Non-Relativistic EFTs of QCD

– weak coupling –

Nora Brambilla

INFN and University of Milano

Quarkonium Working Group

Motivations

- Competitive source of some SM parameters:

  m_t, m_b, m_c, $\alpha_{\rm s}$, ...

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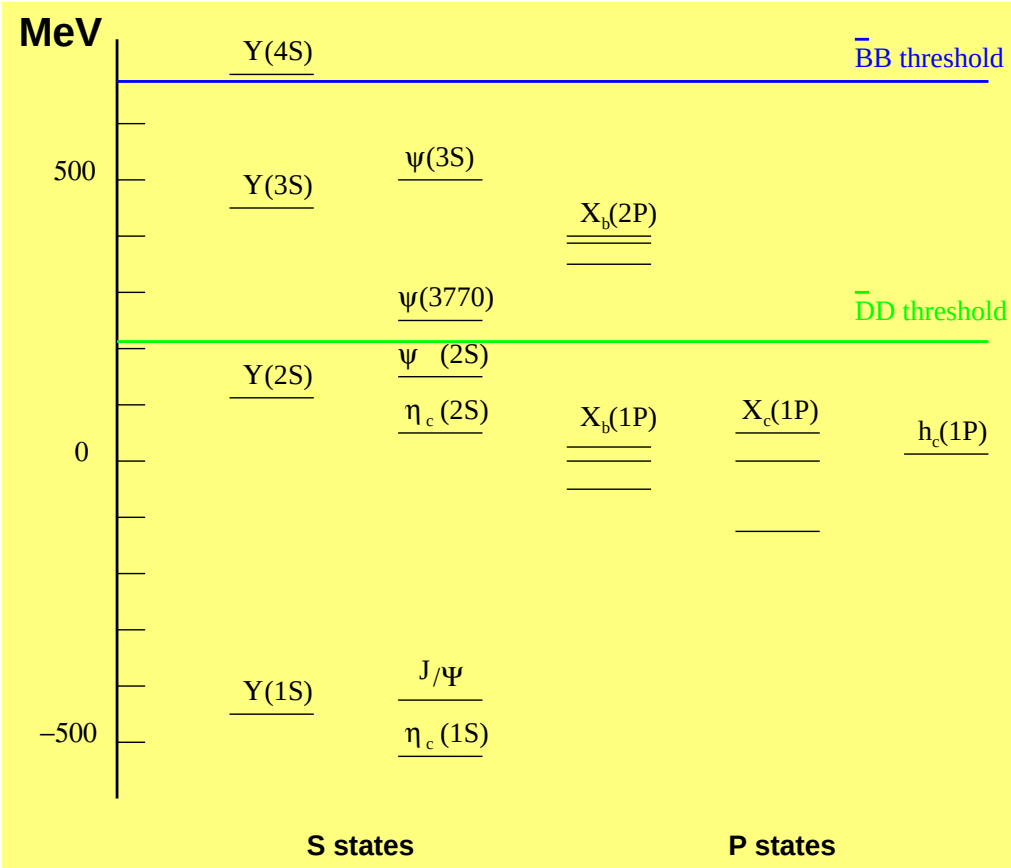
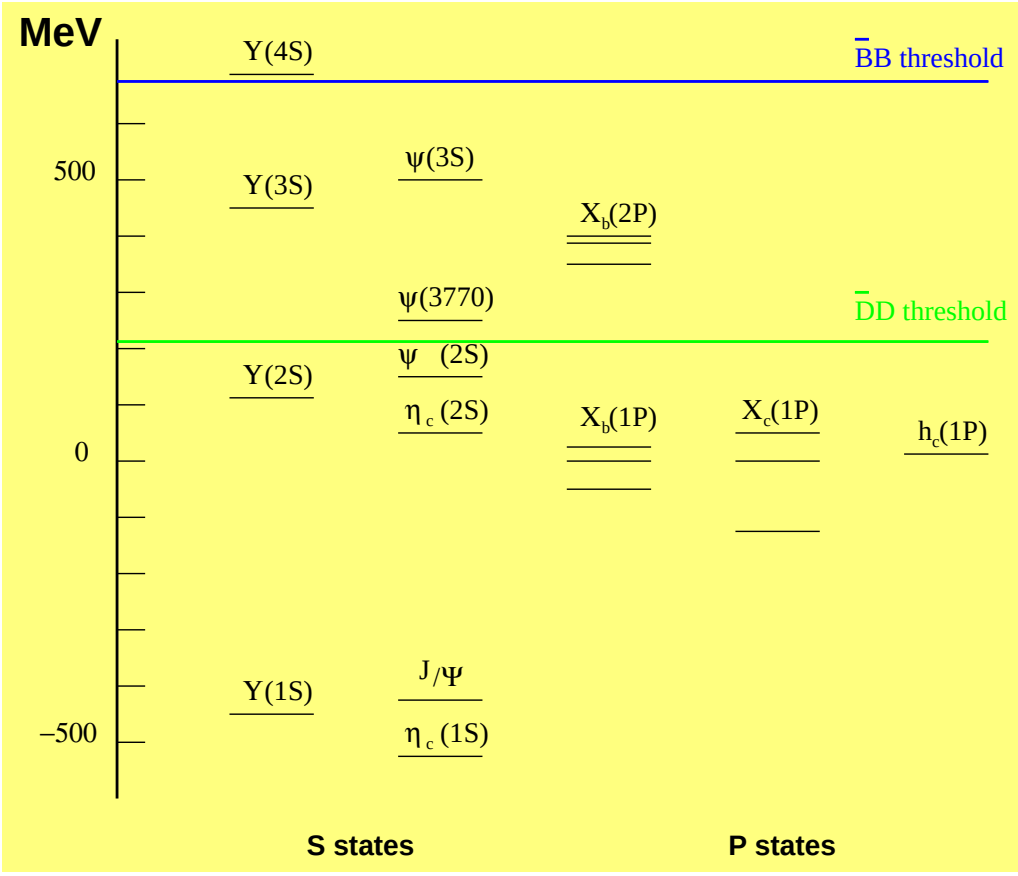
Quarkonium Scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

Quarkonium Scales



The mass scale is perturbative:

$m_b \simeq 5$ GeV, $m_c \simeq 1.5$ GeV

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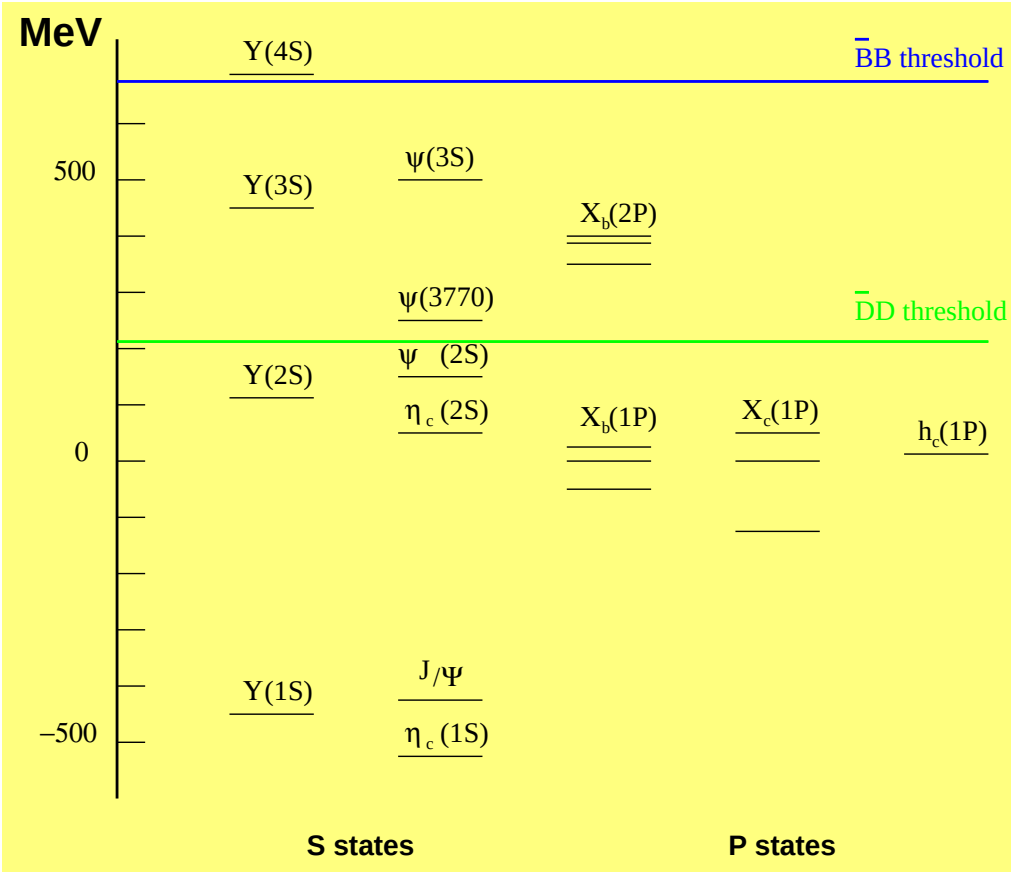
Quarkonium Scales



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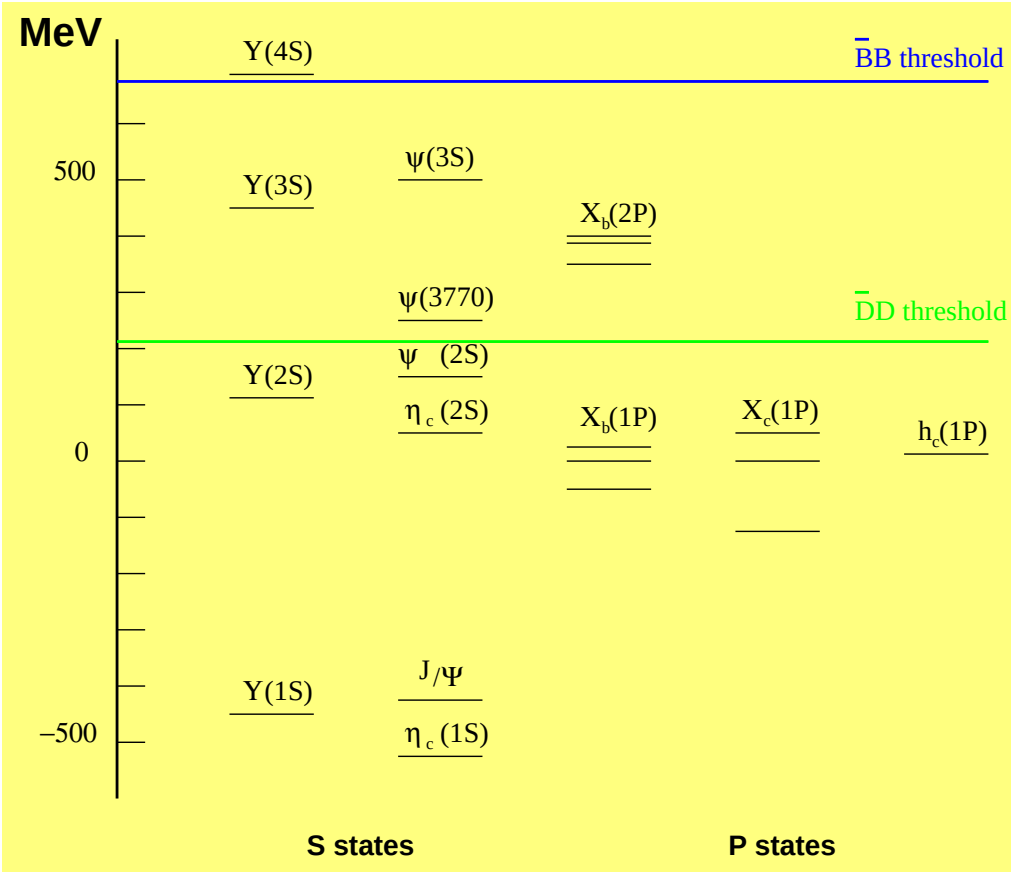
The system is non-relativistic:

$\Delta_n E \sim mv^2$, $\Delta_{\rm fs} E \sim mv^4$

$v_b^2 \simeq 0.1$, $v_c^2 \simeq 0.3$

Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

Quarkonium Scales



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$v_b^2 \simeq 0.1$, $v_c^2 \simeq 0.3$

The dynamical scales are:

$r \sim 1/mv$, $E \sim mv^2$ $\quad v \ll 1$

Non-relativistic bound states are characterized by at least three energy scales, hierarchically ordered by the quark velocity.

