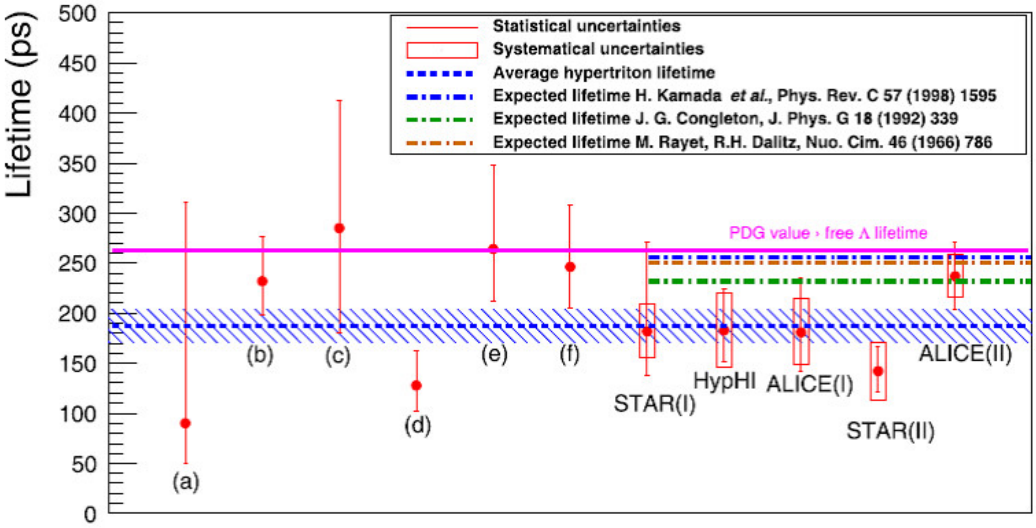

FIG. 2. Lifetime of Λ hypertriton summarized in Ref. [19]. Experimental data labeled as (a)–(f) were obtained in bubble chamber and emulsion experiments.

at FAIR would show high accuracy in the hypertriton-lifetime measurement, it is important to measure the lifetime with various reactions or methods from a point of minimizing systematic error which might be appeared depending on experimental techniques. Now, new experiments are planned to directly measure the hypertriton lifetime by the (γ, K^+) and (π^-, K^0) reactions at respectively ELPH [21] and J-PARC [22].

The fact of small binding energy of ${}^3_\Lambda$H contradict the short lifetime in a framework of the ΛN and ΛNN interactions that were constructed mainly by Λ hypernuclear energies measured in the old experiments. It is of great significance to re-measure physics quantities with modern experimental techniques in which the systematic uncertainty is well controlled and understood. We are proposing a precise measurement of Λ binding energy of ${}^3_\Lambda$H with the $(e, e'K^+)$ missing-mass spectroscopy established at JLab [23].

B. Charge symmetry breaking effect in the $A = 4$ hypernuclei

It is known that the strong interactions between baryons that consist of u and d quarks, i.e. nucleons, are (almost) flavor blind and have charge symmetry. However, it was found that the charge symmetry is considerably broken (CSB) between a nucleon and a Λ which includes a s quark. The CSB was experimentally observed in the $A = 4$ iso-doublet Λ hypernuclear system (${}^4_\Lambda$He and ${}^4_\Lambda$H). Figure 3 shows the Λ binding energies of the 0^+ and 1^+ states in ${}^4_\Lambda$He and ${}^4_\Lambda$H. There

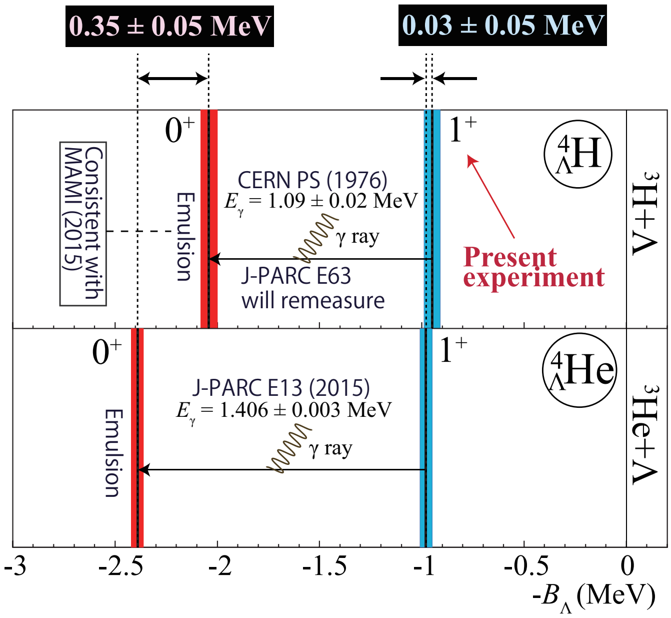

FIG. 3. The Λ binding energies of $A = 4$ iso-doublet Λ hypernuclei.The present experiment aims to measure the absolute value of B_Λ ($^4_\Lambda$H; 1^+) using the electron beam missing-mass spectroscopy established at JLab [1].

is a large binding energy difference for the ground state being $\Delta B_\Lambda(^4_\Lambda\text{He} - ^4_\Lambda\text{H};\ 0^+) = 350 \pm 50$ keV which was obtained from old emulsion experiments. After the Coulomb correction, about 400-keV energy is attributed to the strong force interaction [24–26]. This difference is larger than that for the case of ordinal nuclear system (^{3}H and ^{3}He) by a factor of about five, and thus the charge symmetry looks to be broken in the ΛN interaction. Recently, MAMI successfully measured $B_\Lambda(^4_\Lambda\text{H};\ 0^+)$ by the decay pion spectroscopy which measured monochromatic pions emitted from two body decays of hypernuclei at rest [5, 6]. The result was consistent with the emulsion experiment, and clarified the existence of the ΛN CSB for the ground state of $A = 4$ hypernuclear iso-doublet.

The energy spacings between 0^+ and 1^+ were measured by Λ hypernuclear γ-ray spectroscopy. The Λ binding energies for the 1^+ state were derived by using the 0^+ energies and the energy spacings measured in respectively the nuclear emulsion experiment and γ-ray spectroscopy. It was believed that the energy difference for the 1^+ state is $\Delta B_\Lambda(^4_\Lambda\text{He} - ^4_\Lambda\text{H};\ 1^+) = 290 \pm 60$ keV according to the old γ-ray measurements using a NaI detector. It showed there are the large CSB for both the 0^+ and 1^+ states. However, J-PARC E13 experiment re-measured $^4_\Lambda\text{He}(1^+)$ by using germanium detector array which had better precision [7], and the data was updated to be $\Delta B_\Lambda(^4_\Lambda\text{He} - ^4_\Lambda\text{H};\ 1^+) = 30 \pm 50$ keV that means there is little binding-energy difference for 1^+. Surprisingly, it turned out the ΛN CSB is spin dependent.

The ΛN-ΣN coupling is considered to be a key issue of the ΛN CSB. However, it is difficult to understand the $A = 4$ iso-doublet hypernuclear system maintaining consistency with low lying energies of other light-Λ hypernuclei even when the ΛN-ΣN coupling is taken into account [27, 28]. There might be further important factors to be considered in the theoretical models. On the other hand, it is necessary to confirm B_Λ for not only $A = 4$ Λ hypernuclei but also particularly light hypernuclei with new experimental techniques as the cases for ${}^4_\Lambda$He (1^+) and ${}^4_\Lambda$H (0^+).

