Study of a triaxially deformed nucleus
using a Lambda particle as a probe
A Letter of Intent

S. N. Nakamura (contact person),
F. Garibaldi, T. Gogami, P. Markowitz, S. Nagao, J. Reinhold, L. Tang, G. M. Urciuoli
on behalf of JLab Hypernuclear Collaboration
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Exective summary

This is a letter of intent of the first experiment to study triaxially deformed nuclei using a Λ particle as a probe. So far, triaxial deformed nuclei have been studied using γ-ray transitions, but the proposed method is based on a completely different experimental approach. A Λ particle in p-orbit of a hypernucleus has a wave function extending in a straight line, which produces shifts in energy levels depending on which axis of the triaxially deformed core nucleus this wave function is oriented. By measuring this difference using the high-resolution spectroscopy of ${}^{27}_{\Lambda}$Mg hypernucleus with the ^{27}Al $(e, e'K^+)^{27}_{\Lambda}$Mg reaction, it is possible to investigate triaxially deformed nuclei, ^{26}Mg, from the inside using a probe of a Λ particle. This aims to establish a completely new application of hypernuclei that has never been explored before.

Setup will be shared with the approved experiments, E12-15-008 and E12-20-013 with installation of HKS (K$^+$ central momentum $p_K = 1.200$ GeV/c), HES ($e', p_{e'} = 0.740$ GeV/c) and PCS in Hall-C. Electron beam of $E_e = 2.240$ GeV with a beam current of 50 μA and ^{27}Al target of 100 mg/cm^2 thick will be used. Calibration beamtime will be also shared with E12-15-008 and E12-20-013 and 672 hours (28 days) of beam time is requested.

1 Introduction

It is known that collective deformation plays important role in sd-shell nuclei. Though direct experimental evidence has not been found yet, triaxial deformation is expected to appear in the ground state of ^{24}Mg and theoretical works were developed in association with the shell gaps $N = Z = 12$ prolate and $N = Z = 14$ oblate region in Nilson diagram [1]. Recently, similar triaxial quadrupole deformation is predicted for ^{26}Mg [2]. While the deformation of ^{24}Mg has been discussed for decades, discussion of ^{26}Mg is not yet clarified since $Z = 12$ and $N = 14$ favor different shapes, prolate and oblate, for protons and neutrons respectively. Behavior of these triaxial deformed nuclei with a Λ was already discussed for ${}^{25}_{\Lambda}$Mg $= \Lambda +$ ^{24}Mg and ${}^{27}_{\Lambda}$Mg $= \Lambda +$ ^{26}Mg [3]. Added Λ makes potential energy surface slightly softer along the triaxial degree of freedom, but the energy surface of the core nuclei will be not changed and triaxially deformed nature will be kept for hypernuclei. So far, deformation property or collective motion of hypernuclei have not been studied well, it is an interesting new field of research.

The genuine hypernuclear states

Similar phenomenon, a Λ in p-shell coupled to deformed nuclei, was theoretically discussed for ${}^{9}_{\Lambda}$Be and ${}^{13}_{\Lambda}$C [4], and an experimental observation exists for ${}^{9}_{\Lambda}$Be [5].

The ^{8}Be nucleus plays a significant role in nuclear synthesis and it can be considered an unbound state composed of two α clusters. When a Λ particle is added to the unbound ^{8}Be nucleus, it becomes bound and forms a ${}^{9}_{\Lambda}$Be hypernucleus, thanks to the glue-like role of the Λ particle.

The ^{8}Be nucleus is prolate deformed and a Λ in p-shell has large overlap, thus deeper bound when Λ's orbit is in parallel to $\alpha - \alpha$ axis while it has less bound when Λ is in perpendicular to $\alpha - \alpha$

axis. Possibility of such states was originally discussed on the basis of the shell model [6] and the on the basis of the cluster model [8, 7] . Figure 1 shows calculated energy levels of ${}^{9}_{\Lambda}$Be for three cluster configurations of ^{8}Be $= \alpha + \alpha$ and Λ. The wave function of a Λ particle in the *s*-orbit is spherically symmetric, so its energy does not depend on the orientation of the $\alpha - \alpha$ cluster axis. However, the wave function of the *p*-orbit is linear, so the binding energy of the Λ particle depends on whether its wave function is parallel ($p^{\Lambda}_{\|}$) or perpendicular ($p^{\Lambda}_{\perp}$) to the axis of the $\alpha - \alpha$ cluster. Since Λ-nucleon interaction is attractive, the larger overlap between the Λ and $\alpha - \alpha$ wave functions ($p^{\Lambda}_{\|}$) results in deeper binding.