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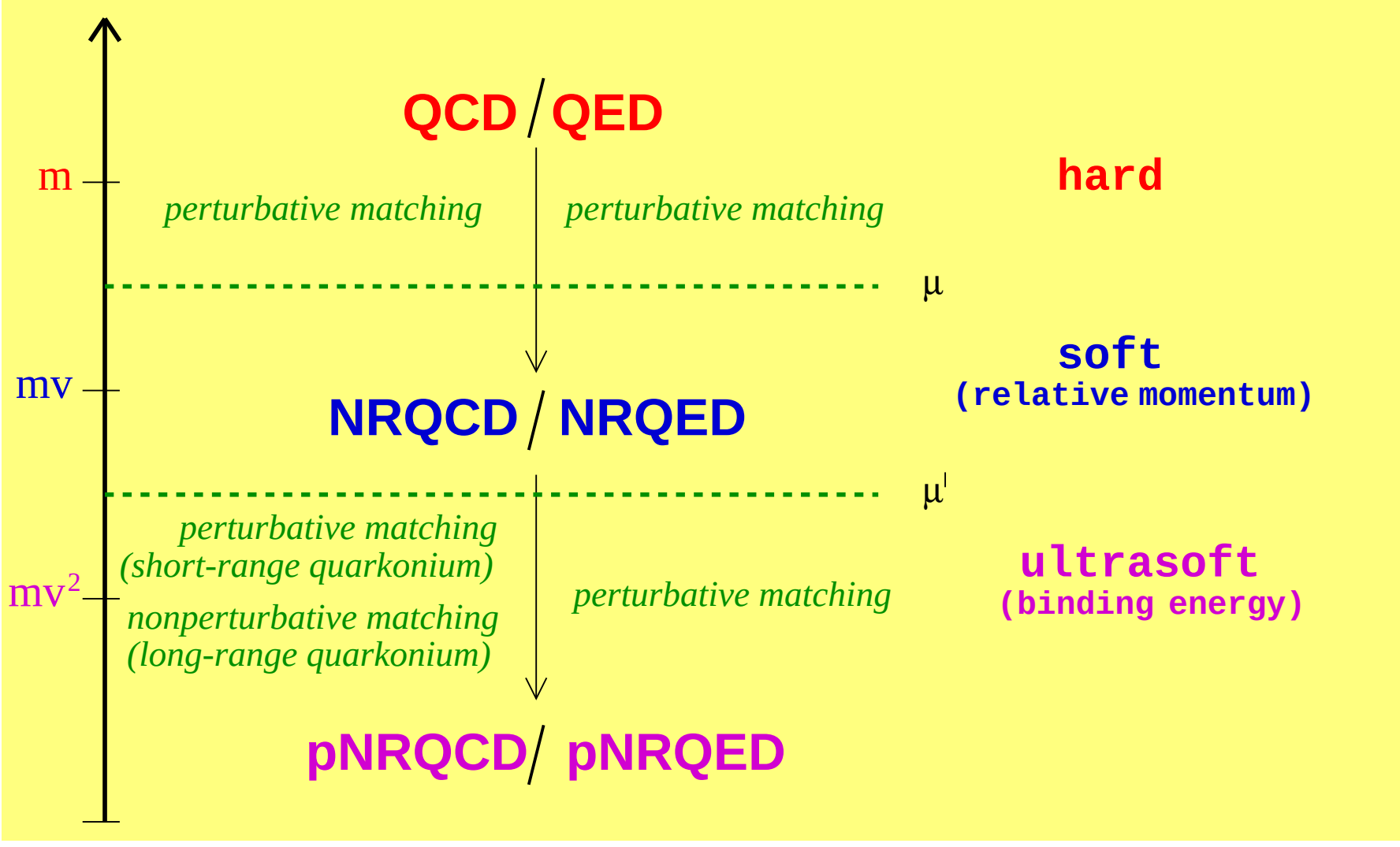
A way to disentangle rigorously the scales is by substituting QCD scale by scale with simpler but equivalent effective field theories.

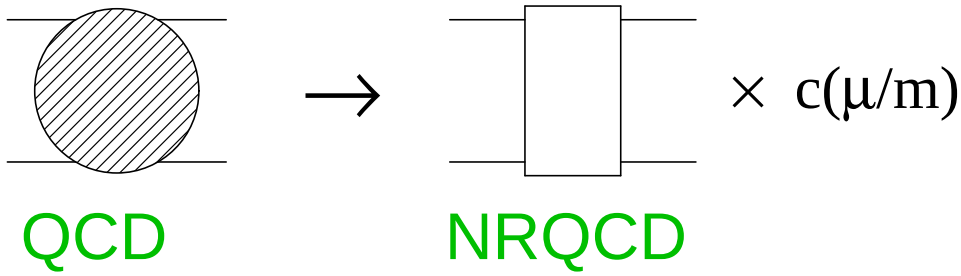
Non-Relativistic EFT



In QCD another scale is relevant: $\Lambda_{\rm QCD}$

NRQCD

Degrees of freedom that scale like m are integrated out:



NRQCD

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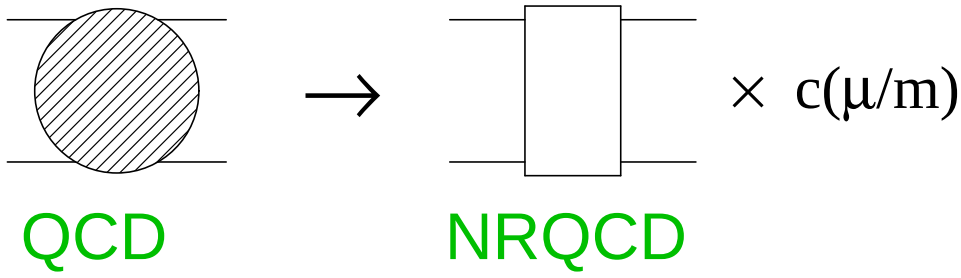

- The matching is perturbative.
- The Lagrangian is organized as an expansion in v and $\alpha_s(m)$.

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \ldots \right) \psi$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \ldots$$

NRQCD

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$$\sim v^2 \qquad \sim v^2(v^4) \qquad \sim v^3$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \ldots$$

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \ldots \right) \psi$$

$$+ \chi^\dagger \left(\qquad \ldots \qquad \right) \chi$$

$$+ \sum_K \frac{f}{m^2} \psi^\dagger \, K \, \chi\chi^\dagger \, K \, \psi + \cdots$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q} \, i\not{D} \, q + \ldots$$

Caswell Lepage 86, Bodwin Braaten Lepage 95, Manohar 97

NRQCD

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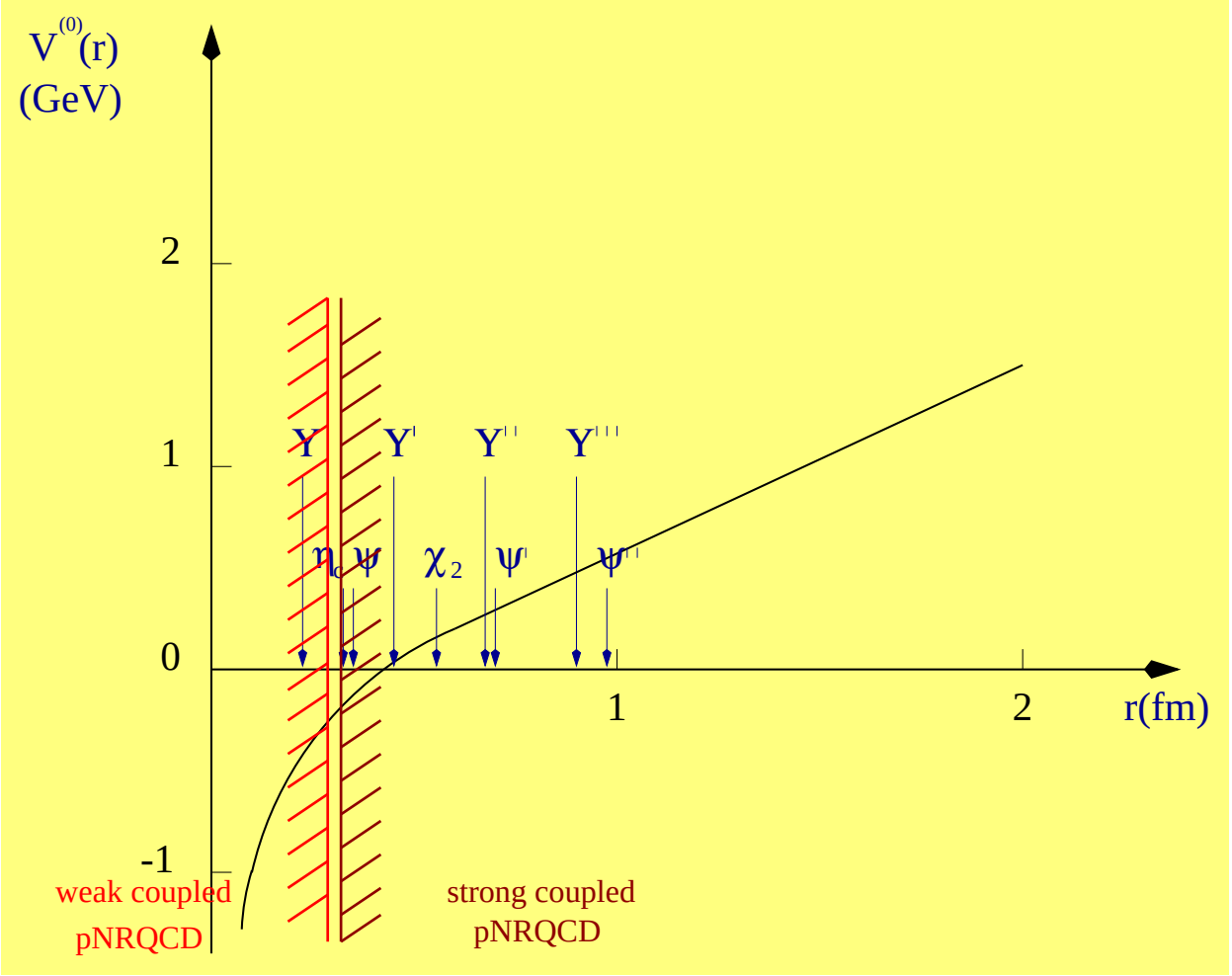
$$1 + (\cdots)\,\alpha_s + \ldots$$

$$f = \mathrm{Re} f + i \mathrm{Im} f$$

$$+ \sum_K \frac{f}{m^2} \psi^\dagger\, K\, \chi\chi^\dagger\, K\, \psi + \cdots$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \ldots$$

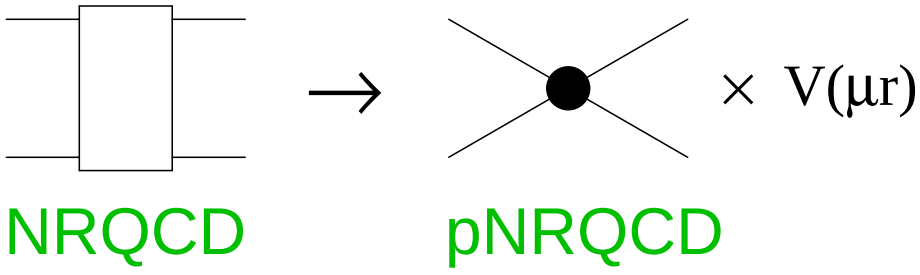
Quarkonium Radius



Godfrey Isgur 85

pNRQCD for $mv \gg \Lambda_{\mathrm{QCD}}$

Degrees of freedom that scale like mv are integrated out:



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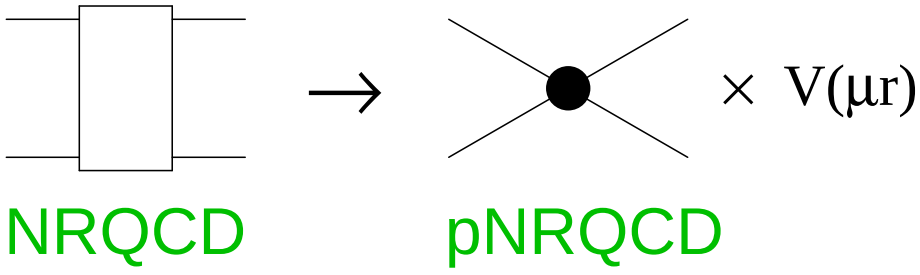

