PROPOSAL TO JLAB PAC49

C12-19-002

High accuracy measurement of nuclear masses of Λ hyperhydrogens

T. Gogami,[1,*] S. N. Nakamura,[2] F. Garibaldi,[3,4] P. Markowitz,[5] J. Reinhold,[5] L. Tang,[6,7]
G. M. Urciuoli,[3] for the JLab Hypernuclear Collaboration, and the JLab Hall A Collaboration

[1]*Department of Physics, Graduate School of Science, Kyoto University, Kyoto, Kyoto 606-8502, Japan*

[2]*Department of Physics, Graduate School of Science, Tohoku University, Sendai, Miyagi 980-8578, Japan*

[3]*INFN, Sezione di Roma, 00185 Rome, Italy*

[4]*Istituto Superiore di Sanità, 00161 Rome, Italy*

[5]*Department of Physics, Florida International University, Miami, FL 33199, USA*

[6]*Department of Physics, Hampton University, Hampton, VA 23668, USA*

[7]*Thomas Jefferson National Accelerator Facility (JLab), Newport News, VA 23606, USA*

(Dated: May 23, 2021)

Members

K. Ebata, T. Gogami (spokesperson), K. Tsutsumi, K. N. Suzuki, E. Umezaki

Department of Physics, Graduate School of Science, Kyoto University, Kyoto, Kyoto 606-8502, Japan

T. Akiyama, T. Fujiwara, K. Itabashi, M. Kaneta, S. Nagao, S. N. Nakamura (spokesperson),
Y. R. Nakamura, K. Okuyama, Y. Toyama, K. Uehara

Department of Physics, Graduate School of Science, Tohoku University, Sendai, Miyagi 980-8578, Japan

N. Lashley, B. Pandey, L. Tang (spokesperson)

Department of Physics, Hampton University, Hampton, VA 23668, USA

P. Markowitz (spokesperson), J. Reinhold (spokesperson)

Department of Physics, Florida International University, Miami, FL 33199, USA

G. M. Urciuoli (spokesperson), F. Garibaldi (spokesperson)

INFN, Sezione di Roma, 00185 Rome, Italy

Y. Fujii

Physics Section, Tohoku Medical and Pharmaceutical University, Sendai, Miyagi 981-8558, Japan

A. Camsonne, S. Covrig Dusa, D. W. Higinbotham, D. Meekins, B. Sawatzky, S. A. Wood

Thomas Jefferson National Accelerator Facility (JLab), Newport News, VA 23606, USA

P. Achenbach

Institute for Nuclear Physics, Johannes Gutenberg-University, D-55099 Mainz, Germany

A. Asaturyan, H. G. Mkrtchyan, A. H. Mkrtchyan, A. Shahinyan

A.I. Alikhanyan National Science Laboratory, Yerevan 0036, Armenia

D. Androić, M. Furic

Department of Physics, University of Zagreb, HR-10000 Zagreb, Croatia

V. M. Rodíguez

División de Ciencias y Tecnología, Universidad Ana G. Méndez, Recinto de Cupey, San Juan 00926, Puerto Rico

M. A. Elaasar

Department of Physics, Southern University at New Orleans,New Orleans, LA 70126, USA

E. Brash

Department of Physics, Computer Science & Engineering, Christopher Newport University, Newport News, VA, USA 23606

S. Širca

Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia

I. Niculescu, G. Niculescu

Department of Physics and Astronomy, James Madison University, Harrisonburg, VA 22807, USA

C. Samanta

Department of Physics & Astronomy, Virginia Military Institute, Lexington, Virginia 24450, USA

M. H. Bukhari

Jazan University, Jazan 45142, Saudi Arabia

M. H. Shabestari

Department of Physics, University of West Florida, FL 32514, USA

P. Gueye

Facility for Rare Isotope Beams, Michigan State University, MI 48824, USA

* Contact person, gogami.toshiyuki.4a@kyoto-u.ac.jp / gogami@jlab.org

EXECUTIVE SUMMARY

Λ binding energies of few body systems are basic inputs for constructing ΛN interaction models. Recent experimental findings about light hypernuclei have deepened our understanding of the baryonic interaction and at the same time raised new puzzles. In order to settle these issues, we are proposing high precision measurements of the binding energies of ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H using the HKS-HRS spectrometer system combined with a newly fabricated magnet PCS in Hall A. A unique measurement proposed here will provide strong constraints to discussions of (1) the conflict between the short lifetime and the small binding energy of hypertriton, and (2) ΛN charge symmetry breaking. In addition, the basic understanding of simple three or four body systems is essential to understand heavier and more dense nuclear objects such as neutron stars. A beamtime of 12 days with a 50 μA beam on cryogenic helium gas targets along with 2.5 days of calibration runs will enable us to measure ${}^{3}_{\Lambda}$H ($1/2^{+}$ or $3/2^{+}$) and ${}^{4}_{\Lambda}$H (1^{+}) with an accuracy of $|\Delta B_{\Lambda}^{\rm total}| = 65$ keV which will be the best accuracy in hypernuclear counting experiments. The beam energy of 4.24 GeV is required with the stability of $\Delta E/E \leq 1 \times 10^{-4}$ (FWHM) which was realized in the previous hypernuclear experiment at Hall A (E12-17-003 performed in 2018).

This is an updated proposal from the last year. The main reason for the C2 classification of the last proposal was that the design of the target system was found to be incompatible with limited space around the target (`https://www.jlab.org/exp_prog/PACpage/PAC48/PAC48_PrelimReportPlus_FINAL.pdf`). We discussed the target design with JLab target group and made a feasible concept. We request beamtime based on the target design in the proposal.

I. WHY IS THE STUDY OF SIMPLEST HYPERNUCLEI NECESSARY, NOW?

Precision spectroscopy of Λ hypernuclei, which was established at JLab after a long effort which started in 2000 [1], is now being used to attempt to solve the puzzle of heavy neutron stars (NS). A neutron star can be considered as a single nucleus with a radius of about 10 km and is one of the most dense objects in the Universe. It has special features including: 1) an isospin asymmetry of almost 1 (number of neutrons $\gg$ number of protons) and 2) its mass number is enormous. So far, theoretical calculations with established baryonic force potential models set an NS mass upper limit of $\sim$1.6 solar masses and thus recent observation of two solar mass NSs have triggered hot discussions (hyperon puzzle). Observation of gravitational waves from a NS merger (GW170817) has started to provide limits on the maximum mass, tidal deformability, and radii of NS [2]. Obtaining detailed information about baryonic forces under high-density and in neutron rich environments has become even more important.

It is commonly understood that an additional repulsive force other than the established baryonic force is necessary to make the soft Equation of State hard enough to support two solar mass NSs. The inclusion of a three-body repulsive force with hyperons is the most promising way to explain this. The isospin dependence of light hypernuclei has been studied by measuring the charge symmetry breaking (CSB) effect of light Λ hypernuclei. Originally CSB of the ΛN interaction was discussed in the 1960s with emulsion and NaI γ-ray measurements but the limited precision of the data prevented further research on it. Recent precise spectroscopy of ${}^{7}_{\Lambda}$He at JLab [3, 4] reignited the CSB study with state-of-the-art experimental techniques in the 21st century. The mass number 4 hypernuclear system, the ground state of ${}^{4}_{\Lambda}$H [5, 6] and the excitation energy of ${}^{4}_{\Lambda}$He [7] were recently measured very precisely. Although new experimental measurements of excitation energy of ${}^{4}_{\Lambda}$H and ground state energy of ${}^{4}_{\Lambda}$He have not yet been made, it is recognized that systematic study of CSB or the isospin dependence of not only s-shell hypernuclei but also p-shell and heavier hypernuclei is important [8]. There has been no experimental study of isospin dependence for heavier hypernuclei though such information is essential to understand the structure of neutron rich, high density objects like NS.

A first attempt to measure binding energies of ${}^{40}_{\Lambda}$K and ${}^{48}_{\Lambda}$K hypernuclei to study isospin dependence of the ΛNN force (E12-15-008) has already been approved by JLab PAC44. Systematic spectroscopy of medium to heavy Λ hypernuclei is also important to extend our knowledge at normal nuclear density (ρ_0) to the density of a NS core, $3-5\rho_0$. Such studies with various targets are now planned with high-intensity, high-resolution π beam at J-PARC [9]. It is noted that a proposal of the $(e, e'K^+)$ spectroscopy with a ^{208}Pb target was approved in PAC 48. Such studies will reveal the existence of the ΛNN repulsive force and, its isospin and mass dependencies to solve the hyperon puzzle. In order to investigate the ΛNN force from hypernuclear energies, the ΛN interaction needs to be precisely known as information on multi-body effects such as the ΛNN interaction can be extracted from residuals after the two body forces (the NN and ΛN interactions) are taken into account. The simplest bound hypernucleus with a Λ hyperon is the hypertriton $^{3}_{\Lambda}$H, and thus the Λ binding energy of $^{3}_{\Lambda}$H plays an important role in constraining the ΛN interaction. It was believed that light hypernuclear systems were reasonably understood based on emulsion data taken in 1960s and various spectroscopic results of hypernuclei. For $^{3}_{\Lambda}$H, however, the small binding energy that was obtained by the emulsion experiment is found to be hard to explain the short lifetime that were recently reported by heavy-ion beam experiments, suggesting we have been missing some important issues. Another mystery of the $A = 3$ hypernuclear system is the possibility of an atomic number zero hypernucleus, $^{3}_{\Lambda}$n. Events that can be interpreted as a bound $nn\Lambda$ state ($^{3}_{\Lambda}$n) were reported by HypHI Collaboration at GSI, Germany [10]. However, the existence of $^{3}_{\Lambda}$n cannot be explained with any available baryonic potential models [11, 12]. In order to investigate the existence of the bound $nn\Lambda$ three-body system, we performed an experiment with the ^{3}H$(e, e'K^+)nn\Lambda$ reaction at JLab Hall A using two HRSs (E12-17-003) [13, 14] with a tritium-gas target [15].

As we described above, a new generation of experiments on heavier hypernuclear systems as well as light hypernuclei are on-going or proposed at JLab. It is now possible to connect discussions of hypernuclei and NS systematically. Precise determination of the ΛN interaction is important for discussions of heavier hypernuclear systems. Here, we request 14.5 days of beamtime using the same spectrometer configuration of HRS-HKS that will be used for experiment E12-15-008 ($^{40,48}_{\Lambda}$K spectroscopy). It will enable us to perform a precise measurement of the hyperhydrogen nuclei $^{3,4}_{\Lambda}$H that are keys for investigations of (A) the contradiction between the short lifetime and small binding energy of $^{3}_{\Lambda}$H, (B) the puzzle of existence of bound $nn\Lambda$ state, and (C) the CSB in the ΛN interaction.

A. Contradiction between the short lifetime and small Λ binding energy of $^{3}_{\Lambda}$H

The Λ binding energy of hypertriton $^{3}_{\Lambda}$H was measured to be $B_{\Lambda} = 130 \pm 50$ keV in an emulsion experiment [16]. The Λ hypertriton is considered to be a loosely bound system of a Λ and a deuteron, and the spatial extent can be simply estimated by the root mean square radius of a two body system as follows:

$$\sqrt{< r^2 >} \simeq \frac{\hbar}{\sqrt{4\mu B_\Lambda}} \tag{1}$$

where, μ is the reduced mass $\mu = m_\Lambda m_d/(m_\Lambda + m_d)$ which is about 76% of the nucleon mass. The root mean square radius $\sqrt{< r^2 >}$ of hypertriton is then evaluated to be about 10 fm which is about five times larger than that of deuteron. Figure 1 shows a calculated density distribution of Λ hypertriton for which only s-waves are included [17]. In such a Λ-halo hypernuclear system, a wave

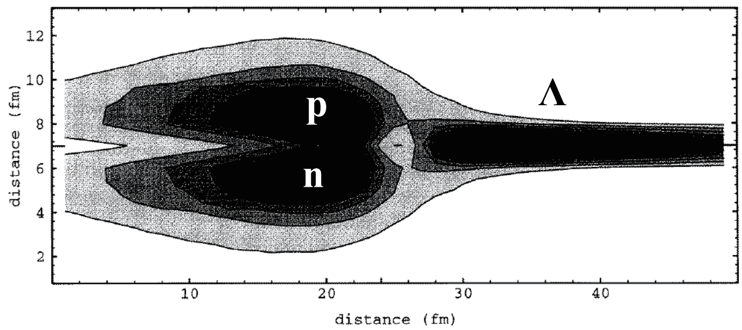

FIG. 1. A theoretical calculated probability distribution of a proton, a neutron and a Λ in Λ hypertriton [17]. Center of mass is fixed at the center of the figure and only s-waves are included for the calculation. The distance between the dense parts of the nucleons and Λ is more than 10 fm, and Λ and nucleons have little overlap.

function overlap between the core nucleus and Λ is small, and the Λ is almost free from interactions due to the core nucleus. Therefore, the hypertriton lifetime is naively expected to be similar to that of a free Λ hyperon. A theoretical calculation by a three-body Faddeev equations with realistic NN and YN interactions predicts that the lifetime of the hypertriton is nearly unchanged from that of a Λ hyperon (shorter by only 3%) [19].