For the $A = 4$ iso-doublet hypernuclei, ${}^4_\Lambda$H (1^+) and ${}^4_\Lambda$He (0^+) remain depending old experimental data taken in 1960s and should be re-measured with modern experimental techniques. However, the ground state of ${}^4_\Lambda$He measured by the emulsion experiment, can be considered more reliable compared to that of ${}^4_\Lambda$H because of higher statistics. Therefore, a re-measurement on $B_\Lambda({}^4_\Lambda\text{H}; 1^+)$ is being waited by priority. There is a plan to measure M1 transition γ-rays ($1^+ \to 0^+$) of ${}^4_\Lambda$H at J-PARC (J-PARC E63) [29], that needs the ground state energy to deduce the excited state energy. The ground state energies that were obtained in the previous experiments are shown in Fig. 4 [5]. The J-PARC E63 experiment was approved to be performed at the K1.1 beam line that

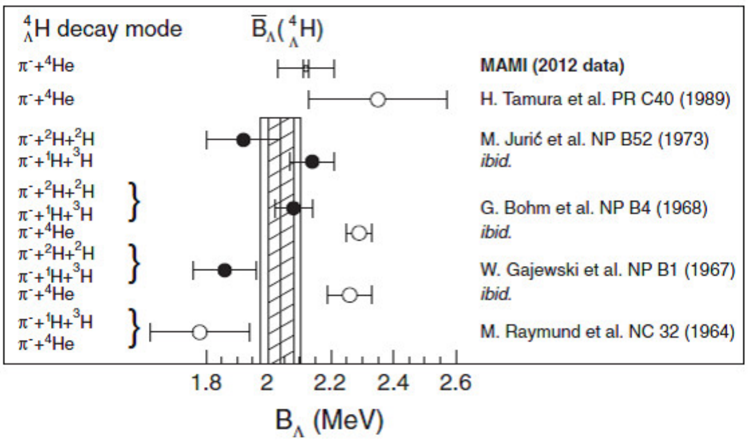

FIG. 4. The ground state Λ binding energy of ${}^4_\Lambda$H (0^+) measured in the past experiments [5]. The γ ray spectroscopy (J-PARC E63) that aims to measure $B_\Lambda({}^4_\Lambda\text{H}; 1^+)$ needs the ground state energy.

will be newly constructed. However, construction of the K1.1 beam line is not specifically on a few years time-line yet. Here, we propose to perform the first direct measurement of the absolute Λ binding energy of ${}^4_\Lambda$H (1^+) by the $(e, e'K^+)$ reaction at JLab.

II. PREVIOUS MEASUREMENT

Previously, hyperhydrogens were investigated with the ${}^{3,4}\text{He}(e, e'K^+){}^{3,4}_{\Lambda}\text{H}$ reaction at JLab Hall C [30, 31]. In the experiment, HMS and SOS spectrometers were used for detection of a scattered electron and a K^+, respectively. While Λ binding energies were not obtained due to a limited

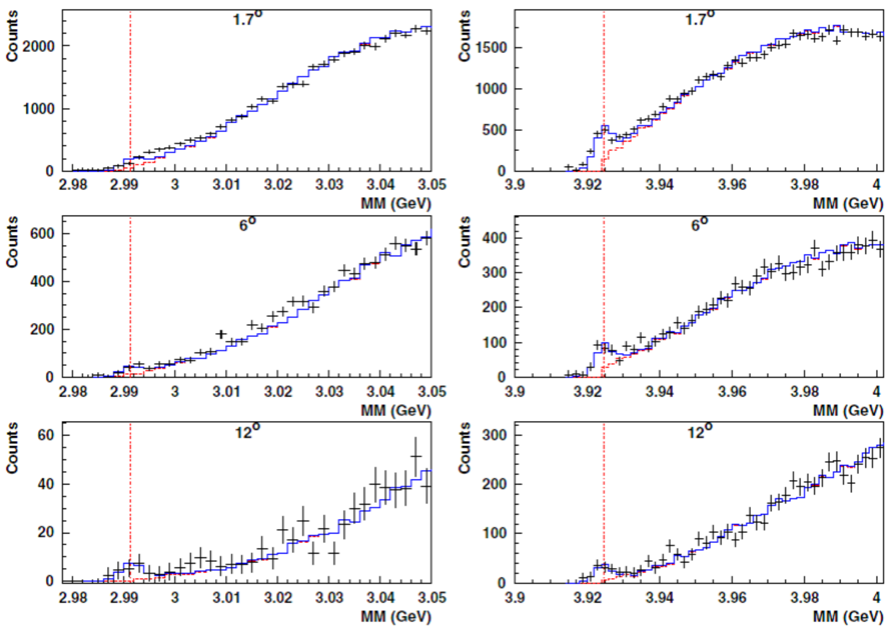

FIG. 5. Missing mass spectra for $^{3}_{\Lambda}$H (left) and $^{4}_{\Lambda}$H (right) obtained at JLab Hall C [31].

missing mass resolution, differential cross sections at the several K^+ scattering angles with respect to a virtual photon direction $\theta_{\gamma^* K}$ were measured [31]. The invariant mass of a virtual photon and the total energy were $Q^2 = 0.35$ GeV^2 and $W = 1.91$ GeV. At the forward scattering angle at which we are proposing the new experiment in Hall A, the production cross sections were obtained to be 5 and 20 nb/sr for $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H, respectively. The previous experiment tells us the differential cross sections for the electroproduction of the Λ hyperhydrogens $^{3,4}_{\Lambda}$H are reasonably large at the small $\theta_{\gamma^* K}$.

III. PROPOSED EXPERIMENT

We present the goal of the experiment and the requested beam time and conditions in Sections III A and III B. Then, the experimental setup and expected results are shown in Sections III C and III D, respectively.

A. The goal of the proposed experiment

1. $^{3}\mathrm{He}(e, e'K^{+})^{3}_{\Lambda}\mathrm{H}$

The ground state binding energy of $^{3}_{\Lambda}\mathrm{H}$ was reported from the emulsion experiments as shown in Table I. Juric *et al.* complied and reanalyzed these data, and deduced the Λ binding energy being $B_{\Lambda}(^{3}_{\Lambda}\mathrm{H}) = 130 \pm 50$ keV [14]. However, it was obtained as the average of results of two-body and three-body decay channels with scattered values. Table I shows $B_{\Lambda}(^{3}_{\Lambda}\mathrm{H})$ for various decay channels and different experiments. Though statistical error of each data set is about 100 keV, there are apparently large systematic errors which were not taken into account for averaging of them, depending on decay modes as seen also in the cases of $^{4}_{\Lambda}\mathrm{H}$, $^{5}_{\Lambda}\mathrm{He}$, $^{9}_{\Lambda}\mathrm{Be}$ and so on (refer to Tables 1 and 3 in [14]). Recently, the Λ binding energies of hypertriton and anti-hypertriton were reported to be $B_{\Lambda}(^{3}_{\Lambda}\mathrm{H}) = 410 \pm 120^{\mathrm{stat.}} \pm 100^{\mathrm{sys.}}$ keV from the STAR collaboration who used the heavy ion collision at RHIC [32]. This is the first precise measurement of the Λ binding energy of hypertriton by means of a counter experiment. The STAR result is apart from that of the emulsion experiment by two sigmas, indicating that the discussion about the Λ hypertriton needs to be reexamined.

