# Light Meson Radiative Decays

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• Radiative transitions have proved their value in the baryon sector, successfully reproducing the magnitudes and relative phases of over 100 helicity amplitudes for photoexcitation of the proton and neutron.

• Calculations of light meson radiative decays have concentrated mainly on ground-state to ground-state decays.

• New and proposed facilities (ISR at BABAR & BELLE, CLEO-C, JLab, Novosibirsk) promise greatly increased statistics and open up the possibility of studying radiative decays of excited light mesons.

Several key radiative widths are found to be large, $\gtrsim$ 500 keV, and offer strong discriminatory power. We shall concentrate on three aspects:

• discrimination between the radial ($2^3S_1$) and orbital ($1^3D_1$) excitations of the $\rho$

• discrimination between these and the $J^{PC} = 1^{--}$ hybrid

• discrimination among different $q\bar{q}$ and glueball mixing scenarios in the scalars

## The Model

Wave functions are taken as Gaussian, that is of the form $\exp(-p^2/\beta^2)$ multiplied by the appropriate polynomial, and the parameter $\beta$ found for each of the $1S$, $1P$, $2S$, $1D$ states by treating it as the variational parameter in the Hamiltonian

$$H = \frac{p^2}{m_q} + \sigma r - \frac{4}{3}\frac{\alpha_s}{r} + C$$

with standard quark-model parameters:
$m_{u,d} = 0.33$ GeV, $m_s = 0.45$ GeV, $\sigma = 0.18$ GeV$^2$, $\alpha_s = 0.5$

The decay at rest of the meson $A$ to the meson $B$ and a photon with three-momentum $\mathbf{p}$ has the form

$$\mathbf{M}_{A\to B} = \mathbf{M}^q_{A\to B} + \mathbf{M}^{\bar{q}}_{A\to B}.$$

with

$$\begin{aligned}\mathbf{M}^q_{A\to B} &= \frac{I_q}{2m_q}\int d^3k\big[Tr\{\phi^\dagger_B(k-\tfrac{1}{2}p)\phi_A(k)\}(2\mathbf{k}-\mathbf{p}) \\ &\qquad - iTr\{\phi^\dagger_B(\mathbf{k}-\tfrac{1}{2}\mathbf{p})\sigma\phi_A(\mathbf{k})\}\times\mathbf{p}\big]\end{aligned}$$

and

$$\begin{aligned}\mathbf{M}^{\bar{q}}_{A\to B} &= \frac{I_{\bar{q}}}{2m_q}\int d^3k\big[Tr\{\phi_A(\mathbf{k})\phi^\dagger_B(\mathbf{k}+\tfrac{1}{2}\mathbf{p})\}(2\mathbf{k}+\mathbf{p}) \\ &\qquad - iTr\{\phi_A(\mathbf{k})\sigma\phi^\dagger_B(\mathbf{k}+\tfrac{1}{2}\mathbf{p})\}\times\mathbf{p}\big]\end{aligned}$$

where $I_q$ and $I_{\bar{q}}$ are isospin factors and $m_q$ is the quark mass.

The differential decay rate is then given by

$$\frac{d\Gamma}{d\cos\theta} = 4p\frac{E_B}{m_A}\alpha I \sum |M_{A\to B}|^2$$

where the sum is over final-state polarisations and $I = I_q^2 = I_{\bar{q}}^2$ is the isospin factor.

The pure electric-dipole ($E1$) transition is well-defined for heavy quarks, but is certainly a bad approximation for light quarks so we include the magnetic quadrupole ($M2$) transition as well.

This approach has a long history of success in the baryon sector even though the $M2$ terms are the same order in $p^2$ as $E1$ corrections such as:
- anomalous magnetic moments of the constituents
- spin-orbit terms
- Thomas precession
- binding effects

The success in the baryon sector suggests that the collective effect of these corrections is small.

Within this "leading multipole" hypothesis there are checks on our procedures.

• Ground-state to ground-state transitions, e.g. $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, given correctly by the model.

• The predicted width for $\Gamma(f_1(1285) \to \gamma\rho)$ is in good accord with experiment.

