HUGS 2018
Jefferson Lab, Newport News, VA
May 29- June 15 2018

Fundamental Symmetries - I

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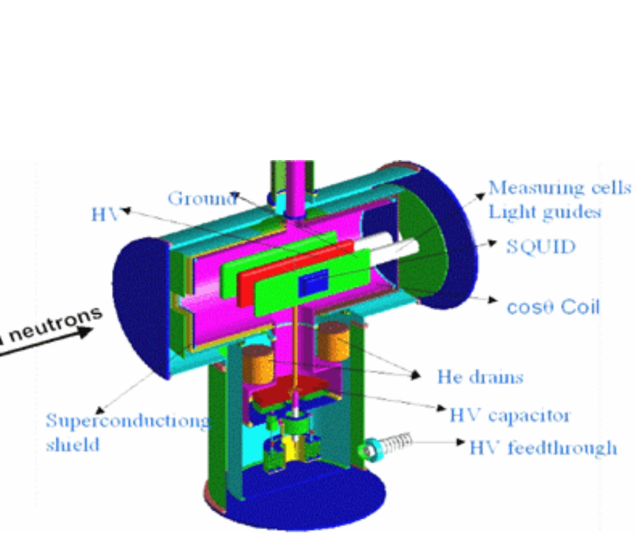
Goal of these lectures

Provide an introduction to exciting physics at the Intensity/Precision Frontier

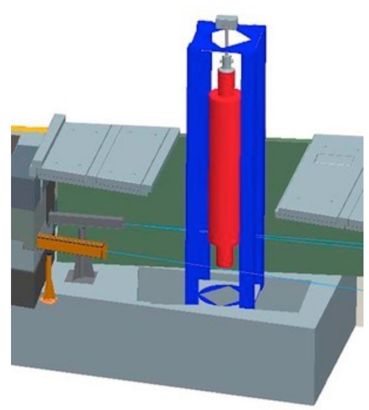
- Searches for new phenomena beyond the Standard Model through precision measurements or the study of rare processes at low energy
- (Research area called "Fundamental Symmetries" by nuclear physicists)

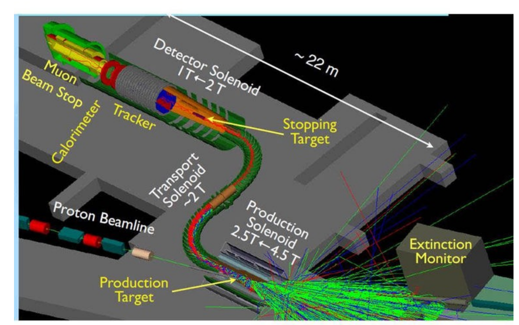


Neutron EDM

nEDM

Nab

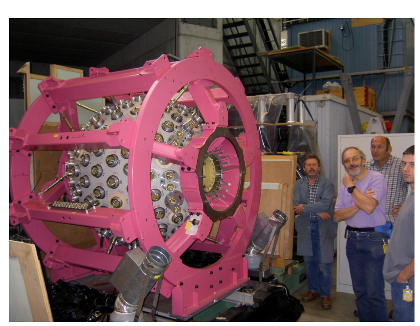


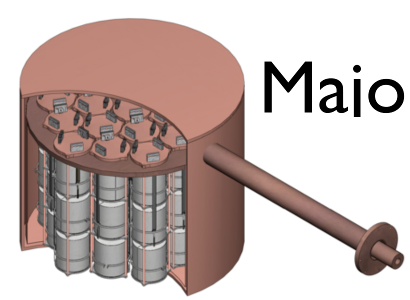


Mu2e

muon g-2

PEN



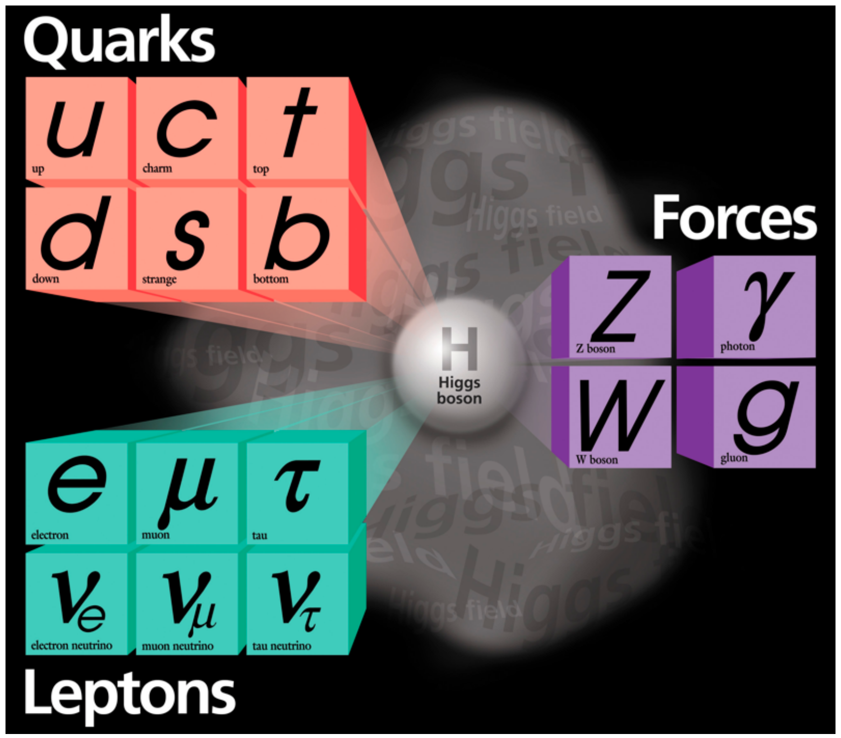


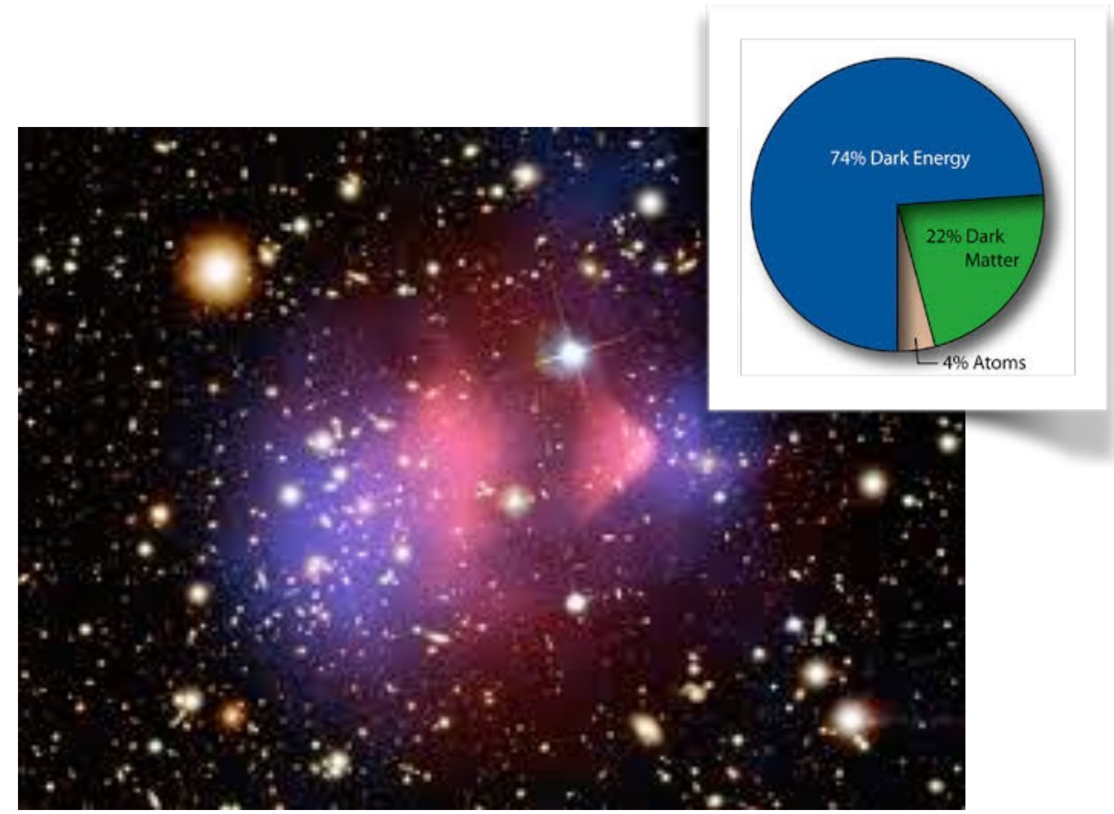
Majorana

Qweak

. . .