It is worth noting that the HKS (JLab E05-115) collaboration measured $^{10}_{\Lambda}\mathrm{Be}$ [33], and the ground-state Λ binding energy was obtained to be $B_{\Lambda} = 8.60 \pm 0.07^{\mathrm{stat.}} \pm 0.16^{\mathrm{sys.}}$ MeV that differs from the result of old emulsion experiment ($B_{\Lambda} = 9.11 \pm 0.22$ MeV [34]) by about 0.5 MeV. In addition, we suggested that Λ binding energy of $^{12}_{\Lambda}\mathrm{C}$ measured in the emulsion experiment has a shift of about a half MeV by a careful comparison between the (π^{+}, K^{+}) and emulsion data [33]. The shift of about a half MeV for the $^{12}_{\Lambda}\mathrm{C}$ binding energy is also addressed by the FINUDA collaboration [35]. The need of the half MeV correction on $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C})$ has a large impact because Λ binding energies for many of hypernuclei were measured by the (π^{+}, K^{+}) experiments in which $B_{\Lambda}(^{12}_{\Lambda}\mathrm{C})$ was used for their energy calibration. Now, the correction is widely used. These updates and series of measurement on B_{Λ} for p-shell hypernuclei by the $(e, e'K^{+})$ experiments at JLab provided new insights into the ΛN interaction research; e.g. resulted in solving a puzzle of the large CSB in p-shell hypernuclear systems [4, 33].

Figure 6 shows the Λ-d rms radius versus the Λ binding energy obtained by using various NN and NΛ interactions [15]. There is a general correlation between Λ-d rms radius and the Λ binding energy. Choice of the interaction model gives a small effect on it. The Λ-d rms radius directly affects the hypertriton lifetime since it corresponds to the wave function overlap between a deuteron

TABLE I. The obtained Λ binding energy of $^{3}_{\Lambda}$H by the emulsion experiments.

Emulsion data	$\pi^- + {}^3$He	$\pi^- + {}^1$H $+ {}^2$H
	Λ binding energy (keV)	
M. Juric (1973) [14]	$+60 \pm 110$ (23 events)	$+230 \pm 110$ (58 events)
G. Bohm (1968) [16]	$+50 \pm 80$ (86 events)	-110 ± 130 (16 events)

and a Λ. If $B_\Lambda = +230+110 = 340$ keV which is the deepest bound case in Table I is taken, the Λ-d

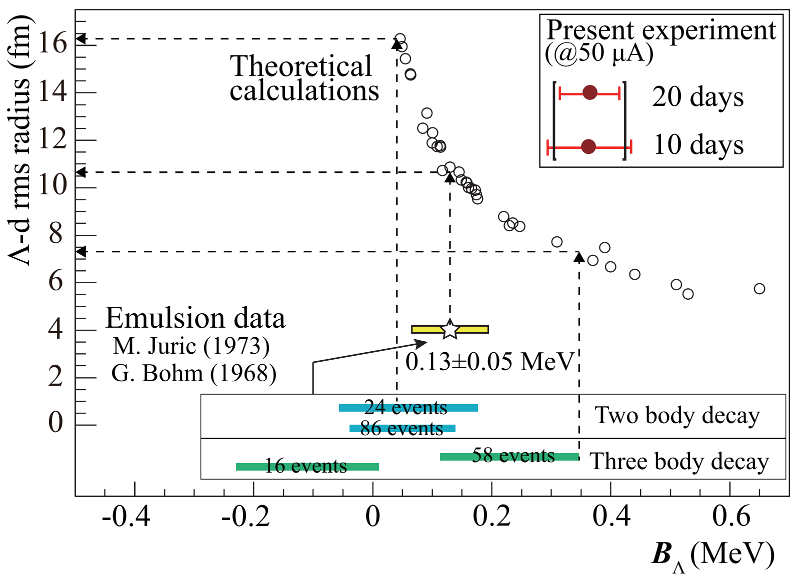

FIG. 6. The Λ-d rms radius versus the Λ binding energy [15]. Experimental data of the emulsion experiments were taken from Refs. [14, 16].

rms radius becomes about 7 fm. This rms radius is much shorter than the case of $B_\Lambda = 130$ keV by more than 30%. There is an idea that such a deep binding energy could explain the short lifetime of the hypertriton [36]. The Λ binding energy is crucial for solving the hypertriton-lifetime puzzle, and needs to be determined with less systematic uncertainty.

We aim to measure the ground state Λ binding energy of $^{3}_{\Lambda}$H with an accuracy of $\Delta B_\Lambda^{\rm stat.} \simeq \pm 70$ keV that can be achieved in ten days with a 50-μA beam impinged on 168-mg/cm^2 gaseous-^{3}He target (the statistical error can be reduced down to $\Delta B_\Lambda^{\rm stat.} = \pm 50$ keV with doubled statistics by increasing either beam intensity or beam time; a low accidental coincidence rate allows for the double luminosity). In addition, there is a possibility that we can measure the first excited state

($J^\pi = 3/2^+$) which has not been observed yet. A theory predicts that the first excited state is produced more than the ground state in the case of the $(e, e'K^+)$ reaction by a factor of about eight [37]. If this is the case, we will be able to determine the Λ binding energy of the first excited state with the statistical uncertainty of less than ± 50 keV.

A precise measurement on the Λ binding energy of the Λ hypertriton that can be realized by the experiment proposed here will be an important key to examine the hypertriton lifetime puzzle. Low energy properties of the ΛN interaction are hard to be investigated by a scattering experiment due to short lifetimes of hyperons. We know that two body systems with a Λ are not bound. Therefore, the Λ binding energy of the Λ hypertriton which is the lightest bound system with a Λ is significant to constrain the ΛN interaction model, and has sensitivity to particularly the singlet s-wave scattering lengths. The accurate binding energy measurement on the Λ hypertriton should be done with an experiment in which a systematic error is well controlled like this proposed experiment.

2. $^4\mathrm{He}(e, e'K^+)^4_\Lambda\mathrm{H}$

As discussed in Section I B, the high precision measurement of $B_\Lambda(^4_\Lambda\mathrm{H};\ 1^+)$ with a modern experimental techniques is awaited to ensure the discussion about $A = 4$ ΛN CSB. The missing-mass spectroscopy with the $(e, e'K^+)$ reaction was established at JLab, and has a unique capability to directly determine $B_\Lambda(^4_\Lambda\mathrm{H};\ 1^+)$ which is complementary to the γ-ray spectroscopy.

We aim to measure $B_\Lambda(^4_\Lambda\mathrm{H};\ 1^+)$ with a statistical error of $\Delta B_\Lambda^{\mathrm{stat.}} = \pm 20$ keV in one day beam time by using 50-μA beam impinged on 312-mg/cm^2 gaseous-^{4}He target. This absolute B_Λ measurement for $^4_\Lambda\mathrm{H}$ (1^+) is unique and complementary to the emulsion + γ-ray measurement. This measurement is totally independent from the $^4_\Lambda\mathrm{H}$ ground state energy measured at MAMI [5, 6].

B. Requesting conditions and beam time

1. Beam

We request a 50-μA beam at $E_e = 4.5$ GeV (two passes) with a bunch frequency of 500 MHz (250 MHz repetition rate will result in worse accidental background rate though it is still acceptable). In order to achieve a sufficient precision in a resulting missing-mass spectrum, the beam energy spread and energy centroid are required to be $\Delta p/p < 1 \times 10^{-4}$ (FWHM). A beam raster with an area of about 2×2 mm^2 would need to be applied to avoid a damage on a target cell due to an overheat.