However, recent heavy-ion beam experiments at GSI, LHC and RHIC consistently showed much shorter lifetime of hypertriton than a free Λ by 10–50%, in disagreement with theoretical predictions. Rappold *et al.* applied statistical analysis to old data including heavy-ion data from GSI, deducing a hypertriton lifetime of 216^{+19}_{-16} ps which is about 18% shorter than that of the Λ [20]. Figure 2 shows the experimental data and theoretical calculations of the Λ hypertriton lifetime [21]. Gal and Garcilazo recently took into account the pion final state interaction in a calculation of the hypertriton decay process [21]. They found that the pion FSI could enhance the decay rate and that a hypertriton lifetime about 20% smaller than of the free Λ is conceivable. The theoretical calculation with the pion FSI is consistent with the world average (206^{+15}_{-13} ps) that includes the

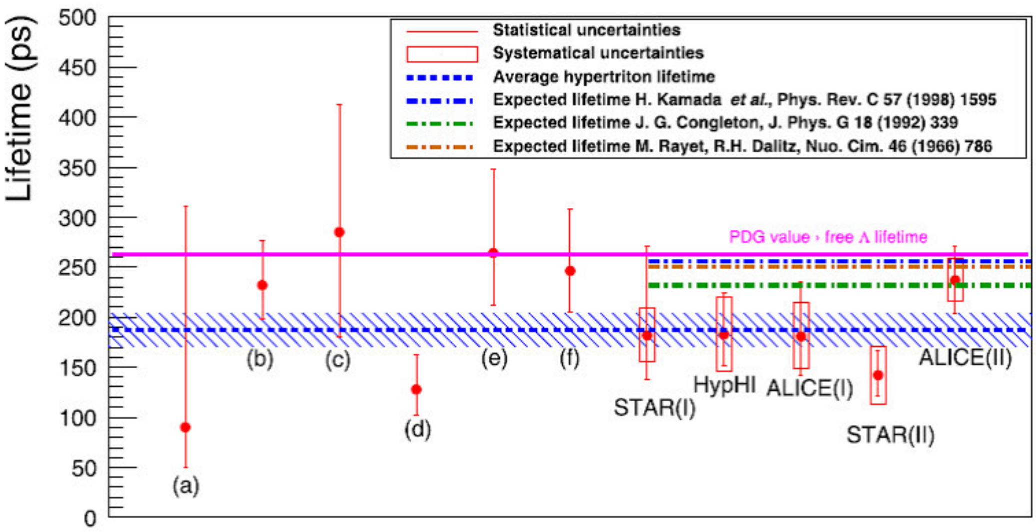

FIG. 2. Lifetime of Λ hypertriton summarized in Ref. [21]. Experimental data labeled as (a)–(f) were obtained in bubble chamber and emulsion experiments.

most recent measurement of hypertriton lifetime by ALICE Collaboration [22] who used a Pb-Pb collision at $\sqrt{S_{NN}} = 5.02$ TeV. In order to experimentally confirm the hypertriton lifetime, new experiments are now being prepared. At FAIR, using a heavy-ion beam, the lifetime of ${}^{3}_{\Lambda}$H will be measured with higher statistics by more than a factor of ten [23] than that obtained in GSI. While the new heavy ion-beam experiment at FAIR would show high accuracy in the hypertriton-lifetime measurement, it is important to measure the lifetime with various reactions or methods from a point of minimizing systematic error which might appear depending on experimental techniques. Now, new experiments are planned to directly measure the hypertriton lifetime with the (γ, K^+) reaction at ELPH [24], and the (π^-, K^0) [25, 26] and (K^-, π^0) [27] reactions at J-PARC.

The fact of the small binding energy of ${}^{3}_{\Lambda}$H contradicts the short lifetime in a framework of the ΛN and ΛNN interactions that were constructed mainly by Λ hypernuclear energies measured in the old experiments. It is of great significance to determine $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ which is the most basic quantity for the ΛN interaction study by using modern experimental techniques in which the systematic uncertainty is well controlled and understood. We are proposing an accurate measurement of the Λ binding energy of ${}^{3}_{\Lambda}$H with the $(e, e'K^+)$ missing-mass spectroscopy established at JLab [28].

B. The connection of ${}^{3}_{\Lambda}$H to the existence of the $nn\Lambda$ state

The Λ hypertriton, an iso-singlet state $I = 0$ in the $A = 3$ system, is considered to be the lightest bound hypernucleus. Iso-triplet states in the $A = 3$ system ($pp\Lambda$, ${}^{3}_{\Lambda}\mathrm{H}^{I=1}$, $nn\Lambda$) are predicted to be unbound. In invariant mass spectroscopy at GSI, events that can be interpreted to be the three body system of two neutrons and a Λ, which is the so called ${}^{3}_{\Lambda}$n ($I = 1$), were found [10]. The observation of the pionic weak decay process indicated the existence of bound $nn\Lambda$ state (${}^{3}_{\Lambda}$n), while the experimental accuracy for the Λ binding energy did not allow for confirming the bound state at the threshold region in the invariant mass spectrum. However, the bound $nn\Lambda$ state cannot be explained while maintaining a reasonable reproduction of other light hypernuclear binding energies with the various interaction models. On the other hand, a resonant state of $nn\Lambda$ is predicted to exist [29, 30]. In order to experimentally investigate the $nn\Lambda$ state, we carried out the E12-17-003 experiment at Hall A in Oct–Nov 2018. A gaseous tritium target was used to produce the $nn\Lambda$ state, and the $nn\Lambda$ energy was measured with the missing mass spectroscopy which is sensitive to a resonant state as well as a bound state. Analyses, particularly with a careful energy calibration, are in progress. The energy level of the iso-triplet state of ${}^{3}_{\Lambda}$H should be similar to that of the $nn\Lambda$ state because these are iso-triplet partners in the $A = 3$ hypernuclear system. Figure 3 shows expected energy levels of ${}^{3}_{\Lambda}$H with an ordinate axis of $-B_{\Lambda}[= M_{\mathrm{HYP}} - (M_d + M_{\Lambda})]$. The first excited state ($3/2^+$) with $I = 0$ and the $I = 1$ state are shown as boxes since energies for these states have not been measured.

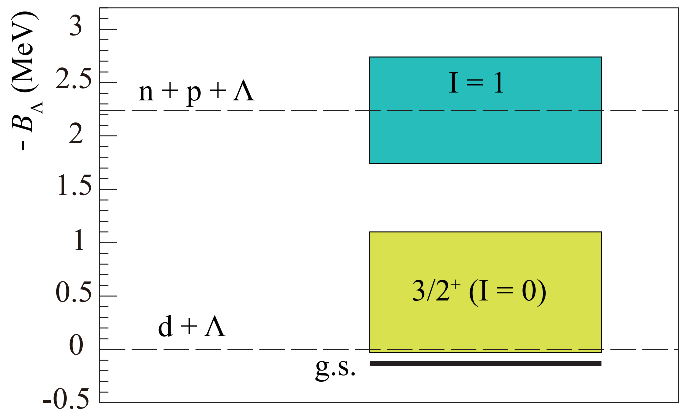

FIG. 3. Expected energy levels for the first excited state ($3/2^+$) with $I = 0$ and the $I = 1$ state in the Λ hypertriton. The energy for the $I = 0$ state energy should be similar to that of $nn\Lambda$ because these are iso-triplet partners in the $A = 3$ hypernuclear system. The measurement of ${}^{3}_{\Lambda}$H ($I = 1$) would pin down the existence of the bound $nn\Lambda$ state.

The proposed experiment in which precise spectroscopy of the $^3\mathrm{He}(e,e'K^+)^3_\Lambda\mathrm{H}$ is possible would determine the energy level of $^3_\Lambda\mathrm{H}$ $(I=1)$ if the production cross section for the iso-triplet state is reasonably large (e.g. more than the order of 0.1 nb/sr). The $^3_\Lambda\mathrm{H}$ $(I=1)$ measurement will give us a strong constraint to the existence of the bound $nn\Lambda$ state. If the energy measurements of both $^3_\Lambda\mathrm{H}$ $(I=1)$ and $nn\Lambda$ states are realized by respectively this proposed experiment and E12-17-003 (and followup experiments), the ΛN CSB could be investigated in the $A=3$ hypernuclear system for the first time.

C. Charge symmetry breaking effect in the $A=4$ hypernuclei

It is known that the strong interactions between baryons that consist of u and d quarks, i.e. nucleons, are (almost) flavor blind and have charge symmetry. However, it was found that the charge symmetry is considerably broken (CSB) between a nucleon and a Λ which includes an s quark. The CSB was experimentally observed in the $A=4$ iso-doublet Λ hypernuclear system ($^4_\Lambda\mathrm{He}$ and $^4_\Lambda\mathrm{H}$). Figure 4 shows the Λ binding energies of the 0^+ and 1^+ states in $^4_\Lambda\mathrm{He}$ and $^4_\Lambda\mathrm{H}$. There

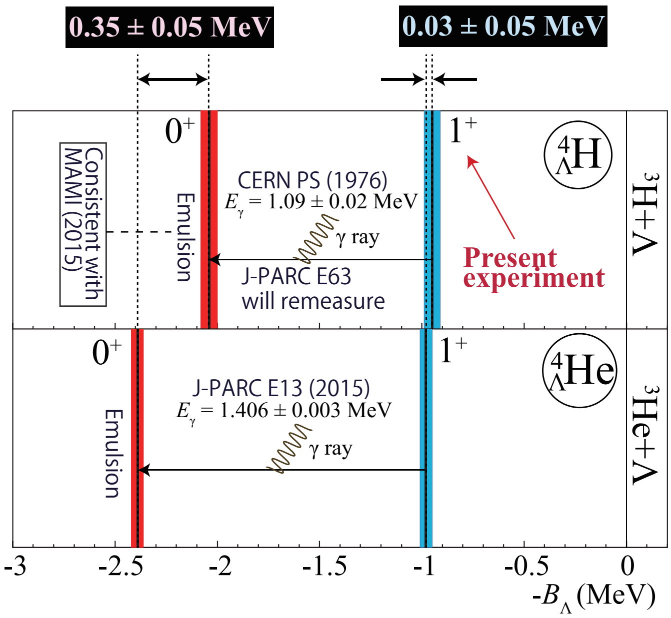

FIG. 4. The Λ binding energies of $A=4$ iso-doublet Λ hypernuclei.The present experiment aims to measure the absolute value of B_Λ ($^4_\Lambda\mathrm{H}$; 1^+) using the electron beam missing-mass spectroscopy established at JLab [1].

is a large binding energy difference for the ground state being $\Delta B_\Lambda(^4_\Lambda\mathrm{He}-^4_\Lambda\mathrm{H};\ 0^+) = 350\pm50$ keV which was obtained from old emulsion experiments. After the Coulomb correction, the energy of about 400 keV is attributed to an effect of the strong interaction [31–33]. This difference is larger than that for the case of an ordinary nuclear system (^{3}H and ^{3}He) by a factor of about five, and thus the charge symmetry looks to be broken in the ΛN interaction. Recently, MAMI successfully measured $B_\Lambda(^4_\Lambda\mathrm{H};\ 0^+)$ with decay pion spectroscopy which measured monochromatic pions emitted from two body decays of hypernuclei at rest [5, 6]. The result was consistent with the emulsion experiment, and clarified the existence of the ΛN CSB for the ground state of $A = 4$ hypernuclear iso-doublet.

