Selection Rules for J^{PC} Exotic Hybrid Meson Decay in Large-N_c

Philip Page

Los Alamos National Laboratory

Workshop on Gluonic Excitations

Jefferson Lab

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Levinson ... NC 1964
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1^{-+} hybrid $\rightarrow \eta\pi, \eta'\pi$

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Formulate in field theory

Iddir... PLB 1988
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1^{-+} hybrid c. $\rightarrow \eta\pi$

Decay amplitudes to lowest two-body threshold

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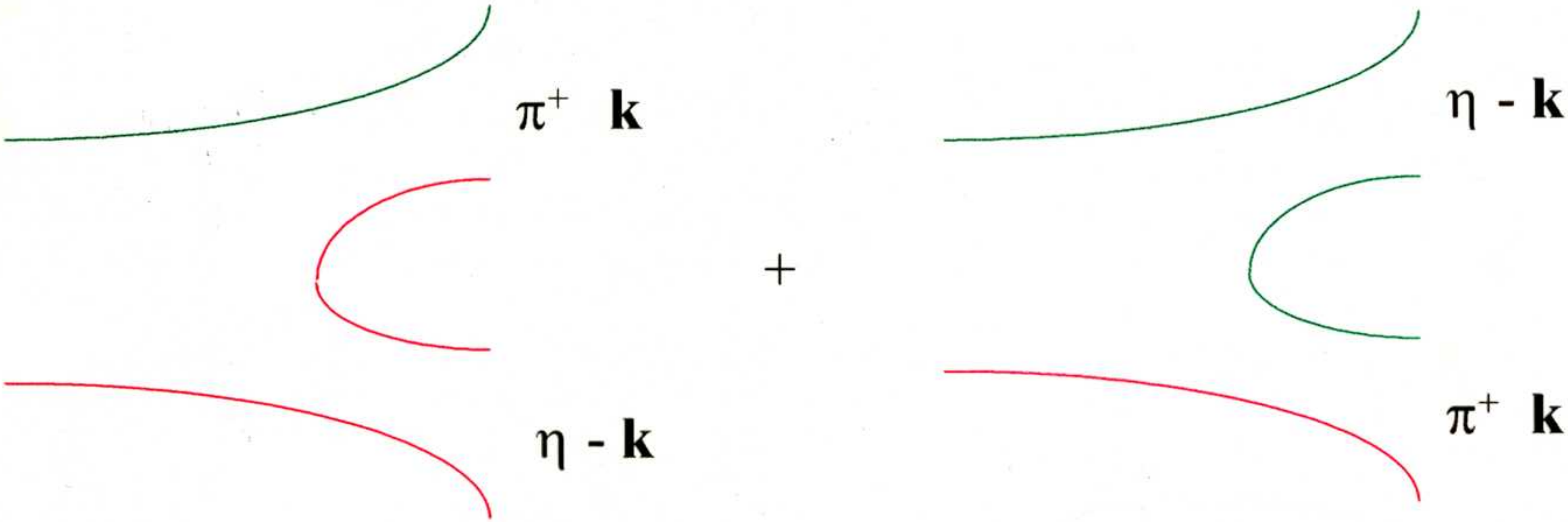
1^{-+} hybrid $\rightarrow \eta\pi, \eta'\pi, \eta(1295)\pi, \eta(1440)\pi$