2. Target

Standard cryogenic systems of gaseous helium-3,4 and LH_2 in Hall A are required for the present experiment. We need a more compact target system than existing ones because of a limitation of space around the target. Therefore, we will design a new target cell as shown in Fig. 9, and install four identical target cells in a vacuum chamber in which the cells will be attached on the same ladder system as the one used for the solid targets of E12-15-008. Three of the target cells will be filled with ^{3}He gas, ^{4}He gas, and liquid hydrogen (LH_2). One cell will be used for an empty run that will help an off-line analysis to subtract background events from the target cell material.

The missing mass would be shifted depending on a position of hypernuclear production point. In order to correct the shift, the position information of production point is necessary event by event. A displacement from the beam center in the x and y directions (vertical to the beam axis) can be derived from applied currents on dipole magnets used for the beam raster. On the other hand, a production position in the z direction (parallel to the beam axis) needs to be reconstructed from a magnetic optics analysis. For a calibration of reconstructed z, we will use a multi-foil carbon target which is an absolute position reference in z direction. The multi-foil target will have three carbon foils with a thickness of 50 mg/cm^2 being aligned 2.5 cm apart from each other.

3. Magnetic spectrometer

We request to use the exactly same spectrometer setup as the E12-15-008 experiment in which the isospin dependence of the ΛN/ΛNN interactions will be investigated through the $^{40,48}\mathrm{Ca}(e,e'K^+)^{40,48}_{\Lambda}\mathrm{K}$ reaction [38]. Existing spectrometers, HRS-L and HKS, combined with a new pair of charge separation dipole magnet (PCSM) will be used for e' and K^+ detection. Central momenta of HRS and HKS are set to be respectively 3.0 and 1.2 GeV/c, and the system covers a kinematical region of the $^{3,4}\mathrm{He}(e,e'K^+)^{3,4}_{\Lambda}\mathrm{H}$ reaction as shown in Fig. 7. Not only Λ but also Σ^0, which will be used for an absolute energy calibration, will be measured with the same spectrometer setting for physics run thanks to the large momentum coverage of HKS ($\Delta p/p_{\mathrm{central}} > \pm 10\%$). Measuring both Λ and Σ^0 masses without a change of spectrometer setting minimizes a systematic error on B_Λ. Another important feature of HKS is a short path length. The path length of PCSM

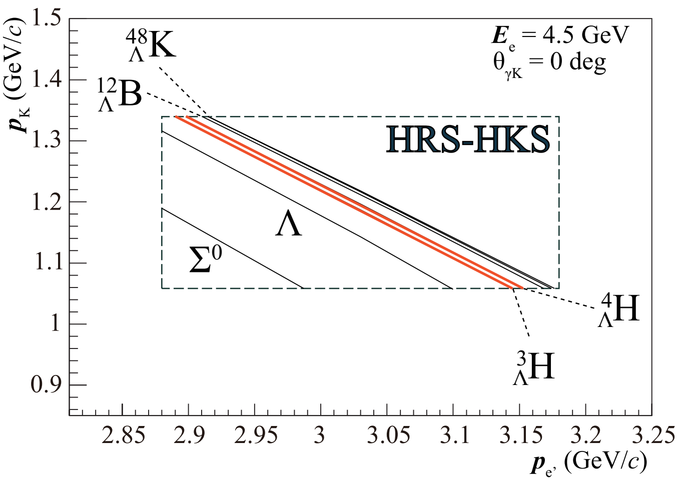

FIG. 7. Momentum acceptance of the HRS-HKS spectrometer system at $E_e = 4.5$ GeV, $\theta_{\gamma K} = 0$ degree.

+ HKS is about 12 m, and thus 26% of K^+s survive at 1.2 GeV/c. This gains a yield of Λ hypernuclei by a factor of more than three compared to the (PCSM+) HRS spectrometer in which K^+s travel more than 23.4 m to be detected.

4. Beam time

Table II shows requested beam time for the present experiment. We request eleven days for ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H production, and one day for calibration data. In total, we request twelve days of beam time.

C. Experimental setup

The experiment is planned to be performed with spectrometers HRS [39] and HKS [40] for respectively e' and K^+ detection as shown in Fig. 8. This experimental setup and kinematics are the same as those in the approved experiment E12-15-008. Electron beams at $E_e = 4.5$ GeV are impinged on a helium target to produce ${}^{3,4}_{\Lambda}$H, and scattered electrons and K^+s with central momenta of 3.0 and 1.2 GeV/c are measured by HRS and HKS, respectively. One of important features of the HRS and HKS spectrometers is a good momentum resolution of $\Delta p/p \leq 2 \times 10^{-4}$ (FWHM). This fact is expected to result in a missing-mass resolution of FWHM = 0.9 MeV which is needed particularly for a separation between the $1/2^+$ and $3/2^+$ states in ${}^{3}_{\Lambda}$H (see also Fig. 11).

What we need to prepare in addition to the experimental setup of E12-15-008 is a cryogenic target system for ^{3}He, ^{4}He, LH$_2$, and a multi-foil carbon target. The LH$_2$ and multi-foil target

TABLE II. Requested beam time.

Mode	Hypernucleus	Target (mg/cm^2)	Beam current (μA)	Beam time (day)	Yield
Physics	${}^{3}_{\Lambda}$H	^{3}He (168)	50	10	840 ($1/2^+$, $3/2^+$)
	${}^{4}_{\Lambda}$H	^{4}He (312)	50	1	470 (1^+)
	Subtotal			**11**	-
Calibration	Λ Σ^0	LH_2 (174)	20	0.5	Λ: 3000 Σ^0: 1000
	-	Multi foil	20	0.1	-
	-	Multi foil + Sieve slit	20	0.2	-
	-	Empty cell	20	0.2	-
	Subtotal			**1**	-
Total				**12**	-

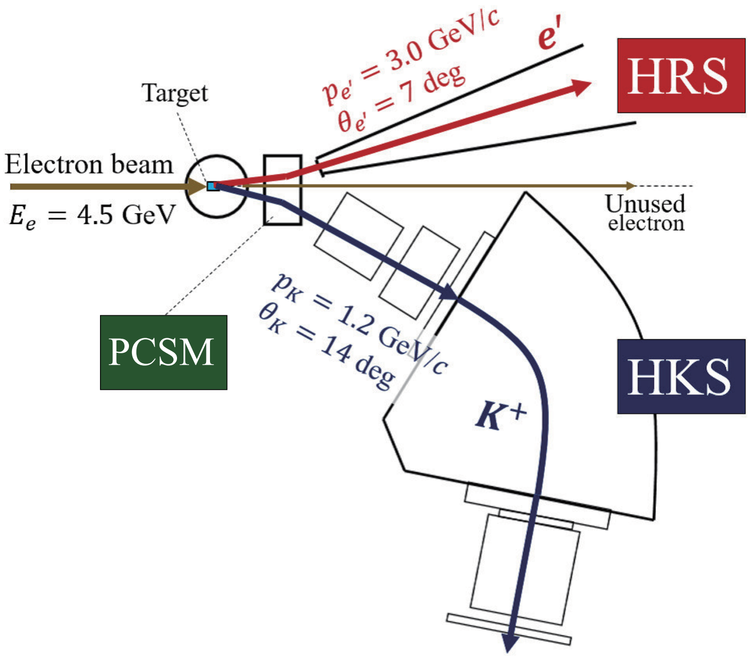

FIG. 8. A schematic of the experimental setup. Electron beams at $E_e = 4.5$ GeV impinged on the helium target which is enclosed in a vacuum chamber located in front of the pair of charge separation magnet (PCSM). The scattered electrons and K^+ with central momenta of 3.0 and 1.2 GeV/c are momentum-analyzed by HRS and HKS, respectively.