• The prediction that $\Gamma(f_2(1270) \to \gamma\rho) \lesssim 0.5\Gamma(f_1(1285) \to \gamma\rho)$ is in qualitative accord with experiment as there is no evidence for the radiative decay of $f_2(1270)$ in either the MARK III or WA102 experiments and both have strong $f_2$ signals.

• From general considerations we can form a positivity constraint among a combination of widths which is satisfied by our explicit model and enables us to draw a more general conclusion, namely that $\Gamma(f_0 \to \gamma\rho) \sim \Gamma(f_1 \to \gamma\rho)$ in agreement with our calculation.

## A Hidden Hybrid?

• The data on $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ and $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$, excluding $\omega\pi$, are completely consistent with $e^+e^- \to a_1\pi$ up to 1.65 GeV and the cross section is large (CLEO-$\tau$, Novosibirsk).

• The data on $e^+e^- \to \pi^+\pi^-$ require a $\rho'$ state ($2\ ^3S_1$) at about 1.45 GeV and there is additional structure at higher mass (CLEO-$\tau$, Novosibirsk, $e^+e^-$).

• The data on $e^+e^- \to \omega\pi$ are consistent with the tail of the $\rho$ plus the ($2\ ^3S_1$) (CLEO-$\tau$, CLEO-B, Novosibirsk, $e^+e^-$).

• There is no strong $\omega\pi$ signal above the $\rho'(1450)$ (CLEO-B, $e^+e^-$).

• The $^3P_0$ model cannot explain the large $4\pi$ cross section, excluding $\omega\pi$:

$$\Gamma(\rho_{2S} \to a_1\pi \to 4\pi) \sim 3 \text{ MeV}$$

$$\Gamma(\rho_{2S} \to h_1\pi \to 4\pi) \sim 1 \text{ MeV}$$

$$\Gamma(\rho_{2S} \to \omega\pi) \sim 115 \text{ MeV}$$

$$\Gamma(\rho_{2S} \to \pi\pi) \sim 68 \text{ MeV}$$

• The cross section for direct’ production of $a_1\pi$ via $\rho$ dominance is also very small.

• One solution is to invoke a vector hybrid as its dominant decay mode is $a_1\pi$.

## But

• The hybrid has no direct $e^+e^-$ coupling. It must be induced by mixing. To get a large $4\pi$ cross section the mixing must be maximal with the $\rho'(1450)$. This then implies a large cross section for $e^+e^- \to \omega\pi$ above the $\rho'(1450)$ which is not observed!

## However

• Pham, Roiesnel & Truong (1978) and Penso & Truong (1980) argued that in the special case of $a_1\pi$ it is incorrect to use naive $\rho$ dominance and that the axial current matrix element (which is dominated by the $a_1$) should be used. This gives a large (non-resonant) cross section for $e^+e^- \to a_1\pi$.

• The $1\,^3D_1$ state (the $\rho'(1700)$) has a large $4\pi$ decay width:

$$\Gamma(\rho_{1D} \to a_1\pi \to 4\pi) \sim 104 \text{ MeV}$$

$$\Gamma(\rho_{1D} \to h_1\pi \to 4\pi) \sim 105 \text{ MeV}$$

• The (rather strong) direct $a_1\pi$ can interfere with the (comparatively weak) $1\,^3D_1$ state boosting it significantly.

## So goodbye hybrid?

• Not necessarily. There will be some mixing between the hybrid and the $1\,^3D_1$ and the direct $a_1\pi$ can boost the mixed states.

## Vector Meson Radiative Decays

• Radiative decays can distinguish between the $2\,^3S_1$ and the $1\,^3D_1$. For example:

- $\Gamma(\rho(1450) \to f_2(1270)\gamma) \sim 700$ keV

  $\Gamma(\rho(1700) \to f_2(1270)\gamma) \sim 140$ keV

  $\Gamma(\rho(1450) \to f_1(1285)\gamma) \sim 350$ keV

- $\Gamma(\rho(1700) \to f_1(1285)\gamma) \sim 1100$ keV

• Radiative decays can separate cleanly the $\phi(1690)$. For example the $f_2(1525)\gamma$ decay of the $\phi(1690)$ provides a unique signature for the $s\bar{s}$ state, albeit with a smallish width:

$\Gamma(\phi(1690) \to f_2(1525)\gamma) \sim 200$ keV

• Radiative decays can resolve the issue of the $J^{PC} = 1^{--}$ hybrid, $\rho_H$. As the $q\bar{q}$ pair in the hybrid is in a spin-singlet state, radiative decays to the spin-triplet $f_2(1270)$ and $f_1(1285)$ will be suppressed. The dominant radiative decay should be to the spin-singlet $b_1(1235)$, which is suppressed for the spin-triplet $\rho(1450)$ and $\rho(1700)$. Specific calculation (Close and Dudek) gives

- $\Gamma(\rho_H(1700) \to b_1(1235)\gamma) \sim 700$ keV

• Vector meson radiative decays can also tell us something about the scalar glueball.

• There are three scalar state when, if we only have $q\bar{q}$ states, there should be two.

• If there is no mixing among the scalars so that the $f_0(1370)$ is pure $n\bar{n}$ and the $f_0(1710)$ is pure $s\bar{s}$ then

$$\Gamma(\rho(1700) \to f_0(1370)\gamma) \sim 900 \text{ keV}$$

$$\Gamma(\rho(1450) \to f_0(1370)\gamma) \sim 65 \text{ keV}$$

$$\Gamma(\phi(1900) \to f_0(1710)\gamma) \sim 190 \text{ keV}$$

• The result of mixing is that the bare $n\bar{n}$ and $s\bar{s}$ states contribute in varying degrees to each of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$

- Three different mixing scenarios have been proposed:

  the bare glueball is lighter than the bare $n\bar{n}$ state

  the bare glueball lies between the bare $n\bar{n}$ state and the bare $s\bar{s}$ state

  the bare glueball is heavier than the bare $s\bar{s}$ state

- Each one affects the radiative decays in a unique way:

| | $\rho(1700)$ | | | $\phi(1900)$ | | |
|---|---|---|---|---|---|---|
| | L | M | H | L | M | H |
| $f_0(1370)$ | 174 | 440 | 603 | 7 | 8 | 31 |
| $f_0(1500)$ | 520 | 301 | 98 | 5 | 35 | 261 |
| $f_0(1710)$ | | | | 173 | 156 | 17 |

- So the relative rates of the radiative decays of the $\rho(1700)$ to $f_0(1370)$ and $f_0(1500)$, and of the $\phi(1900)$ to $f_0(1500)$ and $f_0(1710)$ change radically according to the particular model for $q\bar{q}$-glueball mixing.
- An important check on this phenomenology is provided by the decay $\omega(1650) \to a_0(1450)\gamma$, predicted width $\sim 610$ keV.

## Scalar Meson Radiative Decays

• A complementary approach to flavour-filtering among the scalars is provided by the radiative decays of the scalars to the ground-state vectors $\rho$, $\omega$ and $\phi$.

| | $\rho(770)$ | | | $\phi(1020)$ | | |
|---|---|---|---|---|---|---|
| | L | M | H | L | M | H |
| $f_0(1370)$ | 443 | 1121 | 1540 | 8 | 9 | 32 |
| $f_0(1500)$ | 2519 | 1458 | 476 | 9 | 60 | 454 |
| $f_0(1710)$ | 42 | 94 | 705 | 800 | 718 | 78 |

• The width of the decay $f_1(1285) \to \rho\gamma$ is measured and provides a good check on the model: $1320 \pm 312$ keV compared to a predicted value of $\sim 1400$ keV.

• The predicted width for the decay $f_2(1270) \to \rho\gamma$ is $\sim 640$ keV. Experimentally this width is small as neither MARK III nor WA102 see it although both have a large $f_2(1270)$ signal.

• The branching fractions for radiative decay of $J/\psi$ to $f_1(1285)$ and $f_2(1270)$ are comparable at $(6.1 \pm 0.9) \times 10^{-4}$ and $(1.30 \pm 0.14) \times 10^{-3}$, so the non-observation of any $f_2(1270)$ signal in the decay $J/\psi \to \gamma(\gamma\rho)$ is meaningful.

• A similar situation holds in central production in high-energy proton-proton interactions and one an deduce an upper limit on $\Gamma(f_2(1270) \to \rho\gamma)$ of 500 keV at 95% confidence level.