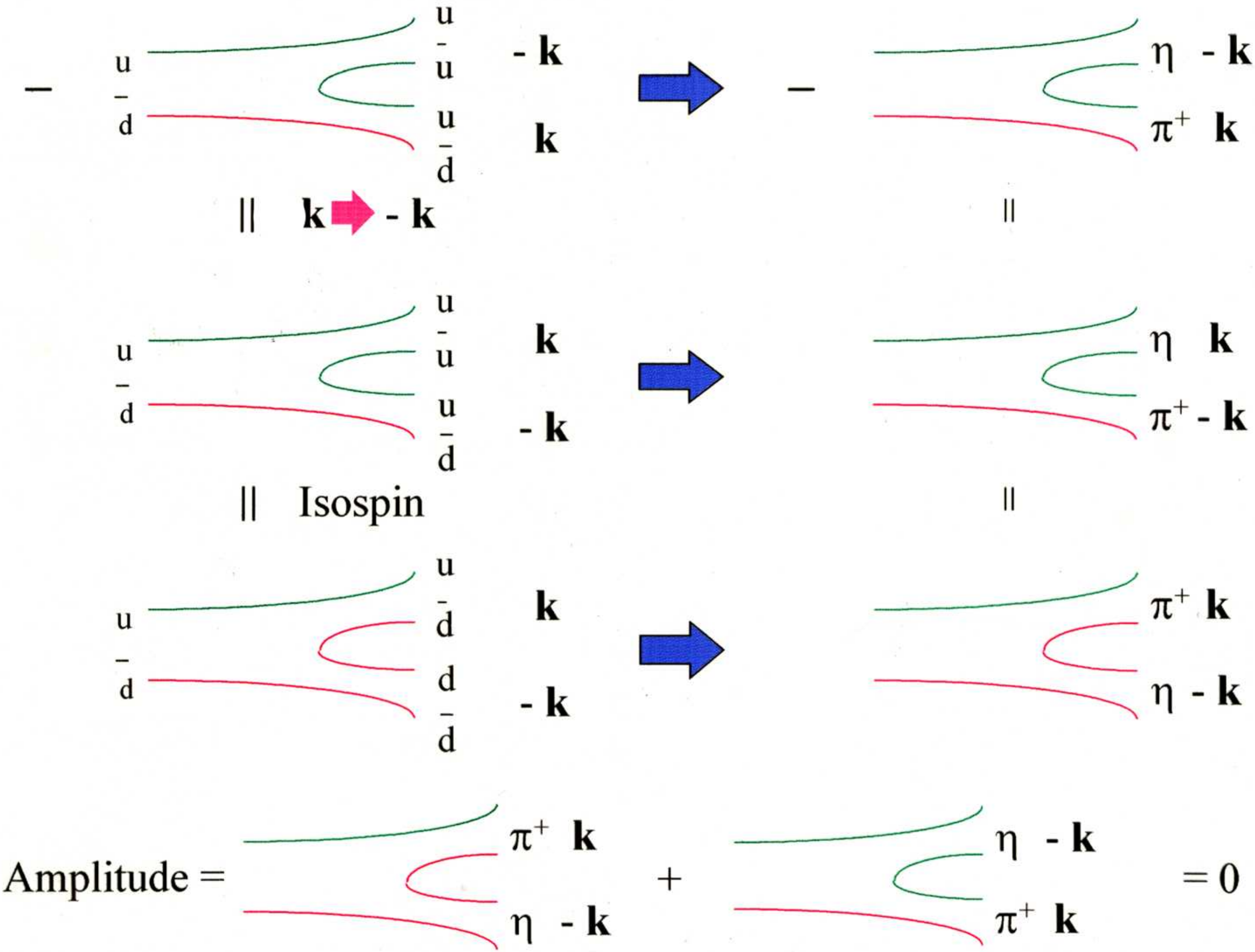
Decay amplitudes in Large-N_c

Page hep-ph 2003

Amplitude =



$1^{-+} \dashrightarrow \eta\pi$ H.J. Lipkin, Phys. Lett. B219 (1989) 99



−
u
d̄
u
ū
u
d̄
- k
k
−
η - k
π⁺ k
||
k
- k
||
u
d̄
u
ū
u
d̄
k
- k
η k
π⁺ - k
||
Isospin
||
u
d̄
u
d̄
d
d̄
k
- k
π⁺ k
η - k
Amplitude =
π⁺ k
η - k
+
η - k
π⁺ k
= 0

1^{-+} Current $\to \eta\pi$ Currents

$$G_\mu(x,y,z) = \langle 0|T(\ \pi^0(x)\ \eta(y)\ H_\mu(z)\ )|0\rangle$$

at $x_0 = t + \delta t \quad y_0 \equiv t \quad z_0 = -\infty \quad \mathbf{z} = \mathbf{0}$

$$G_\mu(x,y,z) = G^S_\mu(x,y,z) + G^A_\mu(x,y,z)$$

$$G_\mu(\mathbf{p},t) \equiv \int d^3x\ d^3y\ e^{i(\mathbf{p}\cdot\mathbf{x} - \mathbf{p}\cdot\mathbf{y})}\ G_\mu(x,y,z)$$

- $G_\mu(-\mathbf{p},t) = \int d^3y\ d^3x\ e^{i(-\mathbf{p}\cdot\mathbf{y} + \mathbf{p}\cdot\mathbf{x})}\ G_\mu(y,x,z)$

