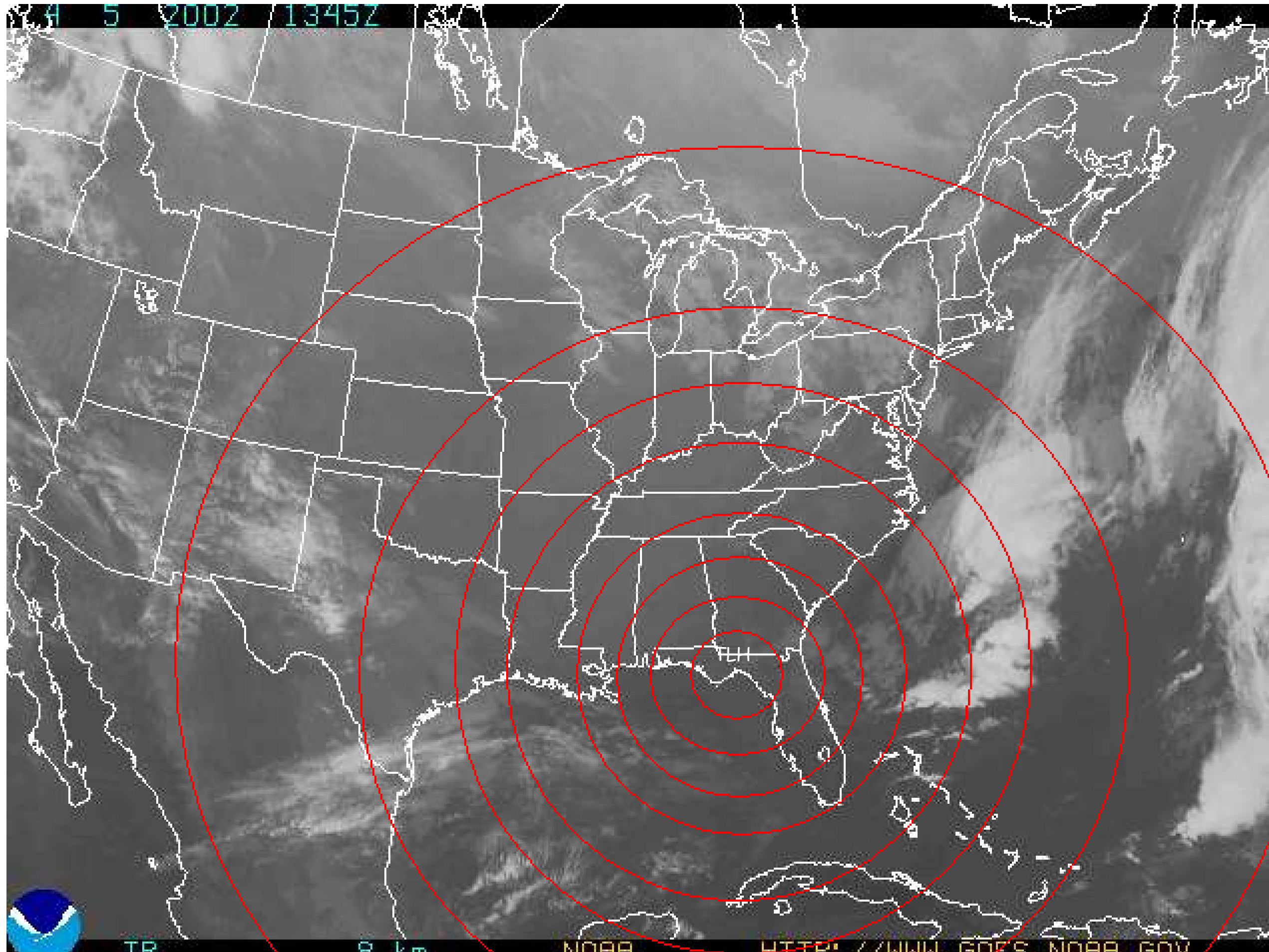
How can you find Tallahassee?

- Head SW
- Stop when humidity(%) = $T(^{o}F)$ = 94 in May
- Try to avoid jokes about elections and bushes



Hybrid Baryons in the Flux-Tube Model

Work done with Philip Page (LANL)

- Meson exotics exist, baryon exotics?
- Bag model hybrids
- Conventional excited baryons
  - Spin, orbital, radial excitations
- Baryon confining potential-lattice results
- Flux-tube model, dynamical glue
  - Analytic results for discretized strings
  - Numerical results, adiabatic potentials
  - Comparison to lattice potentials
- Quantum numbers, masses of light hybrids

The Cork Model

Up Charm Top

Down Strange Bottom

Baryon exotics?

- Conventional mesons:
  - Consider NR bound states of quark and antiquark:
  - **J**=**L**+**S**, $P=(-1)^{L+1}$
  - $C=(-1)^{L+(S+1)+1}=(-1)^{L+S}$, for self-conjugate mesons
  - Certain quantum numbers excluded → exotics!
- Baryon exotics… don't exist!
  - No *C*-parity
  - *All* half-integral J with both parities possible with qqq
  - **J**=**L**+**S**, $\mathbf{L}=\mathbf{l}_\rho+\mathbf{l}_\lambda$, S=1/2 or 3/2
  - $P=(-1)^{l\rho+l\lambda}$
- Baryons with excited glue—should exist!
- Model-dependent concept, context is potential model, adiabatic picture

First theoretical results on hybrid baryons

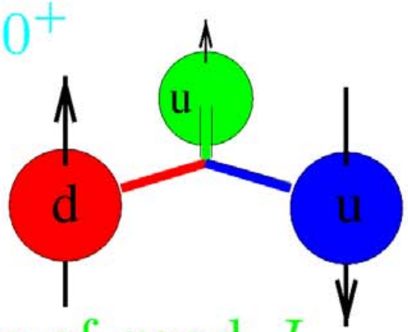
- Bag model hybrids, with constituent gluon $(qqq)_8g$
    - **TED BARNES** & F.E. Close (**1983**); Golowich, Haqq & Karl (**1983**); Carlson & Hansson (**1983**); Duck and Umland (**1983**)
  - transverse electric (lowest energy) gluon eigenmode of vector field in spherical cavity
    - $L^{\pi}=1^{+}$ gluon, quarks in S-wave spatial ground state
    - Mixed exchange symmetry color wvfns of $(qqq)_8$
    - $S_{qqq}=1/2$ gives flavor(J^P) = $N1/2^+$, $N3/2^+$, $\Delta1/2^+$, $\Delta3/2^+$
    - $S_{qqq}=3/2$ gives $N1/2^+$, $N3/2^+$, $N5/2^+$
  - Bag qqq Hamiltonian + gluon K.E. + color-Coulomb energy + interactions:
    - $O(\alpha_s)$ one-gluon exchange, gluon Compton effect

First theoretical results on hybrid baryons…

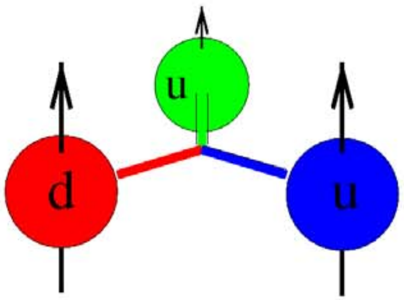
- Bag model hybrids:
  - Lightest $N1/2^+$ state between $P_{11}(1440)$ (Roper) and $P_{11}(1710)$
  - $N1/2^+$ and $N3/2^+$ are 250 MeV heavier, all Δ's heavier still
  - Problems with phenomenology…where is extra P_{11} state?
- QCD sum rules
    - Kisslinger and Li (1995)
  - Also predict lightest hybrid ~1500 MeV