The energy spacings between 0^+ and 1^+ were measured by Λ hypernuclear γ-ray spectroscopy. The Λ binding energies for the 1^+ state were derived by using the 0^+ energies and the energy spacings measured in respectively the nuclear emulsion experiment and γ-ray spectroscopy. It was believed that the energy difference for the 1^+ state is $\Delta B_\Lambda(^4_\Lambda\mathrm{He} -^4_\Lambda \mathrm{H};\ 1^+) = 290 \pm 60$ keV according to the old γ-ray measurements using a NaI detector. It showed there are the large CSB for both the 0^+ and 1^+ states. However, J-PARC E13 experiment re-measured $^4_\Lambda\mathrm{He}(1^+)$ by using germanium detector array which had better precision [7], and the data was updated to be $\Delta B_\Lambda(^4_\Lambda\mathrm{He} -^4_\Lambda \mathrm{H};\ 1^+) = 30 \pm 50$ keV meaning that there is little binding energy difference for 1^+. Surprisingly, it appears that ΛN CSB is spin dependent.

The ΛN-ΣN coupling is considered to be a key issue of ΛN CSB. However, it is difficult to understand the $A = 4$ iso-doublet hypernuclear system maintaining consistency with low lying energies of other light-Λ hypernuclei even when the ΛN-ΣN coupling is taken into account [34, 35]. There might be further important factors to be considered in the theoretical models. On the other hand, it is necessary to confirm B_Λ for not only $A = 4$ Λ hypernuclei but also particularly light hypernuclei with new experimental techniques as the cases for $^4_\Lambda$He (1^+) and $^4_\Lambda$H (0^+).

As knowledge of $A = 4$ iso-doublet hypernuclei $^4_\Lambda$H (1^+) and $^4_\Lambda$He (0^+) depends on old experimental data taken in 1960s, they should be re-measured with modern experimental techniques. However, the ground state of $^4_\Lambda$He measured by the emulsion experiment, can be considered more reliable compared to that of $^4_\Lambda$H because binding energies obtained from two body decay and three body decay processes are consistent. Therefore, a re-measurement on $B_\Lambda(^4_\Lambda\mathrm{H};\ 1^+)$ should be the priority. There is a plan to measure M1 transition γ-rays ($1^+ \to 0^+$) of $^4_\Lambda$H at J-PARC (J-PARC E63) [36]. This will need the ground state energy to deduce the excited state energy. Figure. 5 shows the ground state energy obtained by a recent result by A1 Collaboration at MAMI and other old measurements [5]. γ ray spectroscopy can determine the energy with a precision of a few keV. However, the energy of the excited state (g.s. + γ-ray energy) will have an uncertainty of about 100 keV due to the error on the ground state energy if the result by the A1 Collaboration at MAMI is used. The J-PARC E63 experiment has been approved for the J-PARC K1.1 beam line that will

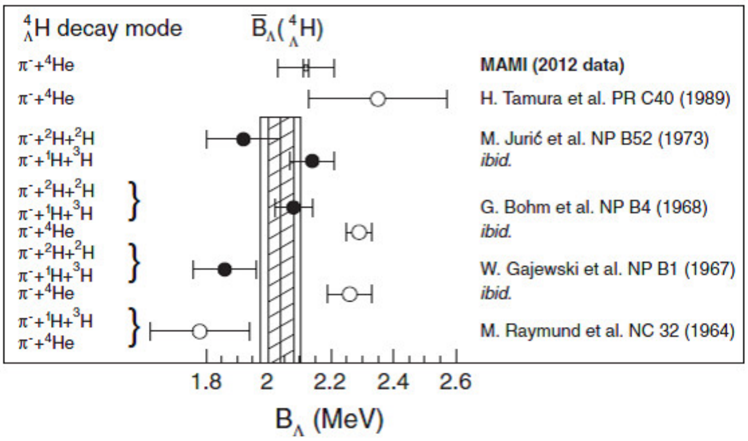

FIG. 5. The ground state Λ binding energy of $^4_\Lambda$H (0^+) measured in the past experiments [5]. The γ ray spectroscopy (J-PARC E63) that aims to measure $B_\Lambda(^4_\Lambda\mathrm{H}; 1^+)$ needs the ground state energy measured in the other experiments.

be constructed. However, the K1.1 beam line will not be built for several years. Therefore, we propose to perform the first direct measurement of the absolute Λ binding energy of $^4_\Lambda$H (1^+) by the $(e, e'K^+)$ reaction at JLab.

II. PREVIOUS MEASUREMENT

Previously, nuclei of Λ hyperhydrogens were investigated with the $^{3,4}\mathrm{He}(e, e'K^+)^{3,4}_{\Lambda}\mathrm{H}$ reaction at JLab Hall C [37, 38]. In the experiment, HMS and SOS spectrometers were used for detection of a scattered electron and a K^+, respectively. While Λ binding energies were not obtained due to a limited missing mass resolution, differential cross sections at the several K^+ scattering angles with respect to a virtual photon direction $\theta_{\gamma^* K}$ were measured [38]. The Q^2 and the total energy $W(=\sqrt{s})$ were $Q^2 = 0.35$ (GeV/c)^2 and $W = 1.91$ GeV respectively, and they are close to those of the proposed experiment. At the forward scattering angle at which we are proposing the new experiment in Hall A, the production cross sections were obtained to be 5 and 20 nb/sr for $^3_\Lambda$H and $^4_\Lambda$H, respectively. The previous experiment tells us that the differential cross sections for the electroproduction of the Λ hyperhydrogens $^{3,4}_{\Lambda}$H are large enough at the small $\theta_{\gamma^* K}$ where we aim to measure in the proposed experiment.

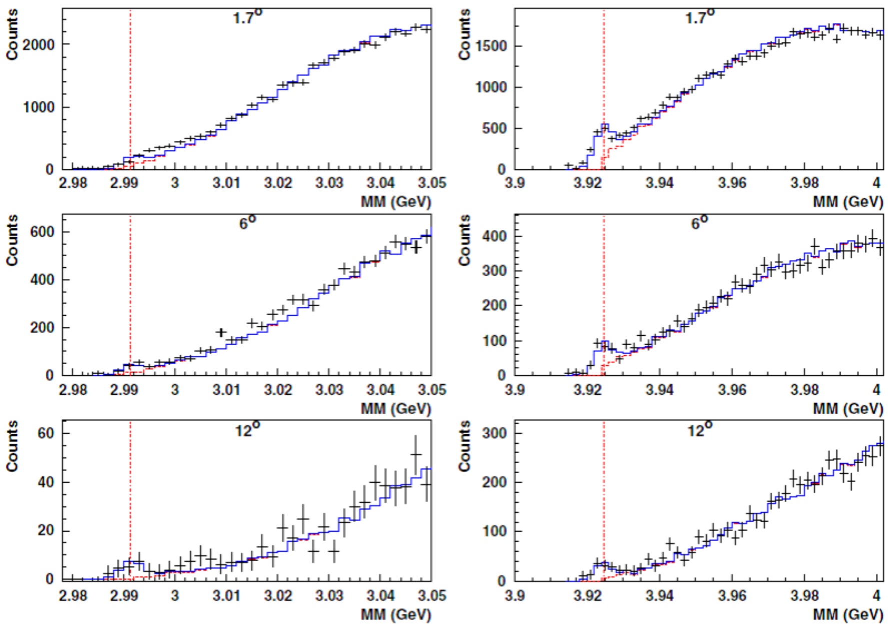

FIG. 6. Missing mass spectra for ${}^{3}_{\Lambda}$H (left) and ${}^{4}_{\Lambda}$H (right) obtained at JLab Hall C [38].

III. PROPOSED EXPERIMENT

We present the goal of the experiment and the requested beamtime and conditions in Sections III A and III B. Then, the experimental setup and expected results are shown in Sections III C and III E, respectively.

A. The goal of the proposed experiment

1. ${}^{3}\mathrm{He}(e, e'K^{+})^{3}_{\Lambda}\mathrm{H}$

The ground state binding energy of ${}^{3}_{\Lambda}$H was reported from the emulsion experiments as shown in Table I. Juric *et al.* complied and reanalyzed these data, and deduced the Λ binding energy as $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H}) = 130 \pm 50$ keV [16]. However, it was obtained from the average of results of two-body and three-body decay channels with scattered values. Table I shows $B_{\Lambda}({}^{3}_{\Lambda}\mathrm{H})$ for various decay channels and different experiments. While the statistical error of each data set is about 100 keV, there are apparently large systematic errors which were not taken into account in the averaging of them, depending on decay modes as seen also in the cases of ${}^{4}_{\Lambda}$H, ${}^{5}_{\Lambda}$He, ${}^{9}_{\Lambda}$Be and so on (refer to Tables 1 and 3 in [16]). Recently, the masses of the hypertriton and anti-hypertriton were reported from the STAR Collaboration who used heavy ion collisions at RHIC [39]. The hyperon binding energy was obtained by averaging the hypertriton and anti-hypertriton energies as $B_\Lambda(^3_\Lambda\mathrm{H}) = 410 \pm 120^{\mathrm{stat.}} \pm 110^{\mathrm{sys.}}$ keV. This is the first high precision measurement on B_Λ ($^3_\Lambda$H) by a counting experiment. However, the hypertriton binding energies obtained from two and three body processes seem to differ with each other by more than two sigmas of the error (∼400 keV) [40] which evoke a similar issue with the emulsion experiments. There may be an additional systematic error to be considered. An accurate measurement of $B_\Lambda(^3_\Lambda\mathrm{H})$ which is the most fundamental ingredients for the study of ΛN interaction is being awaited.

It is worth noting that the HKS (JLab E05-115) Collaboration measured $^{10}_\Lambda$Be [41], and the ground-state Λ binding energy was obtained to be $B_\Lambda = 8.60 \pm 0.07^{\mathrm{stat.}} \pm 0.16^{\mathrm{sys.}}$ MeV differing from the result of old emulsion experiment ($B_\Lambda = 9.11 \pm 0.22$ MeV [42]) by about 0.5 MeV. In addition, we suggested that Λ binding energy of $^{12}_\Lambda$C measured in the emulsion experiment has a shift of about a half MeV by a careful comparison between the (π^+, K^+) and emulsion data [41]. The shift of about a half MeV for the $^{12}_\Lambda$C binding energy is also addressed by the FINUDA Collaboration [43]. The need of the half MeV correction on $B_\Lambda(^{12}_\Lambda\mathrm{C})$ has a large impact because Λ binding energies for many of hypernuclei were measured with (π^+, K^+) experiments in which $B_\Lambda(^{12}_\Lambda\mathrm{C})$ was used for their energy calibration. Now, the correction for the hypernuclear energies is widely used for construction and tests of the ΛN interaction models [44, 45]. These updates and a series of measurements on B_Λ for p-shell hypernuclei by the $(e, e'K^+)$ experiments at JLab provided new insights into the ΛN interaction research; e.g. resulted in solving a puzzle of the large CSB in p-shell hypernuclear systems [4, 41].

TABLE I. The obtained Λ binding energy of $^3_\Lambda$H by the emulsion experiments.