- The matching is perturbative

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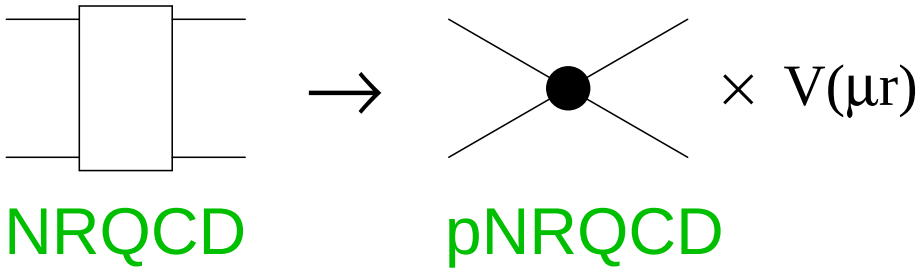


- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}$, mv^2

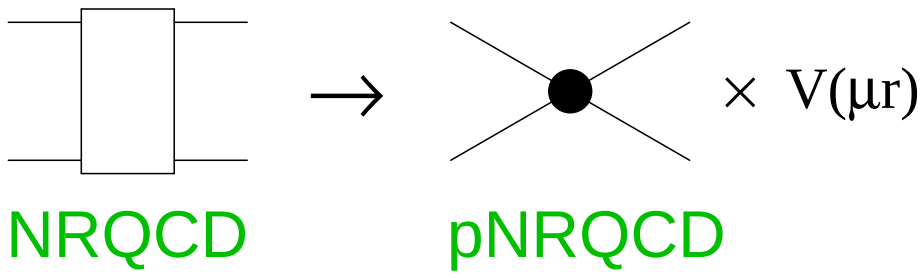
  momentum $\lesssim mv$

  $\Rightarrow$ i) singlet S ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}$, mv^2

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

Degrees of freedom that scale like mv are integrated out:



- Power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:

  $$A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$$

  Non-analytic behaviour in $r \to$ matching coefficients V

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

$$\theta(T)\, e^{-iTH_s}$$

$$\theta(T)\, e^{-iTH_o} \left(e^{-i \int dt\, A^{\rm adj}} \right)$$

pNRQCD for $mv \gg \Lambda_{\rm QCD}$

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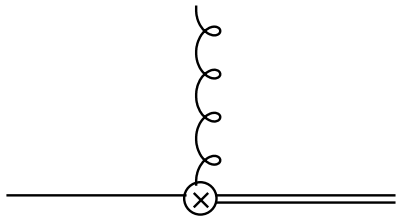
LO in r

$$+ V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

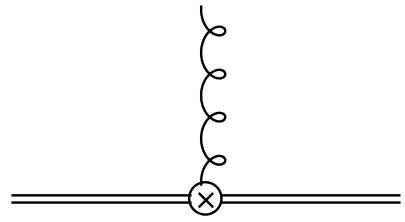
NLO in r

Pineda Soto 97, Brambilla Pineda Soto Vairo 99, 00, 03

pNRQCD for $mv \gg \Lambda_{\rm QCD}$



$O^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{S}$

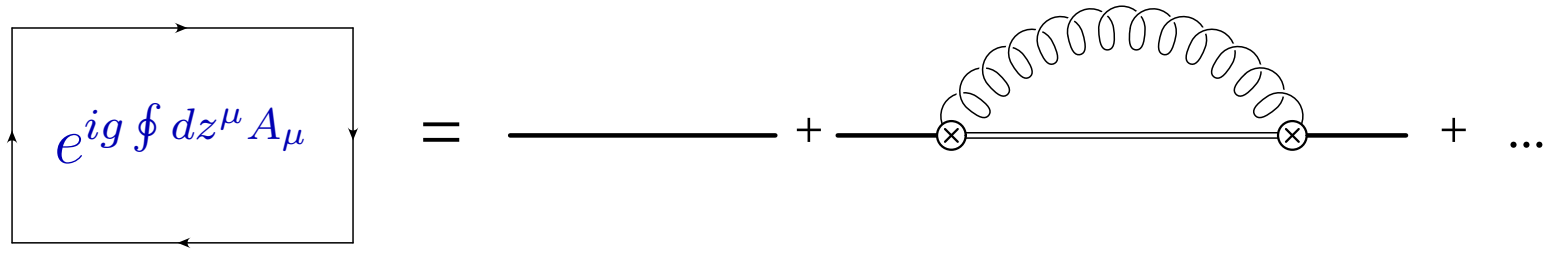


$O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$

$$+V_A \mathrm{Tr}\left\{\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{O}\right\}$$
$$+\frac{V_B}{2} \mathrm{Tr}\left\{\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E}\right\}$$

NLO in r

The Static Potential



NRQCD pNRQCD

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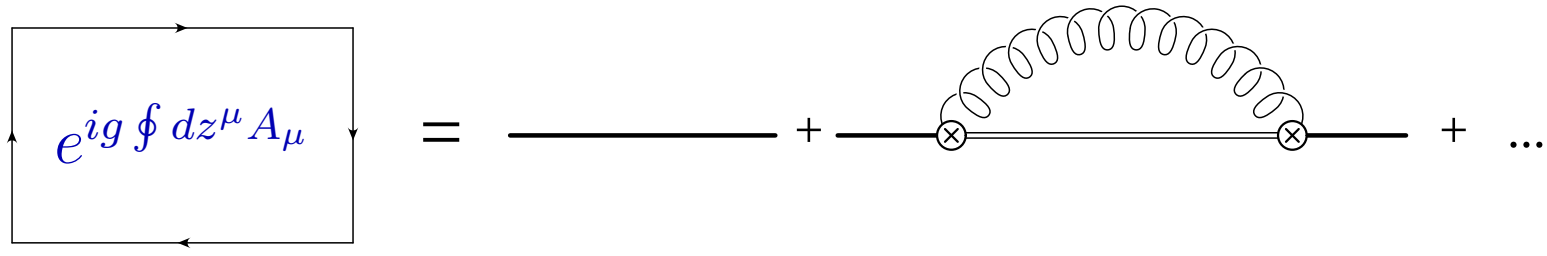
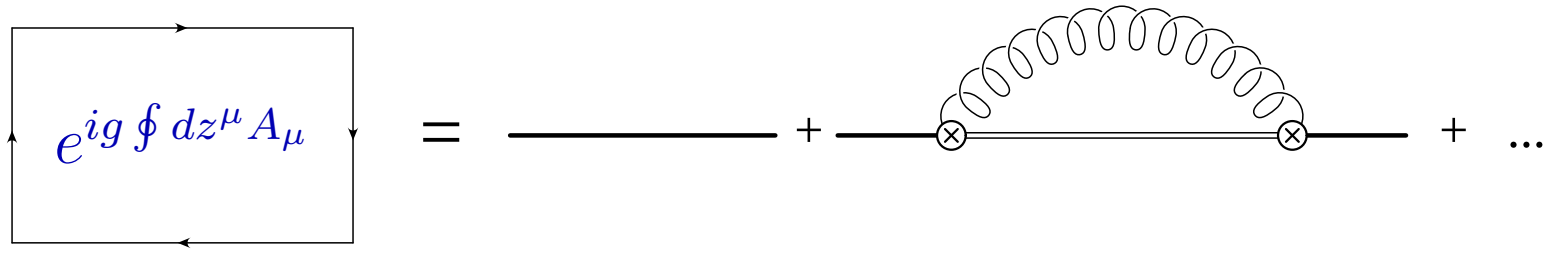


NRQCD pNRQCD

$$V_s(r) = \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle$$

$$+i\frac{g^2}{N_c}\int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathrm{Tr}(r\cdot E(t)\, r\cdot E(0))\rangle + \ldots$$

The Static Potential



NRQCD pNRQCD

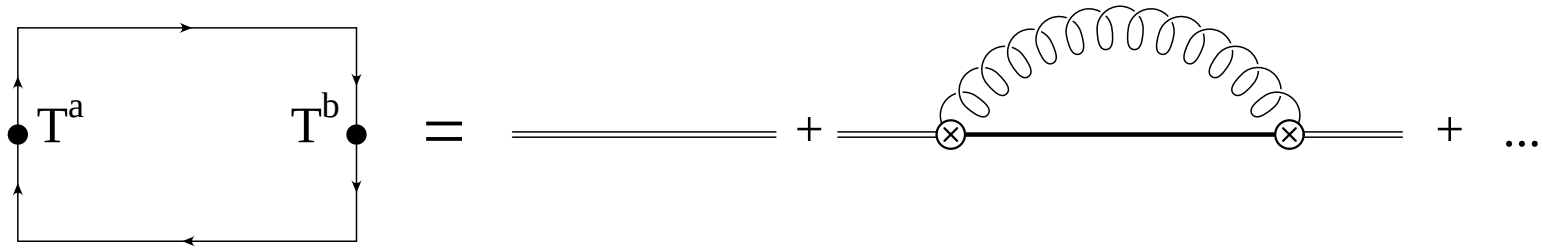
$$V_s(r,\mu) = -C_F \frac{\alpha_{V_s}(r,\mu)}{r}$$

$$\alpha_{V_s}(r,\mu) = \alpha_s(r)\left[1 + \tilde{a}_1 \alpha_s(r) + \tilde{a}_2 (\alpha_s(r))^2 + \frac{\alpha_s^3}{\pi}\frac{C_A^3}{12}\ln \mu r\right]$$

$\tilde{a}_1$ Billoire 80, $\tilde{a}_2$ Schröder 99, Peter 97

3 loop LL Brambilla Pineda Soto Vairo 99

The Static Octet Potential



NRQCD pNRQCD

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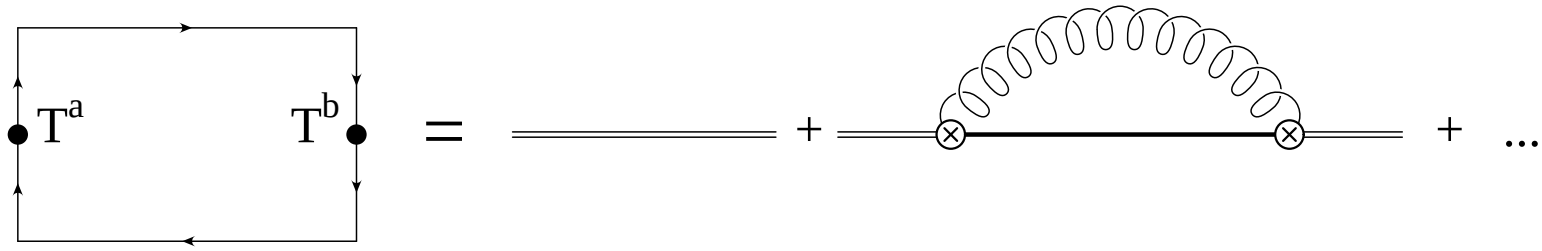
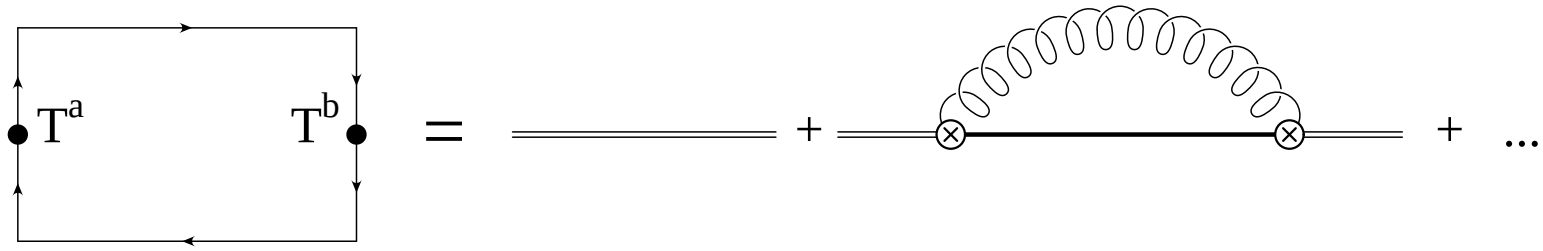


