(e,e'p) and Nuclear Structure

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Outline

- Introduction
- Background
  - Experimental
  - Theoretical
- Nuclear Structure
- Medium-modified nucleons
  - Cross sections
  - Polarization transfer
- Studies of the reaction mechanism
- Few-body nuclei
  - The deuteron
  - $^{3,4}\text{He}$
A(e,e'p)B

Known:  e and A
Detect:  e' and p
Infer:  \( p_m = q - p = p_B \)
\((e, e'p)\) - Schematically

\[ e' \quad \gamma_v \quad e \quad \rightarrow \quad A \quad \leftrightarrow \quad B \quad + \quad p \]

\[ B = \begin{cases} 
A-1 & i.e. \text{bound} \\
A-2 & \text{Etc.} \\
N & \end{cases} \]
In ERL$_e$: \[ Q^2 \equiv -q_\mu q^\mu = q^2 - \omega^2 = 4ee' \sin^2 \theta / 2 \]

Missing momentum: \[ p_m = q - p = p_{A-1} \]

Missing mass: \[ \epsilon_m = \omega - T_p - T_{A-1} \]
Some (Very Few) Experimental Details …
“accidental” (uncorrelated)

“real” (correlated)
C(x) = \left[ C(x) \cap \text{Real} \right] - \frac{\Delta_r}{\Delta_a} \times \left[ C(x) \cap \text{Accidental} \right]
Accidentals Rate = \( R_e \times R_p \times \Delta \tau / DF \)
\[ \propto I^2 \Delta \tau / DF \]

Reals Rate = \( R_{eep} \)
\[ \propto I \]

\( S:N = \frac{\text{Reals}}{\text{Accidentals}} \propto \frac{DF}{\Delta \tau \times I} \)

Compromise:
Optimize \( S:N \) and \( R_{eep} \)
Extracting the cross section

\[
\left\langle \frac{d^6\sigma}{d\Omega_e d\Omega_p dp_e dp_p} \right\rangle = \frac{\text{Counts}}{N_e N_N \Delta\Omega_e \Delta\Omega_p \Delta p_e \Delta p_p}
\]
Some Theory ...
Cross Section for A(e,e'p)B in OPEA

\[ d\sigma_{\text{lab}} = \frac{1}{\beta} \frac{m_e}{e} \sum_{i f} \left| M_{fi} \right|^2 \left[ \frac{m_e}{e'} \frac{d^3 k'}{(2\pi)^3} \right] \left[ \frac{m}{E} \frac{d^3 p}{(2\pi)^3} \right] \times (2\pi)^4 \delta^4 (P + P_{A-1} - Q - P_A) \]

where

\[ M_{fi} = \frac{4\pi\alpha}{Q^2} \left\langle k' \lambda' | j_\mu | k \lambda \right\rangle \left\langle B_p | J^\mu | A \right\rangle \]

Current-Current Interaction
Square of Matrix Element

\[
\sum \left| M_{fi} \right|^2 = \left( \frac{4\pi\alpha}{Q^2} \right)^2 \sum \langle k' \lambda' | j_\mu | k \lambda \rangle^* \langle k' \lambda' | j_\nu | k \lambda \rangle 
\times \sum \langle Bp | J^\mu | A \rangle^* \langle Bp | J^\nu | A \rangle
\]

\[\eta_{\mu\nu}\]

\[\mathcal{W}_{\mu\nu}\]
Cross Section in terms of Tensors

\[ \frac{d^6 \sigma}{d\Omega_{e} d\Omega_{p} dp d\omega} = \sigma_{M} \eta_{\mu \nu} W^{\mu \nu} \]

- Mott cross section
- Electron tensor
- Nuclear tensor
Consider Unpolarized Case

Lorentz Vectors/Scalars

3 indep. momenta: \( Q, P_i, P \) \((P_{A-1} = Q + P_i - P)\)

6 indep. scalars: \( P_i^2, P^2, Q^2, Q \cdot P_i, Q \cdot P, P \cdot P_i \)

\[ = m^2 \]

\[ = M_A^2 \]
Nuclear Response Tensor

\[ W^{\mu \nu} = X_1 g_{\mu \nu} + X_2 q^\mu q^\nu + X_3 p_i^\mu p_i^\nu \]
\[ + X_4 p^\mu p^\nu + X_5 q^\mu p_i^\nu + X_6 p_i^\mu q^\nu \]
\[ + X_7 q^\mu p^\nu + X_8 p^\mu q^\nu + X_9 p^\mu p_i^\nu \]
\[ + X_{10} p_i^\mu p^\nu \]
\[ + (PV \text{ terms like } \varepsilon_{\mu \nu \rho \sigma} q^\rho p_\sigma) \]

\[ X_i \] are the response functions
Impose Current Conservation

\[ S^\nu \equiv q_\mu W^{\mu\nu} = 0 \]

\[ T^\mu \equiv q_\nu W^{\mu\nu} = 0 \]