Emulsion data	$\pi^- + {}^3\mathrm{He}$	$\pi^- + {}^1\mathrm{H} + {}^2\mathrm{H}$
	Λ binding energy (keV)	
M. Juric (1973) [16]	$+60 \pm 110$ (23 events)	$+230 \pm 110$ (58 events)
G. Bohm (1968) [18]	$+50 \pm 80$ (86 events)	-110 ± 130 (16 events)

Figure 7 shows the Λ-d rms radius versus the Λ binding energy obtained by using various NN and ΛN interactions [17]. There is a general correlation between the Λ-d rms radius and the Λ binding energy, and there is a small effect from the choice of the interaction models if the binding energies are similar. The Λ-d rms radius directly affects the hypertriton lifetime since it corresponds

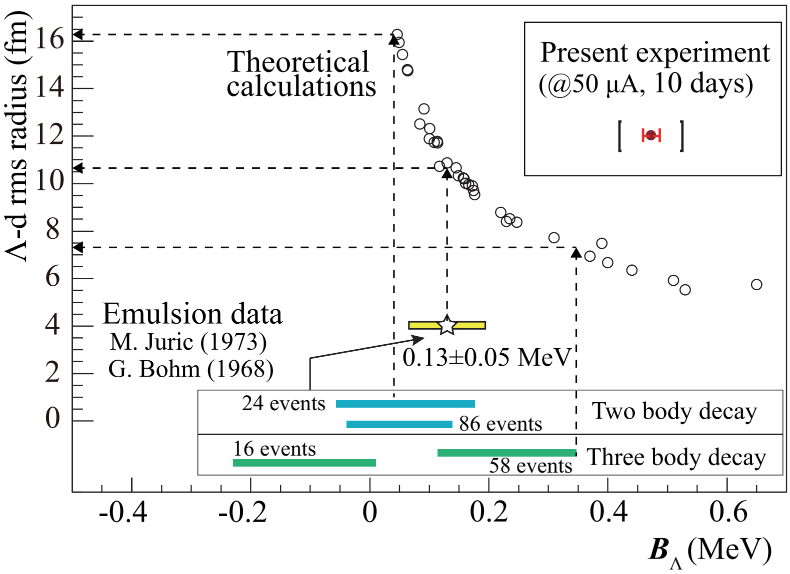

FIG. 7. The Λ-d rms radius versus the Λ binding energy taken from Ref. [17] (open circles). Experimental data of the emulsion experiments for two and three body decays [16, 18] are shown blue and green boxes. The empirical value $B_\Lambda = 130 \pm 50$ keV is represented by a star with a box. Dashed lines are eye guides to show how much the Λ-d rms radius changes with the assumed B_Λ of hypertriton. We aim to determine B_Λ with an accuracy of $|\Delta B_\Lambda^{\rm total}| = 65$ keV.

to the wave function overlap between a deuteron and a Λ. If $B_\Lambda = +230 + 110 = 340$ keV which is the deepest bound case in Table I is taken, the Λ-d rms radius becomes about 7 fm. This rms radius is much shorter than the case of $B_\Lambda = 130$ keV by more than 30%. There is an idea that such a deep binding energy could explain the short lifetime of the hypertriton [46]. A recent theoretical calculation with the pion final state interaction shows a quantitative prediction of the lifetime as a function of the binding energy, and it shows that the larger binding energy may explain the short lifetime [47]. The Λ binding energy is crucial for solving the hypertriton-lifetime puzzle, and needs to be determined with a less uncertainty.

The spin-parity J^π of the ground state of ${}^3_\Lambda$H was deduced to be $J^\pi = 1/2^+$ from the branching ratio of the two body π^- decay mode to the all of decay modes with a π^- [48, 49]. Therefore, the ground state binding energy of ${}^3_\Lambda$H is dominated by the spin-singlet S-wave $p\Lambda$ interaction. For the analysis of $N\Lambda$ scattering data [50], $B_\Lambda({}^3_\Lambda\mathrm{H};\mathrm{g.s.})$ was used as a constraint on the relative strength of the spin-singlet to the spin-triplet [51, 52]. The spin-singlet scattering length was obtained to be $a_s = -1.8^{+2.3}_{-4.2}$ fm [50]. Figure 8 shows the correlation between the scattering length a_s and $B_\Lambda({}^3_\Lambda\mathrm{H};\mathrm{g.s.})$ by various theoretical predictions. All of these predictions of the scattering length are within the error of the scattering experiment. The result from the emulsion experiment

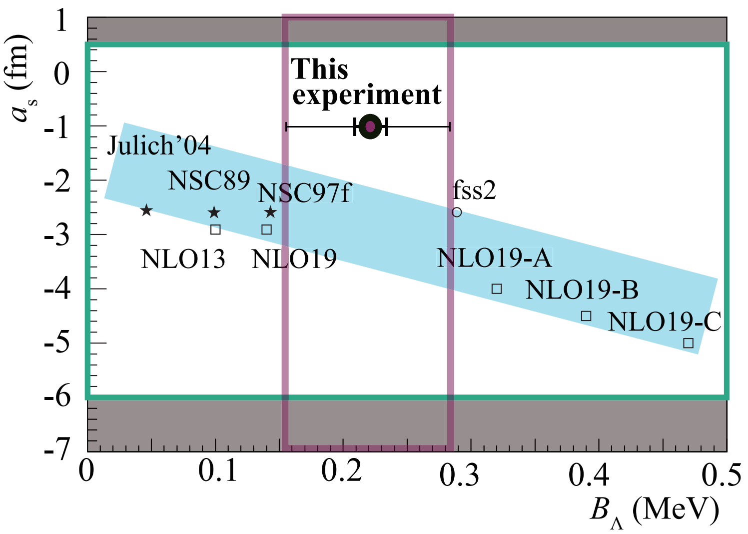

FIG. 8. The spin-singlet Λp scattering length a_s as a function of $B_\Lambda({}^3_\Lambda\text{H};\text{g.s.})$ for various interaction models [51–53]. The accuracy aimed in the proposed experiment is represented as error bars ($|\Delta B_\Lambda^{\text{stat.}}| =$ 20 keV, $|\Delta B_\Lambda^{\text{sys.}}| < 60$ keV).

(130±50 keV) supports interactions of the chiral Effective Field Theory (NLO13 and NLO19 with a cut off parameter of $\Lambda = 500$ MeV) [51] and Nijmegen Soft-Core models (NSC89 and NSC97f) [52]. On the other hand, the recent result of the STAR Collaboration which is larger binding energy ($410 \pm 120 \pm 110$ keV) is more preferable for the interactions of the SU(6) quark model (fss2) [53] and the chiral EFT for which the scattering length is adjusted (NLO19-A,B,C) [51]. The proposed experiment aims to determine the $B_\Lambda({}^3_\Lambda\text{H};\text{g.s.})$ with an accuracy of $|\Delta B_\Lambda^{\text{total}}| = 65$ keV, and to pin down the validation of the interaction models as shown in Fig. 8. In addition, our measurement would be a constraint on the spin-singlet $p\Lambda$ scattering length with a precision of about ±1.5 fm.

There is a possibility that we can measure the first excited state ($I = 0$, $J^\pi = 3/2^+$) which has not been observed yet. A theory predicts that the first excited state ($3/2^+$) is produced more than the ground state in the case of the $(e, e'K^+)$ reaction by a factor of about eight [54] if the $3/2^+$ state does exist. If this is the case, we will be able to determine the Λ binding energy of the $3/2^+$ state instead of the ground state ($J^\pi = 1/2^+$). The $B_\Lambda({}^3_\Lambda\text{H}; 3/2^+)$ will be a strong constraint for the spin-triplet Λp interaction because the $3/2^+$ state is dominated by the S-wave spin-triplet partial wave. Furthermore, the iso-triplet state ($I = 1$, $J^\pi = 1/2^+$) may be observed. An expected accuracy for the ${}^3_\Lambda\text{H}^{I=1}$ state is $|\Delta B_\Lambda^{\text{stat.}}| \simeq 90$ keV assuming the (γ^*, K^+) cross section is 0.5 nb/sr that corresponds to a smaller cross section than that for the ground state by an order of magnitude. The precise measurement of ${}^3_\Lambda\text{H}^{I=1}$ will be fateful about the existence of bound $I = 1$ three-body nuclei with a Λ hyperon such as the $nn\Lambda$ state.

We aim to measure the Λ binding energy of ${}^{3}_{\Lambda}$H with an accuracy of $|\Delta B_{\Lambda}^{\mathrm{stat.}}| = 20$ keV using 10 days of beamtime with a 50-μA beam on a gaseous ^{3}He target. The precise measurement on the Λ binding energy of the Λ hypertriton that can be realized by the experiment proposed here will be an important key to examine the hypertriton lifetime puzzle. Low energy properties of the ΛN interaction are hard preecisely investigate by scattering experiments due to experimental difficulties originating from the short lifetimes of hyperons. We know that two body systems with a Λ are not bound. Therefore, the Λ binding energy of the Λ hypertriton which is the lightest bound system with a Λ is significant to constrain the ΛN interaction model. The accurate binding energy measurement of the Λ hypertriton should be done with an experiment in which systematic errors are well controlled, and we propose here the best measurement for the purpose. Furthermore, we have a chance to observe excited states (the iso-singlet $J^{\pi} = 3/2^{+}$ and iso-triplet $J^{\pi} = 1/2^{+}$ states). The precise measurements of the excited states of the Λ hypertriton presently only possible at JLab. Experiments, such as an emulsion experiment and a heavy ion experiment, in which particles from weak decay processes are detected have little sensitivity to the excited state. The proposed experiment is very unique way to provide one of fundamental quantities for studies of the baryon interaction and strangeness nuclear physics.

2. $^{4}\mathrm{He}(e, e'K^{+})^{4}_{\Lambda}\mathrm{H}$

As discussed in Section I C, the high precision measurement of $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H};\ 1^{+})$ with a modern experimental techniques is a priority to ensure the discussion about $A = 4$ ΛN CSB. Missing-mass spectroscopy with the $(e, e'K^{+})$ reaction which was established at JLab has a unique capability to directly determine the absolute energy of $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H};\ 1^{+})$. The high precision measurement of $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H};\ 1^{+})$ is possible thanks to (i) the high quality electron beam at JLab, (ii) the high resolution spectrometer system HRS + HKS, and (iii) the large amplitude of spin flip probability due to the (virtual) photon reaction. The excited state (1^{+}) which we aim to measure and the ground state (0^{+}) can not be resolved by an expected missing mass resolution in the proposed experiment. However, a strong selectivity of the 1^{+} state production is expected for the $(e, e'K^{+})$ reaction due to the fact of (iii), and the production cross section of the 1^{+} state is predicted to be larger than that of the 0^{+} state by more than two orders of magnitude at the forward K^{+} scattering angle whcih we will cover. The absolute energy of $B_{\Lambda}({}^{4}_{\Lambda}\mathrm{H};\ 1^{+})$ measured by the proposed experiment will be complementary with the combined information $[\Delta E(1^{+} - 0^{+}) + B_{\Lambda}(0^{+})]$ of γ-ray spectroscopy

$[\Delta E(1^+ - 0^+)]$ and the decay pion spectroscopy or emulsion experiment $[B_\Lambda(0^+)]$.

We aim to measure $B_\Lambda({}^4_\Lambda\mathrm{H};\, 1^+)$ with a total error of $|\Delta B_\Lambda^{\mathrm{total}}| = 65$ keV ($|\Delta B_\Lambda^{\mathrm{stat.}}| = 20$ keV, $|\Delta B_\Lambda^{\mathrm{sys.}}| < 60$ keV) in two days of beamtime by using a 50-μA beam on a gaseous ^{4}He target.

B. Requesting conditions and beamtime

1. Beam

We request a 50-μA beam at $E_e = 4.24$ GeV (two passes) with a bunch frequency of 500 MHz (250 MHz repetition rate will result in worse accidental background rate though it is still acceptable). In order to achieve a sufficient precision in the resulting missing-mass spectrum, the beam energy spread and energy centroid are required to be $\Delta p/p \leq 1 \times 10^{-4}$ (FWHM) which was already confirmed to be feasible in the $nn\Lambda$ search experiment (E12-17-003, Hall A) in 2018. A beam raster with an area of about 2×2 mm^2 would need to be applied to avoid a damage on a target cell due to an overheat.