New physics: why?



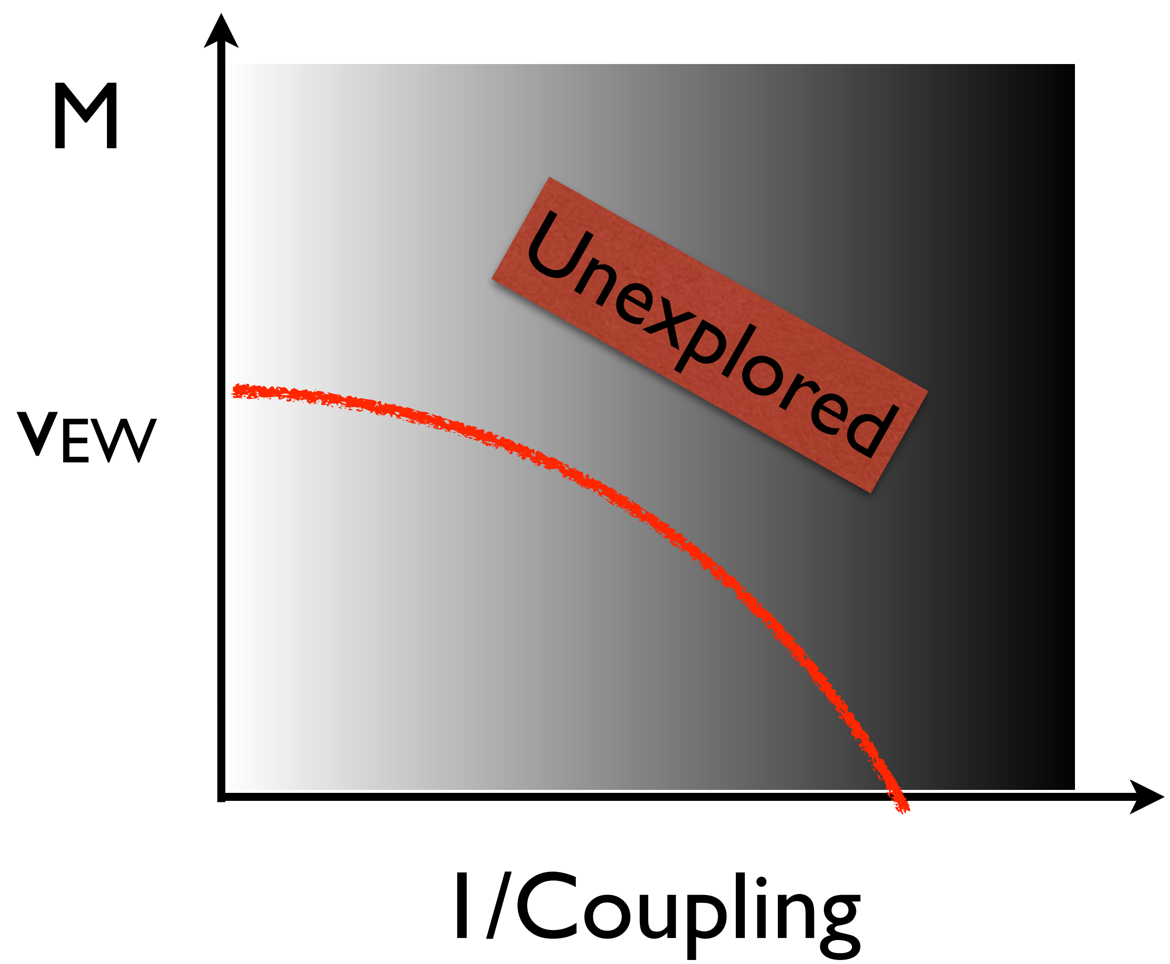


No Matter, no Dark Matter, no Dark Energy

While remarkably successful in explaining phenomena over a wide range of energies, the SM has major shortcomings

New physics: where?

- New degrees of freedom: Heavy? Light & weakly coupled?



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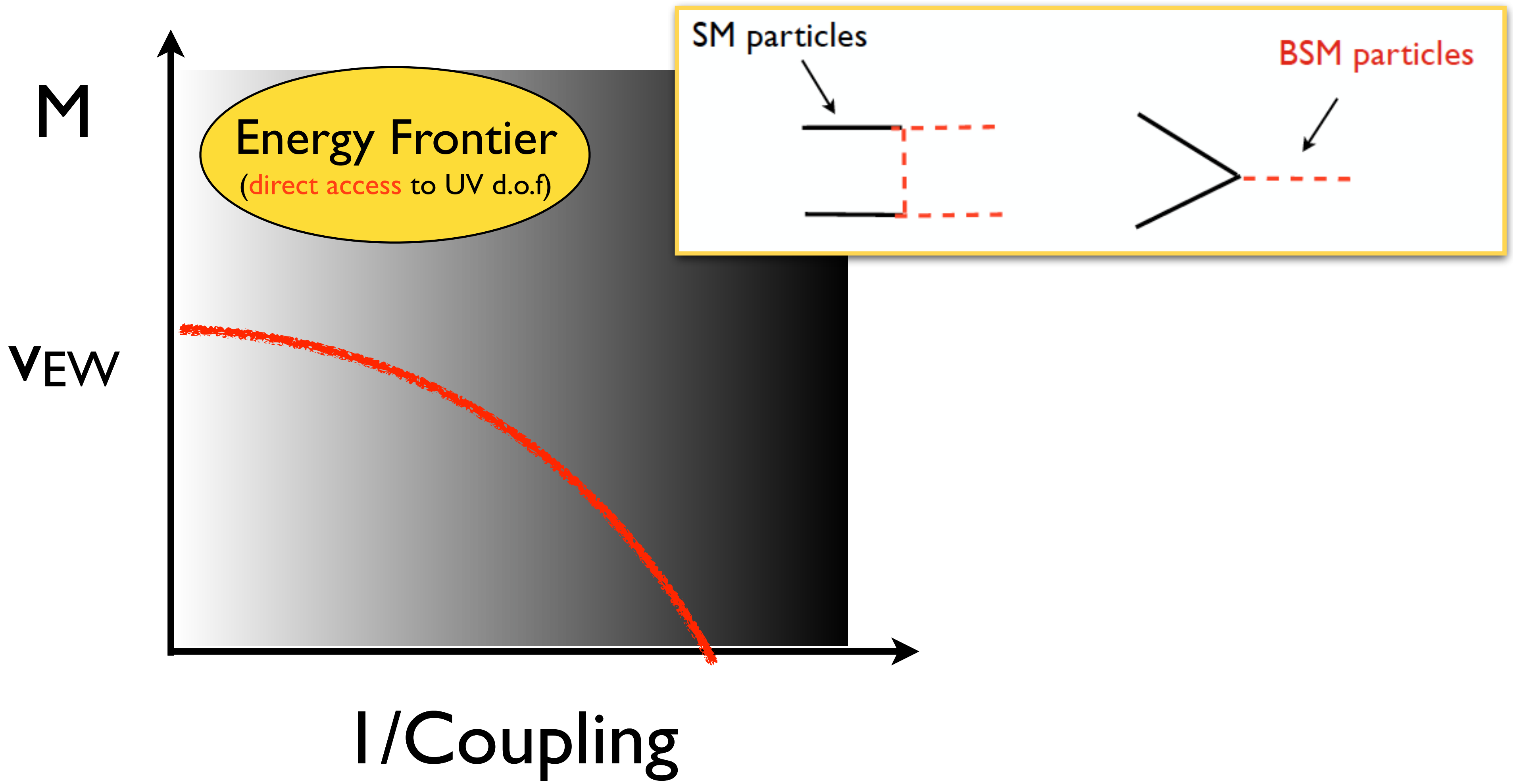

- Two experimental approaches

New physics: how?