Then \[ q_\nu S^\nu = 0, \quad p_\nu S^\nu = 0, \quad p_{i\nu} S^\nu = 0 \]

\[ q_\mu T^\mu = 0, \quad p_\mu T^\mu = 0, \quad p_{i\mu} T^\mu = 0 \]

Get 6 equations in 10 unknowns

4 independent response functions
Putting it all together …

\[
\left( \frac{d^6 \sigma}{d\Omega_e d\Omega_p dp d\omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M \left[ v_L R_L + v_T R_T + v_{LT} R_{LT} \cos \phi_x + v_{TT} R_{TT} \cos 2\phi_x \right]
\]

with

\[
\sigma_M = \frac{\alpha^2 \cos^2 \theta/2}{4e^2 \sin^4 \theta/2}
\]

\[
v_L = \left( \frac{Q^2}{q^2} \right) \quad v_T = \frac{Q^2}{2q^2} + \tan^2 \theta/2
\]

\[
v_{TT} = \frac{Q^2}{2q^2} \quad v_{LT} = \frac{Q^2}{q^2} \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2}
\]
The Response Functions

Use spherical basis with z-axis along \( q \):

\[
J_{fi}^0 \equiv J_{fi}^z = \frac{\omega}{q} \rho_{fi}
\]

\[
J_{fi}^{\pm 1} \equiv \pm \frac{1}{\sqrt{2}} (J_{fi}^x \pm iJ_{fi}^y)
\]

Nuclear 4-current

\[
R_L = \left| \rho_{fi}(\vec{q}) \right|^2 = \left( \frac{\vec{q}}{\omega} \right)^2 \left| J_{fi}^0(\vec{q}) \right|^2
\]

\[
R_T = \left| J_{fi}^{+1}(\vec{q}) \right|^2 + \left| J_{fi}^{-1}(\vec{q}) \right|^2
\]

\[
R_{TT} = 2 \text{ Re} \left\{ J_{fi}^{+1}(\vec{q}) J_{fi}^{-1}(\vec{q}) \right\}
\]

\[
R_{LT} = -2 \text{ Re} \left\{ \rho_{fi}(\vec{q}) \left( J_{fi}^{+1}(\vec{q}) - J_{fi}^{-1}(\vec{q}) \right) \right\}
\]
Response functions depend on scalar quantities

\[
\begin{align*}
Q \cdot P_i &= \omega M_A \\
P \cdot P_i &= E M_A \\
Q \cdot P &= \omega E - q p \cos \theta_{pq}
\end{align*}
\]

In lab:

Can choose: \( Q^2, \omega, \varepsilon_m, p_m \)

Note: no \( \phi_x \) dependence in response functions
Including electron and recoil proton polarizations

\[
\left( \frac{d^6 \sigma}{d\Omega_e d\Omega_p dp d\omega} \right)_{LAB} = \frac{pE}{(2\pi)^3} \sigma_M \left\{ \nu_L (R_L + R^n_L S_n) + \nu_T (R_T + R^n_T S_n) \right. \\
+ \nu_{LT} [(R_{LT} + R^n_{LT} S_n) \cos \varphi_x + (R^l_{LT} S_l + R^t_{LT} S_t) \sin \varphi_x] \\
+ \nu_{TT} [(R_{TT} + R^n_{TT} S_n) \cos 2\varphi_x + (R^l_{TT} S_l + R^t_{TT} S_t) \sin 2\varphi_x] \\
+ h\nu_{LT'} [(R_{LT'} + R^n_{LT'} S_n) \sin \varphi_x + (R^l_{LT'} S_l + R^t_{LT'} S_t) \cos \varphi_x] \\
+ h\nu_{TT'} (R^l_{TT'} S_l + R^t_{TT'} S_t) \}
\]

with \( \nu_{LT'} = \frac{Q^2}{q^2} \tan \theta/2 \quad \nu_{TT'} = \tan \theta/2 \sqrt{\frac{Q^2}{q^2} + \tan^2 \theta/2} \)

and other \( \nu \)'s defined as before.
Extracting Response Functions

For instance: \( R_{LT} \) and \( A_\phi (=A_{LT}) \)

\[
\sigma_{\text{eep}} = K\sigma_M [\nu_L R_L + \nu_T R_T + \nu_{LT} R_{LT} \cos \varphi_x + \nu_{TT} R_{TT} \cos 2\varphi_x ]
\]

\[
R_{LT} = \frac{\sigma_{\text{eep}}(\varphi_x = 0) - \sigma_{\text{eep}}(\varphi_x = \pi)}{2K\sigma_M \nu_{LT}}
\]

\[
A_\phi = \frac{\sigma_{\text{eep}}(\varphi_x = 0) - \sigma_{\text{eep}}(\varphi_x = \pi)}{\sigma_{\text{eep}}(\varphi_x = 0) + \sigma_{\text{eep}}(\varphi_x = \pi)}
\]
Plane Wave Impulse Approximation (PWIA)