Conventional excited baryons

- proton wavefunction $\Psi = C_A \sum \psi\, \chi\, \phi$
  - $\star$ symmetric spatial wavefunction ψ, $L_q^P = 0^+$
  - $\star$ χ : $S = \frac{1}{2}$; ϕ : $I = \frac{1}{2}$



- $L_q^P = 0^+ \otimes S = \frac{1}{2} \rightarrow J^P = \frac{1}{2}^+$
- sum makes Ψ totally antisymmetric and state of good J
- spin excitation: spin-3/2 baryon
  - $\star$ χ : $S = \frac{3}{2}$; ϕ : $I = \frac{3}{2}$



- $L_q^P = 0^+ \otimes S = \frac{3}{2} \rightarrow J^P = \frac{3}{2}^+$

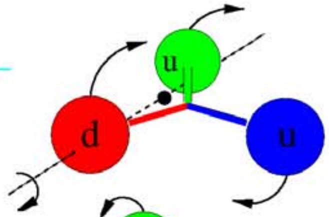
$\rightarrow$ one $0\hbar\omega$ state: Δ^{++} seen in $\pi^+ p$, and Δ^0 in $\pi^- p$

…Conventional excited baryons

- orbital excitations: can orbitally excite either of two relative coordinates
- $L^P = 1^- \otimes \left\{ S = \frac{1}{2} \text{ or } S = \frac{3}{2} \right\} \rightarrow J^P = \left\{ \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^- \right\}$

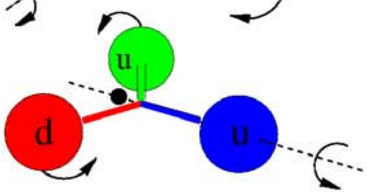
  $\rightarrow$ seven $1\hbar\omega$ states: $2\ N\frac{1}{2}^-$; $\Delta\frac{1}{2}^-$; $2\ N\frac{3}{2}^-$; $\Delta\frac{3}{2}^-$; $N\frac{5}{2}^-$

  $\rightarrow$ all seen in $\pi N \rightarrow \pi N$



- sixteen $2\hbar\omega$ states with $L^P = 1^+$ and $L^P = 2^+$:

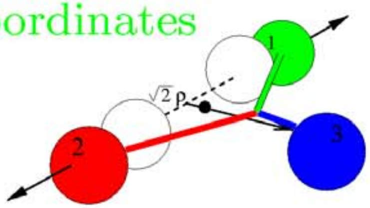
  $\rightarrow$ some seen in $\pi N \rightarrow \pi N$

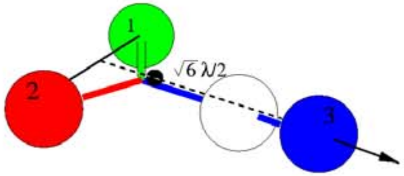


- radial excitations: radially excite either of two relative coordinates
- $L^P = 0^+ \otimes \left\{ S = \frac{1}{2} \text{ or } S = \frac{3}{2} \right\} \rightarrow J^P = \left\{ \frac{1}{2}^+, \frac{3}{2}^+ \right\}$

  $\rightarrow$ five $2\hbar\omega$ states: $2\ N\frac{1}{2}^+$, $N\frac{3}{2}^+$, $\Delta\frac{1}{2}^+$, $\Delta\frac{3}{2}^+$

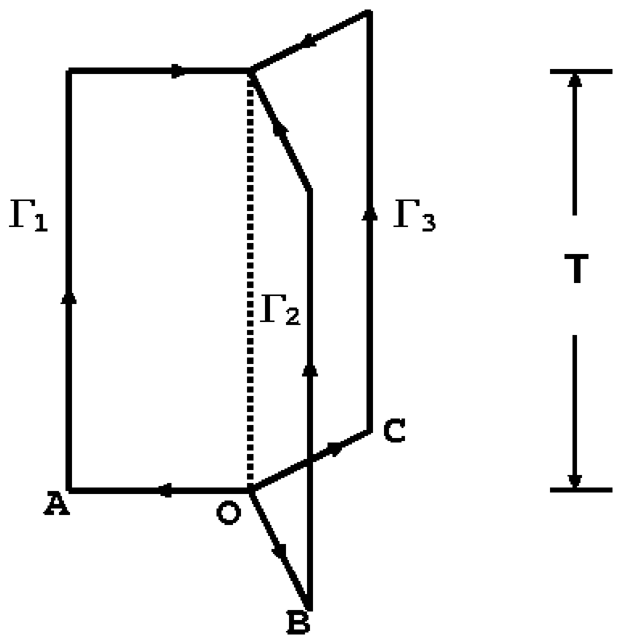
  $\rightarrow$ some seen in $\pi N \rightarrow \pi N$





How should we treat confinement?

- Quenched lattice measurement of QQQ potential
- Takahashi, Matsufuru, Nemoto and Suganuma, PRL**86** (2001) 18.
- Measure potential with 3Q-Wilson loop (static quarks) for 0<t<T
- Also fit $Q\bar{Q}$ potential to compare σ and Coulomb terms



… How should we treat confinement?

- Fit 16 QQQ configurations to

$$V_{3Q} = -A_{3Q}\sum_{i<j}\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3Q}L_{\min} + C_{3Q}$$

- L_{min} = min. length Y-shaped string _
- 3Q, QQ string tensions similar
- Coulomb terms in OGE ratio $\frac{1}{2}$

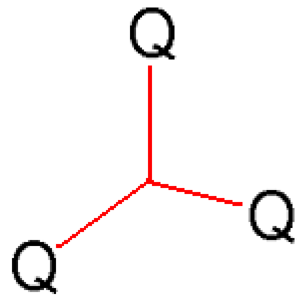
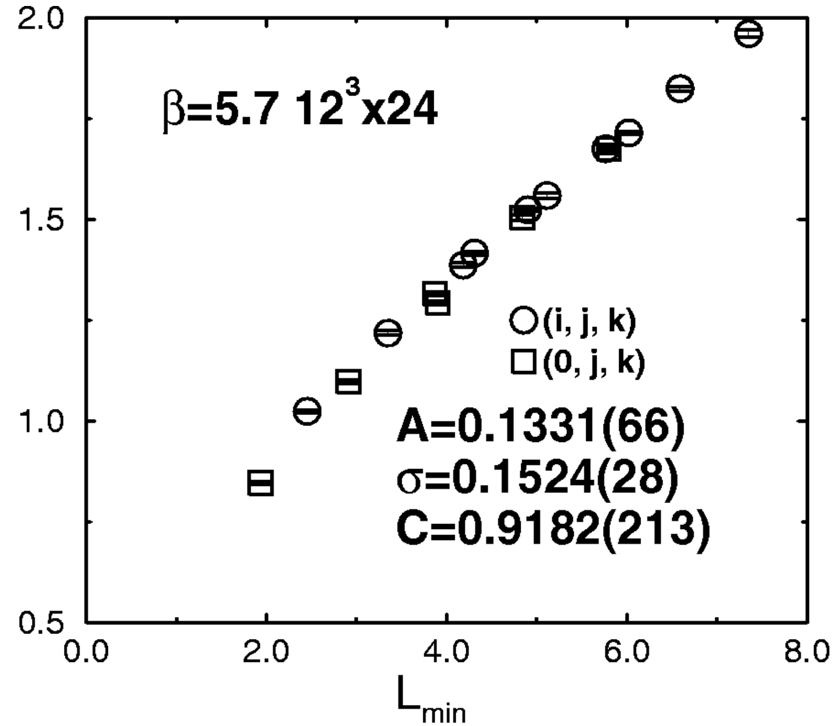


TABLE I. The coefficients in Eq. (6) for the $3Q$ potential and those in Eq. (5) for the Q-$\bar{Q}$ potential in the lattice unit.