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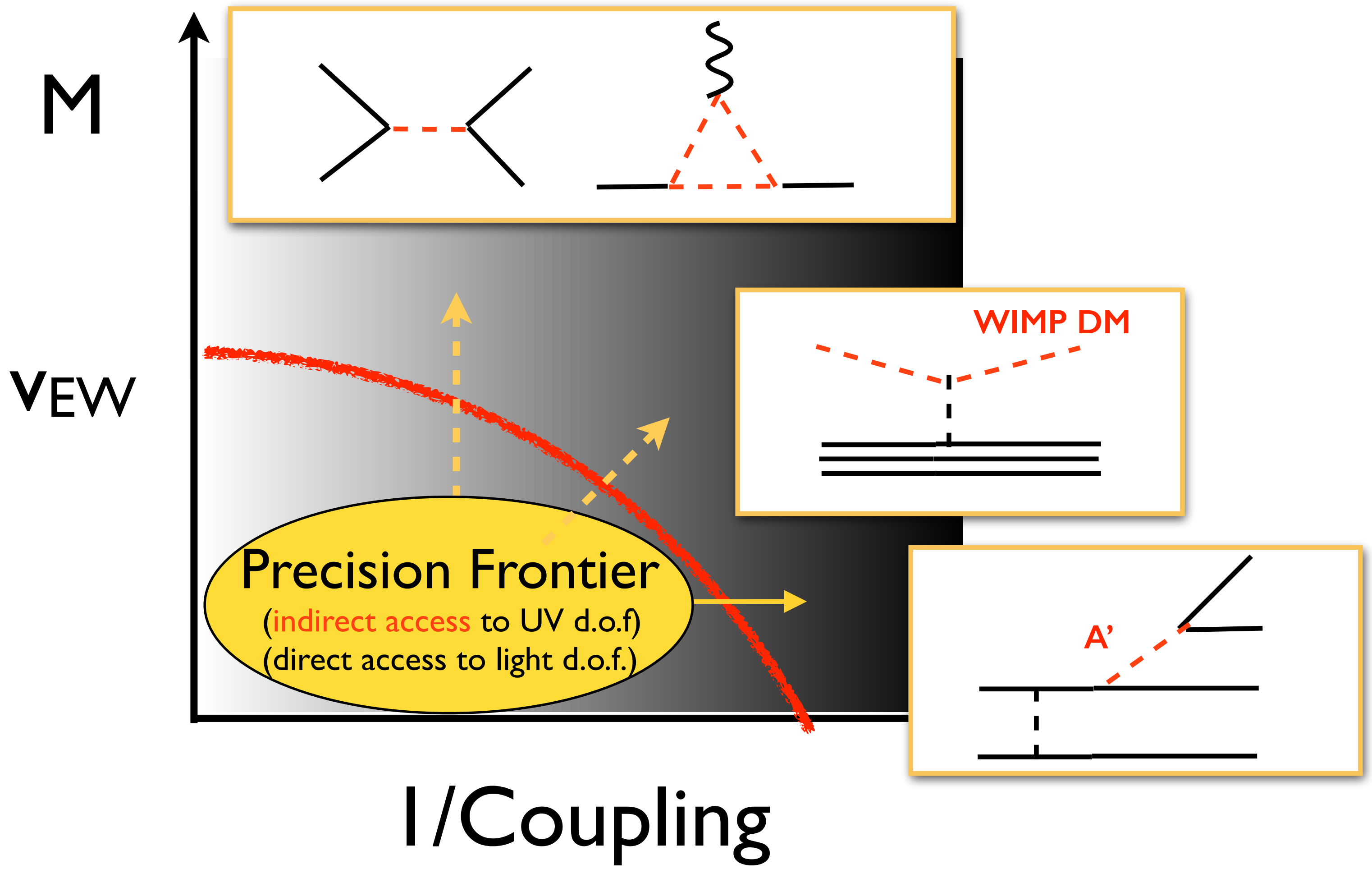

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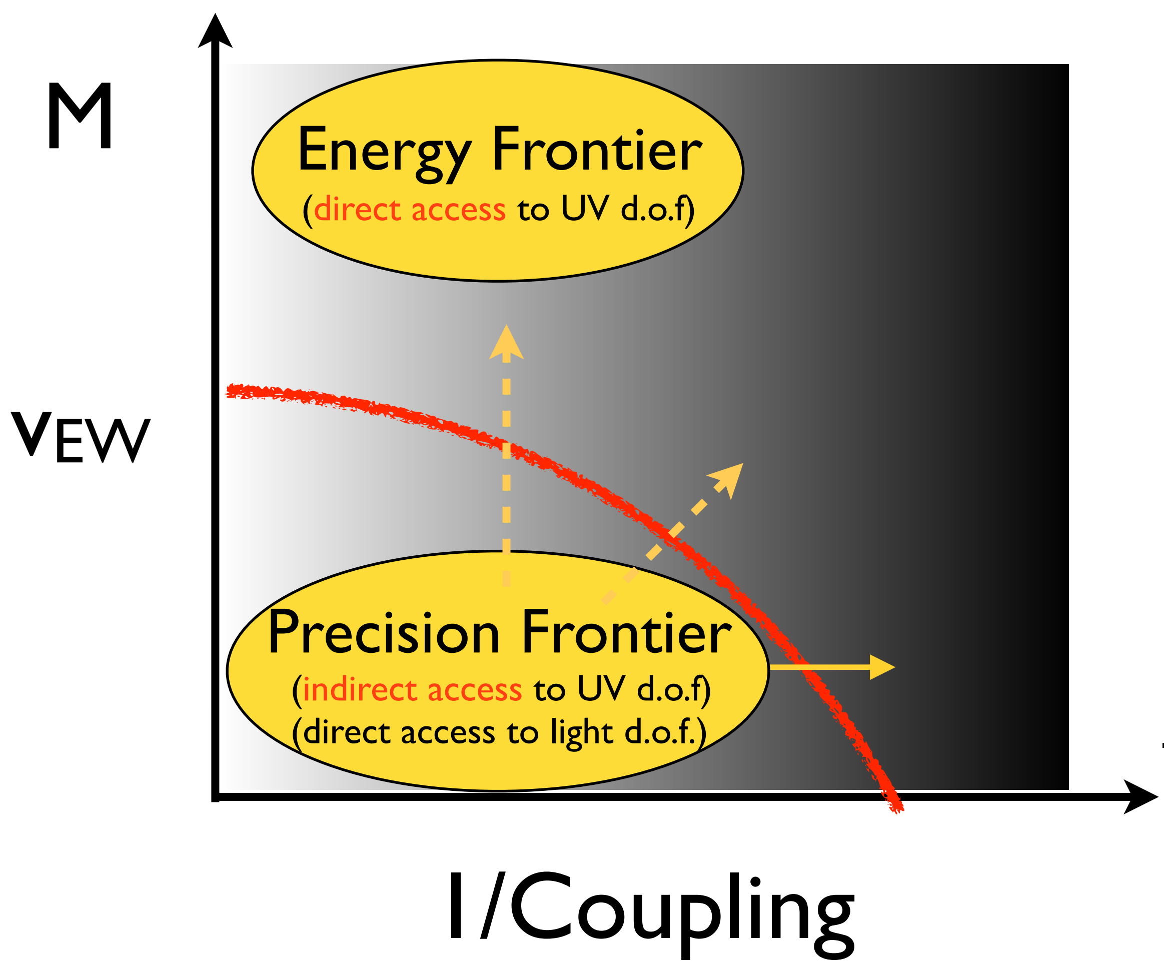

- EWSB mechanism
- Direct access to heavy particles
- ...

- L and B violation
- CP violation (w/o flavor)
- Flavor violation: quarks, leptons
- Heavy mediators: precision tests
- Neutrino properties
- Dark sectors
- …

- Two experimental approaches, both needed to reconstruct BSM dynamics: structure, symmetries, and parameters of $\mathcal{L}_{BSM}$