$q - p = p_{A-1} = p_m = -p_0$
The Spectral Function

In nonrelativistic PWIA:

\[
\frac{d^6 \sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S(p_m, \varepsilon_m)
\]

For bound state of recoil system:

\[
\rightarrow \frac{d^5 \sigma}{d\omega d\Omega_e d\Omega_p} = K' \sigma_{ep} \left| \Phi(p_m) \right|^2
\]
The Spectral Function, cont’d.

\[ S(\vec{p}_0, E_0) = \sum_f \left| \langle B_f | a(\vec{p}_0) | A \rangle \right|^2 \delta(E_0 - \varepsilon_m) \]

where \( \vec{p}_0 = -\vec{p}_m = \) initial momentum

\[ E_0 = E - \omega = \] initial energy

Note: S is not an observable!
Elastic Scattering from a Proton at Rest

Before

\[ (\omega, q) \] \[ \rightarrow \] \[ p \]

After

\[ (\omega + m, q) \]

Proton is on-shell \( \Rightarrow \)

\[ (\omega + m)^2 - q^2 = m^2 \]

\[ \omega^2 + 2m\omega + m^2 - q^2 = m^2 \]

\[ \omega = Q^2 / 2m \]
Scattering from a Proton, cont’d.

\[
\langle p, s_f \mid J^\mu \mid p - q, s_i \rangle = \overline{U}_f \Gamma^\mu U_i
\]

Vertex fcn

\[
\Gamma^\mu = \gamma^\mu
\]

point proton

structure/anomalous moment
Scattering from a Proton, cont’d.

Vertex fcn:
\[ \Gamma^\mu = \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_v}{2m} \kappa F_2(Q^2) \]

Sachs FF’s
\[ \begin{align*}
G_E(Q^2) &= F_1(Q^2) - \tau\kappa F_2(Q^2) \\
G_M(Q^2) &= F_1(Q^2) + \kappa F_2(Q^2)
\end{align*} \]

with \( \tau = \frac{Q^2}{4m^2} \)

\( G_E \) and \( G_M \) are the Fourier transforms of the charge and magnetization densities in the Breit frame.
Form Factor

\[ \Delta \varphi_1 = \vec{k} \cdot \vec{r} \]

\[ \Delta \varphi_2 = -\vec{k}' \cdot \vec{r} \]

Phase difference:

\[ \Delta \varphi = (\vec{k} - \vec{k}') \cdot \vec{r} = \vec{q} \cdot \vec{r} \]

Amplitude at \( q \):

\[ F(q) = \int d\vec{r} A(\vec{r}) e^{i\vec{q} \cdot \vec{r}} \]
Cross section for $ep$ elastic

\[
\frac{d\sigma}{d\Omega} = f_{\text{rec}} \sigma_M \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau \tan^2 \frac{\theta}{2} G_M^2 \right]
\]

However, (e,e'p) on a **nucleus** involves scattering from **moving** protons, *i.e.* **Fermi motion.**
Elastic Scattering from a Moving Proton

\[ (\omega, q) \rightarrow (E, p) \]

Before

\[ \begin{align*}
(\omega + E)^2 - (q+p)^2 &= m^2 \\
\omega^2 + 2E\omega + E^2 - q^2 - 2p \cdot q - p^2 &= m^2
\end{align*} \]

After

\[ Q^2 = 2E\omega - 2p \cdot q \]

\[ \omega \frac{(E/m)}{(Q^2/2m) + p \cdot q / m} \]
Cross section for ep elastic scattering off moving protons

Follow same procedure as for unpolarized (e,e'p) from nucleus

We get same form for cross section, with 4 response functions …
Response functions for ep elastic scattering off moving protons

\[ R_L = \left[ \frac{(E_0 + E)}{2m} \right]^2 W_1 - \frac{\vec{q}^2}{4m^2} W_2 \]

\[ R_T = 2 \tau W_2 + \frac{\vec{p}^2 \sin^2 \theta_{pq}}{m^2} W_1 \]

\[ R_{LT} = -\frac{(E_0 + E)}{m^2} \left| \vec{p} \right| \sin \theta_{pq} W_1 \]

\[ R_{TT} = \frac{\vec{p}^2 \sin^2 \theta_{pq}}{m^2} W_1 \]

with

\[ W_1 = F_1^2 + \tau(\kappa F_2)^2 \]

\[ W_2 = (F_1 + \kappa F_2)^2 \]
Quasielastic Scattering

For $E \approx m$:

$$\omega \approx \left(\frac{Q^2}{2m}\right) + \frac{p \cdot q}{m}$$

If we “quasielastically” scatter from nucleons within nucleus:

Expect peak at:

$$\omega \approx \left(\frac{Q^2}{2m}\right)$$

Broadened by Fermi motion:

$$\frac{p \cdot q}{m}$$
Electron Scattering at Fixed $Q^2$

\[
\frac{d^2\sigma}{d\omega d\Omega}
\]

**Nucleus**

Elastic

Quasielastic $\Delta$

Deep Inelastic $N^*$

\[
\frac{Q^2}{2M} \quad \frac{Q^2}{2m} \quad \frac{Q^2}{2m} + 300 \text{ MeV}
\]

**Proton**

Elastic

$\Delta$

Deep Inelastic $N^*$

\[
\frac{Q^2}{2m} \quad \frac{Q^2}{2m} + 300 \text{ MeV}
\]
Quasielastic Electron Scattering

\[ \frac{\alpha^2}{\alpha^2} (10^{-3} \text{ m}^2/\text{MeV}) \]

\[ 250 \quad 300 \quad 350 \quad 400 \]

\[ E' (\text{MeV}) \]

\[ 6^\text{Li} \quad 12^\text{C} \quad 24^\text{Mg} \]

\[ 40^\text{Ca} \quad 58^\text{Ni} \quad 89^\text{Y} \]

\[ 118^\text{Sn} \quad 181^\text{Ta} \quad 208^\text{Pb} \]


Nuclear Structure
First, a bit of history:
The first \((e,e'p)\) measurement

\(^{12}\text{C}(e,e'p)\)

\(^{27}\text{Al}(e,e'p)\)

Frascati Synchrotron, Italy


\[ \text{FIG. 2. Electron-proton coincidence counting rate per } 10^{11} \text{ equivalent quanta at 550 MeV as a function of the incident energy. The dashed lines indicate the contributions of the various shells and the background as explained in the text.} \]
(e,e'p) advantages over (p,2p)

- Electron interaction relatively weak: OPEA is reasonably accurate.
- Nucleus is very transparent to electrons: Can probe deeply bound orbits.

However: ejected proton is strongly interacting. The “cleanness” of the electron probe is somewhat sacrificed.

FSI must be taken into account.
Recall, in nonrelativistic PWIA:

\[ \frac{d^6 \sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S(p_m, \varepsilon_m) \]

where \( q - p = p_m = -p_0 \)

FSI destroys simple connection between the measured \( p_m \) and the proton initial momentum (not an observable).
Final State Interactions (FSI)

\[ \mathbf{q} - \mathbf{p} = \mathbf{p}_{A-1} \neq \mathbf{p}_0 \]
Distorted Wave Impulse Approximation (DWIA)

Treat outgoing proton distorted waves in presence of potential produced by residual nucleus (optical potential).

\[
\frac{d^6 \sigma}{d\omega d\Omega_e dp d\Omega_p} = K \sigma_{ep} S^D(p_m, \epsilon_m, p)
\]

“Distorted” spectral function
Optical potential is constrained by proton elastic scattering data.

Problems with this approach:

• Residual nucleus contains hole state, unlike the target in $p+A$ scattering.

• Proton scattering data is surface dominated, whereas ejected protons in $(e,e'p)$ are produced within entire nuclear volume.
100 MeV data is significantly overestimated by DWIA near 2\textsuperscript{nd} maximum.

Fig. 1. Experimental momentum distributions for the 2p\textsubscript{3/2} and 2p\textsubscript{1/2} transitions in the $^{90}\text{Zr}(e,e'p)^{89}\text{Y}$ reaction. The curves correspond to DWIA calculations for the two proton energies (set I in table 1).

\[ \rho_{\alpha}^{\text{th}}(p_m, \vec{p}) = S_{\alpha} \left\{ \int_{r_c=0}^{\infty} \chi^{(-)*}(\vec{r}_p, \vec{p}) \right. \\
\times \left. \exp(iq \cdot \vec{r}_p) \psi_{\alpha}(\vec{r}_p) \mathrm{d}\vec{r}_p \right\}^2 \]

At \( p_m \approx 160 \text{ MeV/c} \), \( \psi \) is probed in nuclear interior.

Fig. 2. Radial sensitivity, \( S(r_c) \), as a function of the lower integration limit \( r_c \) (see text). For reference the radial dependence of the charge density (\( \rho \)) and of the \( 2p \) wave functions (\( \psi_{2p} \)) are indicated as well (not to scale).

Adjusting optical potential renders good agreement while maintaining agreement with p+A elastic.