	σ	A	C
$3Q$	0.1524(28)	0.1331(66)	0.9182(213)
Q-$\bar{Q}$	0.1629(47)	0.2793(116)	0.6203(161)

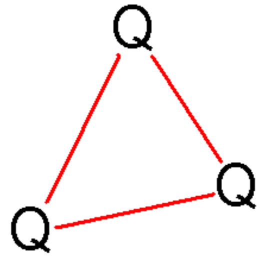


- σ is in lattice units a^{-2}
- Meson string tension 0.89 GeV/fm (a=0.19 fm)

… How are the quarks confined?

- Also tried fit to function

$$V_{3Q} = -A_\Delta \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_\Delta \sum_{i<j} |\mathbf{r}_i - \mathbf{r}_j| + C_\Delta$$



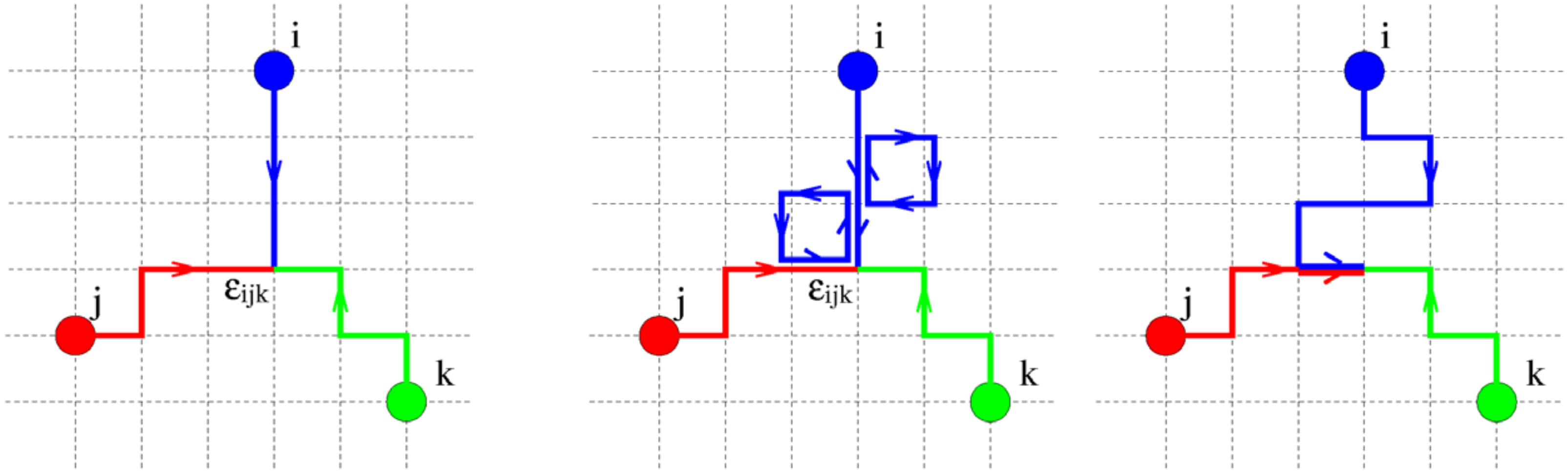
- Fit worse: χ^2 per d.f. 3.8 ⇨10.1
- Result is a reduced string tension $\sigma_\Delta = 0.53\,\sigma$
  - Simply a geometrical factor
  - Perimeter P satisfies $1/2 < L_{min}/P < 1/(3)^{1/2} = 0.58$
  - Accidentally close to $<\Lambda_i.\Lambda_j>_{baryons}$ / $<\Lambda_i^*.\Lambda_j>_{mesons} = 1/2$

    ⇨ but confinement is **not** (colored) vector exchange!

⇨string-like potential + color Coulomb good for QQQ baryons

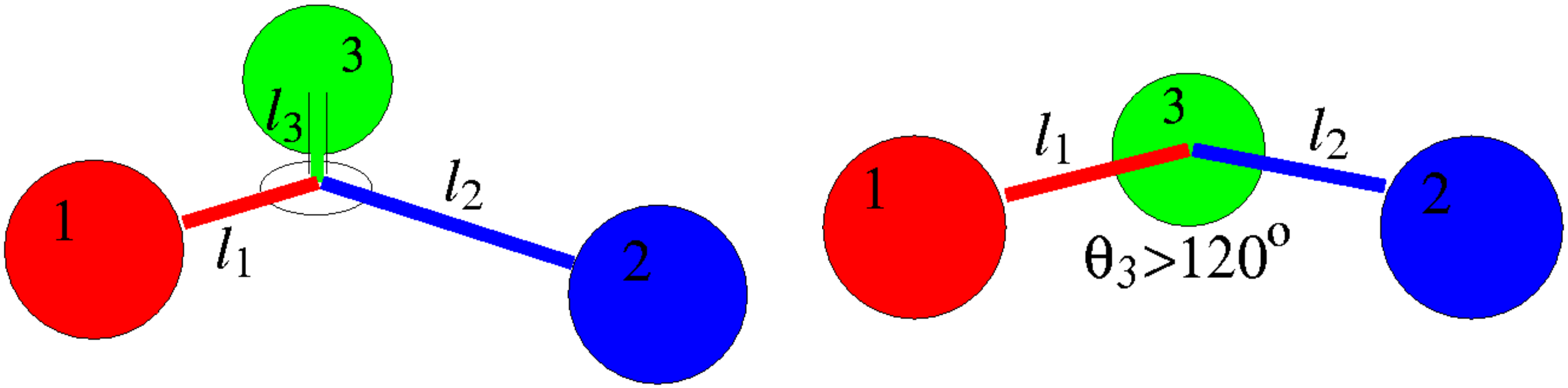
⇨Model with flux-tube for qqq baryons

Flux-tube model



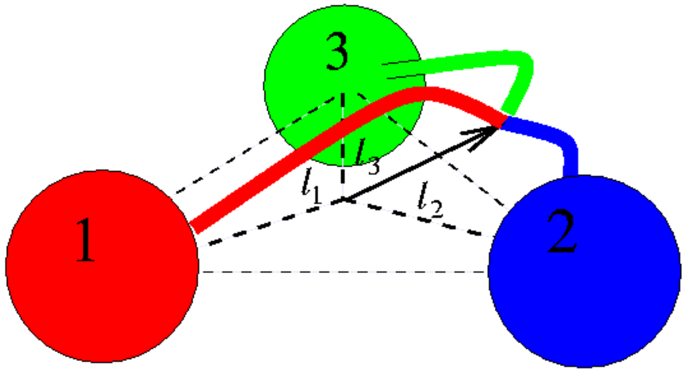
- Based on strong-coupling lattice QCD
  - Color fields confined to narrow tubes, energy ∝ length
  - Junction, to maintain global color gauge invariance
  - Plaquette operator from lattice action:
    - Moves tubes transverse to their original orientations
    - Moves junction

Model confining interaction



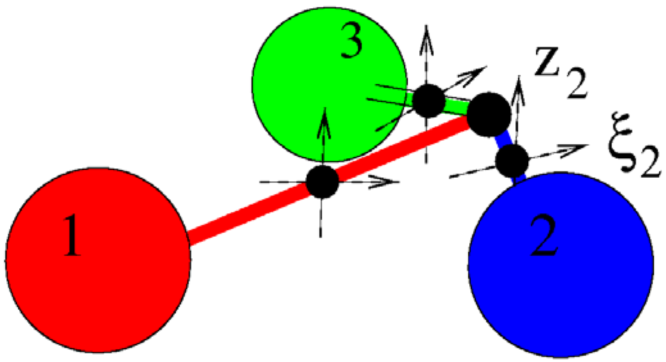
- Flux tubes, combined with adiabatic approx.
  - confining interaction: minimum length string
  - $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$
    - note σ is meson string tension
    - linear at large q-junction separations
  - Conventional baryon states:
    - Solve for qqq energies in this confining potential
    - With additional interactions between quarks…