(under $\mathbf{x} \leftrightarrow \mathbf{y}$) $= \int d^3x\ d^3y\ e^{i(\mathbf{p}\cdot\mathbf{x} - \mathbf{p}\cdot\mathbf{y})}$

$\times \{G^S_\mu(x,y,z) - G^A_\mu(x,y,z)\} = G^S_\mu(\mathbf{p},t) - G^A_\mu(\mathbf{p},t)$

Need final states to be at the same time t

- $$G_\mu(-\mathbf{p},t) = \int d^3(-x)\, d^3(-y)\, e^{i(\mathbf{p}\cdot\mathbf{x}-\mathbf{p}\cdot\mathbf{y})}$$
$$\times\, G_\mu((-\mathbf{x},t),(-\mathbf{y},t),z)$$
$$= -G_\mu(\mathbf{p},t) = -G_\mu^S(\mathbf{p},t) - G_\mu^A(\mathbf{p},t)$$

Parity: $G_\mu((-\mathbf{x},t),(-\mathbf{y},t),(\underline{0},-\infty)) = -G_\mu(x,y,z)$

$$G_\mu^S(\mathbf{p},t) - G_\mu^A(\mathbf{p},t) = -G_\mu^S(\mathbf{p},t) - G_\mu^A(\mathbf{p},t)$$

$$\Rightarrow G_\mu^S(\mathbf{p},t) = 0 \quad \forall\ \mathbf{p},t$$

Quantum Field Theoretic Study of 1^{-+} current $\rightarrow \eta\pi$

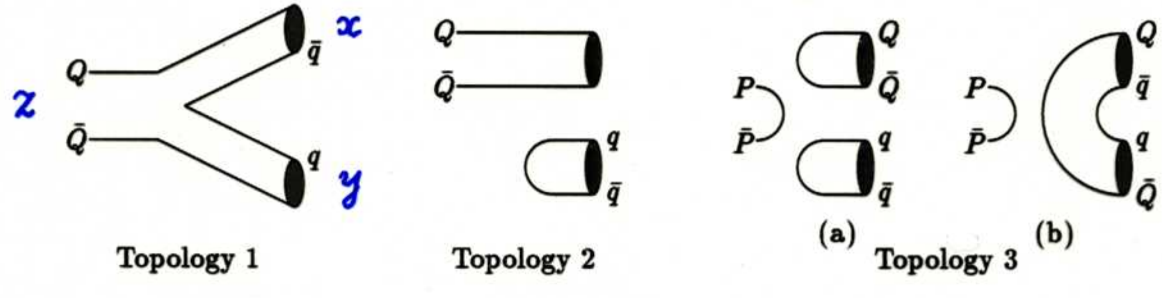
$$G_\mu(x,y,z) \sim \langle 0|\, T(\, \bar{u}(x)\, R(x)\, u(x)$$

$$\times\, \bar{u}(y)\, Q(y)\, u(y)\;\; \bar{u}(z)\, P(z)\, u(z)\;)\;|0\rangle$$

symmetric under $x \leftrightarrow y$ if $R(x) = Q(x)$

Examples:

$R(x) = \gamma_5 \quad Q(y) = \gamma_5 \quad P(z) = F^a_{\mu\nu}(z)\, \frac{\lambda^a}{2}\, \gamma^\nu$



Topology 1 symmetric if $R(x) = Q(x)$

Field symmetrization selection rule

Hybrid Current to $\eta\pi$ Amplitude

$$G_\mu^{nOZI}(\mathbf{p},t) = G_\mu(\mathbf{p},t) = \int d^3x\, d^3y\, e^{i(\mathbf{p}\cdot\mathbf{x}-\mathbf{p}\cdot\mathbf{y})}$$

$$\times \langle 0|\pi^0(x)\,\eta(y)\,H_\mu(z)|0\rangle$$

$$\int_{-\infty}^{\infty} dt\, G_\mu^{nOZI}(\mathbf{p},t)\, e^{iEt}$$

$$= \sum_n (2\pi)^4\, \delta^3(\mathbf{p}_n)\, \delta(E_n - E)$$

$$\times \langle 0|\left(\int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}}\, \pi^0(\mathbf{x},0)\right)\eta(0)|n\rangle\, \langle n|H_\mu(z)|0\rangle$$