NRQCD pNRQCD

$$V_o(r) = \lim_{T\to\infty} \frac{i}{T} \ln \frac{\langle \square \rangle}{\langle \phi^{\rm adj}_{ab} \rangle}$$

$$+ i \frac{g^2}{N_c} \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \phi^{\rm adj}_{aa'}\, \mathbf{r}\cdot\mathbf{E}^{a'}(t')\, \mathbf{r}\cdot\mathbf{E}^{b'}(t)\, \phi^{\rm adj}_{b'b} \rangle / \langle \phi^{\rm adj}_{ab} \rangle + \ldots$$

The Static Octet Potential



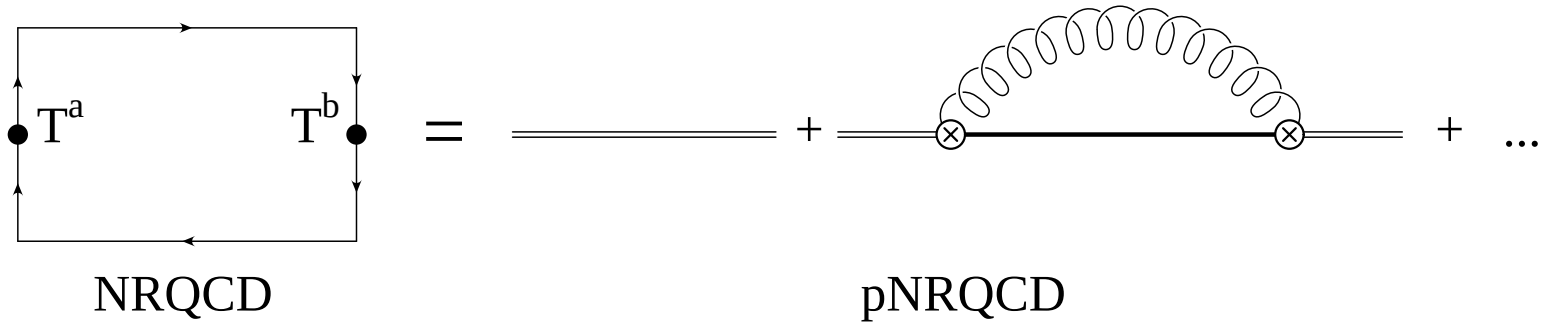
NRQCD pNRQCD

$$V_o(r,\mu) = \left(\frac{C_A}{2} - C_F\right)\frac{\alpha_{V_o}(r,\mu)}{r}$$

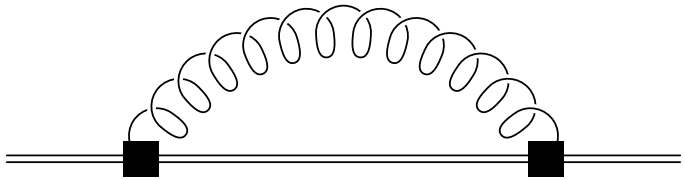
$$\alpha_{V_o}(r,\mu) = \alpha_s(r)\left[1 + \tilde{b}_1\alpha_s(r) + \tilde{b}_2(\alpha_s(r))^2 + \frac{\alpha_s^3}{\pi}\frac{C_A^3}{12}\ln\mu r\right]$$

$\tilde{b}_2$ Schröder 99 03, 3 loop LL Brambilla Pineda Soto Vairo 99

The Static Octet Potential

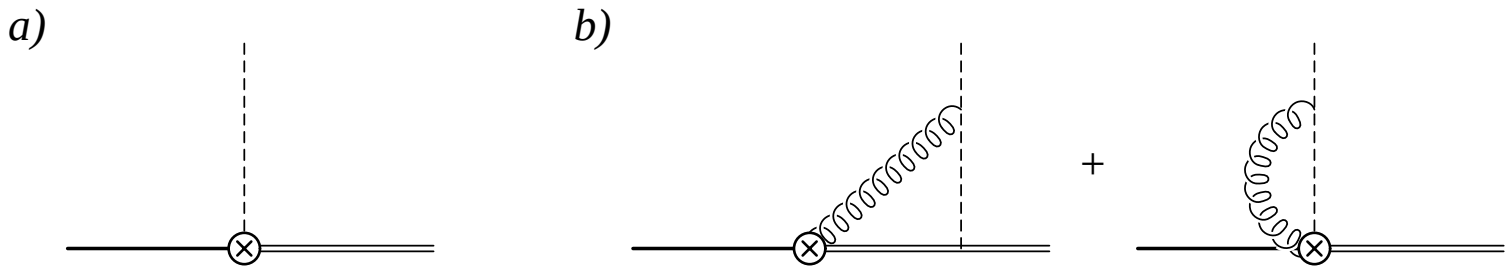


The octet matching formula holds only in perturbation theory:

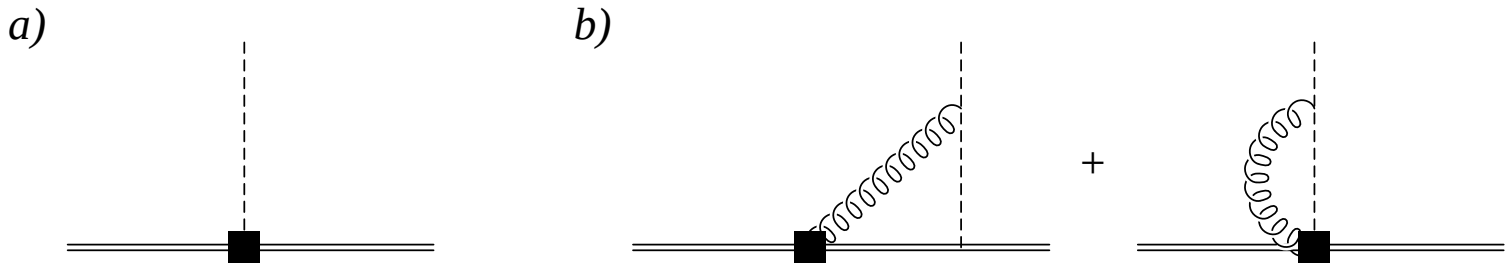
- *It is not gauge invariant.*
-  $\sim V_B^2 r^2$

  and contact terms $\sim r^2$ *would contribute to the potential if* $\Lambda_{\rm QCD}$ *is considered.*

Ultrasoft 1-loop running



$$V_A(r,\mu) = 1 + 0 \times \alpha_s \ln \alpha_s$$

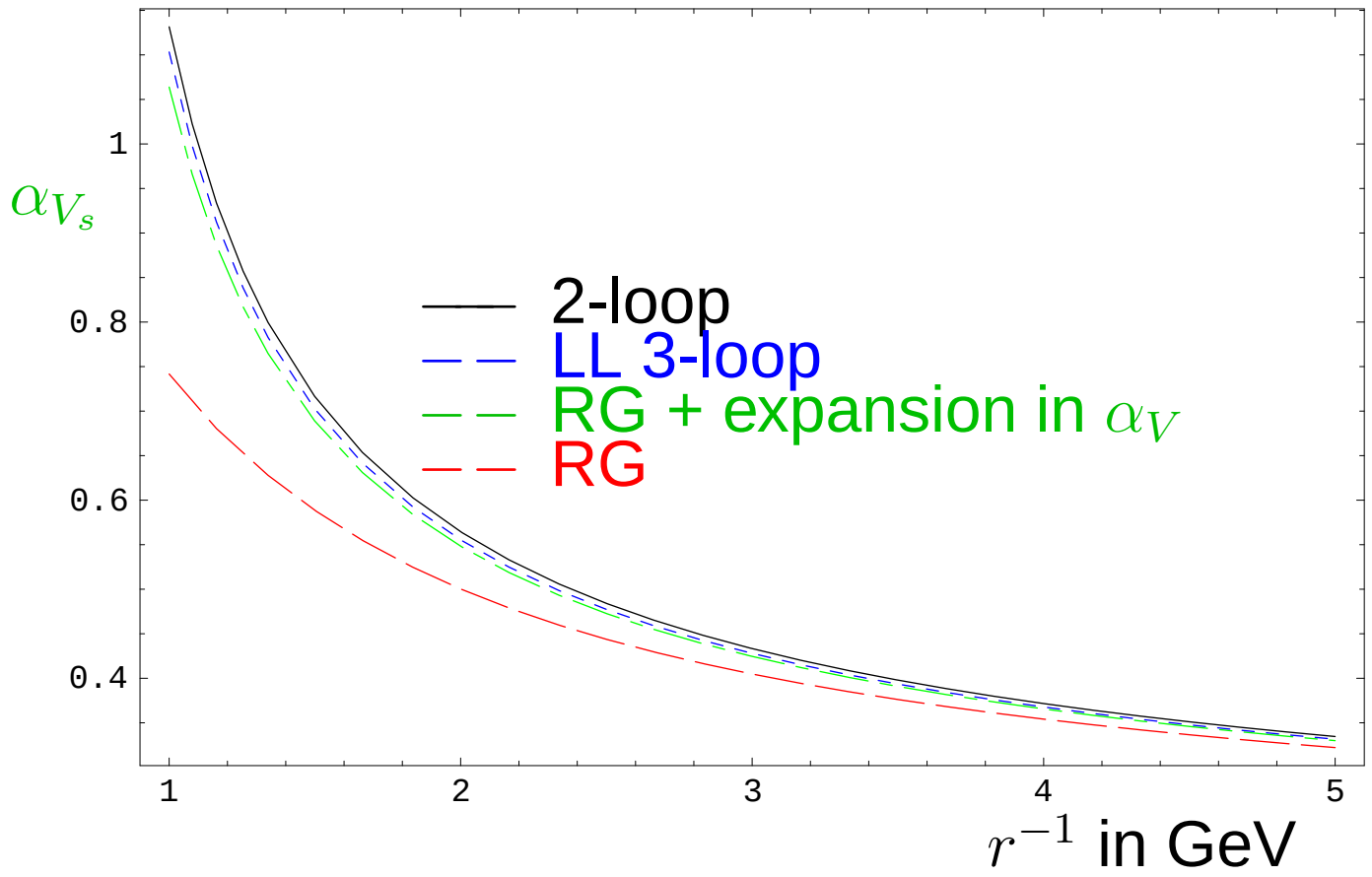


$$V_B(r,\mu) = 1 + 0 \times \alpha_s \ln \alpha_s$$

Summing Logs

$$
\begin{cases}
\mu \dfrac{d}{d\mu} \alpha_{V_s} = \dfrac{2}{3} \dfrac{\alpha_{\mathrm{s}}}{\pi} V_A^2 \left[\left(\dfrac{C_A}{2} - C_F \right) \alpha_{V_o} + C_F \alpha_{V_s} \right]^3 \\
\mu \dfrac{d}{d\mu} \alpha_{V_o} = \dfrac{2}{3} \dfrac{\alpha_{\mathrm{s}}}{\pi} V_A^2 \left[\left(\dfrac{C_A}{2} - C_F \right) \alpha_{V_o} + C_F \alpha_{V_s} \right]^3 \\
\mu \dfrac{d}{d\mu} \alpha_{\mathrm{s}} = \alpha_{\mathrm{s}} \beta(\alpha_{\mathrm{s}}) \\
\mu \dfrac{d}{d\mu} V_A = 0 \\
\mu \dfrac{d}{d\mu} V_B = 0
\end{cases}
$$

Summing Logs



$(\mu = \alpha_s(r)/r;\ N_f = 4)$

Pineda Soto 00

Summing $(\alpha_s \beta_0)^n$

V_s is affected by renormalons: $V_s(renormalon) = C_0 + C_2 r^2 + \ldots$

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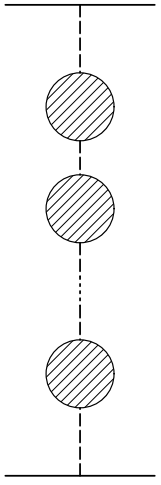
$$V_s \underset{\text{large}\,\beta_0}{\simeq} -\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\mathbf{qr}}}{\mathbf{q}^2} 4\pi C_F\, \alpha_s(q)^{1\,\text{loop}}$$

$$\underset{1/r \gg \mu \gg \Lambda}{=} -\alpha_s(\mu) \sum_{n=0}^{\infty} \int^{\mu} \frac{d^3q}{(2\pi)^3} \frac{\{1 + i\mathbf{qr} - (\mathbf{qr})^2/2 + \ldots\}}{\mathbf{q}^2}$$

$$\times 4\pi C_F \left\{ \frac{-\beta_0 \alpha_s(\mu)}{4\pi} \ln\left(\frac{q^2}{\mu^2}\right) \right\}^n$$