Fig. 3. Same as fig. 1, but for the modified optical potential (set II in table 1).

\[ ^{12}\text{C}(e,e'p)^{11}\text{B} \]

Fig. 9. Missing energy spectra from \(^{12}\text{C}(e,e'p)\), (a) \(0 \leq P \leq 36\ \text{MeV/c} \), (b) \(80 \leq P \leq 180\ \text{MeV/c} \) and (c) \(0 \leq P \leq 60\ \text{MeV/c} \) for \(20 \leq E \leq 60\ \text{MeV} \).

Fig. 10. Momentum distribution from $^{12}\text{C}(e,e'p)^{11}\text{B}$; (a) $15 \leq E \leq 21.5$ MeV and (b) $30 \leq E \leq 50$ MeV. The solid and dashed lines represent DWIA and PWIA calculations respectively, with normalization obtained by a fit to the data.

Fig. 1. Excitation-energy spectrum of the reaction $^{12}\text{C}(e,e'p)^{11}\text{B}$ at a central value of the missing momentum $p_m = 29 \text{ MeV/c}$. The spectrum has been sorted in 100 keV bins.

$^{12}\text{C}(e,e'p)^{11}\text{B}$

NIKHEF-K
Amsterdam

Fig. 2. Same as fig. 1 but for $p_m = 172\text{ MeV}/c$.

$^{12}\text{C}(e,e'p)^{11}\text{B}$

NIKHEF-K
Amsterdam

$^{12}\text{C}(e,e'p)$

Bates Linear Accelerator

MAMI
Mainz, Germany

\[ \sigma_{\text{red}}(E, p_m) \text{ (GeV}^{-1}\text{ GeV/c)}^{-3} \]

\[ ^{16}\text{O}(e, e'p) \]

\[ 330 \leq p_m \leq 570 \text{ MeV/c} \]

MAMI Mainz, Germany

Factorization violated.

DWIA calculations underpredict at high $p_m$.

Neglected MEC’s & relativistic effects.

Offshell effects uncertain at high $p_m$.

FIG. 1. The reduced cross section of the reaction $^{208}$Pb($e, e'p$) at an average missing momentum of 340 MeV/c, showing the knock out of valence protons to discrete states in $^{207}$Tl, labeled by their spin, parity, and excitation energy. The solid curve is the result of a fit to the spectrum.


$^{208}\text{Pb}(e,e'p)$

AmPS NIKHEF-K
Amsterdam

FIG. 2. Missing-momentum distributions for the transitions to the $\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^-, \frac{5}{2}^+$, and $\frac{7}{2}^+$ states in the reaction $^{208}\text{Pb}(e, e'p)$ at excitation energies of 0.00, 0.35, 1.35, 1.68, and 3.47 MeV, respectively. The present data are represented by solid circles, the plus marks have been measured by Quint [16]. The solid curves are knockout calculations in the distorted-wave impulse approximation. The calculations including correlations as proposed by Pandharipande [8], Ma and Wambach [10], and Mahaux and Sartor [12] are represented by dash–double-dotted, dashed, and dot-dashed curves, respectively.

Long-range correlations important.
SRC and TC less so, but expected to grow with $\varepsilon_m$. 
Some of the lessons learned:

• (e,e'p) sensitive probe of single-particle orbits.

• Proton distortions (FSI) must be accounted for to reproduce shape of spectral function. Energy dependence of FSI breaks factorization.

• Missing strength in valence orbits, even after accounting for FSI

• At high $P_m$ significant discrepancies found relative to calculations.
Where does the “missing” strength go?

One possibility:

Detected

recoils

populates high $\varepsilon_m$
SRC dominate high $k (=p_m)$ and are related to large values of $\varepsilon_m$.  

Similar shapes for few-body nuclei and nuclear matter at high $k (=p_m)$.  

Medium-Modified Nucleons
Searching for Medium Effects on the Nucleon …

In parallel kinematics:

\[
\frac{d^6 \sigma}{d\omega_e d\omega_p dp d\omega} = \frac{pE}{(2\pi)^3} \sigma_M [\nu_L R_L + \nu_T R_T]
\]

Can write \( ep \) elastic cross section as:

\[
\frac{d\sigma}{d\Omega} = f_{rec} \sigma_M [\nu_L k_L G_E^2 + \nu_T k_T G_M^2]
\]

with \( k_L = \frac{|\vec{q}|^2}{Q^2} \) and \( k_T = \frac{Q^2}{2m^2} \)
Relate $R_T/R_L$ to in-medium proton FF’s

$$R_G \equiv \frac{m|\vec{q}|}{Q^2} \sqrt{\frac{2R_T}{R_L}} \rightarrow \frac{\tilde{G}_M}{\tilde{G}_E}$$

This relies on (unrealistic) model assumptions!