TABLE III. Basic parameters of the present experiment.

Beam	$\Delta p/p$	$< 1 \times 10^{-4}$ FWHM
	E_e	4.5 GeV
PCSM + HRS (e')	D(PCSM) + QQDQ	
	$\Delta p/p$	$\simeq 2 \times 10^{-4}$ FWHM
	$p_{e'}$	3.0 GeV/$c \pm 4.5\%$
	$\theta_{ee'}$	7.0 ± 1.5 deg
	Solid angle $\Omega_{e'}$	5 msr
PCSM + HKS (K^+)	D(PCSM) + QQD	
	$\Delta p/p$	$\simeq 2 \times 10^{-4}$ FWHM
	p_K	1.2 GeV/$c \pm 10\%$
	θ_{eK}	14.0 ± 4.5 deg
	Solid angle Ω_K	3 msr
	Optical length	12 m
	K^+ survival ratio	26%

will be used for calibrations of an energy scale and a z reconstruction analysis. A thick target deteriorate the missing-mass resolution due to momentum straggling and multiple scattering. In order to keep the effects of the momentum straggling and multiple scattering on the missing-mass resolution to be small enough compared to those of the momentum and angular resolutions of the spectrometer system, the target thickness is designed to be less than a few hundred mg/cm^2 including cell material for the present experiment. Figure 9 shows the schematic of a target cell of ^{3,4}He. A diameter of the target cell which defines a target length in z is set to be 50 mm so that background events from the cell wall can be separated clearly in off-line analysis by using information of a reconstructed reaction position in z ($\Delta z_{\rm react} \simeq$ 15-mm FWHM at 16 degrees [39]). Using events in $|z_{\rm react}| \leq 12$ mm for analyses, about 2σ events that come from the target cell are rejected. The 24-mm length in z corresponds to 174, 312 and 168 mg/cm^2 assuming the densities are 72, 130 and 70 mg/cm^3, respectively [39].

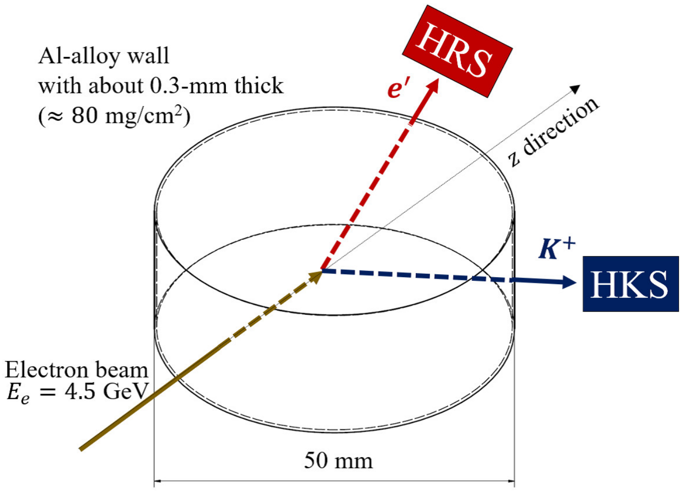

FIG. 9. A schematic of a target cell for a gaseous helium (^{3,4}He) and a liquid hydrogen (LH$_2$). A diameter of the target cell which defines the target length in the beam direction (z-direction) is 50 mm. If events in $|z_{\mathrm{react}}| \leq 12$ mm are used for analyses, about 2σ events ($\Delta z_{\mathrm{react}} \simeq$ 15-mm FWHM [39]) coming from the target cell are rejected. The 24-mm length in z corresponds to 174, 312 and 168 mg/cm^2 assuming the densities are 72, 130 and 70 mg/cm^3, respectively [39].

D. Expected results

1. *Missing-mass resolution and yield*

The missing mass resolutions was estimated by a Monte Carlo (MC) simulation in which magnetic field maps generated by the finite element calculation software, Opera3D (TOSCA), were used. At first in the simulation, momentum vectors of scattered electrons and K^+s at a production point were calculated event by event with the kinematics of the $^{3,4}\mathrm{He}(e, e'K^+)^{3,4}_{\Lambda}\mathrm{H}$ reaction taking into account the energy straggling effect of incident electrons in the target. The scattered electrons and K^+s were generated in the MC simulators, PCSM + HRS and PCSM + HKS, according to the above kinematics calculation. The scattered electrons and K^+s were measured at detection planes of the spectrometers taking into account realistic position and angular resolutions of the particle detectors. Then, the information of particle positions and angles at the detection planes were converted into momentum vectors at the target point by using backward transfer matrices to reconstruct a missing mass. Reconstructed momentum and angular resolutions in the PCSM + HRS and PCSM + HKS simulators are summarized in Table IV. As a result of a reconstruction of the missing mass in the simulation, the resolution was obtained to be $\Delta E_{\Lambda} = 0.9$ MeV FWHM.

TABLE IV. Resolutions of momenta and angles which were reconstructed by backward transfer matrices taking into account particle detector resolutions in the Monte Carlo simulations [41].

Spectrometer system	$\Delta p/p$ (FWHM)	$\Delta\theta$ (mrad)
PCSM + HRS	1.7×10^{-4}	0.23
PCSM + HKS	2.6×10^{-4}	1.94

The yield of hypernuclei ($N_{\rm HYP}$) was estimated as follows:

$$N_{\rm HYP} = \Gamma^{\rm int} \times N_{\rm beam} \times N_{\rm target} \times \left(\frac{d\sigma}{d\Omega}\right) \times \Omega_K \times \epsilon \tag{2}$$

where $N_{\rm beam}$, $N_{\rm target}$, $\left(\frac{d\sigma}{d\Omega}\right)$, Ω_K, and ϵ are the number of incident electrons, the number of target nuclei (cm^{-2}), differential cross section of the ($\gamma^{(*)}$,K^+) reaction (cm^2 msr^{-1}), the solid angle of the K^+ spectrometer (msr), and total experimental efficiency (DAQ, detector, analysis, K^+ survival ratio etc.) being 20% [$\simeq 0.26$ (K^+ survival ratio) $\times$ 0.75] for the present estimation. The $\Gamma^{\rm int}$ is the virtual photon flux integrated over the PCSM + HRS acceptance (notations can be found in [1]):

$$\begin{aligned} \Gamma^{\rm int} &= \iint_{\rm HRS} \frac{\alpha}{2\pi^2 Q^2} \frac{E_\gamma}{1-\epsilon} \frac{E_{e'}}{E_e} dp_{e'} d\theta_{e'} && (3) \\ &= 3.2 \times 10^{-5} \quad (\text{/electron}). && (4) \end{aligned}$$

The differential cross sections of the ^{3}He(γ,K^+)$^3_\Lambda$H and ^{4}He(γ,K^+)$^4_\Lambda$H reactions with a similar $\sqrt{s}$ and scattering angle ($\theta \simeq 0$ deg) to those of our experiment were measured in a past experiment at JLab Hall C (E91-016) [30], although the Λ binding energies were not determined in this experiment because the energy resolution was not enough. The differential cross sections were obtained to be about 5 and 20 nb/sr for the $^3_\Lambda$H and $^4_\Lambda$H production [31]. By the photoproduction, excited states are preferably produced since the spin-flip amplitude is large. Figure 10 shows the differential cross section of ^{4}He($e,e'K^+$)$^4_\Lambda$H predicted by DWIA calculation [42]. At the forward K^+ scattering angle with respect to a photon direction, the differential cross section of the first excited state (1^+) is predicted to be larger than that of the ground state (0^+) by two orders of magnitude. In the case of $^3_\Lambda$H photoproduction, the $1/2^+$ and $3/2^+$ states are expected to have similar cross sections, although there is a different theoretical prediction that the $3/2^+$ state has a larger cross section than that of $1/2^+$ by some amount [37]. The proposing experiment can discriminate these models by cross section information which cannot obtained by the γ-ray measurement. Expected yield per day with a beam current of 50 μA is summarized in Table V.