This ($p^{\Lambda}_{\|}$) state is referred to as a genuine hypernuclear (or super-symmetric) state. It is termed as such because a normal nucleon cannot occupy this orbital due to Pauli exclusion principle, whereas a Λ particle can, as it is exempt from the Pauli exclusion imposed by nucleons. The $p^{\Lambda}_{\perp}$ state does not have overlap of Λ and nucleons and thus this is an analog state of ^{9}Be which is $\alpha - \alpha - n$.

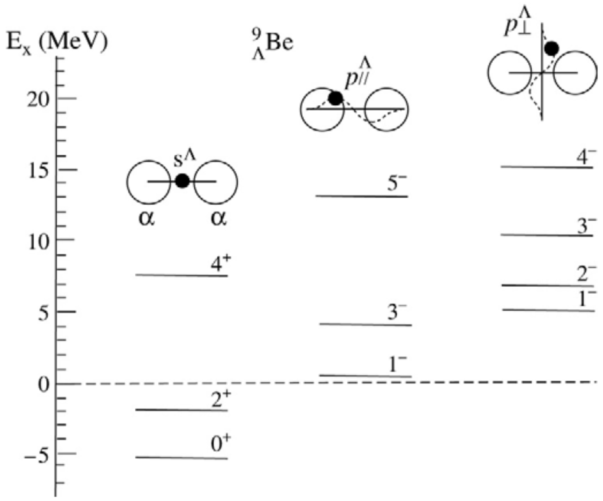

Figure 1: Theoretical calculated energy levels of ${}^{9}_{\Lambda}$Be. Three cluster configurations for ^{8}Be $= \alpha + \alpha$ and Λ are shown [9].

An initial experimental indication of these states was observed at BNL-AGS [10] and confirmed at KEK-PS through the ^{9}Be$(\pi^{+}, K^{+})^{9}_{\Lambda}$Be reaction [5]. Figure 2 shows the experimental result at KEK-PS.

Peaks of #1 and #2 correspond to a Λ in *s*-orbit coupled to ^{8}Be(0^{+}) and ^{8}Be(2^{+}) states. Peaks #3 and #4 were interpreted as the *genuine hypernuclear* states, the Λ in $p^{\Lambda}_{\|}$ coupled to ^{8}Be(1^{-}) and ^{8}Be(3^{-}). Further detailed analysis of *genuine hypernuclear* states was hindered by the limited energy resolution of the (π^{+}, K^{+}) reaction spectroscopy.

Investigation of triaxially deformed nuclei by a Λ probe

This characteristic of Λ hypernuclei can be utilized for the study of triaxially deformed nuclei. As depicted in Figure 3, the binding energy of a Λ in the *p*-orbit depends on the orientation of its wave function along the symmetry axes of a triaxially deformed nucleus. Theoretical investigations have been conducted for ${}^{25}_{\Lambda}$Mg [11] and ${}^{27}_{\Lambda}$Mg [12, 13], revealing the existence of three rotational bands coupled with the Λ in the *p*-orbit.

Figure 4 shows the calculated energy levels for ${}^{27}_{\Lambda}$Mg using the Anti-symmetric Molecular Dynamics with an extension to include a hyperon (Hyper-AMD). In the case that the ${}^{26}_{\Lambda}$Mg core nucleus exhibits triaxial deformation, distinct three bands corresponding to different orientations of Λ's wave functions in the *p*-orbit will emerge. By observing this characteristic feature, the triaxially deformed nature of the