Rigorous Quantum Field Theory Argument

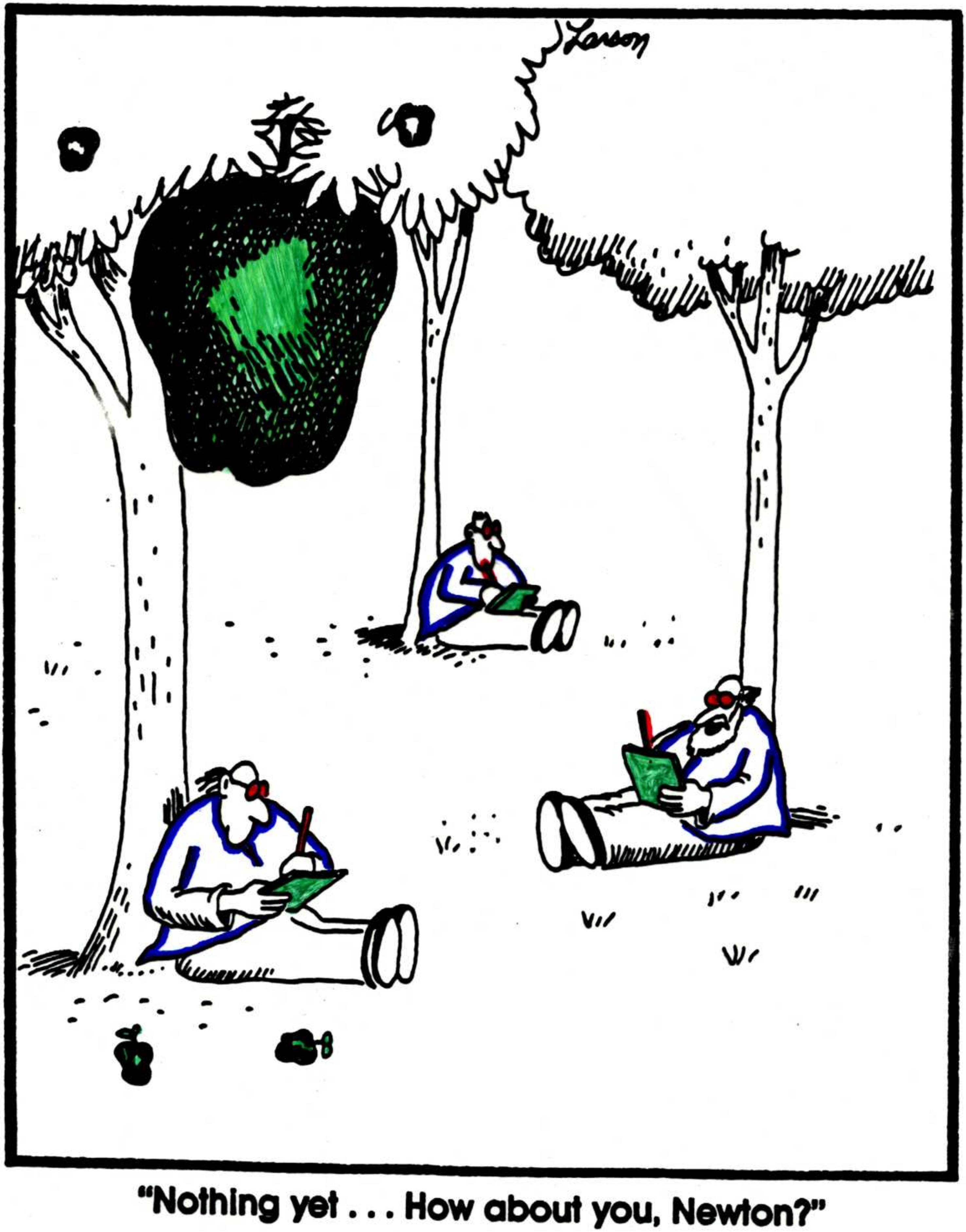
P.R. Page, Phys. Rev. D64 (2001) 056009

If 1^{-+} is a hybrid meson current, then the amplitude $1^{-+} \to \eta\pi$ does not arise from OZI allowed contributions

Interim Summary

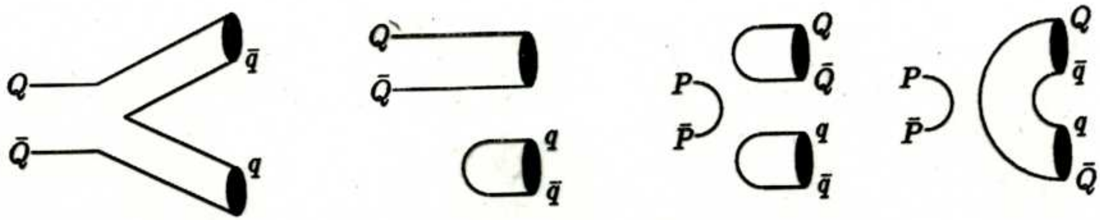
- Close & Lipkin's intuitive non-field theoretic argument for decay amplitudes generalized to rigorous quantum field theoretic argument for Green's functions

- From Green's functions rigorous results for decay to lowest two-body threshold $\eta\pi$ follow



"Nothing yet . . . How about you, Newton?"