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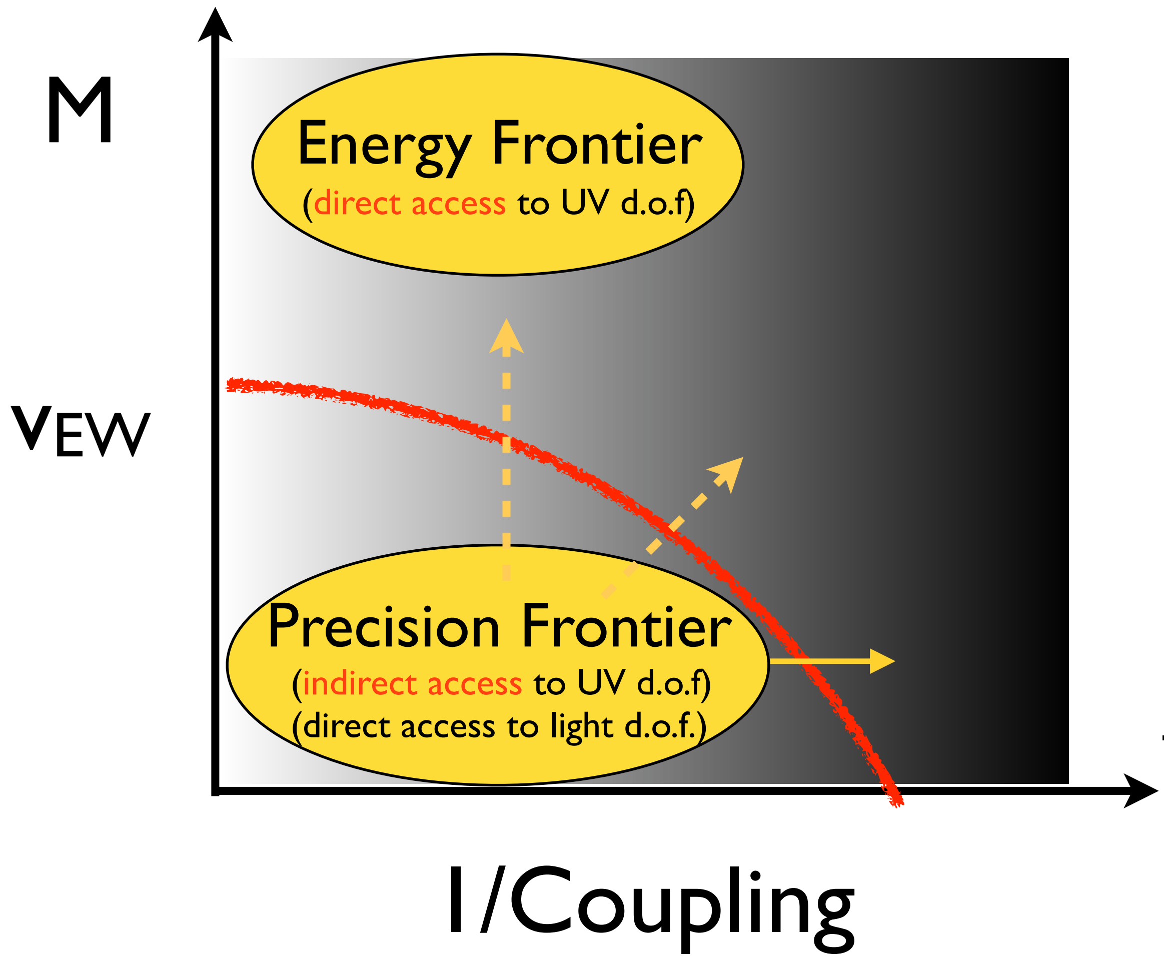

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Nuclear Science Fundamental Symmetry experiments
play a prominent role at the Precision Frontier

Plan of the lectures

- Review symmetry and symmetry breaking

- Introduce the Standard Model and its symmetries

- Beyond the SM: an effective theory perspective and overview

- Discuss a number of "worked examples"
    - Precision measurements: charged current (beta decays); neutral current (Parity Violating Electron Scattering).
    - Symmetry tests: CP (T) violation and EDMs; Lepton Number violation and neutrino-less double beta decay.

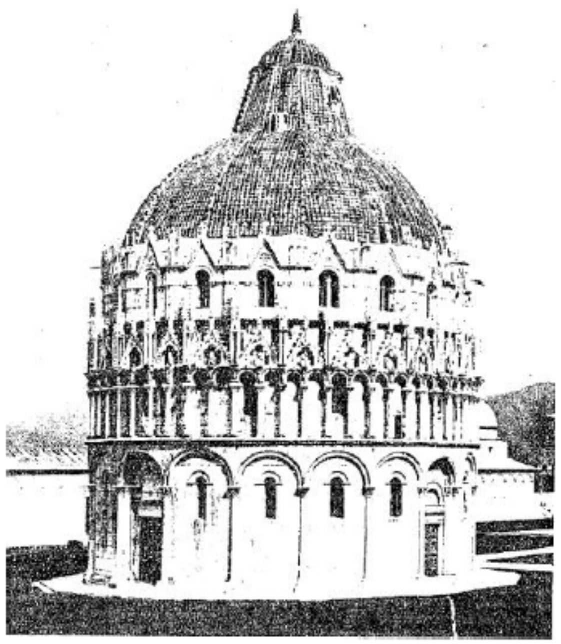
Symmetry
and
symmetry breaking

What is symmetry?

- "A thing** is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before" (Feynman paraphrasing Weyl)

  **An object or a *physical law*

Translational symmetry



Rotational symmetry

Images from H. Weyl, "Symmetry". Princeton University Press, 1952

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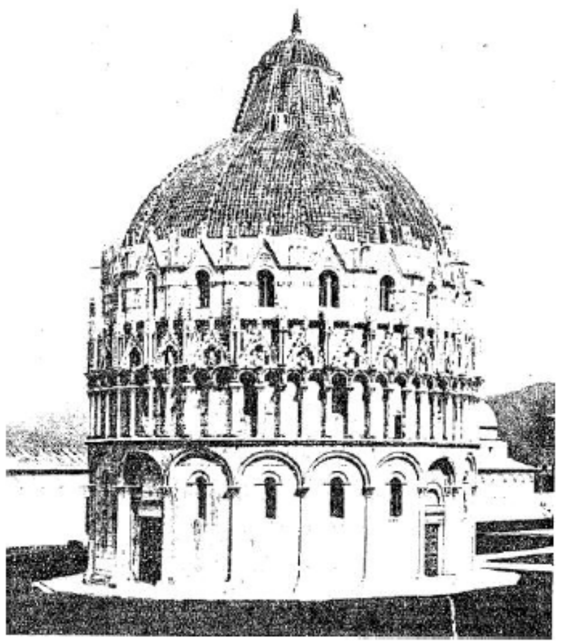

Rotational symmetry

Images from H. Weyl, "Symmetry". Princeton University Press, 1952

- "A *symmetry transformation* is a change in our point of view that does not change the results of possible experiments" (Weinberg)

What is symmetry?

- A transformation of the dynamical variables that leaves the action unchanged (equations of motion invariant)

$$\boxed{\begin{gathered} q(t) \to q'(t) = R[q(t)] \\ \int_D dt\, \mathcal{L}[q(t), \dot{q}(t)] = \int_D dt\, \mathcal{L}[q'(t), \dot{q}'(t)] \end{gathered}}$$

$$\frac{d}{dt}\frac{\delta \mathcal{L}}{\delta \dot{q}_i} = \frac{\delta \mathcal{L}}{\delta q_i}$$

What is symmetry?

- A transformation of the dynamical variables that leaves the action unchanged (equations of motion invariant)

$$x \to x' \qquad \phi(x) \to \phi'(x') = R\phi(x)$$

$$\int_D d^4x\, \mathcal{L}[\phi(x), \partial_\mu \phi(x)] = \int_{D'} d^4x'\, \mathcal{L}[\phi'(x'), \partial_\mu \phi'(x')]$$

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))} = \frac{\delta \mathcal{L}}{\partial \phi_i(x)}$$

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$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))} = \frac{\delta \mathcal{L}}{\partial \phi_i(x)}$$

- Symmetry transformations have mathematical "group" structure: existence of identity and inverse transformation, composition rule

Examples of symmetries

- Space-time symmetries
  - Continuous (translations, rotations, boosts: Poincare')

$$x \to x' = \Lambda x - a \qquad \Lambda : \quad t^2 - \mathbf{x}^2 = t'^2 - \mathbf{x}'^2$$

$$\psi(x) \to \psi'(x) = S(\Lambda)\, \psi(\Lambda^{-1} x)$$

Examples of symmetries

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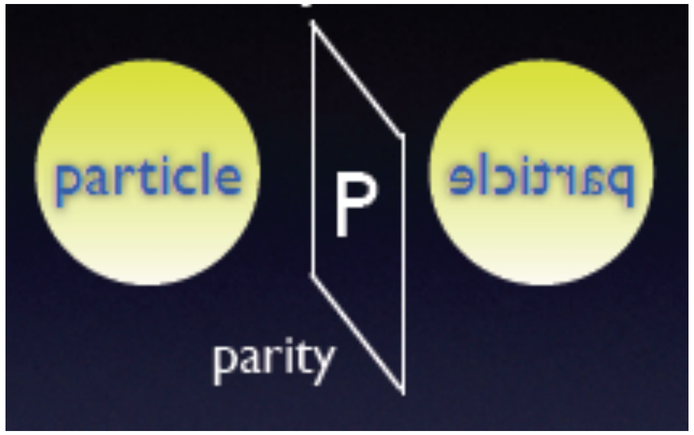
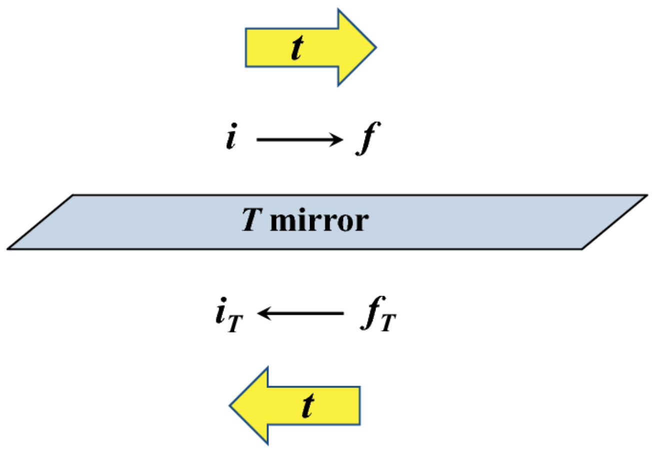
$$x \to x' = \Lambda x - a \qquad \Lambda : \quad t^2 - \mathbf{x}^2 = t'^2 - \mathbf{x}'^2$$