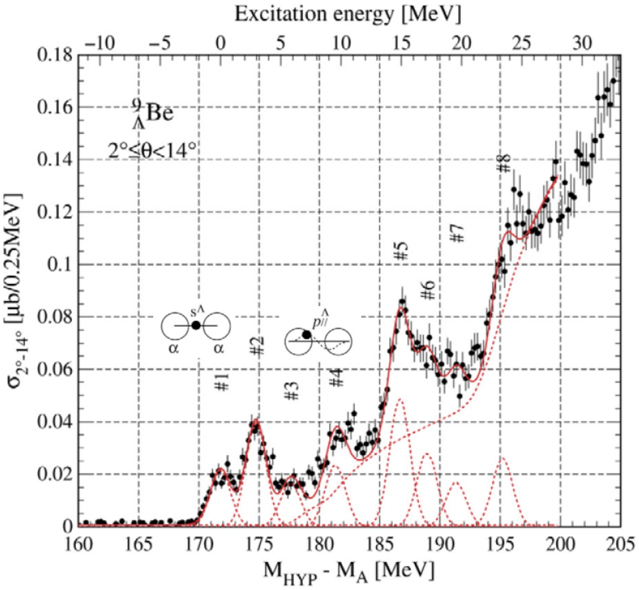

Figure 2: Energy spectrum obtained by the (π^+, K^+) reaction at KEK-PS [5]

^{26}Mg nucleus can be convincingly confirmed. Precise spectroscopy of $^{27}_{\Lambda}$Mg with sub-MeV resolution using the $(e, e'K^+)$ reaction, which is only possible at JLab, would provide valuable insights into the triaxial deformation of ^{26}Mg and reveal characteristic features of hypernuclear states.

2 Experimental setup

The proposed experiment aims to obtain high precision mass spectroscopy of $^{27}_{\Lambda}$Mg hypernuclei produced by the $(e, e'K^+)$ reaction to study triaxial deformation of the core nucleus, ^{26}Mg by the Λ probe. We will employ a newly constructed charge separation magnets (PCS), the High resolution Kaon Spectrometer (HKS) and the High resolution Electron Spectrometer (HES) in Hall-C, as schematically illustrated in Figs. 5 and 6. The same setup will be shared with the campaign of hypernuclear spectroscopic experiments: E12-15-008 (currently on the jeopardy list, to be re-approved), E12-20-013 (spectroscopic study of $^{208}_{\Lambda}$Tl), new letter of intents (spectroscopy of light hypernuclei: $^{6}_{\Lambda}$He, $^{9}_{\Lambda}$Li, $^{11}_{\Lambda}$Be; decay pion spectroscopy of electroproduced hypernuclei). It is worth noting that calibration data can be shared with other experiments in the hypernuclear study campaign conducted in Hall-C.

The PCS magnets separate the scattered electrons and electro-produced kaons at small forward angles to sufficiently large spectrometer angles, while allowing the post-beam to be directly transported to the dump. It also minimizes the chance for the high rate backgrounds (electrons and positrons) at near zero degrees to enter either of the two spectrometers. The collaboration has demonstrated the technique successful in avoiding the background from e' and K$^+$ accidental coincidences by maintaining sufficiently low singles rates at each of the two spectrometers under high luminosity conditions. The PCS magnets were designed and constructed in Japan and successfully shipped to JLab in 2022.

The HKS spectrometer that was successfully used in the previous Hall C experiments will be used as the kaon spectrometer. It features both a momentum resolution $(\Delta p/p)$ of a few 10^{-4} (FWHM). It was already proven in the previous hypernuclear experiments in Hall-C that the excellent detector system (triple layers of silica aerogel Čerenkov counters and double layers of water Čerenkov counters) cleanly identified kaons with the separation powers of 4.7×10^{-4} and 1.9×10^{-4} for 1.2 GeV/c π^+ and p, respectively[14].

For the scattered electron spectrometer, the HES which was also used successfully in the previous