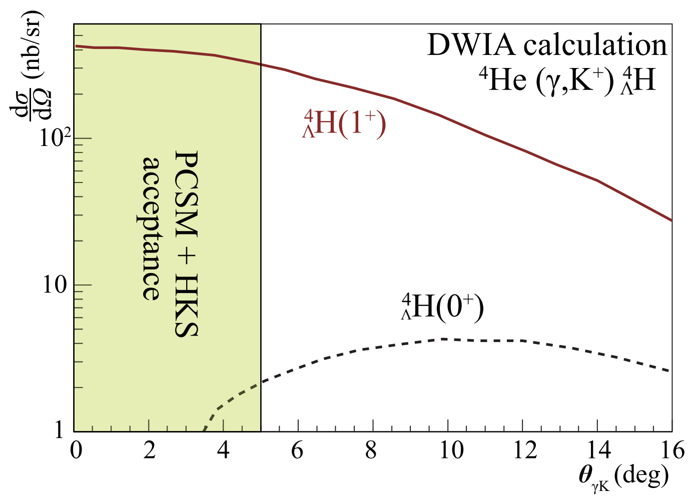

FIG. 10. A theoretical prediction of the differential cross section of ^{4}He$(\gamma, K^+)^4_\Lambda$H by DWIA [42].

TABLE V. Expected yields for $^3_\Lambda$H and $^4_\Lambda$H in the present experiment.

Hypernucleus		Target (mg/cm^2)	Cross section (nb/sr)	Yield per day at 50 μA
$^3_\Lambda$H	$1/2^+$	^{3}He	5	42
	$3/2^+$	(168)		42
$^4_\Lambda$H	0^+	^{4}He	-	-
	1^+	(312)	20	470

2. Accidental background

Accidental $e'K^+$-coincidence events would be a background in a resulting missing-mass spectrum. To estimate the accidental background events, real data which were taken in the last hypernuclear experiment at JLab Hall C (E05-115) were used. With a 0.2-g/cm^2 ^{7}Li target at $I_e = 32$ μA in the E05-115 experiment [43], counting rates in the K^+ spectrometer HKS were:

$$K^+ : \pi^+ : p = 300 (\equiv R_K^{\text{ref}}) : 25000 : 34000 \quad \text{Hz}. \tag{5}$$

The HKS has Cherenkov counters with radiation media of aerogel ($n = 1.05$) and water ($n = 1.33$) to reject π^+s and protons. The Cherenkov counters reduced the fractions of π^+s and protons down to 0.5% and 10%, respectively, at the trigger level. In off-line analysis, π^+s and protons could be reduced to 4.7×10^{-4} and 1.9×10^{-4} by using information on light yields in the Cherenkov counters and reconstructed particle-mass squared [44]. The most important off-line analysis for K^+ identification (KID) is a time-of-flight (TOF) analysis. The TOF from the target to the timing counter was 10 m in HKS, and the TOF resolution was $\sigma = 0.26$ ns. Thus, a time separation of K^+s from π^+s and protons at 1.2 GeV/c were more than 6σ and 20σ respectively when an event selection was applied to select the $e'K^+$ coincidence with a time gate of ± 1 ns [1]. Therefore, accidental coincidence events in the missing mass spectrum originated mainly from $e'K^+$ coincidence which was made by quasi-free Λ and $\Sigma^{0,-}$ production. The KID performance of HKS in the present experiment without any an additional detector will be the same and enough as we achieved in E05-115 since the central momentum is the same.

The K^+ rate in the present experiment R_K is estimated assuming the production-cross section of quasi-free Λ is proportional to $A^{0.8}$:

$$R_K = R_K^{\rm ref} \times \frac{0.1}{0.2} \times \frac{50}{32} \times \frac{\Gamma^{\rm int}}{\Gamma^{\rm int}_{\rm ref}} \times \frac{A^{0.8}}{7^{0.8}} \times \frac{7}{A} \quad \left[\frac{\rm Hz}{(100\ {\rm mg/cm^2})(50\ \mu{\rm A})}\right] \tag{6}$$

$$= 234 \times \frac{\Gamma^{\rm int}}{\Gamma^{\rm int}_{\rm ref}} \times \left(\frac{7}{A}\right)^{0.2} \quad \left[\frac{\rm Hz}{(100\ {\rm mg/cm^2})(50\ \mu{\rm A})}\right] \tag{7}$$

where the rate is normalized to be per a target-thickness of 100 mg/cm^2 and per a beam current of 50 μA. The $\Gamma^{\rm int}_{\rm ref}(= 5.67 \times 10^{-5})$ is the integrated virtual photon flux in E05-115. Similarly, e' rates in HRS were estimated from that in the E05-115 experiment ($R^{\rm ref}_{e'} = 2.2 \times 10^6$ Hz) assuming a major contribution comes from Bremsstrahlung process [45]. For the estimation, a rate-reduction effect due to the smaller acceptance of HRS compared to that of the e' spectrometer in E05-115 (HES) was also taking into account. The expected singles rates of scattered electrons and K^+s in HRS and HKS are summarized in Table VI. The accidental $e'K^+$-coincidence background observed

TABLE VI. Expected singles rates in HRS (e') and HKS (K^+) at $I_e = 50$ μA. The expected number of events of the accidental $e'K^+$- coincidence background in a resulting missing-mass spectrum is shown in the last column.

Target	Rate		Accidental background
(mg/cm^2)	K^+ (Hz)	e' (Hz)	(/MeV/day)
^{3}He (168)	262	1079	1.5
^{4}He (312)	459	1597	5.4

in the missing mass spectrum of ^{7}Li$(e, e'K^+)^7_\Lambda$He in E05-115 was about 6.5 [(nb/sr)/0.375 MeV] ($\equiv h^{\rm acc}_{\rm ref}$) [4]. For the present experiment, the accidental coincidence background is estimated as follows:

$$h^{\rm acc} = h^{\rm acc}_{\rm ref} \times \frac{R_{e'} R_K}{R^{\rm ref}_{e'} R^{\rm ref}_K} \times \left(\frac{I_e}{50} \times \frac{t}{100}\right)^2 \quad \left[\frac{\rm (nb/sr)}{0.375\ {\rm MeV}}\right] \tag{8}$$

where $R_{e'}$ and R_K are singles rates per 50-μA beam current per 100-mg/cm^2 target thick. The I_e and t are the beam current (μA) and areal density of target (mg/cm^2), respectively. The number of events of the accidental $e'K^+$-coincidence which was evaluated by Eq. (8) is shown in the last column of Table VI.