Hybrid baryons



- Fix quark positions $\mathbf{r}_i$, allow flux tubes to move
  - Junction moves relative to its equilibrium position
  - Strings move transverse to their equilibrium directions
- Ground state of string defines adiabatic potential
  - $V_B(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=\sigma(l_1+l_2+l_3)=\sigma L_{min}$, plus zero point motion
- First excited state defines new adiabatic potential
  - $V_H(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$
- Hybrids: solve for qqq motion in this modified potential
- With Philip Page: PRD**69** (1999) 111501, PRC**66** (2002) 065204

Discretized strings



- Simplest model; one bead m_i per string + junction bead, m_j
  - Take $m_i = \sigma l_i$
  - Allow m_J to differ from m_i
- 9 degrees of freedom:
  - string-bead transverse motions ξ_i, z_i
  - junction position **r** relative to equilibrium position

String excitation energies

- Correct for CM motion due to bead and junction motion
  - Simplest correction to adiabatic approx
  - Effective masses m_i^{eff} & m_J^{eff} depend on quark masses: in limit of infinite number of beads:

$$m_J^{eff} = b\Sigma_i l_i \left(1/3 - b\Sigma_i l_i / \left[4 \Sigma_i (b\, l_i + M_i) \right] \right)$$

String excitation energies

- Diagonalize 9x9 Hamiltonian in small oscillations approximation
- String Hamiltonian:

$$V = V_{\text{junction}}(\vec{r}) + V_{\text{bead}}(\xi_i, z_i) \longrightarrow \text{beads and junction decoupled}$$

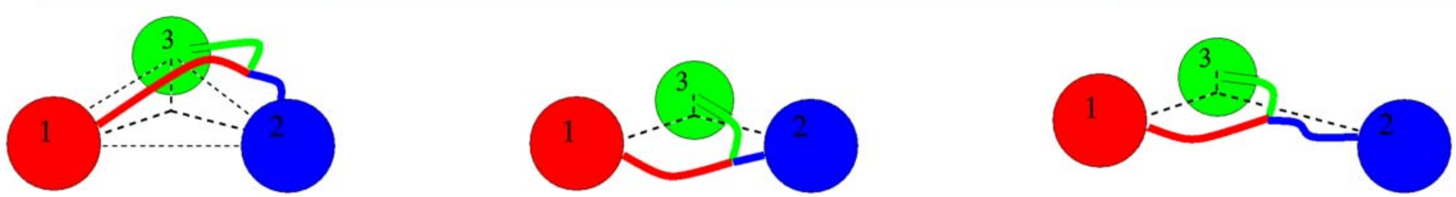
$$T = T_{\text{junction}}(\dot{\vec{r}}) + T_{\text{bead}}(\dot{\xi}_i, \dot{z}_i) + T_{\text{bead}-\text{bead}} + T_{\text{bead}-\text{junction}}$$

$$\longrightarrow \text{couples beads to each other and junction}$$

Approximate excited string energies

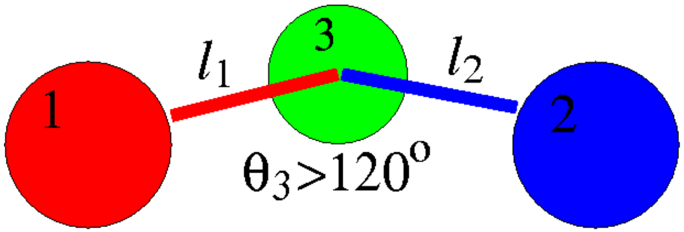
- Good approximation to *first* excited mode energy if ignore junction-bead coupling
  - Non-Interacting below (compared to exact)
  - First excited state is *always* in-plane motion
  - With $m_J = m_q = 0.33$ GeV, string energies, in GeV:

l_i(fm)	E_1(NI)	E_1	E_2(NI)	E_2	E_3(NI)	E_3
0.5, 0.5, 0.5	0.614	0.607	0.614	0.607	0.868	0.828
0.5, 0.5, 0.1	0.623	0.616	1.069	0.985	1.069	1.005
0.5, 1.0, 0.1	0.520	0.483	0.544	0.534	0.544	0.590



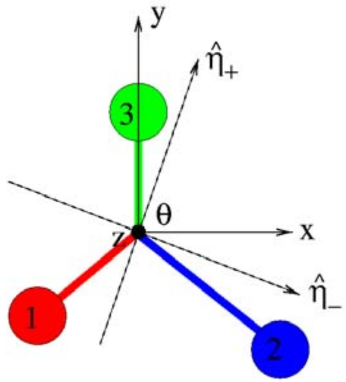
Adiabatic potentials

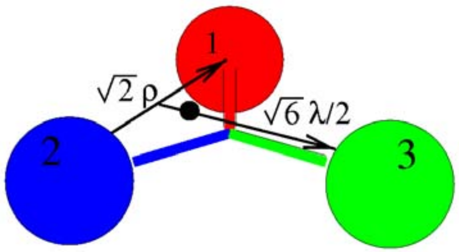
- Results of analytic work:
  - First excited state: look only at junction motion
  - Individual strings follow junction, add to m_J^{eff}
  - Evaluate m_J^{eff} in limit of large number of beads
  - Generate $V_H = E_1(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ for qqq in hybrid
- Numerical work: V_B and V_H found by variational calculation
  - Small oscillations approximation singular when any $l_i \rightarrow 0$
  - Contains term like $|l_i|$
  - Shortest string has l=0 when:



  - Analytic and numerical results agree when l_i all large

…Adiabatic potentials





- one mode always $\hat{\eta}_z = \hat{z}$: $T_{\rm junction}(\dot{\vec{r}}) + V_{\rm junction}(\vec{r})$ even under $z \rightarrow -z$
- trial wavefunctions: ground state, $1^{\rm st}$ excited state anistropic oscillators

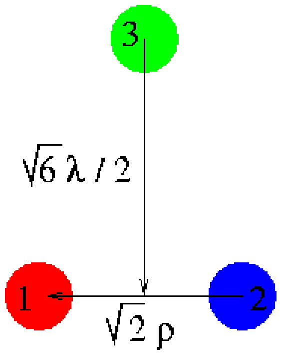
$$\Psi_B(\vec{r}) = \left(\frac{\alpha_+\alpha_-\alpha_z}{\sqrt{\pi}}\right)^{\frac{3}{2}} \exp\left\{-\left[(\alpha_+\hat{\eta}_+\cdot\vec{r})^2 + (\alpha_-\hat{\eta}_-\cdot\vec{r})^2 + (\alpha_z z)^2\right]/2\right\}$$