HYBRID DECAYS



Topology 1	Topology 2	Topology 3	
N_c	1	$\frac{1}{N_c}$	1

$$\int_{-\infty}^{\infty} dt\; G_{\mu}^{nOZI}(\mathbf{p},t)\; e^{iEt}$$

$$= \sum_{n} (2\pi)^4\; \delta^3(\mathbf{p}_n)\; \delta(E_n - E)$$

$$\times \langle 0|\left(\int d^3x\; e^{i\mathbf{p}\cdot\mathbf{x}}\; \pi^0(\mathbf{x},0)\right)\eta(0)|n\rangle\; \langle n|H_{\mu}(z)|0\rangle$$

Keep only leading contributions on R.H.S. :

	$\langle 0\|\pi^0\eta\|n\rangle$	$\langle n\|H_{\mu}\|0\rangle$
n one-particle σ	$\sqrt{N_c}$	$\sqrt{N_c}$
n two-particle $\sigma_1\sigma_2$	N_c	1

σ 1^{-+} hybrid meson, σ_i 0^{-+} (hybrid) meson

$$\langle 0|\; \pi^0(\mathbf{x},0)\; \eta(0)\; |\sigma\; \mathbf{0}\rangle = \mathcal{O}\left(\frac{1}{\sqrt{N_c}}\right)$$

$$\langle \sigma_1\mathbf{k}\; \sigma_2 - \mathbf{k}|H_{\mu}(z)|0\rangle = \mathcal{O}\left(\frac{1}{N_c}\right)$$

Coupling of 1^{-+} to two 0^{-+} particles

$$\langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}\ \mathsf{out}|H_\mu(z)|0\rangle =$$

$$\sum_n \langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}\ \mathsf{out}|n\ \mathsf{in}\rangle\, \langle n\ \mathsf{in}|H_\mu(z)|0\rangle$$

Leading contributions (σ 1^{-+} hybrid meson)

	$\langle \sigma_1 \sigma_2 \| n\rangle$	$\langle n \| H_\mu \| 0\rangle$
n one-particle σ	$\frac{1}{\sqrt{N_c}}$	$\sqrt{N_c}$
n two-particle	1	1

$$\langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}\ \mathsf{out}|H_\mu(z)|0\rangle - \langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}\ \mathsf{in}|H_\mu(z)|0\rangle$$

$$= \sum_\sigma 2\pi i \delta(\ldots)\, \langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}|T|\sigma\, \mathbf{0}\rangle\, \langle \sigma\, \mathbf{0}\ \mathsf{in}|H_\mu(z)|0\rangle$$

$$\langle \sigma_1 \mathbf{k}\, \sigma_2 - \mathbf{k}|\, T\, |\sigma\, \mathbf{0}\rangle = \mathcal{O}(1/N_c^{\frac{3}{2}})$$

Large-N_c Decay Phenomenology

P.R. Page, hep-ph/0303170

Large-N_c: No four-quark $\Rightarrow$ π_1 hybrid mesons.

Decay amplitude of $\{1, 3, 5 \ldots\}^{-+}$ hybrid to $\eta\pi^0, \eta'\pi^0, \eta'\eta, \eta(1295)\pi^0$, $\pi(1300)^0\pi^0, \eta(1440)\pi^0$, $a_0(980)^0\sigma$, $f_0(980)\sigma$ (hybrid) mesons is $\mathcal{O}(1/N_c^{\frac{3}{2}})$ (usually $\mathcal{O}(1/\sqrt{N_c})$).

- Same suppression as OZI rule.
- Important if decay otherwise large.
- Results extended to charged states using isospin.

$\pi_1(1600) \rightarrow \eta\pi$ $\surd$, $\eta'\pi$ $\times$?, $\eta(1295)\pi$ $\surd$?, $\eta(1440)\pi$, $a_0(980)\sigma$ suppressed.