  - Discrete (Parity, Time-reversal)

$$t' = t \quad \mathbf{x}' = -\mathbf{x} \qquad\qquad t' = -t \quad \mathbf{x}' = \mathbf{x}$$





  - Local (general coordinate transformations)

Examples of symmetries

- "Internal" symmetries
  - Continuous

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$$

Dirac matrices

$$\psi(x) \;\rightarrow\; e^{i\epsilon}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

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U(1)

$$\begin{pmatrix} n \\ p \end{pmatrix} \rightarrow e^{i\epsilon^a\,\sigma^a/2} \begin{pmatrix} n \\ p \end{pmatrix}$$

SU(2) - isospin

(if $m_n = m_p$)

Examples of symmetries

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U(1)

  - Discrete: Z_2 , charge conjugation, …

$$\phi(x) \rightarrow -\phi(x) \qquad\qquad \mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi^2)$$

  - Local (gauge)

$$\psi(x) \;\rightarrow\; e^{i\epsilon(x)}\,\psi(x) \qquad\qquad \mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi$$

U(1)

?

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U(1)

Leftover piece: $-\bar{\psi}\gamma_\mu\psi\,\partial^\mu\epsilon$

Examples of symmetries

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$$\phi(x) \rightarrow -\phi(x) \qquad \mathcal{L} = \partial_\mu\phi\partial^\mu\phi - V(\phi^2)$$

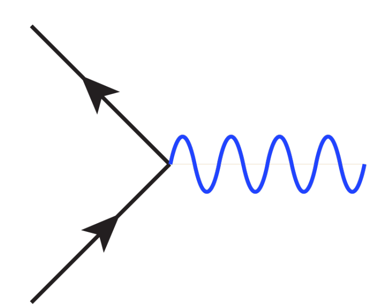
  - Local (gauge)

$$\psi(x) \rightarrow e^{i\epsilon(x)}\psi(x)$$

$$A^\mu \rightarrow A^\mu + \frac{1}{g}\partial^\mu\epsilon$$

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu\partial^\mu - m\right)\psi + gA^\mu\bar{\psi}\gamma_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

U(1)



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Implications of symmetry

- If a state is realized in nature, its "transformed" is also possible
- Time evolution and transformation commute: for a given initial state, transformed of the evolved = evolved of the transformed

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- In Quantum Mechanics
  - Symmetries represented by (anti)-unitary operators U_S (Wigner)

$$|\langle a|U_S^\dagger U_S|b\rangle|^2 = |\langle a|b\rangle|^2$$

  - U_S commutes with Hamiltonian $[U_S, H] = 0$

  - Classification of the states of the system, selection rules, …

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- Continuous symmetries imply conservation laws

Symmetry	Conservation law
Time translation	Energy
Space translation	Momentum
Rotation	Angular momentum
U(1) phase	Electric charge
...	...

$$\psi(x) \rightarrow e^{i\epsilon}\,\psi(x)$$

Emmy Noether

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Rotation	Angular momentum
U(1) phase	#particles - #anti-particles
…	…

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$$\begin{aligned} x^\mu &\to x^\mu + \delta x^\mu \\ \phi(x) &\to \phi(x) + \delta\phi(x) \end{aligned}$$

$$\begin{aligned} \delta x^\mu &= \epsilon^a A^\mu_a \\ \delta\phi(x) &= \epsilon^a (M_a \phi - A^\mu_a \partial_\mu \phi) \end{aligned}$$

$$\frac{d}{dt}\int d^3x J^0_a(x) = 0$$

$$\boxed{\begin{gathered} \partial_\mu J^\mu_a = 0 \\ J^\mu_a = -\frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_i(x))}\,\frac{\delta\phi_i}{\delta\epsilon^a} - \mathcal{L}\,\frac{\delta x^\mu}{\delta\epsilon^a} \end{gathered}}$$

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Implications of symmetry

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- Symmetry principles strongly constrain or even dictate the form of the laws of physics
    - General relativity
    - …
    - Gauge theories

Discrete symmetries in QM

- Parity $\quad \mathbf{x} \to -\mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to \mathbf{s}$
  - Implemented by unitary operator $P\psi(\mathbf{x}) = \psi(-\mathbf{x})$
  - If [H,P] = 0, P cannot change in a reaction; expectation values of P-odd operators vanish

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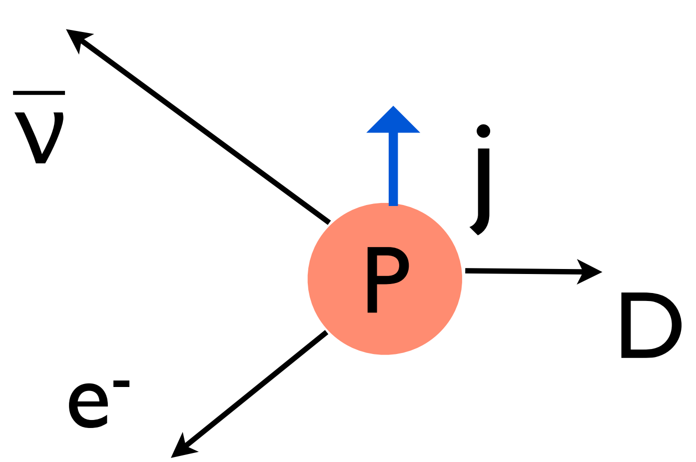
Simple problem: in polarized nuclear beta decay, which of the correlation coefficients a,b,A,B signals parity violation?

$$d\Gamma \propto F(E_e) \left\{ 1 + b\,\frac{m_e}{E_e} + a\,\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \frac{\langle \vec{J} \rangle}{|\vec{J}|} \cdot \left[A\,\frac{\vec{p}_e}{E_e} + B\,\frac{\vec{p}_\nu}{E_\nu} + \cdots \right] \right\}$$



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- Time reversal $\quad t \to -t \qquad \mathbf{x} \to \mathbf{x} \qquad \mathbf{p} \to -\mathbf{p} \qquad \mathbf{s} \to -\mathbf{s}$
  - Implemented by anti-unitary operator $T\psi(\mathbf{x}) = U\,\psi^*(\mathbf{x})$: U flips the spin
  - If H is real in coordinate representation, T is a good symmetry ([T,H]=0)

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- Charge conjugation $\quad |p\rangle \leftrightarrow |\bar{p}\rangle$

  - Particles that coincide with antiparticles are eigenstates of C, e.g. $C|\gamma\rangle = -|\gamma\rangle$
  - C-invariance ([C,H]=0) → C cannot change in a reaction. From EM decay $\pi^0 \to \gamma\gamma$, deduce C-transformation of π^0

Discrete symmetries in QFT

- In the free theory: P, T and C transformations are symmetries
- They can be implemented by (anti)unitary operators
- On the states:

$$U_P|r,\mathbf{p}\rangle = \eta_P|r,-\mathbf{p}\rangle$$

$$U_T|r,\mathbf{p}\rangle = \eta_T S_{rr'}|r',-\mathbf{p}\rangle$$

$$\begin{aligned} U_C^\dagger b(r,\mathbf{p})U_C &= d(r,\mathbf{p}) \\ U_C^\dagger d(r,\mathbf{p})U_C &= b(r,\mathbf{p}) \end{aligned}$$