Nonetheless …
$^2\text{H}(e,e'p)n$

$^6\text{Li}(e,e'p)$

J.E. Ducret et al.,

J.B.J.M. Lanen et al.,

NIKHEF-K
Amsterdam
$^{12}\text{C}(e,e'p)$ and $^{12}\text{C}(e,e')$

G. Van der Steenhoven et al.,
FIG. 3. $R_G = \sqrt{W_\gamma A M_p^2/W_L Q^2}$ for $^{12}$C (solid) from the measurements of this experiment with $^6$Li ($p$ shell: open squares [3], open circles [25], and $s$ shell: open triangles [3], open circles [25]) and $^{12}$C ($p$ shell: open cross [1], open triangles [15], and $s$ shell: open cross [1]). The top panel is for the $p$ shell region and bottom panel is for the $s$ shell region. The inner error bar represents that statistical error and the outer error bar includes the systematic error. The dashed line represents $R_G$ for the free proton with the dipole electric and Ref. [18] magnetic form factor while the dotted lines represent the one sigma error band of the recent proton results of Ref. [27].

However, large FSI effects can mimic this behavior ...
FSI calculations for $^{16}\text{O}$ 1p$_{3/2}$

Data for $^{12}\text{C}$ 1p$_{3/2}$

Another, less model-dependent, method ...

Polarization Transfer
Proton Polarization and Form Factors

Free $\bar{e}p$ scattering

$$I_0 P_x' = -2 \sqrt{\tau(1 + \tau)} G_E G_M \tan\left(\frac{\theta_e}{2}\right)$$

$$I_0 P_z' = \frac{e + e'}{m} \sqrt{\tau(1 + \tau)} G_M^2 \tan^2\left(\frac{\theta_e}{2}\right)$$

$$I_0 = G_E^2 + \tau G_M^2 \left[1 + 2(1 + \tau) \tan^2\left(\frac{\theta_e}{2}\right)\right]$$

$$\frac{G_E}{G_M} = -\frac{P_x'}{P_z'} \cdot \frac{e + e'}{2m} \tan\left(\frac{\theta_e}{2}\right)$$

Polarization Transfer in Hall A

\[ \vec{e} \rightarrow ^1\text{H and (}^2\text{H or }^4\text{He)} \rightarrow e' \rightarrow \text{spectrometer} \]

\[ \vec{p} \rightarrow \text{spectrometer + FPP} \]
Measuring the Proton Polarization: FPP
Density Dependent Form Factors

Quark-Meson Coupling Model (QMC):

\[
\overline{G}_\alpha (Q^2) = \frac{\int d^3 r w_\alpha (r) G(Q^2, \rho_B (r))}{\int d^3 r w_\alpha (r)}
\]

For (e,e'p)

\[
w_\alpha = \exp(iq \cdot \vec{r}) \chi^{(-)} (\vec{p}', \vec{r})^* \phi_\alpha (r)
\]

Quark-Meson Coupling Model

$^2\text{H}(\vec{e}, e' \vec{p})n$

$^4\text{He}(\vec{e}, e' \vec{p})^3\text{H}$

Calculations by Arenhövel

RDWIA calculations by Udias et al.
Induced Polarization – $^4$He

JLab E93-049

$P_y = 0$ in PWIA: test of FSI
$^{16}\text{O}(\vec{e},e'\vec{p})^{15}\text{N}$ at $Q^2 = 0.8\text{ (GeV/c)}^2$

Studies of the Reaction Mechanism
Correlations and Interaction Currents

Correlations

MEC’s

IC’s
Off-shell Effects

Vertex function is not well defined. The “Gordon identity” leads to alternative forms, equivalent only when proton is on-shell.
$^{12}$C(e,e'p) L/T Separations

$Q^2 = 0.15 \text{ GeV}^2$

$Q^2 = 0.64 \text{ GeV}^2$


Bates Linear Accelerator

JLab Hall C
Excess transverse strength at high $\varepsilon_m$. Persists, though perhaps declines, at higher $Q^2$.

\[^{6}\text{Li}(e,e'p)\] T/L Ratio

DWIA (dashed) fails to describe overall strength.

Scaling transverse amplitude in DWIA (solid) gives good agreement → deduce scale factor, \(\eta\).

NIKHEF-K
Amsterdam

$^6\text{Li}(e,e'p)$ T/L Ratio


NIKHEF-K Amsterdam
The L/T separations suggest

- Additional transverse reaction mechanism above 2-nucleon emission threshold.
- MEC’s primarily transverse in character. Suggestive of two-body current.