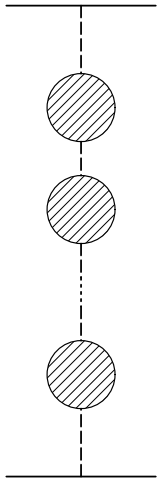


Summing $(\alpha_s \beta_0)^n$

V_s is affected by renormalons: $V_s(renormalon) = C_0 + C_2 r^2 + \ldots$

$$C_0 \simeq -2 \frac{C_F \alpha_s(\mu)}{\pi} \mu \sum_{n=0}^{\infty} n! \left(\frac{\beta_0 \alpha_s(\mu)}{2\pi} \right)^n$$

$$\Rightarrow \delta C_0 \sim \Lambda_{\rm QCD}$$



$$C_2 \simeq \frac{1}{9} \frac{C_F \alpha_s(\mu)}{\pi} \mu^3 \sum_{n=0}^{\infty} n! \left(\frac{\beta_0 \alpha_s(\mu)}{6\pi} \right)^n$$

$$\Rightarrow \delta C_2 \sim \Lambda_{\rm QCD}^3$$

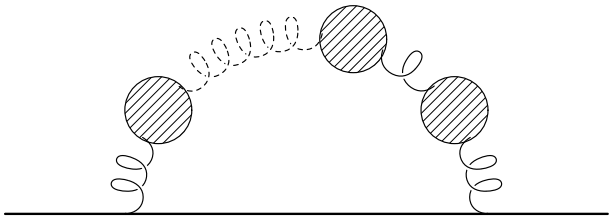
$$1/r \gg \mu \gg \Lambda_{\rm QCD}$$

Aglietti Ligeti 95

Summing $(\alpha_s \beta_0)^n$

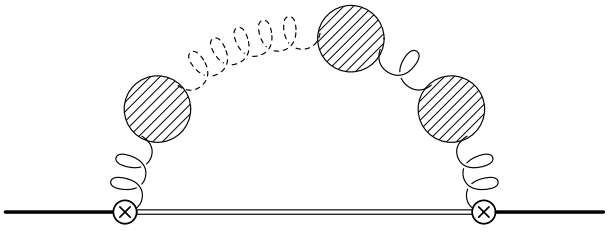
V_s is affected by renormalons: $V_s(renormalon) = C_0 + C_2 r^2 + \ldots$

The $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon cancels against the pole mass.

$2\times$  $= -C_0$

Beneke 98, Pineda 98, Hoang Smith Stelzer Willenbrock 99

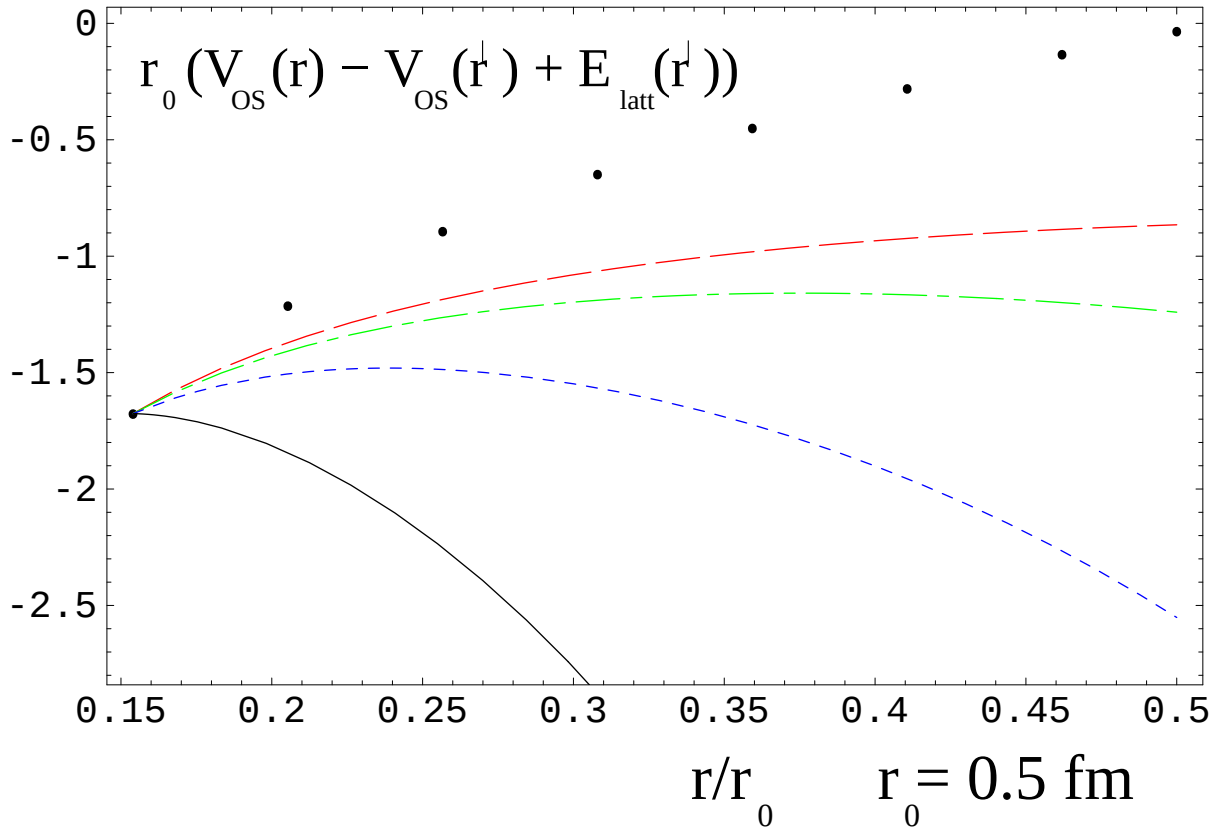
The $\mathcal{O}(\Lambda^3_{\rm QCD})$ renormalon cancels in pNRQCD.

 $= -C_2 r^2$

Brambilla Pineda Soto Vairo 99

Static potential vs lattice QCD

Renormalon subtraction (RS) is crucial in comparing the perturbative static potential with lattice data.



NNLL + 3 loop est.

NNLO

NLO

LO

$$\alpha_s = \alpha_s(1/r)$$
$$\nu_f = \nu_{us} = 2.5\, r_0^{-1}$$
$$r' = 0.15399\, r_0$$

Pineda 02

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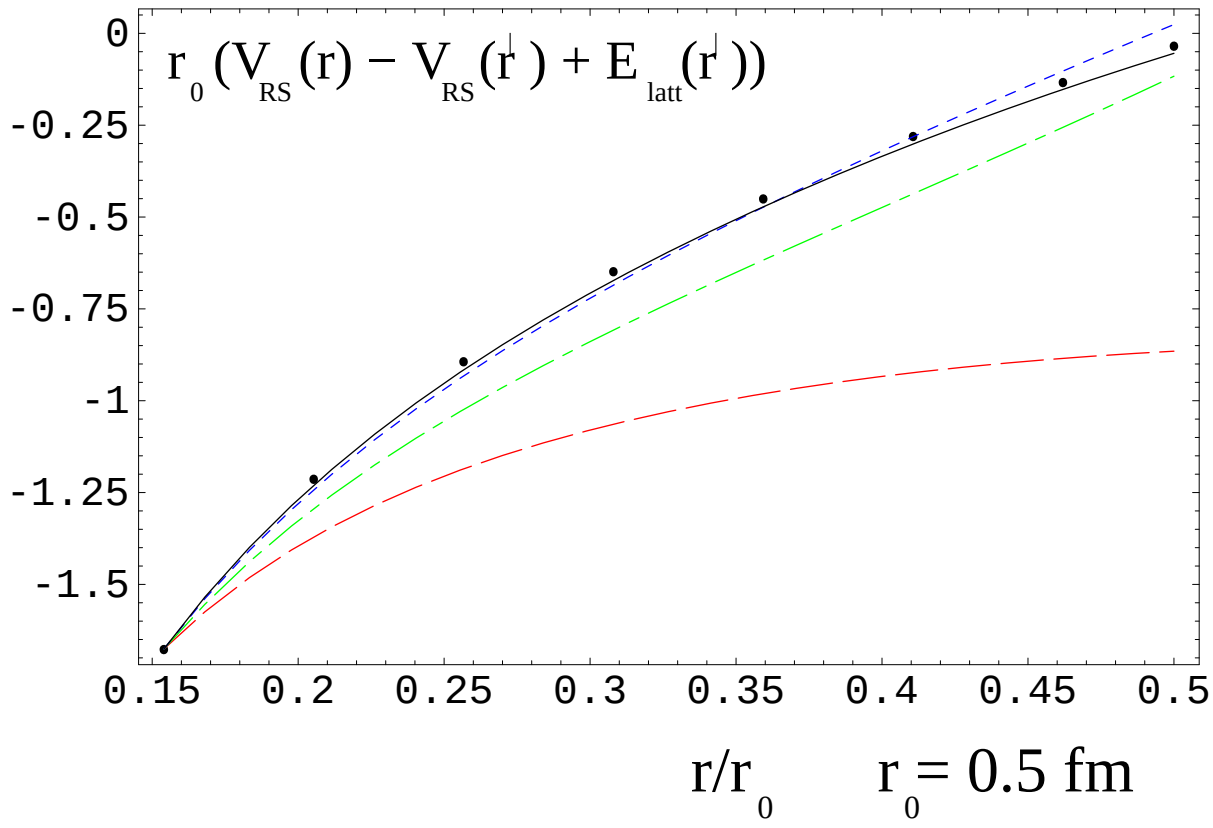

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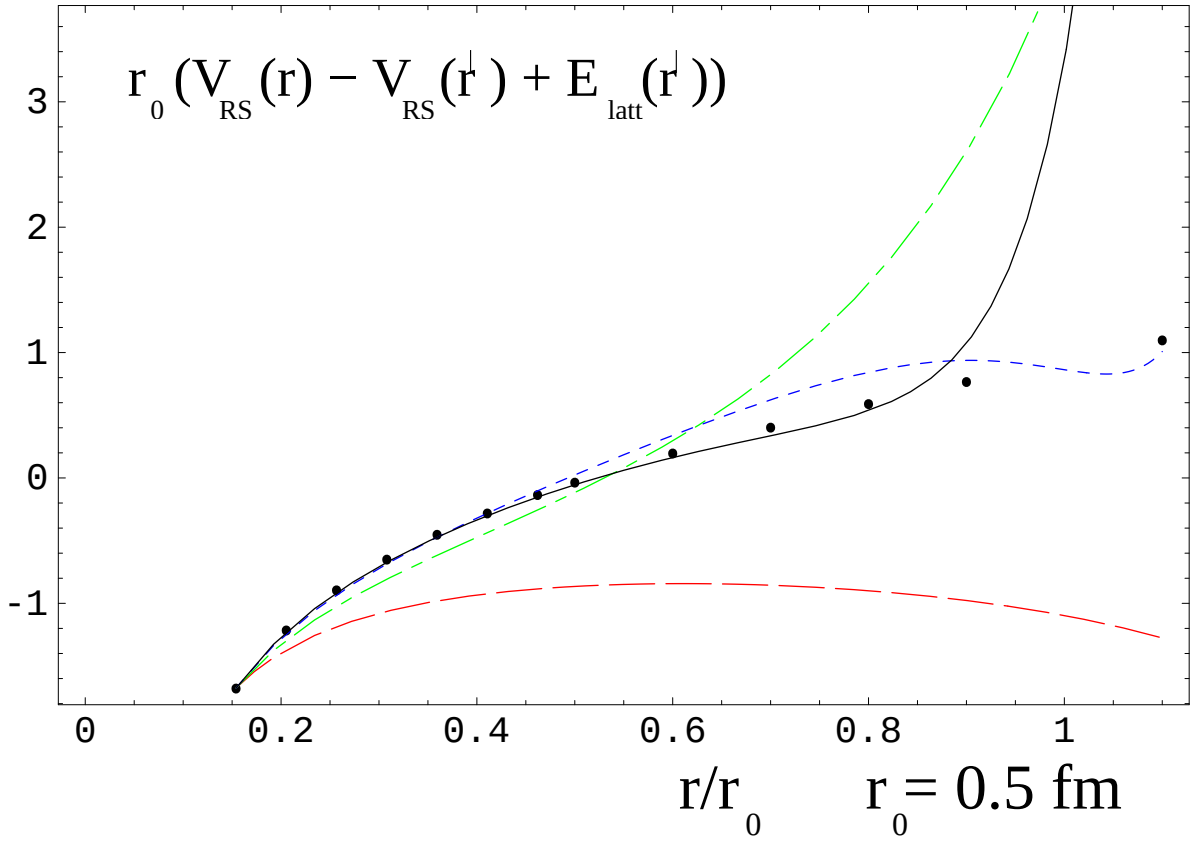
Pineda 02

Static potential vs lattice QCD

No signal of short range non-perturbative effects.