η_A= phases

r = spin label

$S_{rr'}$ reverses spin

b (d) = (anti)particle annihilation operator

Discrete symmetries in QFT

- In the free theory: P, T and C transformations are symmetries
- They can be implemented by (anti)unitary operators
- On the fields:

Scalar field

$$
\begin{aligned}
U_P^\dagger \phi(t,\mathbf{x}) U_P &= \eta_P \phi(t,-\mathbf{x}) \\
U_T^\dagger \phi(t,\mathbf{x}) U_T &= \eta_T \phi(-t,\mathbf{x}) \\
U_C^\dagger \phi(t,\mathbf{x}) U_C &= \eta_C \phi^\dagger(t,\mathbf{x})
\end{aligned}
$$

Vector field

$$
\begin{aligned}
U_P^\dagger V_\mu(t,\mathbf{x}) U_P &= \eta_P \, g_{\mu\mu} \, V_\mu(t,-\mathbf{x}) \\
U_T^\dagger V_\mu(t,\mathbf{x}) U_T &= \eta_T \, g_{\mu\mu} \, V_\mu(-t,\mathbf{x}) \\
U_C^\dagger V_\mu(t,\mathbf{x}) U_C &= \eta_C \, V_\mu^\dagger(t,\mathbf{x})
\end{aligned}
$$

Spin 1/2:

$$
\begin{aligned}
U_P^\dagger \psi(t,\mathbf{x}) U_P &= \eta_P \, \gamma_0 \, \psi(t,-\mathbf{x}) \\
U_T^\dagger \psi(t,\mathbf{x}) U_T &= \eta_T \, (i\gamma_5\gamma_0\gamma_2) \, \psi(-t,\mathbf{x}) \\
U_C^\dagger \psi(t,\mathbf{x}) U_C &= \eta_C \, (i\gamma_2) \, \psi^*(t,\mathbf{x})
\end{aligned}
$$

Discrete symmetries in QFT

- In the free theory: P, T and C transformations are symmetries
- They can be implemented by (anti)unitary operators
- On fermion bilinears:

Bilinear	P	C	T	CP	CPT
$\overline{\psi}_1\psi_2$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_1\psi_2$	$\overline{\psi}_2\psi_1$	$\overline{\psi}_2\psi_1$
$\overline{\psi}_1\gamma_5\psi_2$	$-\overline{\psi}_1\gamma_5\psi_2$	$\overline{\psi}_2\gamma_5\psi_1$	$-\overline{\psi}_1\gamma_5\psi_2$	$-\overline{\psi}_2\gamma_5\psi_1$	$\overline{\psi}_2\gamma_5\psi_1$
$\overline{\psi}_1\gamma_\mu\psi_2$	$\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma_\mu\psi_1$	$\overline{\psi}_1\gamma^\mu\psi_2$	$-\overline{\psi}_2\gamma^\mu\psi_1$	$-\overline{\psi}_2\gamma_\mu\psi_1$
$\overline{\psi}_1\gamma_\mu\gamma_5\psi_2$	$-\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$	$\overline{\psi}_1\gamma^\mu\gamma_5\psi_2$	$-\overline{\psi}_2\gamma^\mu\gamma_5\psi_1$	$-\overline{\psi}_2\gamma_\mu\gamma_5\psi_1$
$\overline{\psi}_1\sigma_{\mu\nu}\psi_2$	$\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma_{\mu\nu}\psi_1$	$-\overline{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\overline{\psi}_2\sigma^{\mu\nu}\psi_1$	$\overline{\psi}_2\sigma_{\mu\nu}\psi_1$

Discrete symmetries in QFT

- In the free theory: P, T and C transformations are symmetries
- They can be implemented by (anti)unitary operators
- In interacting theory one uses the above definitions and checks whether they leave action invariant
- Individual C, P, and T are not necessarily symmetries, but CPT is!

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CPT theorem: hermitian & Lorentz invariant Lagrangian transforms as

$$\mathcal{L}(x) \rightarrow \mathcal{L}(-x)$$

CPT invariance!   CP violation is equivalent to T violation

Symmetry breaking

- Three known mechanisms

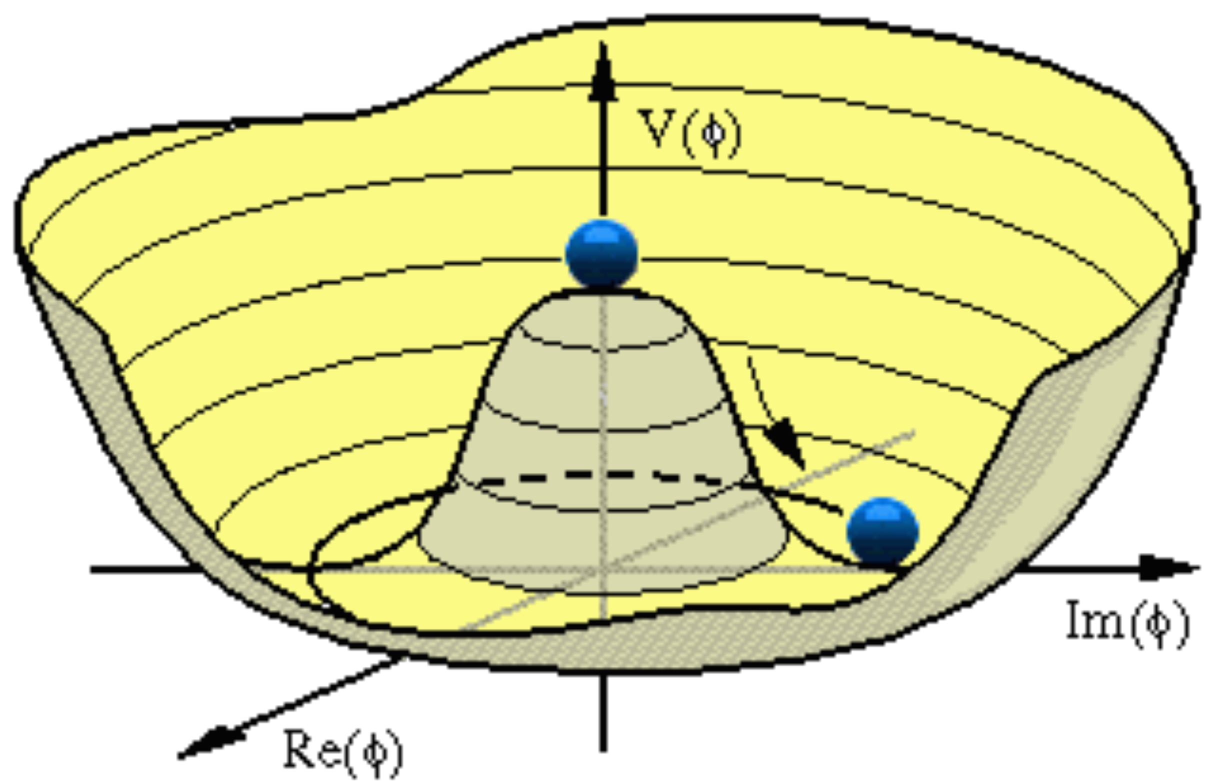
  - Explicit symmetry breaking
    - Symmetry is approximate; still very useful (e.g. isospin)

  - Spontaneous symmetry breaking
    - Equations of motion invariant, but ground state is not

  - Anomalous (quantum mechanical) symmetry breaking
    - Classical invariance but no symmetry at QM level

Spontaneous symmetry breaking

- Action is invariant, but ground state is not
- Continuous symmetry: degenerate physically equivalent minima
- Excitations along the valley of minima → massless states in the spectrum (Goldstone Bosons)



- Many examples of Goldstone bosons in physics: phonons (sound waves) in solids; spin waves in magnets; pions in QCD

Anomalous symmetry breaking

- Action is invariant, but path-integral measure is not!