Reminiscent of …
T/L anomaly in inclusive (e,e'):

$^{12}\text{C}(e,e'p)$ in "Dip Region"


Data from: Bates Linear Accelerator
$^{12}\text{C}(e,e'p)$

"Delta"

Between dip and Δ

Peak of Δ

H. Baghaei et al.,

Bates Linear Accelerator

L.B. Weinstein et al.,

Bates Linear Accelerator

Quasielastic

$Q^2=0.30$

$Q^2=0.48$

$Q^2=0.58$
$^{12}\text{C}(e,e'p) \quad q=990 \text{ MeV}/c, \quad \omega=475 \text{ MeV}$

$$\frac{d^6\sigma}{d\Omega_e d\Omega_p d\omega d\epsilon_m} = \sum_{l=0}^{l_{\text{max}}} \alpha_l(\epsilon_m) P_l \left( \frac{\omega - \omega_0}{\Delta\omega/2} \right)$$

For $60<\epsilon_m<100 \text{ MeV}$, continuum cross section increases strongly with $\omega$.

Large continuum strength continues up to 300 MeV.

Figure adapted from J.H. Morrison et al., Phys. Rev. C 59, 221 (1999).
$^{12}\text{C}(e,e'p)$ $q=970$ MeV/c, $\omega=330$ MeV

\[ d^6\sigma = \frac{\prod_{l=0}^{l_{\text{max}}} \alpha_l(\varepsilon_m) P_l \left( \frac{\omega - \omega_0}{\Delta \omega/2} \right) }{d\Omega_e d\Omega_p d\omega d\varepsilon_m} \]

Continuum strength increases strongly with $\omega$.
Continuum cross section is smaller at high $\varepsilon_m$.

Figure adapted from J.H. Morrison et al., Phys. Rev. C 59, 221 (1999).
$^{12}\text{C}(e,e'^p)$

For $\omega < \omega_{QE}$, spectroscopic factors consistent with naïve expectations.

Bates Linear Accelerator

Large discrepancy for $1p_{3/2}$.
Relativistic effects predicted to be small here.
Two-body currents responsible??


| Circles (solid) – NIKHEF-K | Crosses (dashed) - Saclay |
$^{16}\text{O}(e,e'p) \quad Q^2=0.8 \text{ GeV}^2$ Quasielastic

Relativistic DWIA gives good agreement with data.

JLab Hall A

$^{16}\text{O}(e, e'p) \quad Q^2 = 0.8 \text{ GeV}^2$ Quasieelastic

Two-body calculations of Ryckebusch et al., give flat distribution, as seen in the data, but underpredict by a factor of two.

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At high energies, $R_{LT}$ interference response function sensitive to relativistic effects.

For example, spinor distortion …
Spinor Distortions

\[ \Psi = \begin{pmatrix} \Psi^+ \\ \Psi^- \end{pmatrix} \]

\[ \Psi^- = \frac{\sigma \cdot p}{E + m + S - V} \Psi^+ \]

**N.R. reduction**

S+V → Mean field
S+V relatively small

**Dirac spinor**

S–V affects lower components
S–V large
$^{16}\text{O}(e,e'p) \ Q^2=0.8 \text{ GeV}^2$ Quasielastic

Sensitive to "spinor distortions"

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Few-body Nuclei …
The Deuteron
Short-distance Structure

For large overlap, nucleons may lose individual identities:

Quark/gluon d.o.f.?
Large FSI/non-nucleonic effects.
Problem at $p_m=0$.

FIG. 2. Separated $f_{00}$ and $f_{11}$ structure functions for this experiment and the NIKHEF experiment of van der Schaar et al. [5]. The NIKHEF data ($q = 380$ MeV/c) are averaged over 5 MeV/c bins in $p_m$. The Bates data ($q = 400$ MeV/c) are averaged over the range of 30 to 70 MeV/c in $p_m$. Only statistical errors are shown.

Blomqvist et al. data cover kinematics beyond $\Delta$. Also neutron exchange diagram important.


Calculations: H. Arenhövel
$^2\text{H}(e,e'p)$ $Q^2=0.23$ GeV$^2$ near $\Delta$

Bonn Electron Synchrotron, Germany


Calculations: Leidemann and Arenhövel
Proton spectator

---

Proton hit (high $p_m$)

```
q -> p -> n
```

Final State

```
p_f -> n_f
```

Neutron hit (low $p_m$)

```
q -> p -> n
```

Final State

```
p_f -> n_f
```
$Q^2 = 0.67$ GeV$^2$ Quasielastic

Large FSI effects.
Also, substantial non-nucleonic effects.

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Final State Interactions
Can be LARGE

\[ p' < p \]

actual
inferred
What do all these data and curves suggest?