Brambilla Sumino Vairo 01, Necco Sommer 01

Sumino 02, Pineda 02, Lee 02 03



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$$r' = 0.15399\, r_0$$

Quarkonium Spectrum at $m\alpha_{\rm s}^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\ \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

Brambilla Pineda Soto Vairo 99

Quarkonium Spectrum at $m\alpha_s^5$

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$$\frac{V_s^{(1)}}{m} = -\frac{C_F C_A D_s^{(1)}}{2mr^2}$$

$$\frac{V_s^{(2)}}{m^2} = -\frac{C_F D_{1,s}^{(2)}}{2m^2}\left\{\frac{1}{r}, \mathbf{p}^2\right\} + \frac{\pi C_F D_{d,s}^{(2)}}{m^2}\delta^{(3)}(\mathbf{r}) + \frac{C_F D_{2,s}^{(2)}}{2m^2}\frac{1}{r^3}\mathbf{L}^2$$

$$+\frac{3C_F D_{LS,s}^{(2)}}{2m^2}\frac{1}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{4\pi C_F D_{S^2,s}^{(2)}}{3m^2}\mathbf{S}^2\delta^{(3)}(\mathbf{r}) + \frac{C_F D_{S_{12},s}^{(2)}}{4m^2}\frac{1}{r^3}S_{12}(\hat{\mathbf{r}})$$

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$$D_s^{(1)} = \alpha_s^2(r)\left\{1+\frac{2}{3}(4C_F+2C_A)\frac{\alpha_s}{\pi}\ln r\mu\right\}+\dots$$

$$D_{1,s}^{(2)} = \alpha_s(r)\left\{1+\frac{4}{3}C_A\frac{\alpha_s}{\pi}\ln r\mu\right\}+\dots \qquad D_{2,s}^{(2)} = \alpha_s(r)+\dots$$

$$D_{d,s}^{(2)} = \alpha_s(r)\left\{1+\frac{\alpha_s}{\pi}\left[\frac{2C_F}{3}+\frac{17C_A}{3}\right]\ln mr+\frac{16}{3}\frac{\alpha_s}{\pi}\left(\frac{C_A}{2}-C_F\right)\ln r\mu\right\}+\dots$$

$$D_{S^2,s}^{(2)} = \alpha_s(r)\left(1-\frac{7C_A}{4}\frac{\alpha_s}{\pi}\ln mr\right)+\dots$$

$$D_{LS,s}^{(2)} = \alpha_s(r)\left(1-\frac{2C_A}{3}\frac{\alpha_s}{\pi}\ln mr\right)+\dots \qquad D_{S_{12},s}^{(2)} = \alpha_s(r)\left(1-C_A\frac{\alpha_s}{\pi}\ln mr\right)+\dots$$

$m\alpha_s^5\ln\alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99

$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

Quarkonium Spectrum at $m\alpha_{\rm s}^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\ \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$$\sim e^{i\Lambda_{\rm QCD}t}$$

Quarkonium Spectrum at $m\alpha_{\rm s}^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\ \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

- $E_n^{(0)} - H_o \ll \Lambda_{\rm QCD} \Rightarrow \delta E_n \simeq \langle n|r^2|n\rangle \int_0^\infty dt \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu)$

  short-range non-perturbative potential

Quarkonium Spectrum at $m\alpha_{\rm s}^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\, \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

- $E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

$E = E^{\rm p.} + E^{n.p.}$

$$\delta E_{nlj}^{p.} = E_n \frac{\alpha_{\rm s}^3}{\pi}\ln\alpha_{\rm s}\left\{\frac{C_A}{3}\left[\frac{C_A^2}{2} + 4C_AC_F\frac{1}{n(2l+1)} + 2C_F^2\left(\frac{8}{n(2l+1)} - \frac{1}{n^2}\right)\right]\right.$$
$$\left. + \frac{3C_F^2\delta_{l0}}{n}\left[C_F + \frac{C_A}{2}\right] - \frac{7}{3}\frac{C_F^2C_A\delta_{l0}\delta_{s1}}{n} - \frac{(1-\delta_{l0})\delta_{s1}}{l(2l+1)(l+1)n}C_{jl}\frac{C_F^2C_A}{2}\right\} + \ldots$$

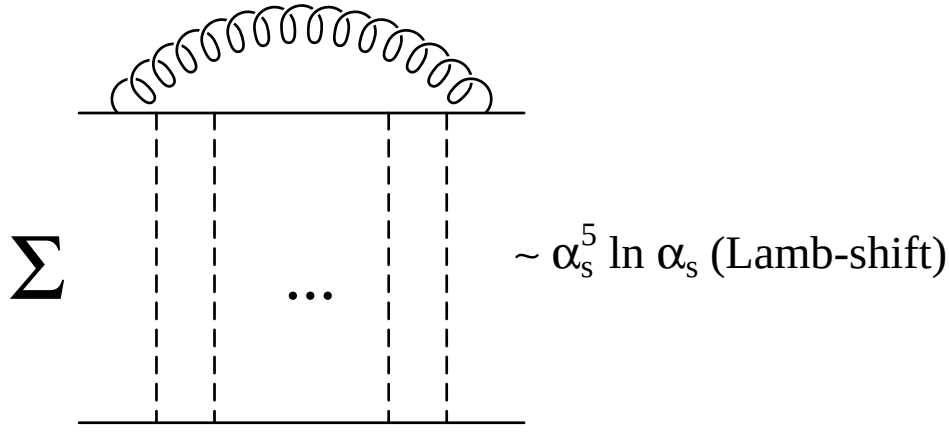
$$\delta E_n^{n.p.} = m\frac{\epsilon_n n^6\langle g^2\,\mathbf{E}^2(0)\rangle}{(mC_F\alpha_{\rm s})^4} + \ldots$$

Voloshin–Leutwyler terms

Quarkonium Spectrum at $m\alpha_{\rm s}^5$

$$E_n = \langle n|V_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\, \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

- $E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



b mass from the Υ system

Ref.	Method	Order	$\overline{m}_b(\overline{m}_b)$ (MeV)
MY99	nonrelativistic Υ sum rules*	NNLO	4200 ± 100
BS99	"	"	4250 ± 80
H00	"	"	4170 ± 50
KS01	low moments sum rules	"	4209 ± 50
BSV01	spectrum, $\Upsilon(1S)$ resonance*	"	$4190 \pm 20 \pm 25 \pm 3$
P01	"	LL N^{3}LO*	$4210 \pm 90 \pm 25$
PS02	"	N^{3}LO*	4346 ± 70

* pole mass vs static potential renormal cancellation

* $\Upsilon(1S)$ mass at N^{3}LO:

$$\frac{\delta E_{\Upsilon(1S)}}{E_{\rm Bohr}} = \alpha_{\rm s}^3(104.819 + 15.297 \log \alpha_{\rm s} + 0.001 a_3)$$

$$\delta m_b(m_b) \simeq 25 \text{ MeV}$$

Melnikov Yelkhovsky 99, Beneke Signer 99, Hoang 00, Kuhn Steinhauser 01, Brambilla Sumino Vairo 01, Pineda 01, Penin Steinhauser 02

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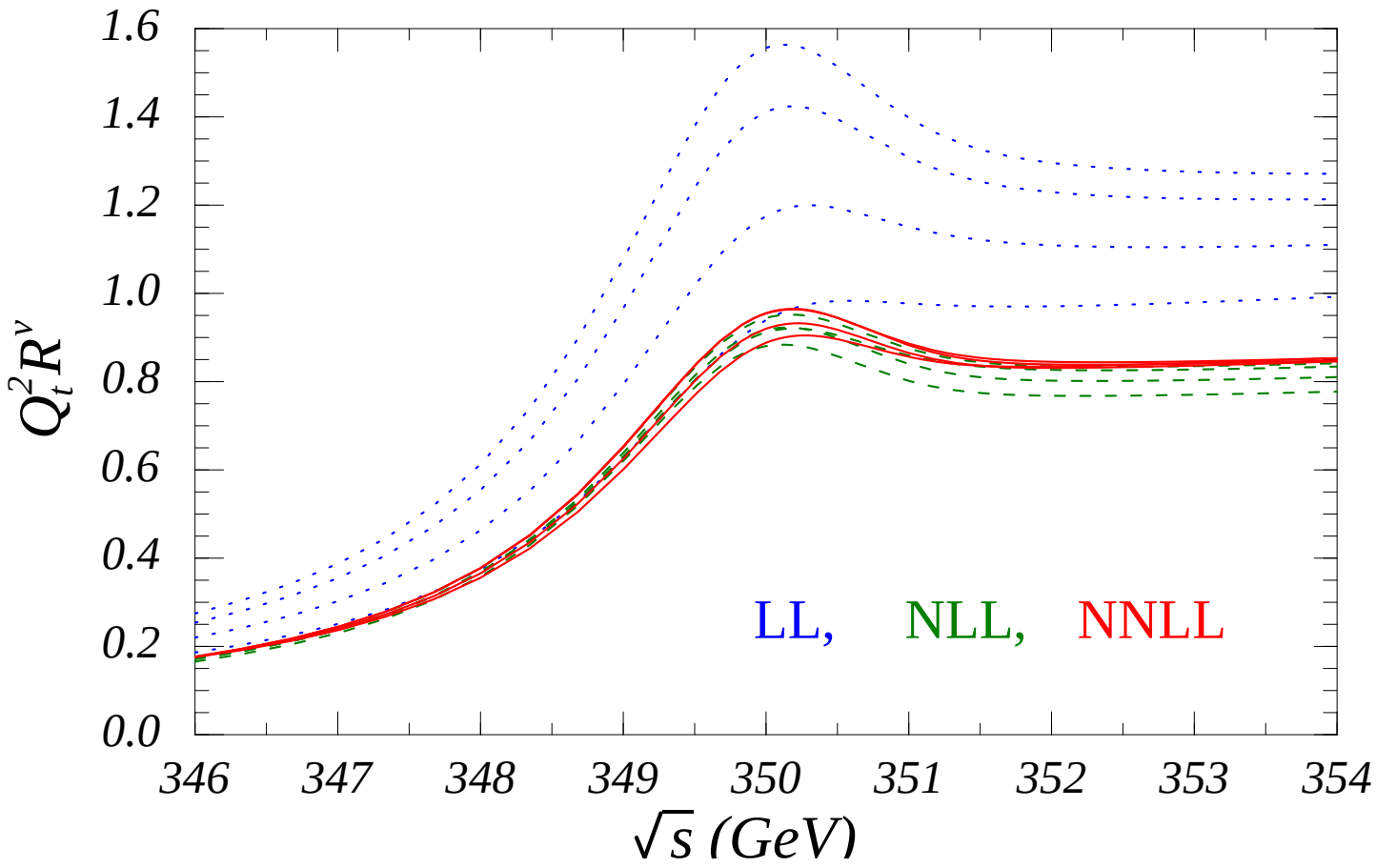
	Method	$\overline{m}_b(\overline{m}_b)$ (MeV)
B system	Inclusive moments	
	(i) lepton spectrum	4310 ± 130
	(ii) γ energy/ had. inv. mass	4220 ± 90
	lattice QCD (stat. limit)	4260 ± 90