• A further check on the phenomenology would be provided by the decay $a_0(1450) \to \omega\gamma$, predicted width $\sim 2100$ keV.

## Single quark transitions

• Independent of details of binding dynamics it is possible to obtain relations among helicity amplitudes, and hence widths, that depend only on the assumption that the mesons are $q\bar{q}$ $P$ and $S$ states. The width for the decays $V \to \gamma f_J$ is

$$\Gamma(V \to \gamma f_J) = 2p\alpha \frac{E_B}{m_A} I \frac{1}{3} \sum_\lambda |A_\lambda|^2$$

and for the decays $f_J \to \gamma V$ is

$$\Gamma(f_J \to \gamma V) = 2p\alpha \frac{E_B}{m_A} I \frac{1}{2J+1} \sum_\lambda |A_\lambda|^2$$

where $A_\lambda$ are the helicity amplitudes.

• In terms of electric dipole and magnetic quadrupole transitions

$$A_0 = \sqrt{2}(E_1 + 2E_R)$$

for ${}^3S_1 \to {}^3P_0$ transitions,

$$A_0 = \sqrt{3}(E_1 + E_R + M) \quad A_1 = \sqrt{3}(E_1 + E_R - M)$$

for ${}^3S_1 \to {}^3P_1$ transitions and

$$\begin{aligned} A_0 &= (E_1 - E_R + 3M) \quad A_1 = \sqrt{3}(E_1 - E_R + M) \\ A_2 &= \sqrt{6}(E_1 - E_R - M) \end{aligned}$$

for ${}^3S_1 \to {}^3P_1$ transitions.

• Consider the decays $V \to \gamma f_J$. For equal phase space and equal form factors the $|M|^2$ term and the cross terms between $E_1$ and $E_R$ can be eliminated and a combination formed proportional to $|E_0|^2 + |E_R|^2 \geq 0$. The resulting inequality is

$$\Gamma(\rho(2S) \to \gamma f_2) + 7\Gamma(\rho(2S) \to \gamma f_0) \geq 3\Gamma(\rho(2S) \to \gamma f_1)$$

Similarly for the decays of pure $n\bar{n}$ states

$$5\Gamma(f_2 \to \gamma\rho) + 7\Gamma(f_0 \to \gamma\rho) \geq 9\Gamma(f_1 \to \gamma\rho)$$

• As $\Gamma(f_1 \to \gamma\rho) \sim 1300$ keV, this latter equation requires that one or other of $f_0$, $f_2$ must have a radiative width $\sim 1000$ keV. As there is no evidence for the $f_2$ decay it follows that $f_0 \to \gamma\rho$ should be large, in line with our specific calculations.

## Summary

• In general, radiative decays are a better probe of meson structure than hadronic decays as the coupling to the charges and spins of constituents gives detailed information on wave functions.

• Specifically, light-quark radiative decays provide a strong discriminatory mechanism and act as a good flavour filter.

Discrimination: $\rho(1450)$, $\rho(1700)$, $\rho_H(1700)$.

Flavour filter: glueball mixing in the scalars $f_0(1370)$, $f_0(1500)$, $f_0(1710)$.

• Specific and general checks of the model are in good agreement with experiment.

• New and proposed facilities promise greatly increased statistics and will allow these decays to be measured.

• Some of the radiative decays may be detectable in present experiments:

E852 sees $\omega(1650) \to \omega\eta$

– $^3P_0$ model predicts $\Gamma(\omega(1650) \to \omega\eta) \sim 13$ MeV

– $\Gamma(\omega(1650) \to a_1(1260)\gamma) \sim 1000$ keV $\sim 8\%$ of $\omega\eta$ width

VES sees $\rho(1450) \to \rho\eta$ and $\rho(1700) \to \rho\eta$

– $^3P_0$ model $\Gamma(\rho(1450) \to \rho\eta) \sim \Gamma(\rho(1700) \to \rho\eta) \sim 25$ MeV

– $\Gamma(\rho(1450) \to f_2(1270)\gamma) \sim 700$ keV

– $\Gamma(\rho(1700) \to f_1(1285)\gamma) \sim 1100$ keV