2. Target

For reasons discussed below, the proposed measurement requires a new target system comprised of both gas cells and solid targets. Some details of the proposed system are given in Sec. III C 2. The proposed cells will contain low density ^{3}He and ^{4}He (H_2) with a length of 20 cm (22 cm) along the beam as shown in Fig. 11. The solid targets would be compatible with those needed for E12-15-008 with an additional optics target.

The missing mass would be shifted depending on a position of hypernuclear production point. In order to correct the shift, the position information of production point is necessary event by event. A displacement from the beam center in the x and y directions (vertical to the beam axis) can be derived from applied currents on raster coils. On the other hand, a production position in the z direction (parallel to the beam axis) needs to be reconstructed from a magnetic optics analysis. For a calibration of reconstructed z, we will use a multi-foil carbon target which is an absolute position reference in z direction. The multi-foil target will have five carbon foils with a thickness of 100 mg/cm^2 being aligned 2.5 cm apart from each other.

We request to use the exactly same spectrometer setup as the E12-15-008 experiment in which the isospin dependence of the $\Lambda N/\Lambda NN$ interactions will be investigated through the $^{40,48}\mathrm{Ca}(e,e'K^+)^{40,48}_{\Lambda}\mathrm{K}$ reactions [55]. The existing spectrometers, (L)HRS [56] and HKS [57], combined with a new pair of charge separation dipole magnets (PCS) will be used for e' and K^+ detection. Construction and an excitation test of the PCS have been completed in a Japanese company TOKIN in March 2020. The vacuum chambers which will be equipped inside PCS were designed and built in TOKIN as well in March 2021. The PCS combined with the vacuum chambers are ready for transport to JLab. The Central momenta of HRS and HKS will be set respectively to 2.74 and 1.20 GeV/c. The system covers the kinematical region of the $^{3,4}\mathrm{He}(e,e'K^+)^{3,4}_{\Lambda}\mathrm{H}$ reaction as shown in Fig. 9. In addition to the Λ, the Σ^0 will will also be used for an absolute energy

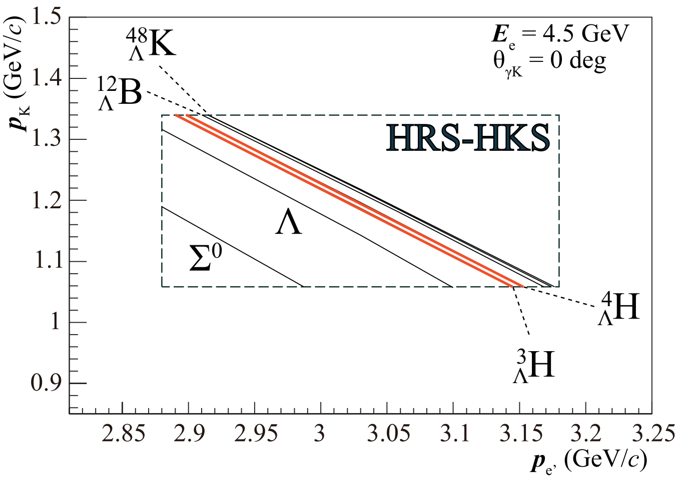

FIG. 9. Correlation between momenta of e' and K^+. A box indicates momentum acceptance of the HRS-HKS spectrometer system at $E_e = 4.5$ GeV, $\theta_{\gamma K} = 0$ degree.

calibration as they will both be in the acceptance at the same spectrometer setting as that for the physics runs thanks to the large momentum coverage of HKS ($\Delta p/p_{\mathrm{central}} > \pm 10\%$). Measuring both Λ and Σ^0 masses without a change of spectrometer setting minimizes the systematic error on B_Λ. Another important feature of HKS is its short path length. The path length of PCS + HKS is about 12 m, and thus 26% of K^+s survive at 1.2 GeV/c. This gains a yield of Λ hypernuclei by a factor of more than three compared to HRS in which K^+s travel more than 23.4 m before detection.

4. *Beamtime*

Table II shows requested beamtime for the present experiment. We request 12 days for ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H productions, and 2.5 days for calibration runs. In total, we request 14.5 days of beamtime.

TABLE II. Requested beamtime of C12-19-002 including calibration runs. Gas target thicknesses shown in the table are after Al-cell events are rejected by applying a cut of reconstructed z_t in the analysis (see Sec. III C 2).

Mode	Hypernucleus	Target (mg/cm^2)	Beam current (μA)	Beamtime (day)	Yield
Physics	${}^{3}_{\Lambda}$H	^{3}He (165)	50	10	600 (1/2$^+$, 3/2$^+$)
	${}^{4}_{\Lambda}$H	^{4}He (228)	50	2	500 (1$^+$)
	Subtotal			**12**	-
Calibration	Λ Σ^0	H_2 (54)	50	1	3500 1150
	${}^{12}_{\Lambda}$B$^{\rm g.s.}$	Multi foil (100 × 5)	50	1	300 × 5
	-	Multi foil + Sieve slit	50	0.2	-
	-	Empty cell	50	0.1	-
	-	Empty cell + Sieve slit	50	0.2	-
	Subtotal			**2.5**	-
Total				**14.5**	-

C. Experimental setup

1. *Spectrometer Setup*

Electron beams at $E_e = 4.24$ GeV will be incident on helium targets to produce ${}^{3,4}_{\Lambda}$H, and scattered electrons and K^+s with central momenta of 2.74 and 1.20 GeV/c will be measured by the HRS and HKS, respectively. One of important features of the HRS and HKS spectrometers is a good momentum resolution which is expected to lead to the missing mass resolution of about 1 MeV FWHM.

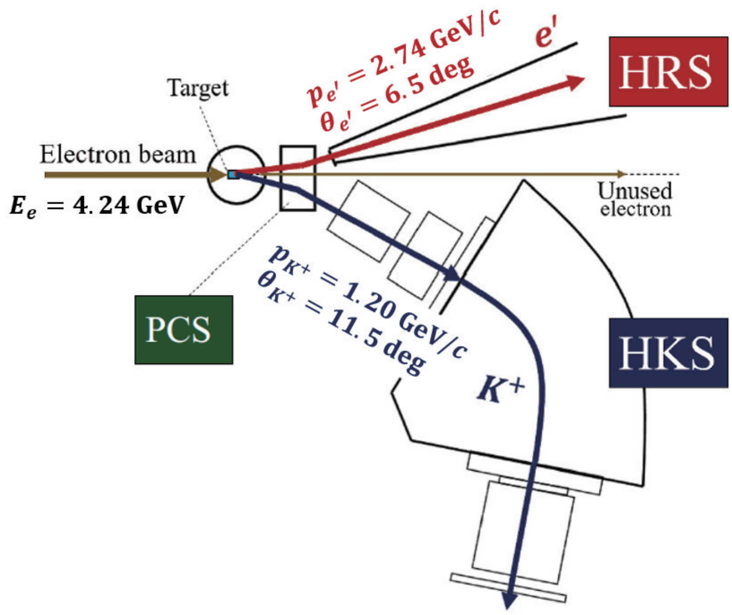

FIG. 10. A schematic of the experimental setup. Electron beams at $E_e = 4.24$ GeV incident on the helium target which is enclosed in a vacuum chamber located in front of the pair of charge separation dipole magnets (PCS). The scattered electrons and K^+ with central momenta of 2.74 and 1.20 GeV/c are momentum-analyzed by the HRS and HKS, respectively.

TABLE III. Basic parameters of the proposed experiment. Momentum resolutions shown in the table were evaluated by a MC simulation taking into account target materials (gas target enclosed in a 0.3-mm thick target cell with the diameter of 200 mm) (see also Table VI).

Beam (e)	$\Delta p/p$	$\leq 1 \times 10^{-4}$ FWHM
	E_e	4.24 GeV
PCS + HRS (e')	D(PCS) + QQDQ	
	$\Delta p/p$	3.2×10^{-4} FWHM
	$p_{e'}$	2.74 GeV/$c \pm 4.5\%$
	$\theta_{ee'}$	6.5 ± 1.5 deg
	Solid angle $\Omega_{e'}$	2.4 msr
PCS + HKS (K^+)	D(PCS) + QQD	
	$\Delta p/p$	5.7×10^{-4} FWHM
	p_K	1.2 GeV/$c \pm 10\%$
	θ_{eK}	11.5 ± 4.5 deg
	Solid angle Ω_K	7 msr
	Optical length	12 m
	K^+ survival ratio	26%

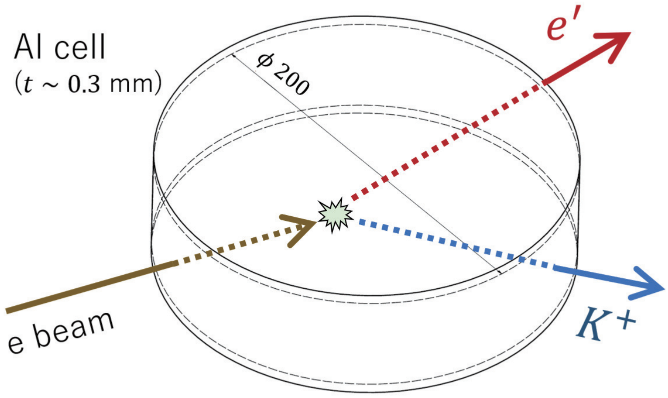

FIG. 11. A schematic of the proposed gas target cell. Note that the axis of the puck-like cell is perpendicular to the beam in vertical direction. The height of each cell is 50 mm.

2. Target Design

a. Target Size Due to the spatial constraints at the pivot which are unique to the hypernuclear configuration, a new target system must be developed. The target system must also meet the requirements of the E12-15-008 experiment to ensure compatibility of the two experiments. The new target system will also require a new scattering chamber to meet these spacial limitations. We propose three 20 cm tuna can style cells filled to 3 atm of H_2, ^{3}He and ^{4}He gas as well as a multi-foil optics target in addition to the targets needed for E12-15-008. The H_2 gas and multi-carbon-foil target will be used for calibrations of energy scale and z reconstruction. A thick target will deteriorate the missing mass resolution due to momentum straggling and multiple scattering. In order to keep the effects of the momentum straggling and multiple scattering on the missing mass resolution small compared to those of the momentum and angular resolutions of the spectrometer system, the target thickness is designed to be less than a few hundred mg/cm^2 including the material of the cell wall (about $\leq$ 500-mg/cm^2 thick target is chosen for the present experiment).

We propose to operate the target in two modes both of which require modest cryogenic support form the ESR. In the calibration mode, the H_2 cell would be filled to a pressure of 3 atm at a temperature of 30 K. In the production mode, the H_2 cell would be evacuated and the helium cell would be further cooled to 12 K while maintaining the same pressure of 3 atm thus significantly increasing the density of the target fluid. The cell from which H_2 was evacuated will be used for an empty-cell runs. Figure 11 shows the schematic of the proposed target cell. The cell wall thickness of 0.3 mm meets the applicable ASME Code requirements for the pressures proposed. The extended length of the cell allows the separation of background events from the cell wall in off-line analysis with a cut in target length. This is shown in Sec. III E, with an expected resolution of the reconstructed z position of about $\Delta z_{\mathrm{react}} = 15$ mm which is consistent with that in Ref. [56]. Using events in $|z_{\mathrm{react}}| \leq 87$ mm (97 mm for H_2) for analyses, about 97.7% of events that come from the target cell can be rejected. The expected target thicknesses are shown in Table IV.

TABLE IV. Expected target thicknesses without density correction from beam heating and including the full 20 cm length. Thicknesses are given for the production mode of 3 atm at 12 K with the exception of the H_2 target which is operated at 3 atm, 30K.

Target	Thickness (mg/cm^2)
^{3}He (+ Al cell)	190 (+ 162)
^{4}He (+ Al cell)	262 (+ 162)
H_2 (+ Al cell)	56 (+ 162)

To position the each target in the beam a motion system similar to that used for PREX is proposed. Like PREX and the tritium target, we propose to mount the gas and solid targets to a heat sink cooled by the ESR. The temperature of the heat sink would be stabilized for H2 running at 30K but for the production mode we would be able to turn this heater off and simply run as cold as ESR coolant will allow. We note that the gas in the cells will not recirculate as with the standard cryotarget. This will lead to larger density reductions at the proposed beam current than is typical. We therefore assume a density reduction of 50% at 50 μA in our rate calculations.