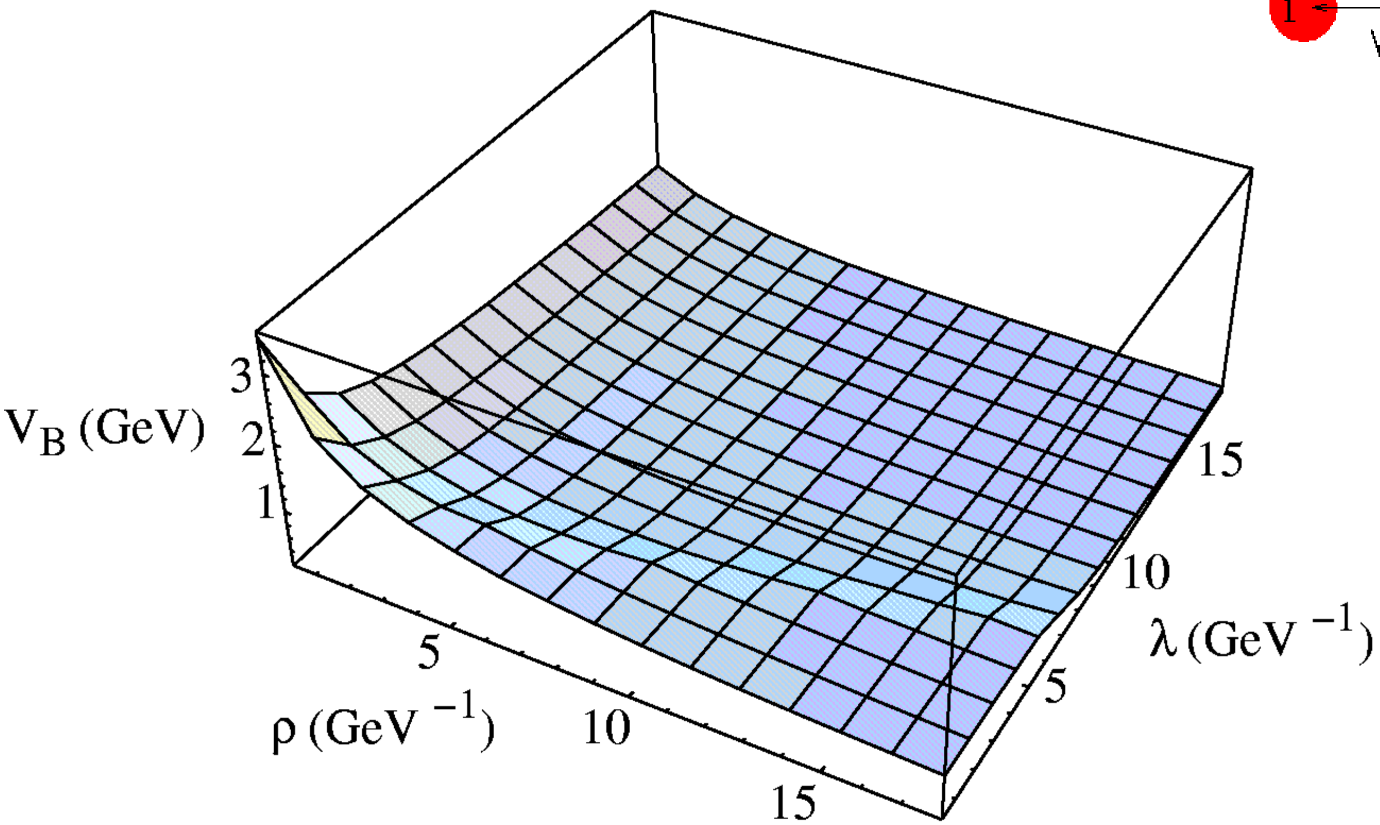
$$\Psi_H(\vec{r}) = \sqrt{2}\alpha_-\hat{\eta}_-\cdot\vec{r}\,\Psi_B(\vec{r})$$

- four variational parameters:
  - ⋆ θ, α_-, α_+, α_z
- for every ρ, λ, $cos(\theta_{\rho\lambda})$:
  - ⋆ independently minimize ground and excited state string energies
    - $\rightarrow$ show plots for $\vec{\rho} \parallel \vec{\lambda}$, $\vec{\rho} \perp \vec{\lambda}$

…Adiabatic potentials

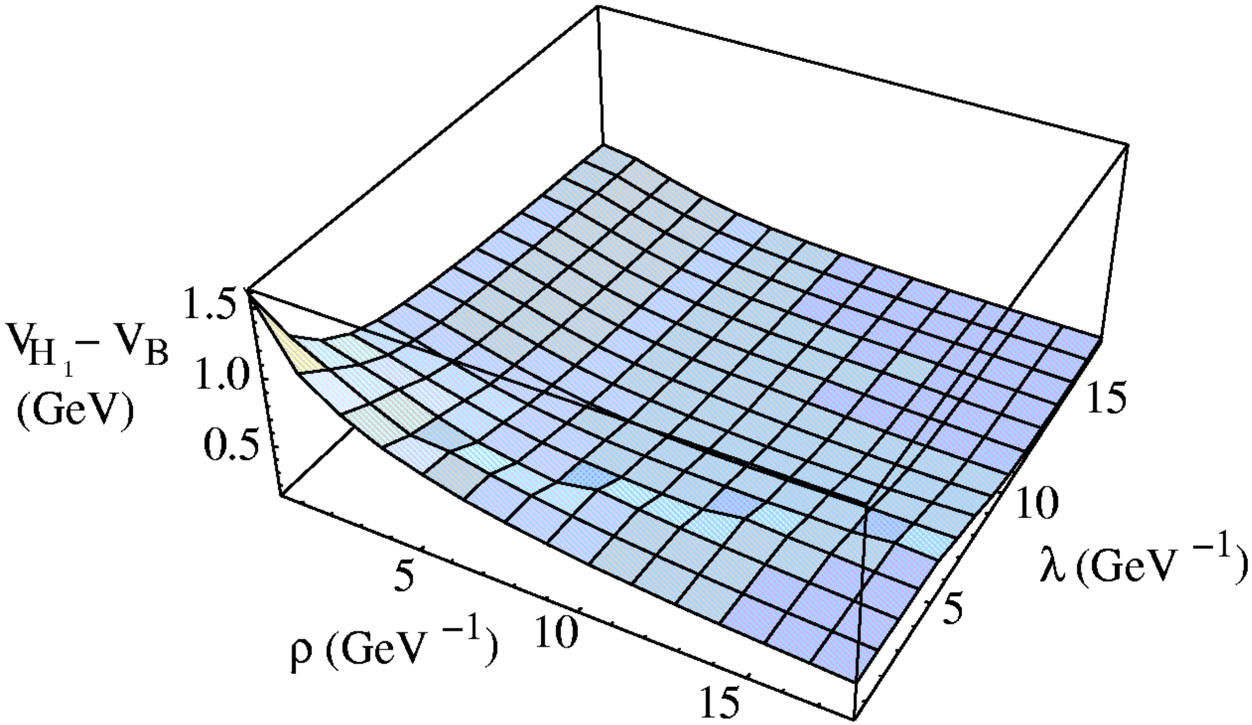
- Baryon potential without the confining term, $V_B - b\Sigma_i l_i$, for $\cos(\theta_{\rho\lambda})=0$; zero-point energy