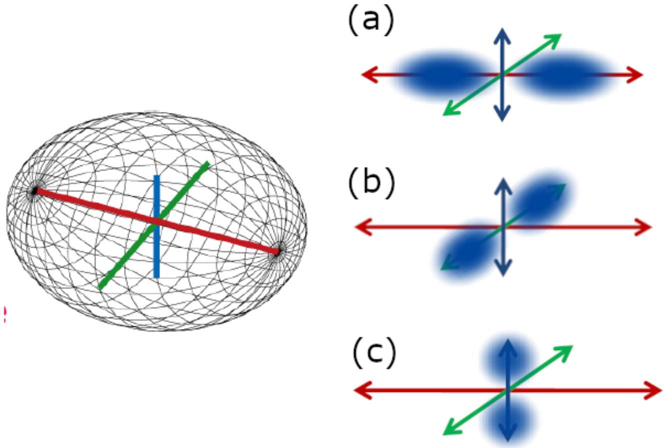

Figure 3: Conceptual picture when a Λ in a p-orbit overlaps three different symmetry axes of a triaxially deformed nucleus. (a) the wave function of Λ is parallel to the long axis of triaxially deformed core nucleus, which results in deep binding. (b) parallel to the middle axis, medium binding, (c) parallel to the short axis, shallow binding [12].

Table 1: Kinematical parameters for the hypernuclear experiments.

Beam	Energy E_e [/(GeV)]	2.240
	Energy stability $\Delta E_e/E_e$	3×10^{-5}
PCS + HES	Central momentum P_e [/(GeV/c)]	0.744
	Central angle $\theta_{e,e'}$ [/(deg)]	8
	Solid angle $\Delta\Omega_{e'}$ [/(msr)]	3.4
	Momentum resolution $\Delta P_{e'}/P_{e'}$	4.4×10^{-4}
PCS + HKS	Central momentum P_K [/(GeV/c)]	1.200
	Central angle θ_K [/(deg)]	15
	Solid angle $\Delta\Omega_K$ [/(msr)]	8.3
	Momentum resolution $\Delta P_K/P_K$	2.9×10^{-4}

Hall-C hypernuclear experiment (E05-115), will be used again. Combination of HKS-HES achieved the excellent energy resolution of 0.54 MeV (FWHM) for the ${}^{12}_{\Lambda}$B missing mass spectrum [15].

The HKS and HES spectrometers have already been used in Hall-C and the newly introduced PCS magnets have exported to the United States, and are currently stored at JLab after the excitation test in Japan. A single scatering chamber with the target holder will be used for all the planned solid targets, including those used for calibrations and the entire system is vacuum connected. The development of the target chamber with the target holder is required. Discussions have begun with the experts of the JLab target group and engineers and no technical issues was found.

3 Expected performance

Performance was evaluated by simulations based on a framework of GEANT4. We already have well established models of HKS, HES and we constructed new models of PCS magnets for this study [16] and results were given in the update documents for E12-15-008 on the jeopardy list [17].

Kinematical parameters for the next campaign of the hypernuclear experiments in Hall-C were summarized in Tab. 1.

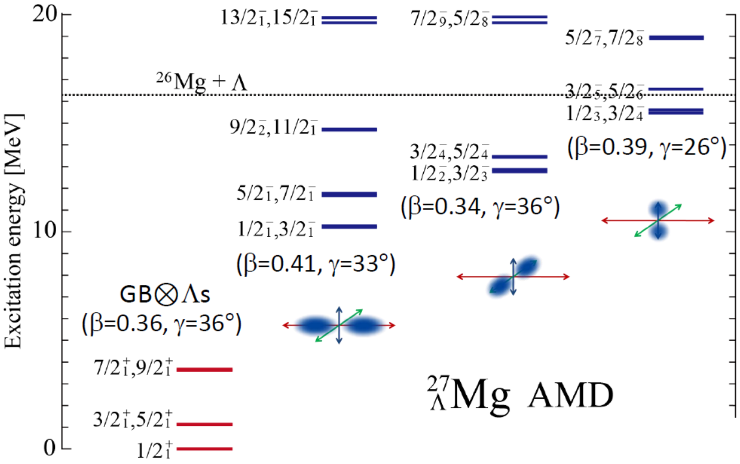

Figure 4: Theoretical expectation of energy levels of ${}^{27}_{\Lambda}$Mg hypernucleus [12, 13]

Table 2: Expected yield and request of beamtime.