While the target ladder, motion system, and scattering chamber will need to be of new design, the standard cryotarget gas handling system can be used with only minor modifications owing to the lower operating pressures. Some of the components fabricated for the PREX/CREX target can also be repurposed. Overall the fabrication of this new target system is expected to require an effort similar to that of the PREX/CREX target.

b. Target Cell Shape We are planning to use a cell of tuna-can type in the proposed experiment so that the path length of scattered particles in the cell can be minimized. In the case of a cell of cigar type which was used for the $nn\Lambda$ production with a tritium gas, the path length is much longer in the target cell because we used a spectrometer setting with a small scattering angle at $\theta_{\mathrm{HRS}} = 13.2$ degrees. The areal density in the cell was about $[0.04\ (\mathrm{cm}) \times 2.8\ (\mathrm{g/cm^3}) \times \frac{1}{\sin\theta_{\mathrm{HRS}}} \simeq]$ 500 mg/cm^2 which is equivalent to about 2000 mg/cm^2 of helium in the radiation length. The large amount of material in the scattered particle path caused deterioration particularly for the angular resolution due to multiple scattering. Missing mass resolution is more sensitive to angular resolution for a light hypernuclear system than that for heavier systems, and thus it is important to minimize multiple scattering for the precise spectroscopy of the proposed experiment. The target cell for the proposed experiment is designed to achieve a more than 80% reduction in path length compared to that in the tritium experiment. In addition, the thickness uniformity of the cigar-cell wall was difficult to be controlled, and it caused large systematic error on B_Λ (the order of a few 100 keV). In order to meet our requirement of the proposed experiment, we decided to use the much simpler shape, tuna-can, which has a less uncertainty on the cell-wall thickness.

D. Trigger Rate

Trigger rates were estimated by a Geant4 Monte Carlo (MC) simulation in which magnetic field maps generated by the finite element calculation software, Opera3D (TOSCA), were used [58]. Singles rates are expected to be dominated by electrons that come from the Bremsstrahlung process in HRS, and protons and π^+s in HKS. In the MC simulation, particles were generated based on the Bremsstrahlung formula [59] and the EPC code [60] for the scattered electrons and hadrons, respectively for the rate estimation. It must be noted that the counting rates in the previous HKS experiment (E05-115, Hall C) were well reproduced by the MC simulation combined with these physics inputs. The expected background particle rates in HRS (e') and HKS (p and π^+) are shown in Tab. V. The accidental coincidence rates for these charged particles were estimated with an assumption of a coincidence window of 150 ns. Our DAQ system allows for a request speed of

TABLE V. Expected background rates by the Geant4 MC simulations of PCS+HRS (e') and PCS+HKS (p and π^+) [58]. The accidental coincidence rates without and with the HKS Cherenkov (PID) detectors are shown in the last two columns.

Target	Thickness (mg/cm^2)	Beam Intensity (μA)	e' (kHz)	p (kHz)	π^+ (kHz)	Coin Rate (kHz)	Coin Rate w/ PID (kHz)
^{3}He + Al cell	190 + 162	50	90.8	163.2	252.5	3.2	0.15
^{4}He + Al cell	262 + 162	50	91.2	201.6	355.9	4.9	0.23
^{12}C	100	50	21.5	56.4	70.1	0.4	0.02
^{40}Ca	100	50	64.5	48.4	70.1	1.2	0.06
^{208}Pb	100	25	97.0	22.2	33.0	0.8	0.04

up to about 2 kHz for the effective data taking. The expected rates of the accidental coincidence were 3.2 and 4.9 kHz for ^{3}He and ^{4}He targets, respectively. If the PID detectors in HKS such as aerogel and water Cherenkov detectors are used for the trigger to suppress p and π^+ in HKS [61], the accidental rates are reduced to be < 0.3 kHz which is well below the acceptable rate of 2 kHz.

It is worth noting that the experiments with the solid targets may not need the HKS PID detectors at the trigger.

E. Expected results

1. Missing-mass resolution and yield

The missing mass resolution was estimated by the MC simulation [62]. It is noted that our Geant4 simulation for the E12-17-003 (the $nn\Lambda$ search experiment in which we used two HRSs at Hall A) could reproduce the missing mass resolution of data for Λ and Σ^0 productions which is about 3.5 MeV (FWHM) [14] as shown in Fig. 12. The same framework of the Geant4 simulation was

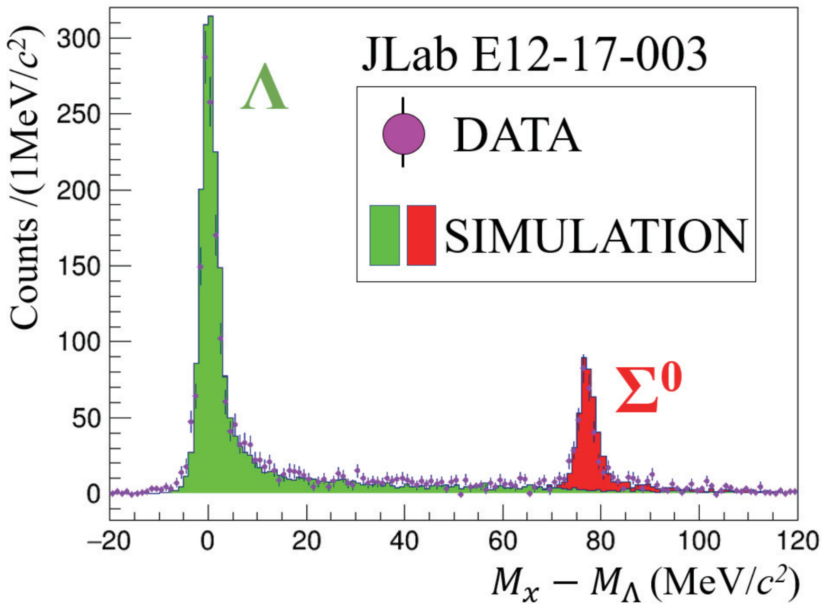

FIG. 12. Comparison of Λ (and Σ^0) spectrum of a Geant4 simulation with real data of E12-17-003 (the $nn\Lambda$ search experiment at Hall A). The Geant4 simulation in which the HRS+HRS spectrometer system was modeled could reasonably reproduce the spectrum of real data. The same framework of Geant4 simulation was applied to the proposed experiment to evaluate an expected missing mass resolution.

applied to a simulation for the proposed experiment with new experimental setup. The scattered electrons and K^+s were generated in the MC simulators, PCS + HRS and PCS + HKS, randomly in a range of the target along the beam direction. The scattered electrons and K^+s were measured at the focal planes of the spectrometers taking into account realistic position and angular resolutions of the particle detectors. Then the particle positions and angles at the focal planes were converted into momentum vectors at the production point by using backward transfer matrices to reconstruct a missing mass. For reconstruction of the momentum vectors at the production point, information of production vertex z was used. In particulary, the HKS needs the z information in addition to the focal plane information because HKS is a horizontal bending spectrometer and the z position (which corresponds to x at target) strongly couples with momentum as well as angle at target. The production vertex z was reconstructed by a backward transfer matrix in HRS. The resolution of the reconstructed z was about 15 mm (FWHM). While the momentum resolution of HKS was obtained to be $\Delta p/p = 1.2 \times 10^{-3}$ FWHM when the the z information was not used, the momentum resolution of HKS was recovered to be $\Delta p/p = 5.7 \times 10^{-4}$ FWHM by using the reconstructed production z position. The resolution of $\Delta p/p = 5.7 \times 10^{-4}$ FWHM is still worse than the original design of HKS by a factor of about three. However, the deterioration of the resolution in HKS does not

TABLE VI. Momentum and angular resolutions of PCS+HRS (e') and PCS+HKS (K^+) obtained by analyses with backward transfer matrices in Monte Carlo simulations by Geant4. The results are shown in FWHM. Expected missing mass resolutions (FWHM) are shown in the last column.

Target Cell Thickness t (mm)	$dp_{e'}/p_{e'}$ ($/10^{-4}$)	dp_K/p_K ($/10^{-4}$)	$\delta x'_{e'}$ ($/10^{-3}$)	$\delta y'_{e'}$ ($/10^{-3}$)	$\delta x'_K$ ($/10^{-3}$)	$\delta y'_K$ ($/10^{-3}$)	$dM_{\rm HYP}$ (MeV/c^2)
0.0 (no target)	2.88	4.48	1.68	1.10	1.32	0.99	1.0
0.3	3.20	5.68	1.90	1.44	1.92	0.97	1.1
1.5	3.84	6.72	2.36	1.90	3.38	0.99	1.3

have a large effect on the the missing mass resolution as the the contribution from HRS is much larger in our kinematics. The expected momentum and angular resolutions of the PCS+HRS and PCS+HKS for the cases without a target and with a target enclosed in Al cell of 0.3 and 1.5-mm thicknesses are summarized in Table VI. As a result, the expected missing mass resolution of the proposed experiment was found to be 1.1 MeV/c^2 FWHM.

The yield of hypernuclei ($N_{\rm HYP}$) was estimated as follows:

$$N_{\rm HYP} = \Gamma^{\rm int} \times N_{\rm beam} \times \left(N_{\rm target} \times f_{\rm density}\right) \times \left(\frac{d\sigma}{d\Omega}\right) \times \Omega_K \times \epsilon \qquad (2)$$

where $N_{\rm beam}$, $N_{\rm target}$, $\left(\frac{d\sigma}{d\Omega}\right)$, and Ω_K are the number of incident electrons, the number of target nuclei (cm^{-2}), the differential cross section of the ($\gamma^{(*)}$,K^+) reaction (cm^2 msr^{-1}), and the solid angle of the K^+ spectrometer (msr). The total experimental efficiency ϵ is the combined K^+ survival and other efficiencies (DAQ, detector, analysis, etc.). 19.5% [$\simeq$ 0.26 (K^+ survival ratio) $\times$ 0.75] is used for this estimate. The $f_{\rm density}$ is a factor to take into account a density reduction of the gas target with a 50 μA beam. We assume $f_{\rm density} = 0.50$. The $\Gamma^{\rm int}$ is the virtual photon

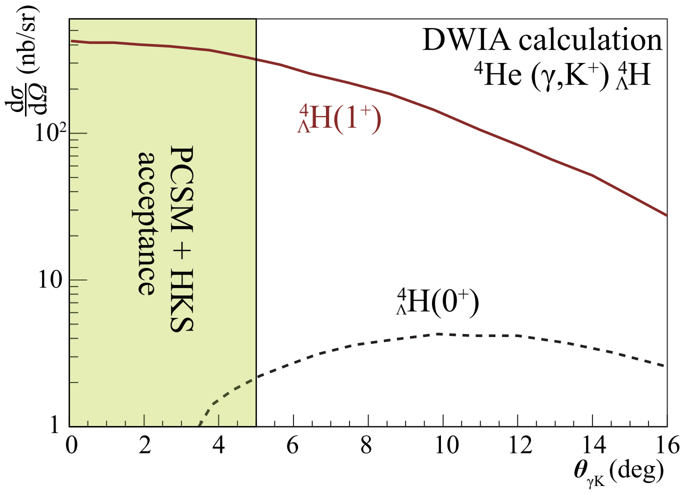

FIG. 13. A theoretical prediction of the differential cross section of $^{4}\mathrm{He}(\gamma, K^{+})^{4}_{\Lambda}\mathrm{H}$ by DWIA [63].

flux integrated over the PCS + HRS acceptance (notations can be found in [1]):

$$\Gamma^{\mathrm{int}} = \iint_{\mathrm{HRS}} \frac{\alpha}{2\pi^{2}Q^{2}} \frac{E_{\gamma}}{1-\epsilon} \frac{E_{e'}}{E_{e}} dp_{e'} d\theta_{e'} \tag{3}$$

$$= 2.0 \times 10^{-5} \quad (/\mathrm{electron}). \tag{4}$$

The differential cross sections of the $^{3}\mathrm{He}(\gamma,K^{+})^{3}_{\Lambda}\mathrm{H}$ and $^{4}\mathrm{He}(\gamma,K^{+})^{4}_{\Lambda}\mathrm{H}$ reactions with a similar $\sqrt{s}$ and scattering angle ($\theta \simeq 0$ deg) to those of our experiment were measured in a past experiment at JLab Hall C (E91-016) [37], although the Λ binding energies were not determined in this experiment because the energy resolution was not enough. The differential cross sections were obtained to be about 5 and 20 nb/sr for the $^{3}_{\Lambda}\mathrm{H}$ and $^{4}_{\Lambda}\mathrm{H}$ production [38]. In photoproduction, excited states are preferentially produced as the spin-flip amplitude is large. Figure 13 shows the differential cross section of $^{4}\mathrm{He}(e, e'K^{+})^{4}_{\Lambda}\mathrm{H}$ predicted by DWIA calculation [63]. At the forward K^{+} scattering angle with respect to a photon direction, the differential cross section of the first excited state (1^{+}) is predicted to be larger than that of the ground state (0^{+}) by two orders of magnitude. In the case of $^{3}_{\Lambda}\mathrm{H}$ photoproduction, the $3/2^{+}$ state is expected to be a larger cross section by a factor of eight if the $3/2^{+}$ state exists [54]. Expected yield per day with a beam current of 50 μA is summarized in Table VII.