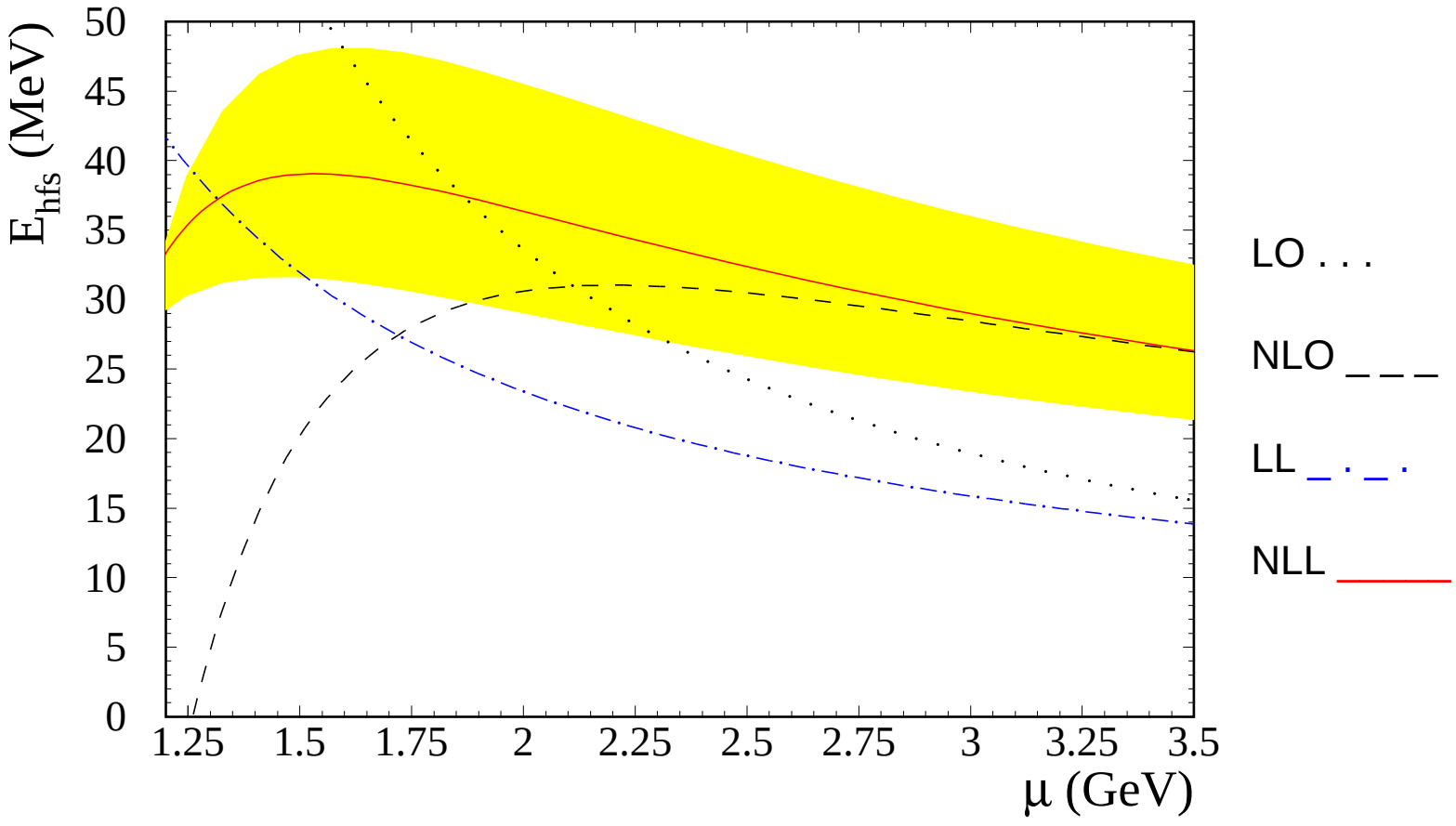
El-Khadra Luke 02

Threshold $t\bar{t}$ cross section



Hoang Teubner 01

η_b mass



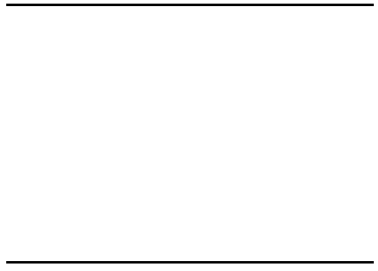
$$M(\eta_b) \simeq 9421 \text{ MeV}$$

Kniehl Penin Pineda Smirnov Steinhauser 03

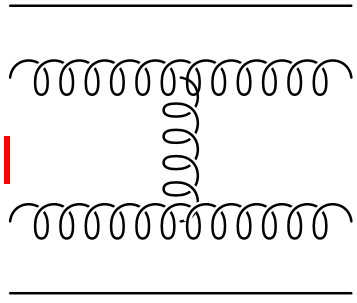
The Static Spectrum

Gluonic excitations between static quarks are of 3 types:

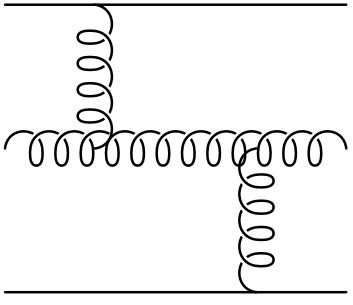
$(Q\bar{Q})_1$



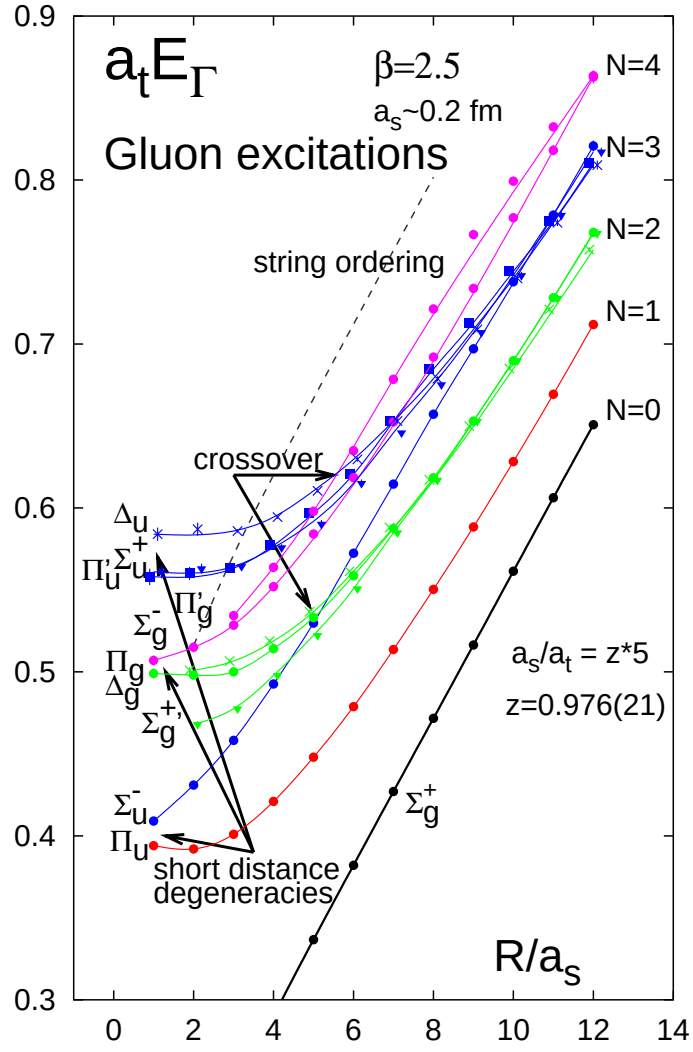
$(Q\bar{Q})_1 +$ Glueball



Hybrid
$(Q\bar{Q})_8 G$



Hybrid Static Energies



Juge, Kuti, Morningstar 98, 02

Hybrids and Gluelumps

At short distance, $1/r \gg \Lambda_{\rm QCD}$ the EFT is pNRQCD:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$
$$+ \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$
$$+ \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \frac{\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O}}{2} + \frac{\mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E}}{2} \right\}$$

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- *At lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.*

Hybrids and Gluelumps

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- *Static hybrids at short distance are called gluelumps and are described by a static adjoint source (O) in the presence of a gluonic field (H):*

$$\mathrm{H}(R, r, t) = \mathrm{Tr}\{\mathrm{O} H\}$$

Hybrids and Gluelumps

At short distance, $1/r \gg \Lambda_{\rm QCD}$ the EFT is pNRQCD:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a}$$
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- *Depending on the glue operator H and its symmetries, the operator* $\mathrm{Tr}\{\mathrm{O}H\}$ *describes a specific gluelump of energy* E_H.

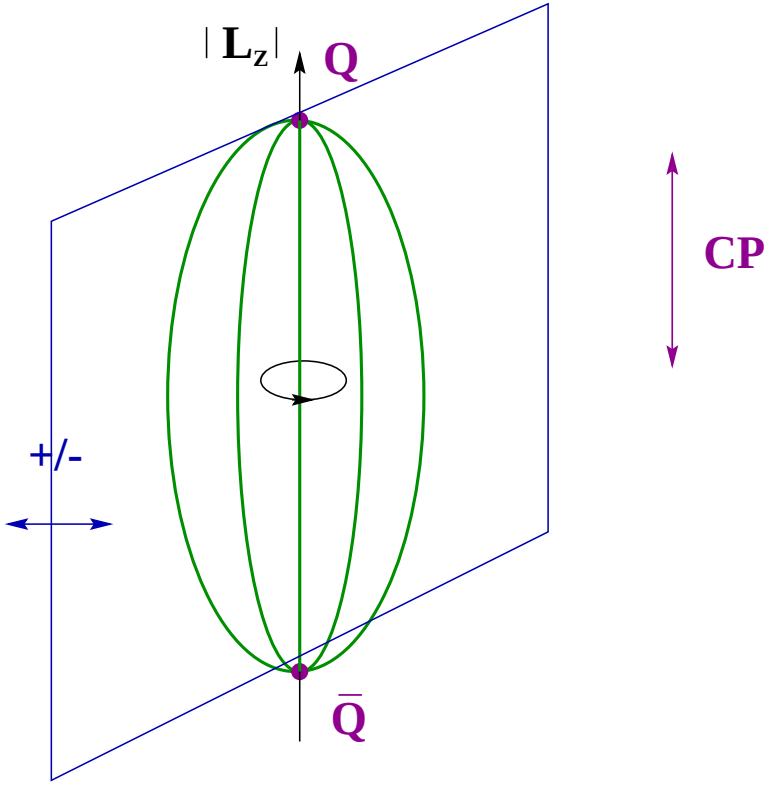
Hybrids and Gluelumps

Symmetries of a diatomic molecule + C.C.

a) $|L_z| = 0, 1, 2, \ldots$
$= \Sigma, \Pi, \Delta \ldots$

b) CP (u/g)

c) Reflection ($+/-$)
(for Σ only)



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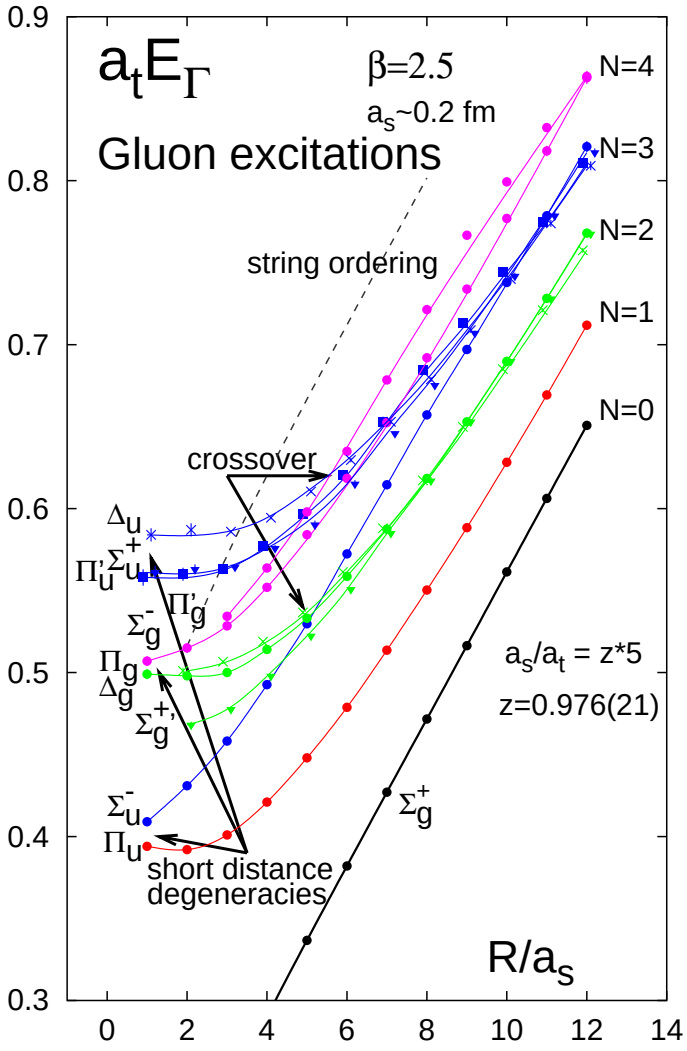
b) CP (u/g)

c) Reflection ($+/-$)
(for Σ only)