Target (Hyper Nucleus)	Beam current (μA)	Target thickness (mg/cm^2)	Assumed cross section (nb/sr)	Expected yield (/h)	Num. of events	Req. beamtime (hours)	B.G. rate (/MeV/h)	S/N	Comments
CH$_2$ (Λ, Σ^0)	2	500	1000	8.62	1000	120	0.03	290	Calibration
^{12}C (${}^{12}_{\Lambda}$B)	50	150	90	6.79	1100	168	1.20	5.67	Calibration
Subtotal						—			Combined with E12-15-008
^{27}Al (${}^{27}_{\Lambda}$Mg)	50	100	10 (p^{Λ})	0.22	150	672	0.78	0.19	Physics
Total						672			

4 Expected yield of hypernuclei and beamtime request

So far, there is no theoretical prediction which gives simultaniously production cross section of ${}^{27}_{\Lambda}$Mg and the excitation energies of Λ in p-orbits for the triaxially deformed ^{26}Mg core nucleus.

Therefore excitation energies of ${}^{27}_{\Lambda}$Mg were estimated by the hyper-AMD calculation as shown in Fig. 7a [12, 13] and production cross section of 10 nb/sr were assumed for the ground state and Λ in each p-orbit which were roughly consistent with the recent shell model prediction.

Figure 7b is an example of shell model prediction [18] and structure of Λ in p-orbits are simpler than Fig. 7a. Analysis of this calculation for the deformation of core nucleus is still under study but we can see the effect of core deformation looks apparently smaller. Result of high precision spectroscopy experiment can answer whether ^{26}Mg is triaxially deformed or not. Even if observed structure in the region of Λ in p-orbits is complicated, high resolution of 0.56 MeV (FWHM) enables us to analyze the data with the above theoretical models varying deformation parameters to extract information of ^{26}Mg core shape.

We will use 100 mg/cm^2 thick of ^{27}Al target to have better resolution than 150 mg/cm^2. We can accumulate 150 events for the state of 10 nb/sr production cross section with the beam time of 672 hours. It gives statistical error of $\sigma < 45$ keV (FWHM 100 keV).

Targets to be used in this experiment, expected yield of hypernuclei and requested beam time are summarized in Table 2.

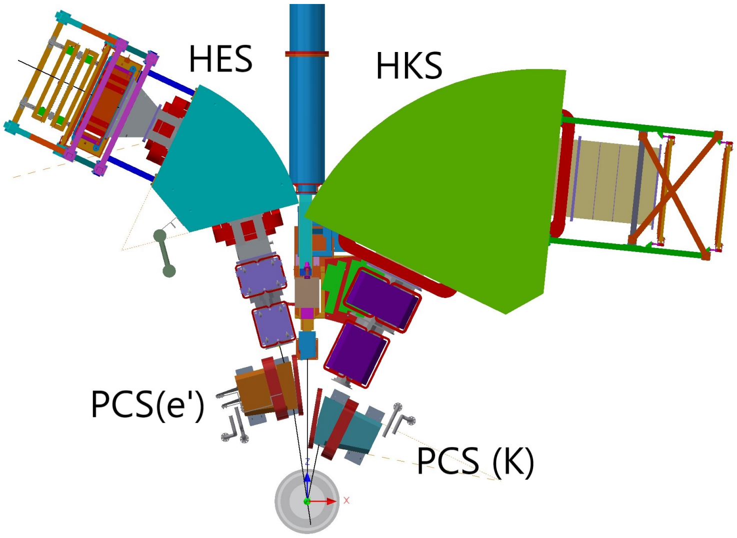

Figure 5: Assumed experimental setup is PCS + HES + HKS. A pair of charge-separation dipole-magnets PCS is installed between a scattering chamber, which encloses the target system, and the spectrometers (HES and HKS).

5 Summary

We proposed the first experiment to study triaxially deformed nuclei using a Λ particle as a probe. By measuring $^{27}\mathrm{Al}(e,e'K^+)^{27}_{\Lambda}\mathrm{Mg}$ reaction with 672 hours of beamtime in Hall-C, we can clearify if the core $^{26}\mathrm{Mg}$ is triaxially deformed or not. Even if observed structure of measured $^{27}_{\Lambda}\mathrm{Mg}$ spectrum in the region of Λ in p-orbits is more complicated than theoretical expectations, high resolution of ~ 0.6 MeV (FWHM) enables us to fit the data with the theoretical models varying deformation parameters to extract information of $^{26}\mathrm{Mg}$ core shape. This is a completely new experimental technique that utilizes the wave function of the Λ particle in the p-orbit, opening up new possibilities for studying hypernuclei.