- Relativistic effects substantial in $A_\phi$ (and $R_{LT}$).
- de Forest “CC1” nucleon cross section gives same qualitative features as more complete calculations $\rightarrow$ here, relativity more related to nucleonic current, as opposed to deuteron structure.
\[ 2 \vec{H}(\vec{e}, e'p) \]

\[ \sigma = \sigma_0 \left[ 1 + P_1^d A_d^V + P_2^d A_d^T + h \left( A_e + P_1^d A_{ed}^V + P_2^d A_{ed}^T \right) \right] \]


\[ D\text{-state important} \]

AmPS
NIKHEF-K
Amsterdam
Lots more d(e,e'p) data on the way!
$^2\text{H}(e,e'p)n$  E01-020 Hall A

**Perpendicular:** $R_{LT}$

$Q^2 : 0.80, 2.10, 3.50 \text{ (GeV/c)}^2$

$x=1$: $p_m$ from 0 to ± 0.5 GeV/c

**Parallel/Anti-parallel**

$Q^2 : 2.10 \text{ (GeV/c)}^2$

vary $x$: $p_m$ from 0 to 0.5 GeV/c

**Neutron angular distribution**

$Q^2 : 0.80, 2.10, 3.50 \text{ (GeV/c)}^2$
$Q^2 = 0.8 \text{ (GeV/c)}^2$

PRELIMINARY

20% error added to statistical error

$^2\text{H(e,e'p)n}$

E01-020 Hall A
$Q^2 = 3.5 \text{ (GeV/c)}^2$

PRELIMINARY

20% error added to statistical error
$^2\text{H}(e,e'p)n$ with JLab 12 GeV upgrade
Hall B data covers large range of $Q^2$ and excitation as well as $\phi$ coverage to separate $R_{LT}$, $R_{LT}'$ and $R_{TT}$. 

Preliminary Hall B E5 Data – $^2$H(e,e'p)
$^{3,4}\text{He}$
$^3\text{He}(e,e'p)$

Calculations by Laget:
- dashed = PWIA
- dot-dashed = DWIA
- solid = DWIA + MEC

Arrows indicate expected position for correlated pair.

C. Marchand et al.,
$^3\text{He}(e,e'p)d$ \quad $^3\text{He}(e,e'p)np$

3BBU similar to $d \rightarrow np$

Large effects from FSI and non-nucleonic currents.

Highest $\rho_m$ shows excess strength.
General features reproduced but not at correct values of $p_m$. 

JLab Hall A
The most direct way to look for correlated nucleons?

Detect both of them

→ JLab Hall B
$^3\text{He}(e,e'pp)n$  Hall B

- **2 GeV**
  - $P_N > 250$ MeV/c
  - Nent = 40432

- **4 GeV**
  - $P_N > 250$ MeV/c
  - Nent = 2529

- **Fast pn pairs**
  - $^3\text{He}(e,e'pp)n$  Hall B
  - **2 GeV**
    - $P_N > 250$ MeV/c
    - Nent = 9591
    - fast pn leading p
  - **4 GeV**
    - $P_N > 250$ MeV/c
    - Nent = 1036
    - fast pn leading p
Hall B

$^3\text{He}(e,e'pp)n$ 2 GeV

$p_{\text{perp}} < 300 \text{ MeV/c}$

Isotropic fast pairs

$\cos(\theta_{nq})$

$\rightarrow$ pair not involved in reaction.

PRELIMINARY
Hall B

$^3$He(e,e'pp)n

Small momentum along q
→ pair not involved in reaction.

Little $Q^2$ or isospin dependence.

2 GeV has acceptance corrections
Direct evidence of NN correlations

Before

After
$^4\text{He}(e,e'p)^3\text{H}$

Data and calculations “corrected” for MEC+IC (Laget).

Longitudinal overpredicted.

Calculations predict $q$ dependence.

$^4\text{He}(e,e'p)^3\text{H}$

$^4\text{He} (e,e'p)^3\text{H}$

Again, calculations predict $q$ dependence.

$^4\text{He}(e,e'p)^3\text{H}$

Minimum filled in by FSI and 2&3-body currents.

FSI: dependence on kinematics

- Actual: $p' < p$
- Inferred: $p' > p$

Large FSI

Small FSI
$^4\text{He}(e,e'p)^3\text{H}$

It looks like the minimum is filled in here as well.

JLab Hall A Experiment E97-111, J. Mitchell, B. Reitz, J. Templon, cospokesmen
Summary

• \((e,e'p)\) sensitive to single-particle aspects of nucleus, but ...

• More complicated physics is clearly important.

• Spectroscopic factors reduced compared to naïve shell model (including FSI corrections).

• Missing strength at least partly due to interaction currents: direct interaction with exchanged mesons or interaction with correlated pairs (spreads strength over \(\varepsilon_m\)).
Summary cont’d.

• After several decades of experimental and theoretical effort, there are still unanswered questions.

• What is the nature of the interaction of the virtual photon with the “nucleon”: medium and offshell effects?

• Handling FSI and other reaction currents still problematic, though realistic calculations are now available for the lighter systems.

• High energy program is underway, pushing to shorter distance scales, emphasizing relativistic effects, …