$$\boxed{\int [d\psi][d\bar{\psi}]\; e^{iS[\psi,\bar{\psi}]}}$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

$$S[\psi,\bar{\psi}] = S[\psi',\bar{\psi}']$$

$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

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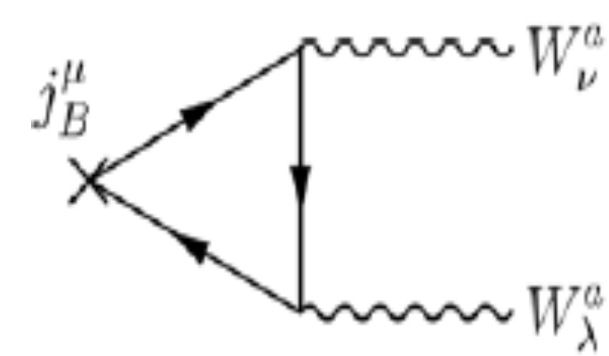
$$\int [d\psi][d\bar{\psi}] = \int [d\psi'][d\bar{\psi}']\,\mathcal{J} \qquad \mathcal{J} \neq 1$$

$$\psi \to \psi' \qquad \bar{\psi} \to \bar{\psi}'$$

- Important example: Baryon (B) and Lepton (L) number in the SM

$$\boxed{\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right)}$$

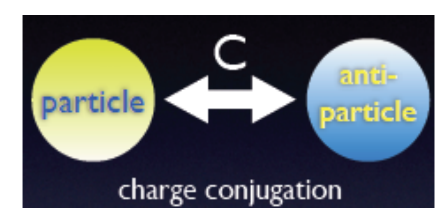


- Only B-L is conserved; B+L is violated; negligible at zero temperature

Symmetry breaking and the origin of matter

- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

1. B (baryon number) violation
   - To depart from initial (post inflation) B=0
2. C and CP violation $\boxed{\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})}$
   - To distinguish baryon and anti-baryon production



3. Departure from thermal equilibrium
   - <B(t)>=<B(0)>=0 in equilibrium

Sakharov '67

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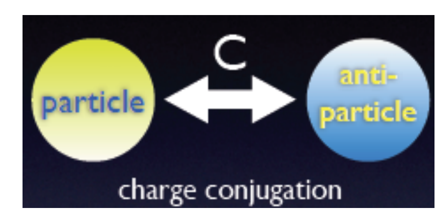

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Symmetry breaking and the origin of matter

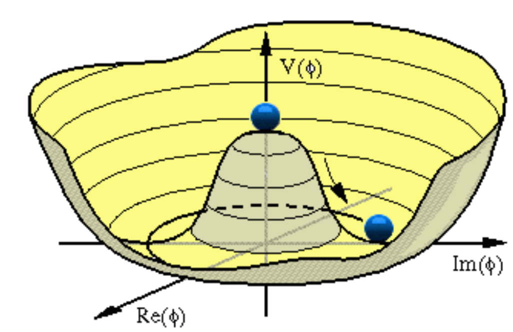
- The dynamical generation of net baryon number during cosmic evolution requires the concurrence of three conditions:

- In weak-scale baryogenesis scenarios (T~100 GeV), the ingredients are tied to all known mechanisms of symmetry breaking:

Sakharov '67

1. B (baryon number) violation — anomalous
2. C and CP violation — explicit
3. Departure from thermal equilibrium — spontaneous (symmetry restoration at hight T: 1st order phase transition?)

$$\langle\phi\rangle \neq 0 \;\Rightarrow\; SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$



More on gauge symmetry

- Classical electrodynamics: $A_\mu \to A_\mu + \partial_\mu \varphi$ does not change **E** and **B**

> This gauge invariance is, of course, an artificial one, similar to that which we could obtain by introducing into our equations the location of a ghost. The equations must then be invariant with respect to changes of coordinates of that ghost. One does not see, in fact, what good the introduction of the coordinate of the ghost does.

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E.Wigner

- Dramatic paradigm shift in the 60's and 70's: gauge invariance requires the existence of spin-1 particles (the gauge bosons)

- Successful description of strong and electroweak interactions

> "Symmetry dictates dynamics"

C. N. Yang

Non abelian gauge symmetry

- Recall U(1) (abelian) example

$$\psi(x) \;\rightarrow\; e^{i\epsilon(x)}\,\psi(x)$$

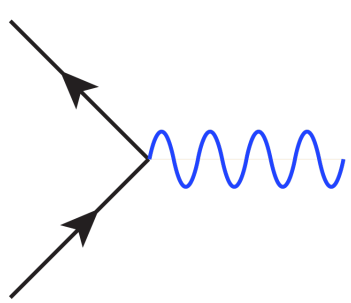
$$A^\mu \;\rightarrow\; A^\mu + \frac{1}{g}\partial^\mu \epsilon$$

$$\boxed{\begin{aligned}\mathcal{L} &= \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi \\ &+ g\,\bar{\psi}\gamma_\mu\,A^\mu\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Form of the interaction:

$$\mathcal{L}_{\text{int}} \;=\; g\,A_\mu\,J^\mu$$



$$J^\mu = \bar{\psi}\gamma^\mu\psi$$

conserved current associated with global U(1)

Non abelian gauge symmetry

- Generalize to non-abelian group G (e.g. SU(2), SU(3), …). $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \dots \end{pmatrix}$

$$\psi(x) \;\rightarrow\; U(x)\,\psi(x) \qquad U(x) = e^{i\epsilon^a(x)T^a} \qquad [T^a, T^b] = i f^{abc} T^c$$

- Invariant dynamics if introduce new vector fields $A_\mu = A_\mu^a\, T^a$ transforming as

$$A^\mu \;\rightarrow\; U\,A^\mu\,U^\dagger - \frac{i}{g}(\partial^\mu U)U^\dagger$$

$$\boxed{\mathcal{L} = \bar{\psi}\,(i\gamma_\mu\partial^\mu - m)\,\psi \,+\, g\,\bar{\psi}\gamma^\mu\,T^a A_\mu^a\,\psi \,-\, \frac{1}{2}\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu})}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] \qquad F_{\mu\nu} \rightarrow U\,F_{\mu\nu}\,U^\dagger$$

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$$\bar\psi\, i\gamma^\mu D_\mu\, \psi \qquad D_\mu \equiv \partial_\mu - igT^a A_\mu^a$$

covariant derivative

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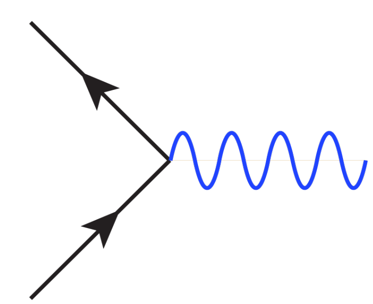


conserved currents associated with global G symmetry

Spontaneously broken gauge symmetry

- Abelian Higgs model: complex scalar field coupled to EM field

$$\phi(x) \to e^{i\epsilon(x)}\phi(x)$$

$$A^\mu \;\to\; A^\mu - \frac{1}{e}\partial^\mu \epsilon$$

$$\boxed{\begin{aligned} \mathscr{L} &= -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi) \\ V(\phi) &= -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^* \phi)^2 \end{aligned}}$$

$$D_\mu = \partial_\mu + ieA_\mu$$

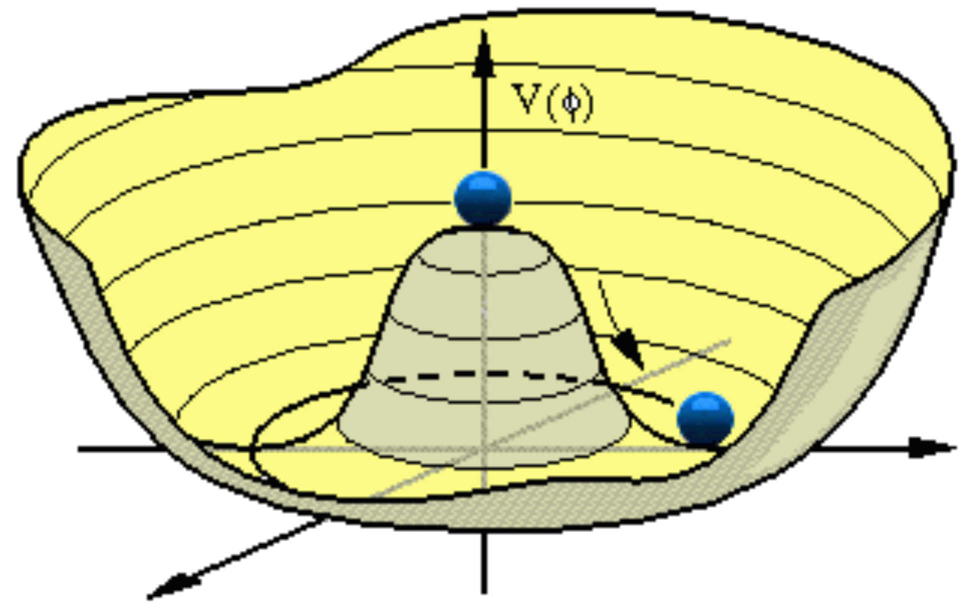
$\mu^2 < 0 \qquad |\langle\phi\rangle| = 0$ QED of charged scalar boson