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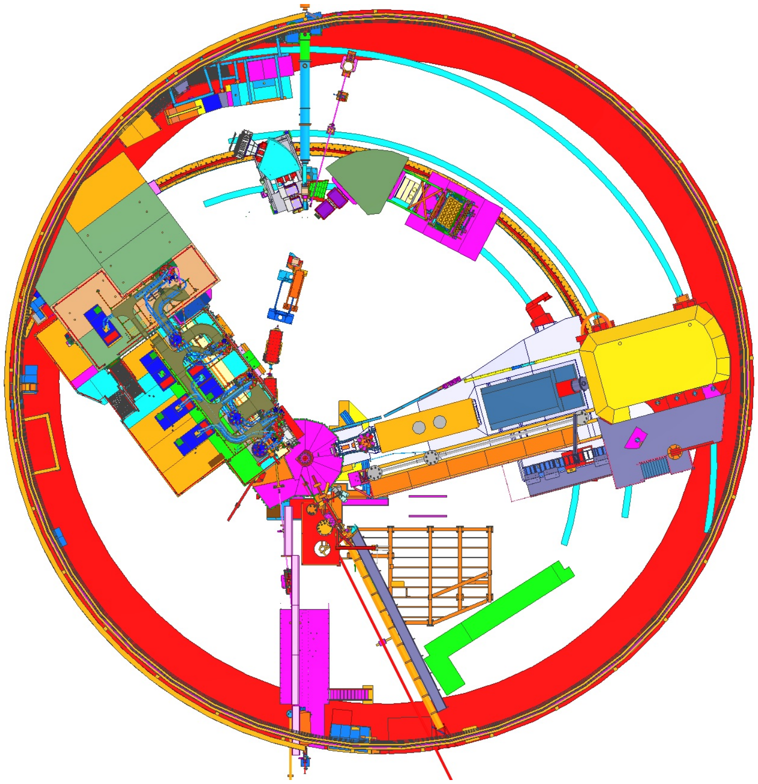

Figure 6: An idea of floor plan of experimental setup in Hall-C. HKS and HES can be fit in Hall-C without physical confliction with the existing spectrometers. A part of beamline and PCS are not drawn in this figure.

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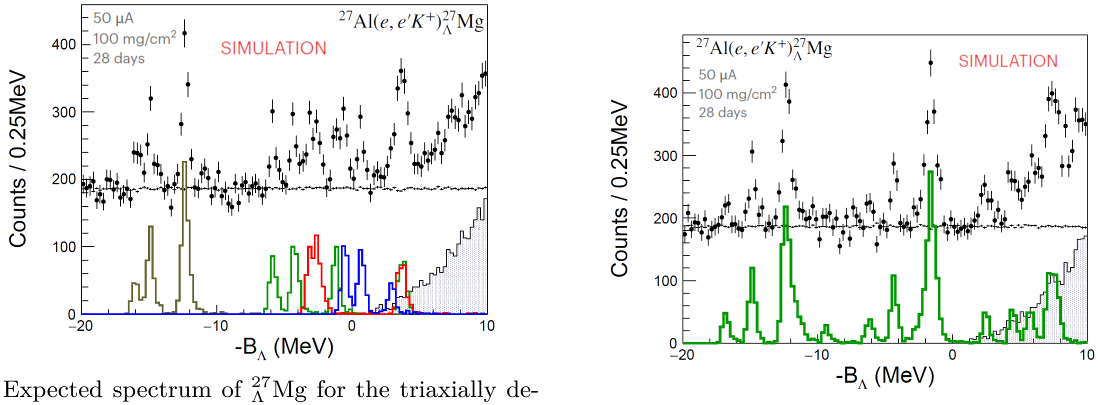

(a) Expected spectrum of $^{27}_{\Lambda}$Mg for the triaxially deformed core ^{26}Mg nucleus based on hyper-AMD calculation [12, 13].

(b) Expected spectrum of $^{27}_{\Lambda}$Mg based on recent shell model calculation [18].

Figure 7: Expected Λ binding energy spectra of $^{27}_{\Lambda}$Mg