3. Statistical error on B_Λ

Figure 11 shows an expected B_Λ spectrum for ${}^3_\Lambda$H. The quasi-free Λ distribution was assumed to be a linear distribution for which the energy resolution of 0.9-MeV FWHM is taken into account as the Gauss distribution. The number of events in the quasi-free Λ relative to that in the bound region is assumed to be the same as reported in the past experiment [30]. The spectrum was fitted by two Gaussian functions for peaks, and linear functions for the quasi-free Λ and accidental $e'K^+$ backgrounds. The fitting simulation with randomly generated dummy data was iterated five hundred times for an estimation of statistical error on B_Λ ($\Delta B_\Lambda^{\rm stat.}$). As a result, It was found that statistical errors are $\Delta B_\Lambda^{\rm stat.} = \pm 70$ keV for both states. If the statistics increased to two times higher by either increasing beam intensity or beam time, the statistical uncertainty is estimated to be $\Delta B_\Lambda^{\rm stat.} = \pm 50$ keV.

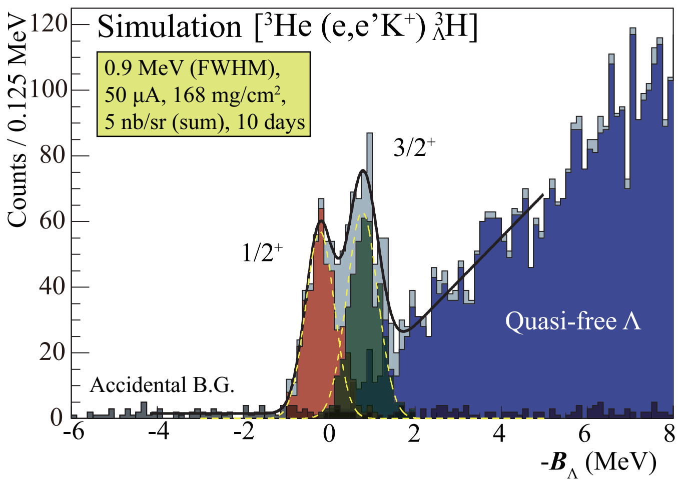

FIG. 11. An expected B_Λ spectrum for the ^{3}He$(e,e'K^+){}^3_\Lambda$H reaction with a 50-μA beam in ten days.

The B_Λ spectrum for the ^{4}He$(e,e'K^+){}^4_\Lambda$H reaction was also estimated (Fig. 12) and ΔB_Λ was evaluated in the same way as simulated for ${}^3_\Lambda$H. As a result, it was found that $\Delta B_\Lambda^{\rm stat.} = \pm 20$ keV for ${}^4_\Lambda$H.

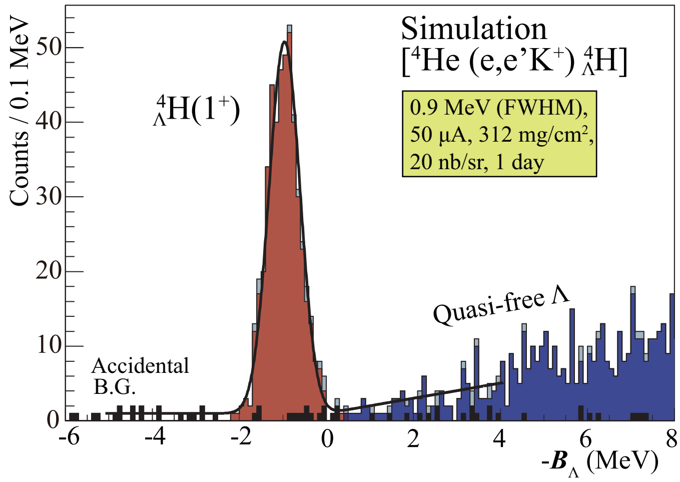

FIG. 12. An expected B_Λ spectrum for the $^4\text{He}(e, e'K^+)^4_\Lambda\text{H}$ reaction with a 50-μA beam in one day.

4. Calibrations and expected B_Λ accuracy

There are four major calibrations to be done to precisely measure Λ binding energy B_Λ:

- Momentum calibration by using elastic scattering

  Elastic scattering data for momentum calibration of each spectrometer will be taken in E12-15-008, and these data can be used for the present data analysis.

- Angle calibration by using a sieve slit

  These data will also be taken in E12-15-008. However, we need additional sieve slit data with the multi-foil target that will help for not only angle but also z reconstruction analyses.

- z reconstruction calibration by using the multi-foil target

  In the present experiment, reconstructed z information will be used for reconstructions of momentum vectors of particles by using backward transfer matrices. The inclusion of the z information in the backward transfer matrices is adopted for analyses of the Λnn data for which we used two HRS spectrometers (JLab E12-17-003). The momentum vector reconstruction using the reconstructed z apparently works well in HRS. In order to check if a similar method is applicable to the HKS analysis, we performed the Monte Carlo simulation in which PCSM and HKS are modeled. Figure 13 shows a distribution of $\Delta p/p$ versus z position obtained in the simulation. There is a linear correlation between momentum and z, and no broadening with larger $\mid z \mid$. It indicates that the momentum vector analysis in the HKS could be done with a linear correction depending on z, which would be simpler than

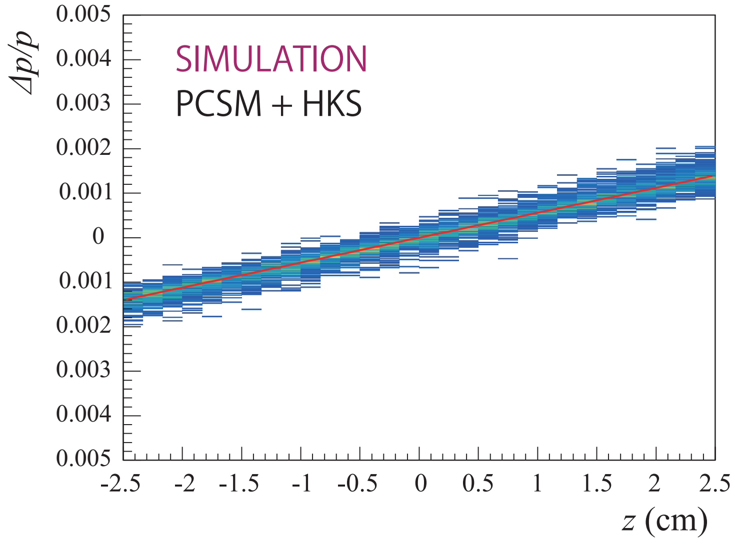

FIG. 13. A correlation between $\Delta p/p$ and the production position z obtained in the Monte Carlo simulation in which PCSM and HKS are modeled.

the HRS analysis for Λnn data. The z dependence was corrected by a linear function which is shown as a red line in Fig. 13. As a result, it was found that the momentum resolution could reach our goal by only the linear correction although a real analysis may need more careful treatment.

The production point z will be reconstructed event by event with transfer matrices for which particle information (position and angle) at focal plane is used. In order to calibrate the reconstructed z position, the multi-foil carbon target which is the absolute position reference in z will be used in the present experiment.