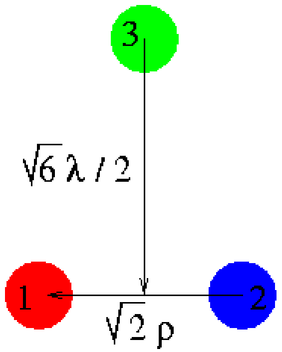




…Adiabatic potentials

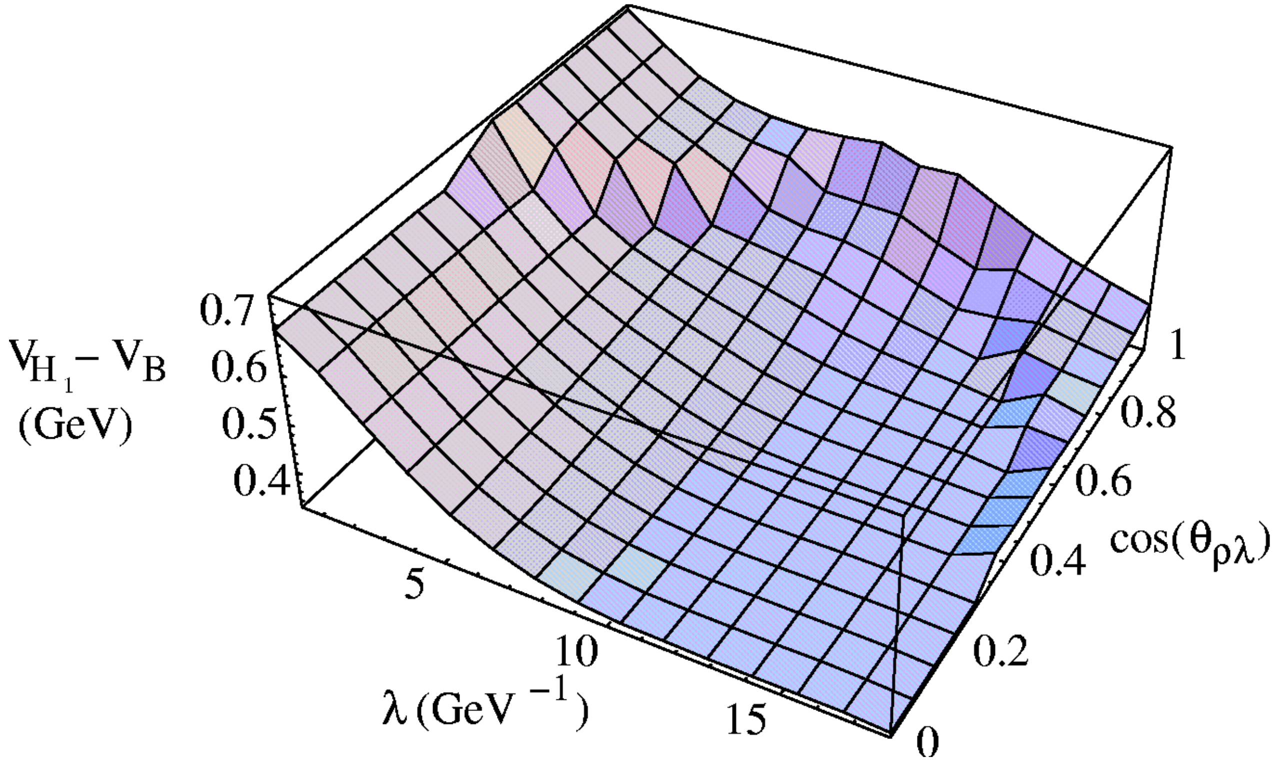
- V_{H1}-V_B for $\cos(\theta_{\rho\lambda})$=0





…Adiabatic potentials

- V_{H1}-V_B for ρ=6.2 GeV^{-1}



Lattice QQQ baryon and hybrid potentials

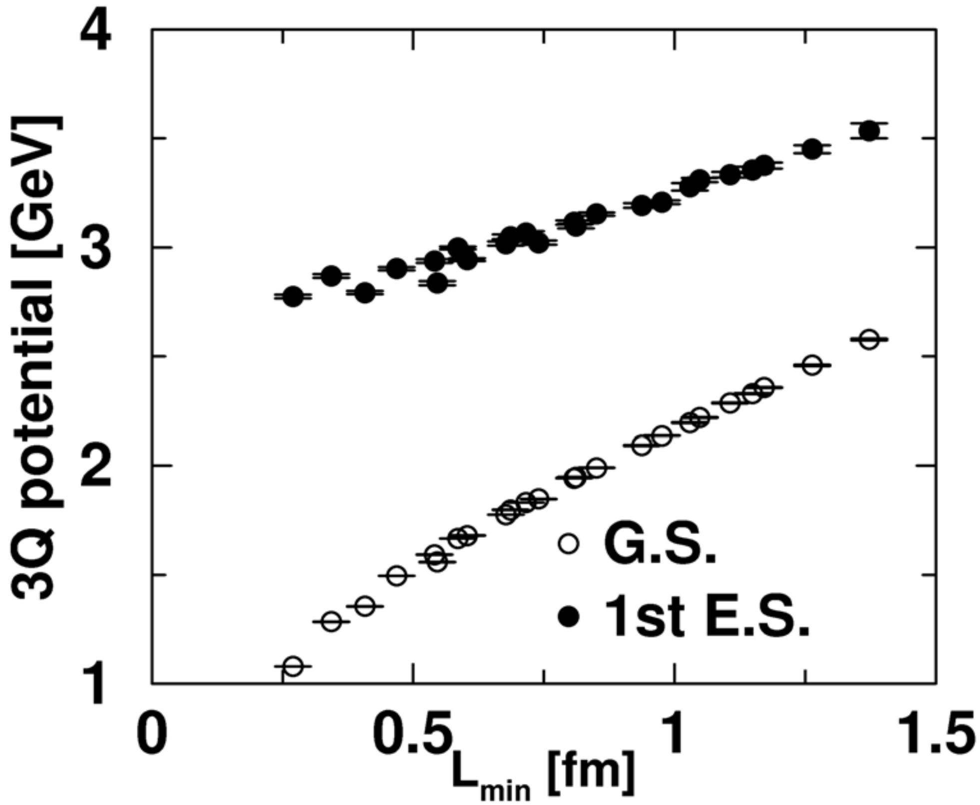
Takahashi & Suganuma
hep-lat/ 0210024

Table 1: The ground-state 3Q potential $V_{3Q}^{\rm g.s.}$ and the 1st excited-state 3Q potential $V_{3Q}^{\rm e.s.}$ in the lattice unit. The label (l,m,n) denotes the 3Q system where the three quarks are put on $(la,0,0)$, $(0,ma,0)$ and $(0,0,na)$ in $\mathbf{R}^3$.

(l,m,n)	$V_{3Q}^{\rm e.s.}$	$V_{3Q}^{\rm g.s.}$	$V_{3Q}^{\rm e.s.} - V_{3Q}^{\rm g.s.}$
(0,1,1)	1.9816(95)	0.7711(3)	1.2104
(0,1,2)	1.9943(72)	0.9682(4)	1.0261
(0,1,3)	2.0252(92)	1.1134(7)	0.9118
(0,2,2)	2.0980(80)	1.1377(6)	0.9603
(0,2,3)	2.1551(87)	1.2686(9)	0.8866
(0,3,3)	2.2125(114)	1.3914(13)	0.8211
(1,1,1)	2.0488(90)	0.9176(4)	1.1312
(1,1,2)	2.0727(75)	1.0686(5)	1.0041
(1,1,3)	2.1023(73)	1.2004(7)	0.9019
(1,1,4)	2.1580(93)	1.3201(10)	0.8380
(1,2,2)	2.1405(72)	1.1907(7)	0.9498
(1,2,3)	2.1899(71)	1.3084(9)	0.8815
(1,2,4)	2.2516(79)	1.4221(12)	0.8296
(1,3,4)	2.2907(91)	1.5260(15)	0.7647
(1,4,4)	2.3807(138)	1.6322(20)	0.7485
(2,2,2)	2.1776(111)	1.2844(10)	0.8932
(2,2,3)	2.2242(96)	1.3882(11)	0.8360
(2,2,4)	2.2799(98)	1.4952(15)	0.7847
(2,3,4)	2.3637(100)	1.5853(18)	0.7784
(2,4,4)	2.4108(137)	1.6836(23)	0.7271
(3,3,3)	2.3408(168)	1.5680(19)	0.7728
(3,3,4)	2.3958(151)	1.6635(22)	0.7323
(3,4,4)	2.4645(177)	1.7565(30)	0.7081
(4,4,4)	2.5245(340)	1.8408(42)	0.6837