2. Accidental background

Accidental $e'K^{+}$-coincidence events will be a background in the resulting missing-mass spectrum. To estimate the accidental background events, real data which were taken in the last

TABLE VII. Expected yields for ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H in the present experiment.

Hypernucleus		Target (mg/cm^2)	Cross section (nb/sr)	Yield per day at 50 μA
${}^{3}_{\Lambda}$H	$1/2^+$	^{3}He	5	60
	$3/2^+$	(165)		
${}^{4}_{\Lambda}$H	0^+	^{4}He	-	-
	1^+	(228)	20	250

hypernuclear experiment at JLab Hall C (E05-115) were used. With a 0.2-g/cm^2 ^{7}Li target at $I_e = 32$ μA in the E05-115 experiment [64], counting rates of K^+, π^+ and p in HKS were:

$$R_K^{\rm ref} : R_\pi^{\rm ref} : R_p^{\rm ref} = 0.3 : 25 : 34 \quad {\rm kHz}. \tag{5}$$

The HKS has Cherenkov counters with radiation media of aerogel ($n = 1.05$) and water ($n = 1.33$) to reject π^+s and protons. The Cherenkov counters reduced the fractions of π^+s and protons down to 0.5% and 10%, respectively, at the trigger level. In off-line analysis, π^+s and protons can be reduced to 4.7×10^{-4} and 1.9×10^{-4} by using information on light yields in the Cherenkov counters and reconstructed particle-mass squared [61]. The most important off-line analysis for the K^+ identification (KID) is a time-of-flight (TOF) analysis. The TOF from the target to the timing counter was 10 m in HKS, and the TOF resolution was $\sigma = 0.26$ ns. Thus, the time separation of (e' - K^+) coincidence from (e' - π^+) and (e' - p) ones at 1.2 GeV/c were more than 6σ and 20σ respectively when an event selection was applied to select the $e'K^+$ coincidence with a time gate of ± 1 ns [1].

K^+ from the quasi-free hyperon production is another background source in addition to p and π^+ for which counting rates in HKS are shown in Sec. III C. The quasi-free K^+ rate R_K was estimated assuming the production-cross section of quasi-free Λ is proportional to $A^{0.8}$:

$$R_K = R_K^{\rm ref} \times \frac{0.1}{0.2} \times \frac{50}{32} \times \frac{\Gamma^{\rm int}}{\Gamma^{\rm int}_{\rm ref}} \times \frac{A^{0.8}}{7^{0.8}} \times \frac{7}{A} \quad \left[\frac{\rm Hz}{(100~{\rm mg/cm^2})(50~\mu{\rm A})}\right] \tag{6}$$

$$= 234 \times \frac{\Gamma^{\rm int}}{\Gamma^{\rm int}_{\rm ref}} \times \left(\frac{7}{A}\right)^{0.2} \quad \left[\frac{\rm Hz}{(100~{\rm mg/cm^2})(50~\mu{\rm A})}\right] \tag{7}$$

where the rate is normalized to a target-thickness of 100 mg/cm^2 and a beam current of 50 μA. The $\Gamma^{\rm int}_{\rm ref}(= 5.67 \times 10^{-5})$ is the integrated virtual photon flux in E05-115. The accidental coincidence background observed in the missing mass spectrum of ${}^{7}{\rm Li}(e, e'K^+){}^{7}_{\Lambda}{\rm He}$ in E05-115 was about 17.3 [(nb/sr) per 1 MeV] ($\equiv h^{\rm acc}_{\rm ref}$) [4]. The accidental coincidence background expected in the

TABLE VIII. Expected accidental coincidence background rates in HRS (e') and HKS (K^+) at $I_e = 50\ \mu$A. The expected rates (counts/MeV/day) with and without the z_t selection are shown in the last two columns.

Target	Rate	Accidental Coincidence Background	
(mg/cm^2)	K^+ (Hz)	w/o z_t cut (/MeV/day)	w/ z_t cut (/MeV/day)
^{3}He (165) + Al (162)	162 + 164	10.9	1.7
^{4}He (228) + Al (162)	211 + 164	14.1	2.5

proposed experiment was estimated as follows:

$$h^{\mathrm{acc}} = h^{\mathrm{acc}}_{\mathrm{ref}} \times \frac{R_{e'} \times (R_K + R_p + R_\pi)}{R^{\mathrm{ref}}_{e'} \times (R^{\mathrm{ref}}_K + R^{\mathrm{ref}}_p + R^{\mathrm{ref}}_\pi)} \quad \text{(nb/sr) per MeV} \tag{8}$$

where $R_{e',K,p,\pi}$ are the expected rates (see Tables V and VIII) multiplied by the survival ratios after the KID by the Cherenkov and TOF detectors. $R^{\mathrm{ref}}_{e'} = 2.2$ MHz, and $R^{\mathrm{ref}}_{K,p,\pi}$ are the rates shown in Eq. (5). The assumed survival ratios by the Cherenkov selections are 0.8, 4.7×10^{-4}, 1.9×10^{-4} for K^+, π and p, respectively which was the performance in E05-115 [61]. The expected accidental coincidence rates are 1.0 and 1.4 counts/MeV/day for ^{3}He and ^{4}He targets including an event contamination from the target cell. We are planning to cut out events from the target cell by using the vertex z_t event selection. It is found that the expected accidental coincidence rates are 0.7 and 1.0 counts/MeV/day respectively for ^{3}He and ^{4}He targets assuming 4.6% of Al-cell events remain with the z_t event selection as shown in Sec. III C 2 (here a safety factor of two is used for the estimation; $4.6\% = 2.3\% \times f_{\mathrm{safety}}$ where $f_{\mathrm{safety}} = 2$). The expected rates (counts/MeV/day) of the accidental coincidence background without and with the z_t selection are summarized in the last two columns of Table VIII.

3. Statistical error on B_Λ

Figure 14 shows an expected B_Λ spectrum for $^3_\Lambda$H. The quasi-free Λ distribution was assumed to be a linear distribution for which the energy resolution of 1.1 MeV (FWHM) was taken into account as the Gauss distribution. The number of events in the quasi-free Λ relative to that in the bound region is assumed to be the same as reported in the past experiment [37]. The spectrum was fitted by a Gaussian function for the peak, and linear functions for the quasi-free Λ and accidental $e'K^+$ backgrounds. The expected statistical error was found to be $|\Delta B^{\mathrm{stat.}}_\Lambda| \simeq 20$ keV. If the first excited state ($3/2^+$) exists, the production cross section with the $(e, e'K^+)$ reaction is larger than that of the ground state by a factor of eight [54]. Figure 15 shows the expected spectrum

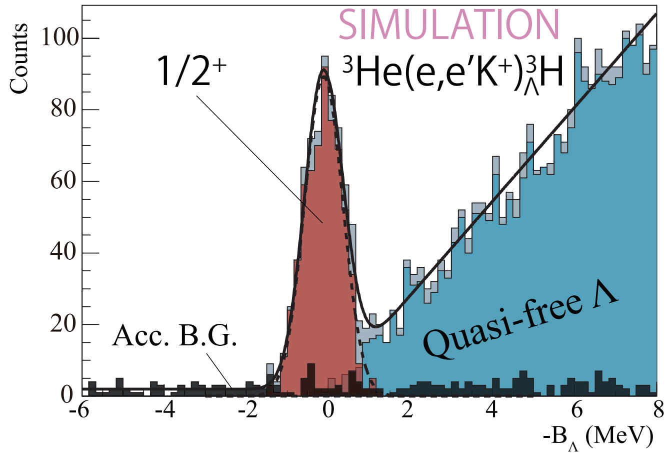

FIG. 14. Expected B_Λ spectrum for the ^{3}He$(e, e'K^+)^3_\Lambda$H reaction with a 50-μA beam for 10 days.

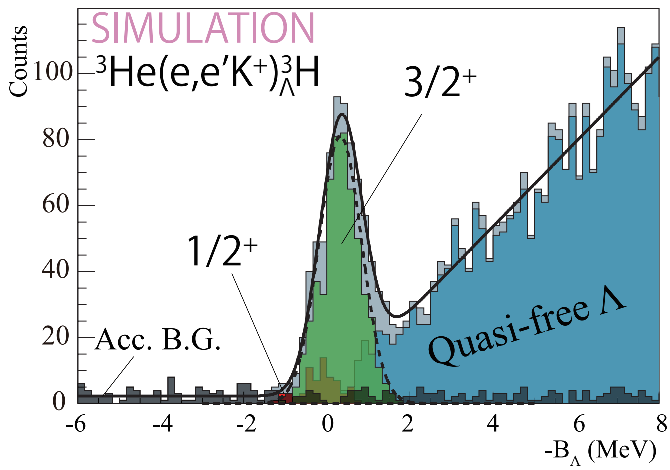

FIG. 15. Expected B_Λ spectrum for the ^{3}He$(e, e'K^+)^3_\Lambda$H reaction with a 50-μA beam for 10 days, assuming the $3/2^+$ does exist. The ratio of production cross section of the $3/2^+$ state to that of the ground state was assumed to be eight [54].

assuming the existence of the $3/2$ state. The Λ binding energy of the $3/2^+$ state would be observed for the first time with a statistical uncertainty of $|\Delta B_\Lambda^{\mathrm{stat.}}| \simeq 20$ keV instead of the ground state observation.

The B_Λ spectrum for the ^{4}He$(e, e'K^+)^4_\Lambda$H reaction was also estimated (Fig. 16) and $\Delta B_\Lambda^{\mathrm{stat.}}$ was evaluated in the same way as simulated for $^3_\Lambda$H. As a result, it was found that $|\Delta B_\Lambda^{\mathrm{stat.}}| \simeq 20$ keV for a measurement on the $^4_\Lambda$H(1^+) state in the proposed experiment.

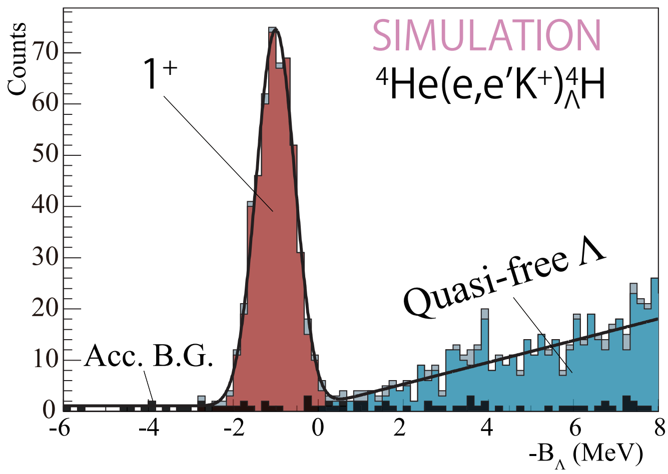

FIG. 16. An expected B_Λ spectrum for the ^{4}He$(e,e'K^+)^4_\Lambda$H reaction with a 50-μA beam for 2 days.