- Absolute energy scale calibration by using Λ and Σ^0 events
  The absolute energy scale will be calibrated by Λ and Σ^0 masses. A polyethylene target will be used for Λ and Σ^0 production in E12-15-008. As shown in the previous item, we will apply the z correction for the missing mass. In order to confirm that the z correction does not deteriorate the mean value of measured B_Λ, data of Λ and Σ^0 production with the liquid hydrogen target in the target cell which is identical with those for gaseous ^{3,4}He targets need to be taken.

We could achieve the systematic error of $\Delta B_\Lambda^{\text{sys.}}$(E05-115)$= \pm 110$ keV in the last hypernuclear experiment [1]. The systematic error was dominated by the momentum calibration by using only reconstructed missing masses of Λ and Σ^0. In order to improve systematic error in the present experiment, we are going to take calibration data for each single-arm spectrometer through elastic scattering reaction. This is expected to improve the systematic error on B_Λ to be less than 100 keV. Therefore, the total error on the Λ binding energy measurement is improved down to be $\Delta B_\Lambda = \sqrt{(B_\Lambda^{\text{stat.}})^2 + (B_\Lambda^{\text{sys.}})^2} \leq \pm 100$ keV in the present experiment.

IV. SUMMARY

We request twelve days of beam time including calibration data for the precision spectroscopy of ${}^{3,4}\text{He}(e, e'K^+)^{3,4}_{\Lambda}\text{H}$ by using the same experimental setup of the approved experiment E12-15-008. The present experiment aims to determine Λ binding energies of ${}^{3}_{\Lambda}\text{H}$ and ${}^{4}_{\Lambda}\text{H}$ with statistical uncertainties of $\Delta B_\Lambda^{\text{stat.}} = \pm 70$ and ± 20 keV, respectively. The systematic error will be improved to be $\Delta B_\Lambda^{\text{sys.}} < \pm 100$ keV compared to that in the last hypernuclear experiment at JLab thanks to better calibrations. These data will be crucial to solve the puzzles of (a) the short-lifetime and small binding energy of Λ hypertriton, and (b) the s-shell ΛN CSB.

---

[1] T. Gogami *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* 900, 69–83 (2018).

[2] L. Rezzolla, E.R. Most and L.R. Weih, *Astr. Jour. Lett.* **852**, L25 (2018).

[3] S.N. Nakamurai *et al.* (HKS (JLab E01-011) Collaboration), *Phys. Rev. Lett.* **110**, 012502 (2013).

[4] T. Gogami *et al.* (HKS (JLab E05-115) Collaboration), *Phys. Rev. C* **94**, 021302(R) (2016).

[5] A. Esser *et al.* (A1 Collaboration), *Phys. Rev. Lett.* **114**, 232501 (2015).

[6] F. Schulz *et al.* (A1 Collaboration), *Nucl. Phys. A* **954**, 149 (2016).

[7] T.O. Yamamoto *et al.*, *Phys. Rev. Lett.* **115**, 222501 (2015).

[8] A. Gal, *Phys. Lett. B* **744**, 352 (2015).

[9] Extension of the J-PARC Hadron Experimental Facility – summary report –, `http://www.rcnp.osaka-u.ac.jp/~jparchua/en/index.html`

[10] C. Rappold *et al.*, *Phys. Rev. C* **88**, 041001(R) (2013).

[11] E. Hiyama *et al.*, *Phys. Rev. C* **89**, 061302(R) (2014).

[12] A. Gal and H. Garcilazo, *Phys. Lett. B* **736**, 93–97 (2014).

[13] L. Tang *et al.* (JLab Hypernuclear Collaboration), *Proposal to JLab PAC45*, E12-17-003, "*An isospin dependence study of the Λ-N interaction through the high precision spectroscopy of Λ-hypernuclei with electron beam*", 2016.

[14] M. Juric *et al.*, *Nucl. Phys. B* **52**, 1–30 (1973).

[15] A. Cobis *et al.*, *J. Phys. G: Nucl. Part. Phys.* **23**, 401–421 (1997).

[16] G. Bohm *et al.*, *Nucl. Phys. B* **4**, 511–526 (1968).

[17] H. Kamada *et al.*, *Phys. Rev. C* **57**, 4 (1998).

[18] C. Rappold *et al.*, *Phys. Lett. B* **728**, 543–548 (2014).

[19] A. Gal and H. Garcilazo, Phys. Lett. B 791, 48–53 (2019).

[20] T.R. Saito *et al.*, *Nucl. Phys. A 954*, 199–212 (2015).

[21] S. Nagao *et al.*, Letter of Intent submitted to ELPH, "A Direct Lifetime Measurement of The Lambda Hypertriton", 2860 (2016).

[22] M. Agnello *et al.*, *Nucl. Phys. A* **954**, 176–198 (2016).

[23] L. Tang *et al.* (HKS (JLab E05-115 and E01-011) Collaborations), *Phys. Rev. C* **90**, 034320 (2014).

[24] R.H. Dalitz and F. Von Hippel, *Phys. Lett.* **10**, 1 (1964).

[25] J.L. Friar and B.F. Gibson, *Phys. Rev. C* **18**, 908 (1978).

[26] A.R. Bodmer and Q.N. Usmani *Phys. Rev. C* **4**, 31 (1985).

[27] Y. Akaishi *et al.*, *Phys. Rev. Lett.* **84**, 16 (2000).

[28] H. Nemura *et al.*, *Phys. Rev. Lett.* **89**, 14 (2002).

[29] H. Tamura *et al.*, *JPS Conf. Proc. 17*, 011004 (2017).

[30] F. Dohrmann et al., *Phys. Rev. C* **76**, 054004 (2007).

[31] F. Dohrmann et al., *Phys. Rev. Lett.* **93**, 242501 (2004).

[32] STAR collaboration, arXiv:1904.10520v1 (2019).

[33] T. Gogami *et al.* (HKS (JLab E05-115) Collaboration), *Phys. Rev. C* **93**, 034314 (2016).

[34] T. Cantwell *et al.*, *Nucl. Phys. A* **236**, 445 (1974).

[35] E. Botta, T. Bressani A. Feliciello, *Nucl., Phys.* **A** *960*, 165–179 (2017).

[36] Y. Akaishi, "Week decay of ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H", 2018 JPS Annual (73th) Meeting, 22aK309-10 (2018).

[37] T. Mart *et al*, *Nucl. Phys. A* **640**, 235-258 (1998); T. Mart and B.I.S. van der Ventel, *Phys. Rev. C* **78**, 014004 (2008).

[38] S.N. Nakamura *et al.* (JLab Hypernuclear Collaboration), *Proposal to JLab PAC44*, E12-15-008, "*An isospin dependence study of the* Λ-*N interaction through the high precision spectroscopy of* Λ-*hypernuclei with electron beam*", 2016.

[39] J. Alcorn *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **522**, 294–346 (2004).

[40] Y. Fujii *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **795**, 351–363 (2015).

[41] G. Aida, *Master's Thesis*, "*Design of a septum magnet for the electro-production spectroscopy of* Λ *hypernuclei: to investigate the isospin dependence of the* ΛNN *interaction*", Tohoku University, Sendai, Japan, 2018 (in Japanese).

[42] T. Motoba, *JPS Conf. Proc.*, 011003 (2017).

[43] T. Gogami, *Doctoral Thesis*, "*Spectroscopic research of hypernuclei up to medium-heavy mass region with the* $(e, e'K^{+})$ *reaction*", Tohoku University, Sendai, Japan, 2014.

[44] T. Gogami *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **729**, 816–824 (2013).

[45] Y. Tsai, *Rev. Mod. Phys.* **46**, 4 (1974).