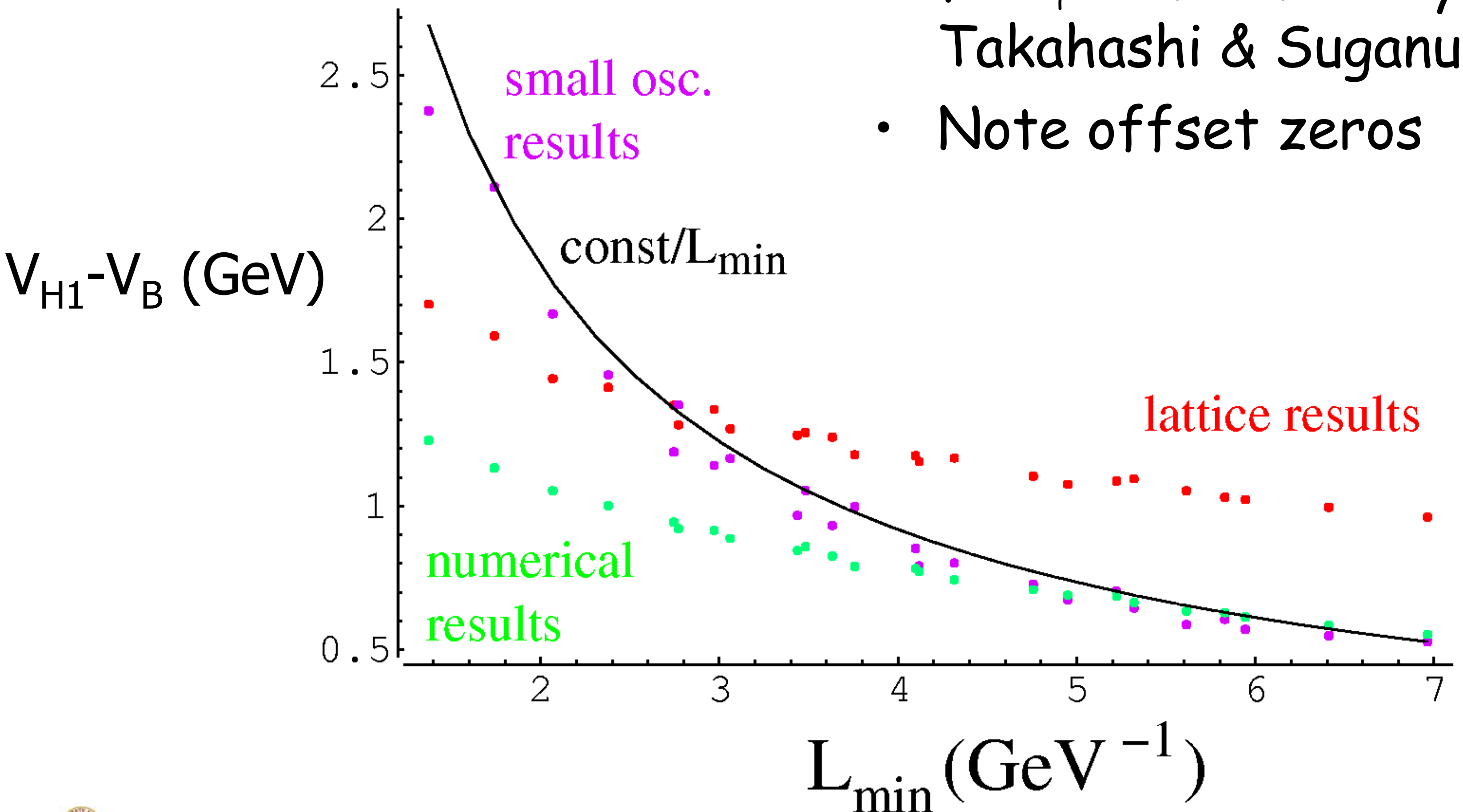
Lattice QQQ baryon and hybrid potentials…

- Calculate L_{min}, plot V_B and V_{H1} vs. L_{min}



Flux tube vs. lattice results

- Difference $V_{H1}-V_B$



- Calculated in model for l_i values used by Takahashi & Suganuma
- Note offset zeros

Hybrid baryon quantum numbers

- Parity of string:
  - Ground state and lightest (in plane) excited state H_1 (also H_2): +ve
  - Out of plane (H_3): -ve
- Quark-label exchange symmetry:
  - $T_{\text{junction}}(\dot{\vec{r}}) + V_{\text{junction}}(\vec{r})$ invariant
  - Excited states both totally S and AS
  - Checked ground state S
- Angular momentum of string:
  - Adiabatic approx breaks rotational invariance
    - Flux wvfn not eigenfunction of $\mathbf{l}$ (junction)
    - But overall wvfn must be eigenfunction of $\mathbf{L}=\mathbf{L}_{qqq}+\mathbf{l}$

…Hybrid baryon quantum numbers

- Expect ground state 0^+ , first excited state 1^+
  - Note: $\Psi_{H1}(\mathbf{r}) \propto \eta_- \cdot \mathbf{r}\ \Psi_B(\mathbf{r})$
  - Since $\eta_- \cdot \mathbf{r}$ lies in plane of quarks, & Ψ_B has $l=0$ to very good approximation:
    - Know $\eta_- \cdot \mathbf{r} \propto aY_{11}(\mathbf{r}) + bY_{1-1}(\mathbf{r})$
    - So m=+1,-1 in body-fixed system
  - If quarks have $L_{qqq}=0$ (lowest energy):
    - M=+1,-1 and so $L=L_{qqq}+l \geq 1$
    - L=1 expected lightest
  - Checked E_{qqq} rises with L_{qqq} in V_{H1}

…Hybrid baryon quantum numbers

- Additional symmetry: parity under reflection in qqq plane – "chirality"
  - Changes sign of z, and out of plane bead coordinates
  - Chirality +1: $\Psi_{H1}(\mathbf{r})$, $\Psi_{H2}(\mathbf{r})$, $\Psi_{B}(\mathbf{r})$
  - Chirality -1: $\Psi_{H3}(\mathbf{r})$ (out of plane)
- Should classify flux wvfns in adiabatic lattice QCD according to:
  - Exchange symmetry
  - Parity
  - chirality

Hybrid baryon masses

- Find quark energies by adding V_{H1}-V_B to usual interquark potential
  - Find lowest energy quark excitations with L_q=0,1,2,...
  - Expand wvfn in large oscillator basis of fixed L_{qqq}
  - Numerical calculations:
    - Spin-averaged L_{qqq}=0 hybrid: 1975 +/- 100 MeV
    - Add 365 MeV with L_{qqq}=1, and 640 MeV with L_{qqq}=2
  - Quantum numbers: $L_{qqq}^P=0^+$ and $l^\pi=1^+$ $\longrightarrow$ $L^P=1^+$
  - Combine with quark spin, and S or AS flux symmetry:

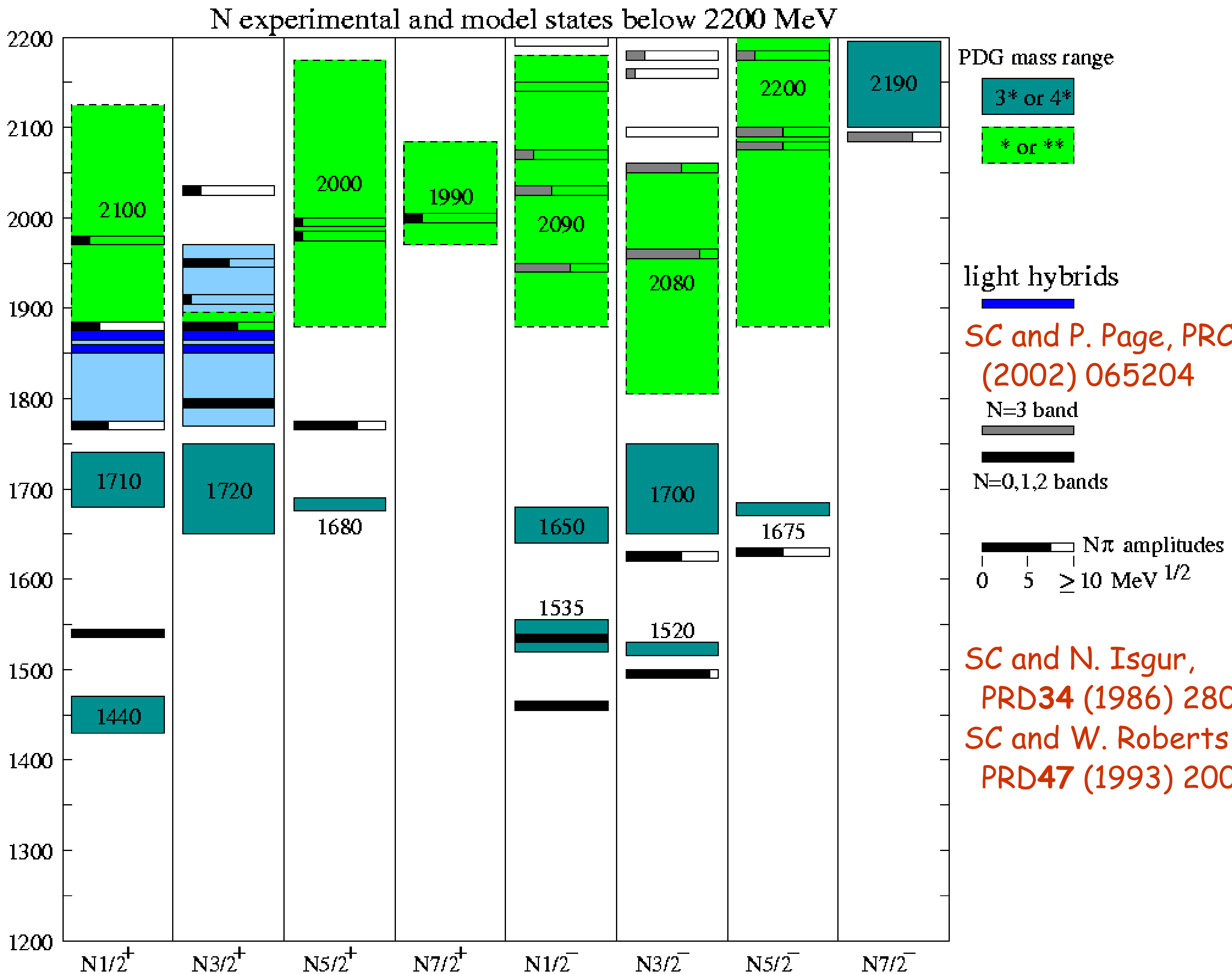
$\rightarrow$ S hybrids $(N,\Delta)^{2S+1}J^P = N^2\frac{1}{2}^+,\ N^2\frac{3}{2}^+,\ \Delta^4\frac{1}{2}^+,\ \Delta^4\frac{3}{2}^+,\ \Delta^4\frac{5}{2}^+$

$\rightarrow$ AS hybrids (flavor-spin AS) $N^2\frac{1}{2}^+,\ N^2\frac{3}{2}^+$

…Hybrid baryon masses

- Add short distance potential from one-gluon exchange
- Color structure same as conventional baryons
  - S_{qqq}=1/2 (N) states: approx. 1870 +/- 100 MeV
  - S_{qqq}=3/2 (Δ) states: approx. 2075 +/- 100 MeV
- Considerably more energetic than bag model constituent gluon $(qqq)_8g$ hybrids
  - Almost same quantum numbers as bag model
  - Bag model (mixed symmetry color for qqq):
    - S_{qqq}=1/2: $N1/2^+$, $N3/2^+$, $\Delta 1/2^+$, $\Delta 3/2^+$
    - S_{qqq}=3/2: $N1/2^+$, $N3/2^+$, $N5/2^+$

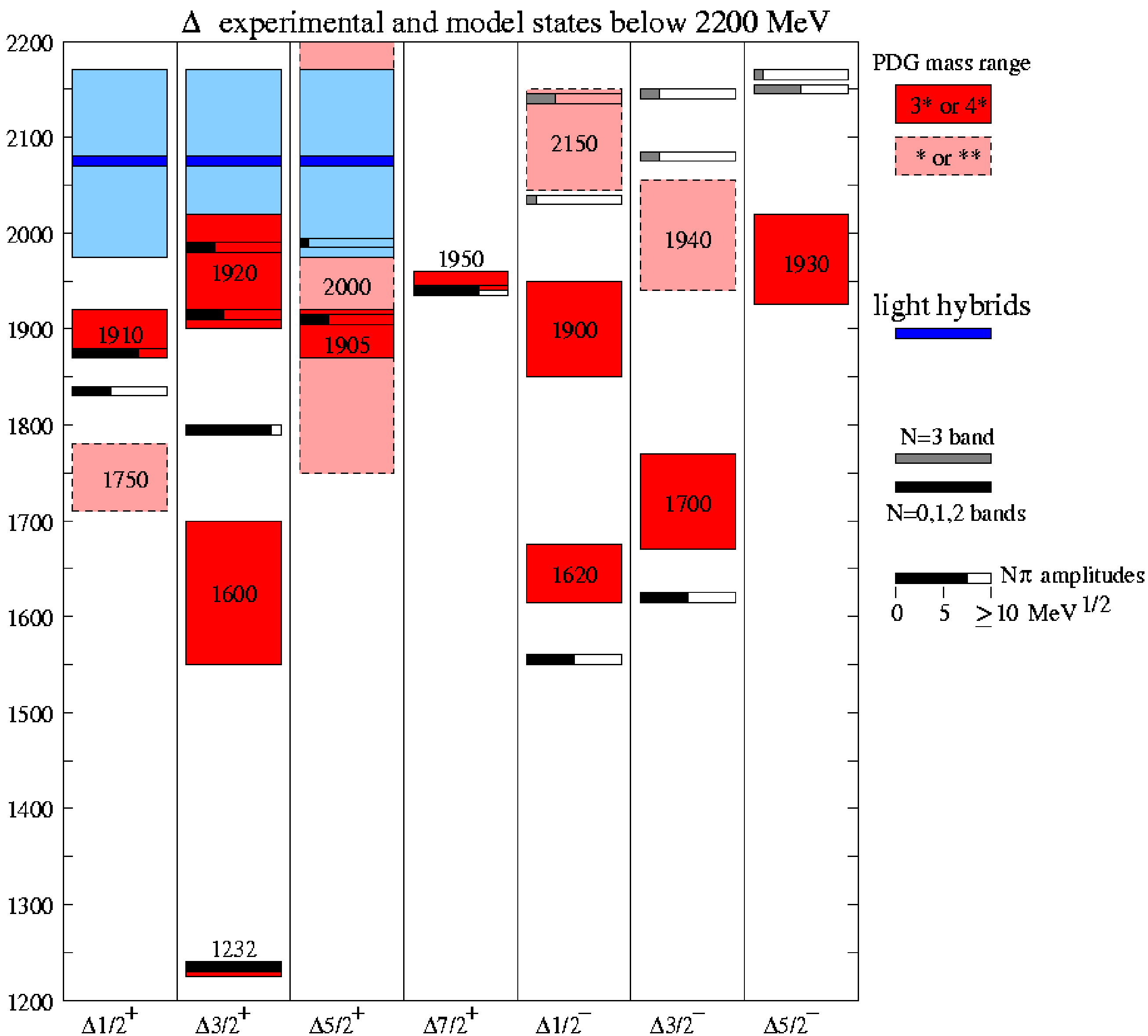
Nucleon flux-tube hybrids



SC and P. Page, PRC**66** (2002) 065204

SC and N. Isgur, PRD**34** (1986) 2809;
SC and W. Roberts, PRD**47** (1993) 2004.

Δ flux-tube hybrids



Conclusions

- Flux-tube model describes collective excitations of glue
- Predictions for light hybrids
  - Masses significantly heavier than 1500 MeV from bag model – consistent with lattice results
  - Positive-parity states with $J^P=1/2^+,3/2^+,5/2^+$
  - Lightest states $N1/2^+$, $N3/2^+$ with usual spin-spin interactions
  - Masses similar to missing conventional states with same quantum numbers

⇨ Strange hybrids, strong and EM couplings with PRP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