4. *Calibrations and expected B_Λ accuracy*

Various calibrations are needed in order to minimize the systematic error on B_Λ. There are five major calibrations and checks to be done as follows:

- **Momentum calibration by using elastic scattering**

  Elastic scattering data for momentum calibration of each spectrometer is planned to be taken in the approved experiment E12-15-008 for $^{40,48}_{\Lambda}$K spectroscopy, and these data can be shared for the present data analysis.

- **Angle calibration by using a sieve slit**

  The sieve slit data is planned to be taken with a solid target in E12-15-008. In addition (or instead), we need sieve slit data with the multi-foil target which will be used for the calibration of the angle measurement which has a dependency on production vertex z.

- **z reconstruction calibration by using the multi-foil target**

  In the present experiment, reconstructed information on the production vertex z will be used for the reconstructions of momentum vectors of particles by using backward transfer matrices. In order to calibrate the reconstructed z position, the multi-foil carbon target, which provides an absolute position reference in z, will be used.

- **Absolute energy scale calibration by using events of Λ and Σ^0**

  The absolute energy scale will be calibrated using the known Λ and Σ^0 masses. A polyethylene target will be used for Λ and Σ^0 production in E12-15-008. As shown above, we will use the reconstructed z in the a missing mass calculation. In order to confirm that the energy measurement is consistent for different z positions, data with Λ and Σ^0 production from hydrogen gas in a target cell larger than that for the gaseous ^{3,4}He targets will be used.

- **Checking the energy scale with different z positions by using $^{12}_{\Lambda}$B**
  The absolute energy scale will be guaranteed by using Λ and Σ^0 production as described above. In order to check the energy scale with a different mass region as well as z positions, we will check the Λ binding energy of $^{12}_{\Lambda}$B hypernuclei produced from different z position of the multi-foil target. This check will be important to minimize the systematic error on final results. The $^{12}_{\Lambda}$B hypernuclear production will also be used for momentum calibration if necessary as was done in the previous hypernuclear experiment at Hall C (E05-115) [1].

There are some factors that could contribute to the systematic error on B_{Λ}. The principal contributions are from the energy scale calibration and the correction of energy loss in the target materials. The systematic error that originates from the energy scale calibration by using Λ and Σ^0 events etc. is expected to be about ± 50 keV taking into account the statistics of the hyperon events and their mass uncertainties. In addition, the uncertainty of the energy loss correction for the the target materials is evaluated to be about ± 25 keV assuming the target cell thickness uniformity is $\sigma = 25$ μm. In total, the systematic error is expected to be $|\Delta B_{\Lambda}^{\rm sys.}| < 60$ keV. The total expected errors on B_{Λ} for $^{3,4}_{\Lambda}$H were then found to be $|\Delta B_{\Lambda}^{\rm total}| = 65$ keV which is the best accuracy in counting experiments [65].

IV. SUMMARY

We request 14.5 days of beamtime including calibration runs for the accurate spectroscopy of $^{3,4}{\rm He}(e, e'K^+)^{3,4}_{\Lambda}{\rm H}$ by using HRS-HKS system which is the same experimental configuration as that for the approved experiment E12-15-008 at Hall A. The present experiment aims to determine Λ binding energies of $^{3}_{\Lambda}$H and $^{4}_{\Lambda}$H with the total error of $|\Delta B_{\Lambda}^{\rm total}| = 65$ keV which is the best accuracy in counting experiments. The proposed experiment will give a strong constraint for the study of the ΛN interaction. Particularly, these data will be crucial to solve the puzzles of (a) the short lifetime and small binding energy of Λ hypertriton, and (b) CSB in the ΛN interaction.

---

[1] T. Gogami *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **900**, 69–83 (2018).

[2] L. Rezzolla, E.R. Most and L.R. Weih, *Astr. Jour. Lett.* **852**, L25 (2018).

[3] S.N. Nakamura *et al.* (HKS (JLab E01-011) Collaboration), *Phys. Rev. Lett.* **110**, 012502 (2013).

[4] T. Gogami *et al.* (HKS (JLab E05-115) Collaboration), *Phys. Rev. C* **94**, 021302(R) (2016).

[5] A. Esser *et al.* (A1 Collaboration), *Phys. Rev. Lett.* **114**, 232501 (2015).

[6] F. Schulz *et al.* (A1 Collaboration), *Nucl. Phys. A* **954**, 149 (2016).

[7] T.O. Yamamoto *et al.*, *Phys. Rev. Lett.* **115**, 222501 (2015).

[8] A. Gal, *Phys. Lett. B* **744**, 352 (2015).

[9] Extension of the J-PARC Hadron Experimental Facility – summary report –, `http://www.rcnp.osaka-u.ac.jp/~jparchua/en/index.html`

[10] C. Rappold *et al.*, *Phys. Rev. C* **88**, 041001(R) (2013).

[11] E. Hiyama *et al.*, *Phys. Rev. C* **89**, 061302(R) (2014).

[12] A. Gal and H. Garcilazo, *Phys. Lett. B* **736**, 93–97 (2014).

[13] L. Tang *et al.* (JLab Hypernuclear Collaboration), *Proposal to JLab PAC45*, E12-17-003, "*An isospin dependence study of the* Λ-*N interaction through the high precision spectroscopy of* Λ-*hypernuclei with electron beam*", 2016.

[14] T. Gogami *et al.*, *"Accurate* Λ *hypernuclear spectroscopy with electromagnetic probe at Jefferson Lab,"* AIP Conf. Proc. **2319**, 080019 (2021).

[15] S.N. Santiesteban *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **940**, 351-358 (2019).

[16] M. Juric *et al.*, *Nucl. Phys. B* **52**, 1–30 (1973).

[17] A. Cobis *et al.*, *J. Phys. G: Nucl. Part. Phys.* **23**, 401–421 (1997).

[18] G. Bohm *et al.*, *Nucl. Phys. B* **4**, 511–526 (1968).

[19] H. Kamada *et al.*, *Phys. Rev. C* **57**, 4 (1998).

[20] C. Rappold *et al.*, *Phys. Lett. B* **728**, 543–548 (2014).

[21] A. Gal and H. Garcilazo, *Phys. Lett. B* **791**, 48–53 (2019).

[22] The ALICE Collaboration, *Phys. Lett. B* **797**, 134905 (2019).

[23] T.R. Saito *et al.*, *Nucl. Phys. A 954*, 199–212 (2015).

[24] S. Nagao *et al.*, Letter of Intent submitted to ELPH, "A Direct Lifetime Measurement of The Lambda Hypertriton", 2860 (2016).

[25] M. Agnello *et al.*, *Nucl. Phys. A* **954**, 176–198 (2016).

[26] A. Feliciello *et al.*, J-PARC Proposal, P74 (2019).

[27] Y. Ma et al., J-PARC Proposal, P73 (2018).

[28] L. Tang *et al.* (HKS (JLab E05-115 and E01-011) Collaborations), *Phys. Rev. C* **90**, 034320 (2014).

[29] I.R. Afnan and B.F. Gibson, *Phys. Rev. C* **92**, 054608 (2015).

[30] H. Kamano et al., *EPJ Web of Conference* **113**, 07004 (2016).

[31] R.H. Dalitz and F. Von Hippel, *Phys. Lett.* **10**, 1 (1964).

[32] J.L. Friar and B.F. Gibson, *Phys. Rev. C* **18**, 908 (1978).

[33] A.R. Bodmer and Q.N. Usmani *Phys. Rev. C* **4**, 31 (1985).

[34] Y. Akaishi *et al.*, *Phys. Rev. Lett.* **84**, 16 (2000).

[35] H. Nemura *et al.*, *Phys. Rev. Lett.* **89**, 14 (2002).

[36] H. Tamura *et al.*, *JPS Conf. Proc. 17*, 011004 (2017).

[37] F. Dohrmann et al., *Phys. Rev. C* **76**, 054004 (2007).

[38] F. Dohrmann et al., *Phys. Rev. Lett.* **93**, 242501 (2004).

[39] The STAR Collaboration, *Nat. Phys.* (2020); `https://doi.org/10.1038/s41567-020-0799-7`.

[40] P. Liu, *Nucl. Phys. A* **982**, 811–814 (2019).

[41] T. Gogami *et al.* (HKS (JLab E05-115) Collaboration), *Phys. Rev. C* **93**, 034314 (2016).

[42] T. Cantwell *et al.*, *Nucl. Phys. A* **236**, 445 (1974).

[43] E. Botta, T. Bressani A. Feliciello, *Nucl., Phys.* **A** *960*, 165–179 (2017).

[44] M.M. Nagels, Th. Rijken, and Y. Yamamoto, *Phys. Rev. C* **99**, 04403 (2019).

[45] M. Isaka, Y. Yamamoto, and T. Motoba, *Phys. Rev. C* **101**, 024301 (2020).

[46] Y. Akaishi, "Week decay of ${}^{3}_{\Lambda}$H and ${}^{4}_{\Lambda}$H", 2018 JPS Annual (73th) Meeting, 22aK309-10 (2018).

[47] A. Pérez-Obiol *et al.*, *Phys. Lett. B* **811**, 135916 (2020).

[48] R.H. Dalitz and L. Liu, *Phys. Rev.* **116**, 5 (1959).

[49] D. Bertrand *et al.*, *Nucl. Phys.* **B 16**, 77–84 (1970).

[50] G. Alexander *et al.*, *Phys. Rev.* **173**, 5 (1968).

[51] H. Le et al., arXiv:1909.0288v1 (2019).

[52] J. Haidenbauer *et al.*, arXiv:1906.11681v1 (2019).

[53] Y. Fujiwara *et al.*, *Phys. Rev. C* **70**, 024001 (2004).

[54] T. Mart *et al*, *Nucl. Phys. A* **640**, 235-258 (1998); T. Mart and B.I.S. van der Ventel, *Phys. Rev. C* **78**, 014004 (2008).

[55] S.N. Nakamura *et al.* (JLab Hypernuclear Collaboration), *Proposal to JLab PAC44*, E12-15-008, "*An isospin dependence study of the* Λ*-N interaction through the high precision spectroscopy of* Λ*-hypernuclei with electron beam*", 2016.

[56] J. Alcorn *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **522**, 294–346 (2004).

[57] Y. Fujii *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **795**, 351–363 (2015).

[58] K. Katayama, *Master's Thesis*, *"Development of HRS-HKS trigger system with FPGA -High precision measurement of hypernuclei at JLab-"*, Kyoto University, Kyoto, Japan, 2021 (in Japanese).

[59] Y.S. Tsai, *Rev. Mod. Phys.* **46**, 815 (1974).

[60] J.W. Lightbody *et al.*, *Computers in Physics* **2**, 57 (1988).

[61] T. Gogami *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **729**, 816–824 (2013).

[62] G. Aida, *Master's Thesis*, "*Design of a septum magnet for the electro-production spectroscopy of* Λ *hypernuclei: to investigate the isospin dependence of the* Λ*NN interaction*", Tohoku University, Sendai, Japan, 2018 (in Japanese).

[63] T. Motoba, *JPS Conf. Proc.*, 011003 (2017).

[64] T. Gogami, *Doctoral Thesis* "*Spectroscopic research of hypernuclei up to medium-heavy mass region with the* $(e, e'K^+)$ *reaction*", Tohoku University, Sendai, Japan, 2014.

[65] T. Toyoda, *Master's Thesis*, *"Gas target basic design for the high accuracy mass measurement of hypertriton at JLab"*, Kyoto University, Kyoto, Japan, 2021 (in Japanese).