$\mu^2 > 0 \qquad |\langle\phi\rangle| = |\phi_0| = \left(\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}$ U(1) spontaneously broken

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)} \left(\phi_0 + \frac{\beta(x)}{\sqrt{2}} \right)$$

$$\phi_0 = \frac{\mu}{\sqrt{\lambda}}$$



$$V(\phi) = -\frac{1}{2}\frac{\mu^4}{\lambda} + \mu^2\beta^2(x) + O(\beta^3(x))$$

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\beta)^2 + e^2\left(\phi_0 + \frac{\beta(x)}{\sqrt{2}}\right)^2 (A_\mu - \partial_\mu\alpha)^2$$

- β(x) describes massive scalar field $m_\beta^2 = 2\lambda\phi_0^2$ (radial mode)
- α(x) (Goldstone) can be removed by a gauge transformation

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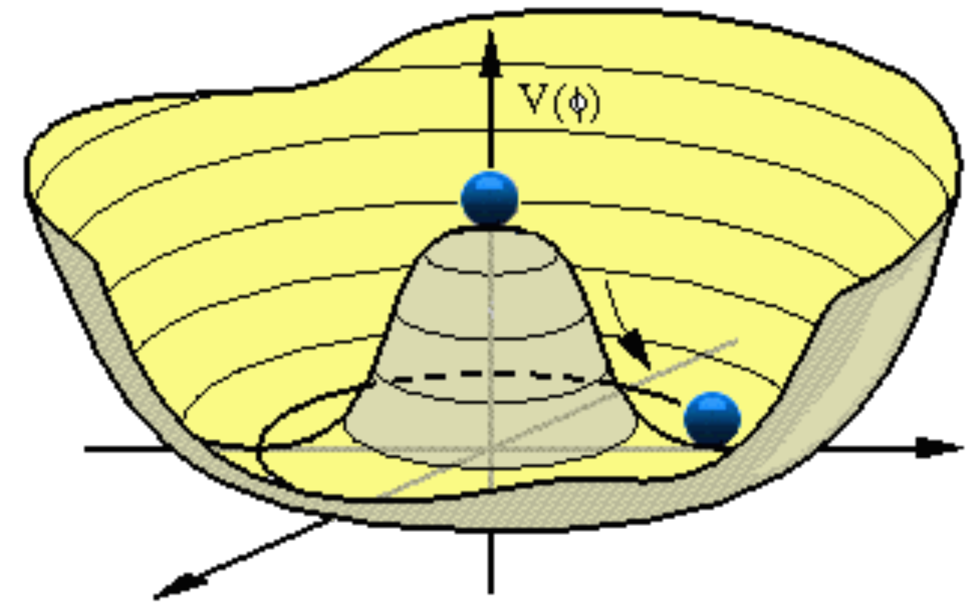

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- β(x) describes massive scalar field $m_\beta^2 = 2\lambda\phi_0^2$ (radial mode)
- α(x) (Goldstone) can be removed by a gauge transformation
- Photon has acquired mass $m_A^2 = 2e^2\phi_0^2$

- Expand around minimum of the potential (in polar representation)

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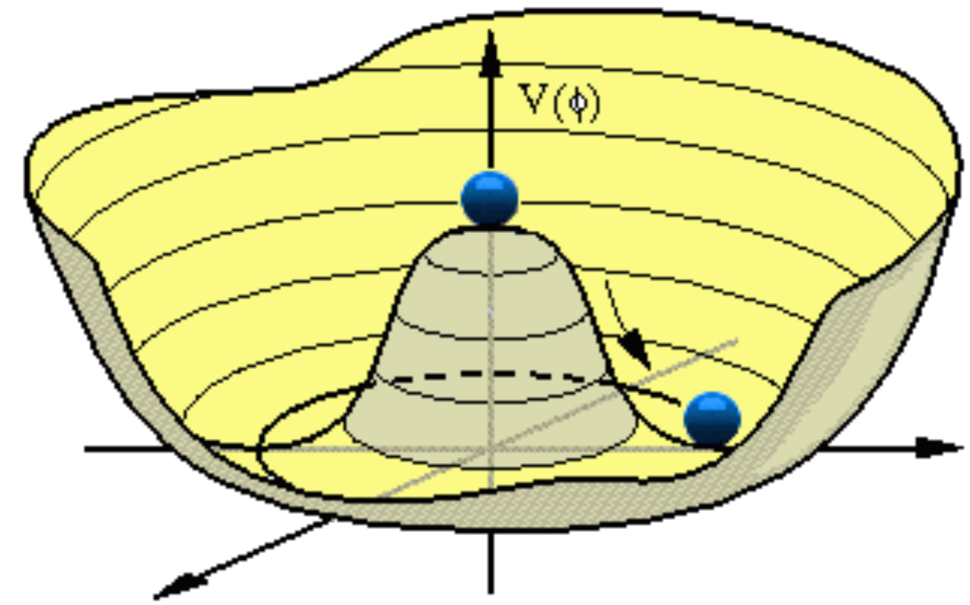

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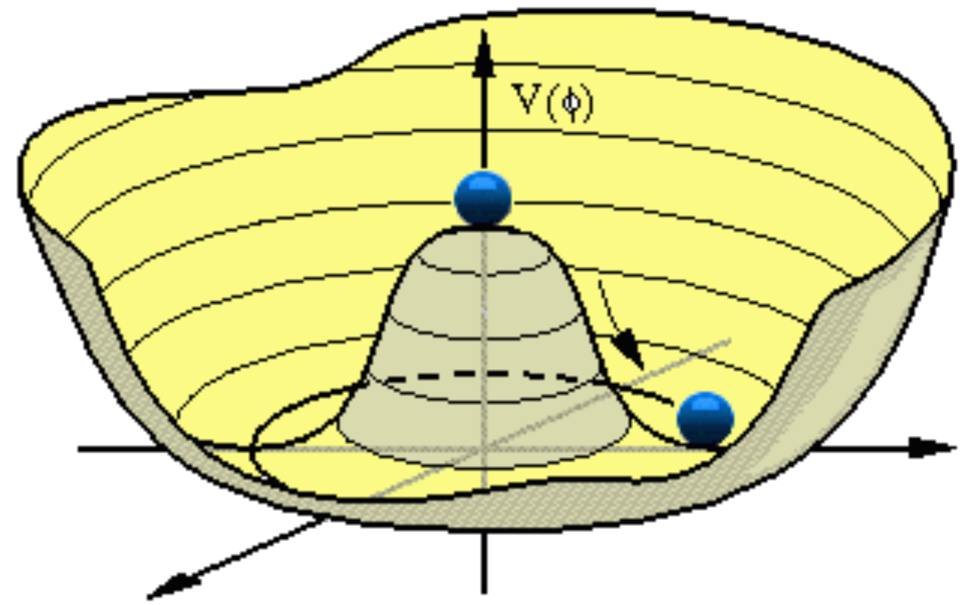
- Count degrees of freedom:
  - Massless vector (2) + complex scalar (2) = 4
  - Massive vector (3) + real scalar (1) = 4

- Expand around minimum of the potential (in polar representation)

$$\phi(x) = e^{-ie\alpha(x)}\left(\phi_0 + \frac{\beta(x)}{\sqrt{2}}\right)$$



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- Higgs phenomenon holds beyond U(1) model: in a gauge theory with SSB, Goldstone modes appear as longitudinal polarization of *massive* spin-1 gauge bosons

Additional material

Lorentz transformation

$$x \to x' = \Lambda x - a \qquad \Lambda : \quad t^2 - \mathbf{x}^2 = t'^2 - \mathbf{x}'^2$$

$$\Lambda = e^{\omega_{\mu\nu}\mathcal{M}^{\mu\nu}} \qquad (\mathcal{M}^{\rho\sigma})^{\mu\nu} = \eta^{\rho\mu}\,\eta^{\sigma\nu} - \eta^{\sigma\mu}\eta^{\rho\nu}$$

$\omega_{\mu\nu}$: real parameters

Six anti-symmetric generators

Spin 0

$$\phi(x) \to \phi'(x) = \phi(\Lambda^{-1}x)$$

Spin 1/2

$$\psi(x) \to \psi'(x) = e^{-\frac{i}{2}\omega_{\mu\nu}\sigma^{\mu\nu}}\,\psi(\Lambda^{-1}x)$$

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$$

Spin 1

$$V_\mu(x) \to V'_\mu(x) = \Lambda^\nu_\mu V_\nu(\Lambda^{-1}x)$$