	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$	
Σ_g^{-}		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$	
Π_g'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^{+}		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^{-}	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π_u'		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

Brambilla Pineda Soto Vairo 99

Hybrids and Gluelumps

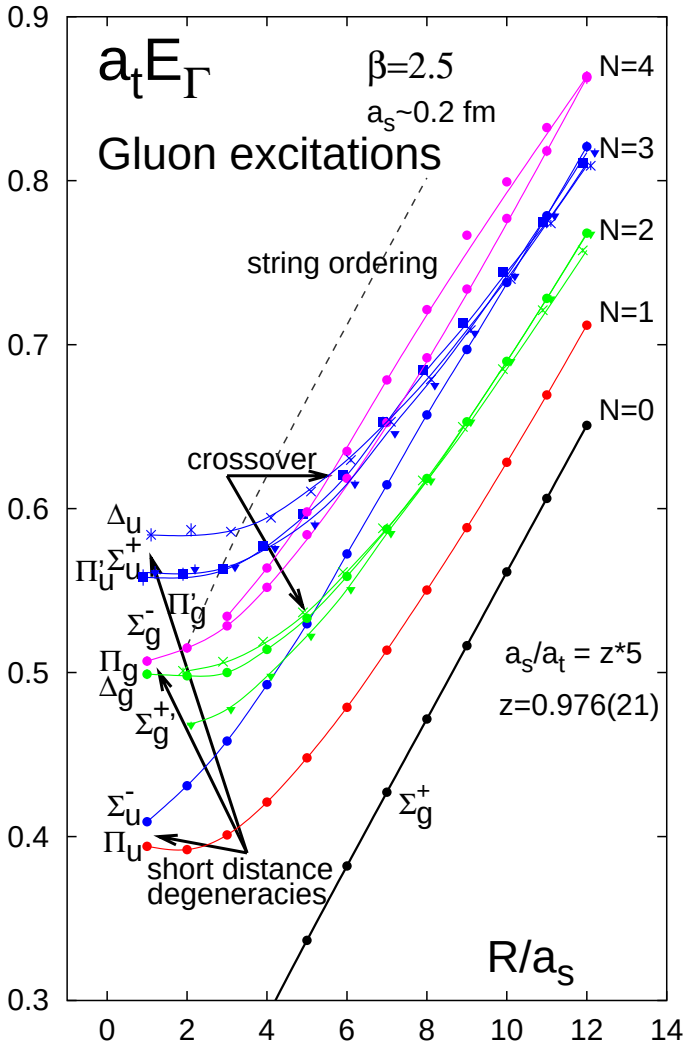


	$L=1$	$L=2$
$\Sigma_g^{+\prime}$	$\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$	
Σ_g^-		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g	$\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$	
Π'_g		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+		$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Σ_u^-	$\mathbf{r}\cdot\mathbf{B}$	
Π_u	$\mathbf{r}\times\mathbf{B}$	
Π'_u		$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u		$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+$ $+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

Juge Kuti Morningstar 98, 02

Brambilla Pineda Soto Vairo 99

Hybrids and Gluelumps



At LO in the multipole expansion

$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle$$

From

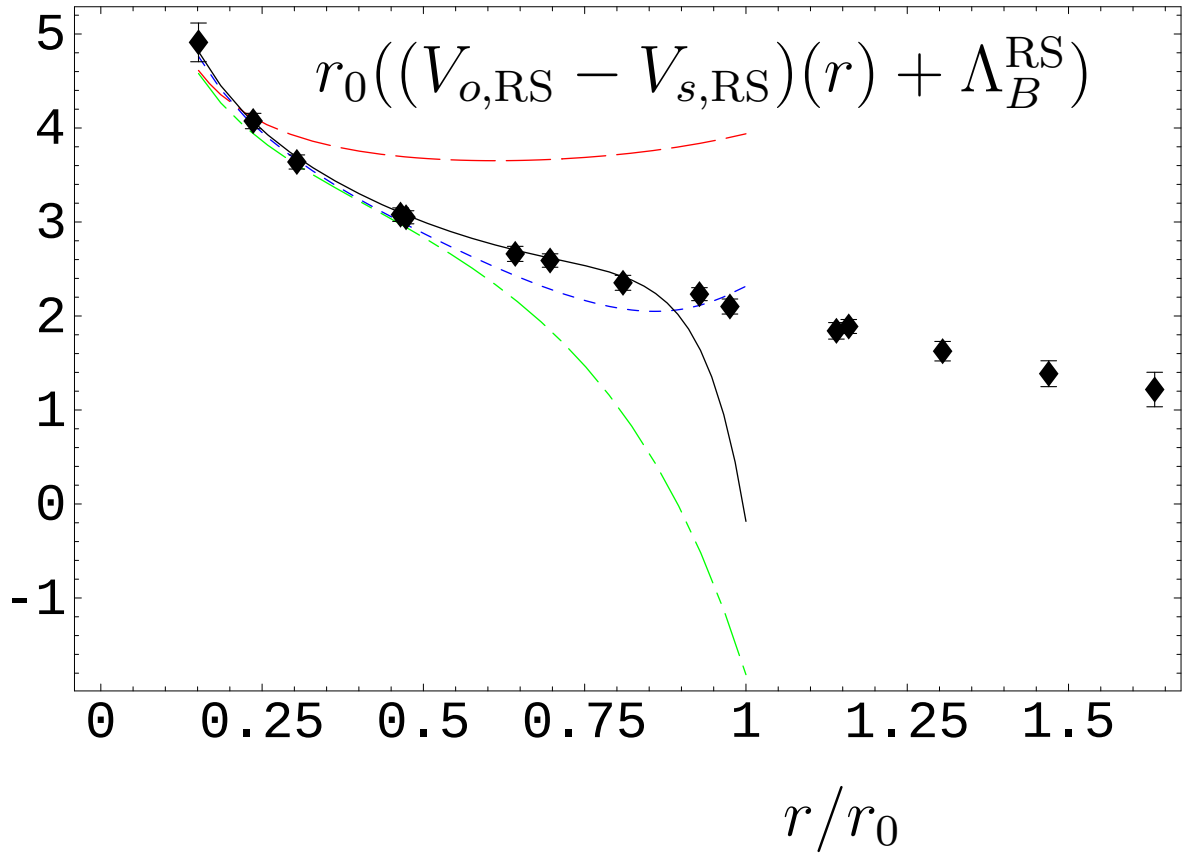
$$\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle^{\rm np} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H$$

Juge Kuti Morningstar 98, 02

Octet potential vs lattice QCD

Renormalon subtraction (RS) is crucial in comparing the perturbative static octet potential with lattice data.



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NNLO

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LO

$$\alpha_{\mathrm{s}} = \alpha_{\mathrm{s}}(1/r)$$
$$\nu_f = \nu_{us} = 2.5\, r_0^{-1}$$

Lattice data of $E_{\Pi_u} - E_{\Sigma_g^+}$

Bali Pineda 03

Octet potential vs lattice QCD

Λ_B correlation length

$$\Lambda_B^{\rm RS}(\nu_f = 2.5\, r_0^{-1}) = [2.25 \pm 0.10({\rm latt.}) \pm 0.21({\rm th.}) \pm 0.08(\Lambda_{\overline{\rm MS}})]\ r_0^{-1}$$

for $\nu_f = 2.5\, r_0^{-1} \approx 1\ \ {\rm GeV}$

$$\Lambda_B^{\rm RS}(1\,\mathsf{GeV}) = [0.887 \pm 0.039({\rm latt.}) \pm 0.083({\rm th.}) \pm 0.032(\Lambda_{\overline{\rm MS}})]\,\mathsf{GeV}$$

Octet potential vs lattice QCD

Higher Gluelump excitations

J^{PC}	H	$\Lambda_H^{\mathrm{RS}} r_0$	Λ_H^{RS}/GeV
1^{+-}	B_i	2.25(39)	0.87(15)
1^{--}	E_i	3.18(41)	1.25(16)
2^{--}	$D_{\{i}B_{j\}}$	3.69(42)	1.45(17)
2^{+-}	$D_{\{i}E_{j\}}$	4.72(48)	1.86(19)
3^{+-}	$D_{\{i}D_jB_{k\}}$	4.72(45)	1.86(18)
0^{++}	$\mathbf{B}^2$	5.02(46)	1.98(18)
4^{--}	$D_{\{i}D_jD_kB_{l\}}$	5.41(46)	2.13(18)
1^{-+}	$(\mathbf{B} \wedge \mathbf{E})_i$	5.45(51)	2.15(20)

Foster Michael 99, Bali Pineda 03

Conclusion

Heavy quarkonium is

- a competitive source for some of the SM parameters: m_t, m_b, m_c, $\alpha_{\rm s}$, ...

- a privileged system to study the interplay of perturbative and non-perturbative QCD.

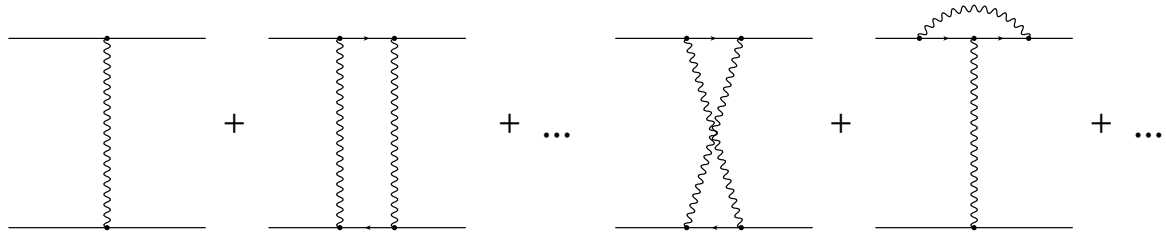
  $\longrightarrow$ *large order perturbation theory vs lattice QCD*

  $\longrightarrow$ *precision physics from lattice QCD*

http://www.qwg.to.infn.it

QWG III workshop: 12-15 October 2004 IHEP Beijing

$\longrightarrow$ *Yellow Report 2004*



$$\underbrace{\alpha\left(1+\frac{\alpha}{v}+\ldots\right)}_{\texttt{Coulomb resummation}} \qquad (\alpha \sim v)$$

Class of diagrams are resummed by Schrödinger equation

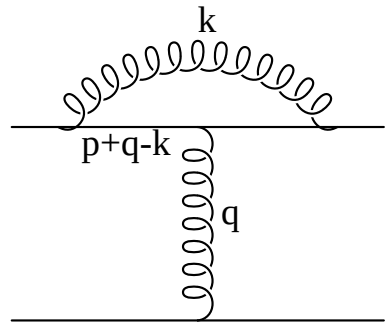
$$\left(\frac{p^2}{2m}+V\right)\phi = E\phi$$

This generates dynamical scales ($\neq$ scattering amplitudes)

$$1/r \sim m\alpha \ll m \qquad\qquad p^2/2m \sim m\alpha^2 \ll m\alpha$$

which mix up in the calculation.

- Technical problem of disentangling the scales which get mixed up in the bound state.



$$q^0 \sim m\, v^2$$
$$\vec{q} \sim m\, v$$
$$k^0 \sim \vec{k}$$

- More fundamental problem of disentangling perturbative from nonperturbative effects.

- In lattice calculations for quantities that involves two very different scales $Q \gg q$ it should hold

$$L^{-1} \ll q \ll Q \ll a^{-1}$$

$a =$ lattice spacing, $L =$ lattice size

c_F

$$c_F = \left(\frac{\alpha_s(m)}{\alpha_s(\mu)}\right)^{\gamma_0/2\beta_0} \left[1 + \frac{\alpha_s(m)}{2\pi}(C_A + C_F) + \frac{\alpha_s(m) - \alpha_s(\mu)}{4\pi}\frac{\gamma_1\beta_0 - \gamma_0\beta_1}{2\beta_0^2}\right]$$

γ_0, γ_1 anomalous dimensions.

Amoros Beneke Neubert 97

$$i\,\frac{g^2}{N_c}\int_0^\infty dt\, e^{-it(V_o-V_s)}\,\langle \mathrm{Tr}(r\cdot E(t)\, r\cdot E(0))\rangle = C_F\,\frac{\alpha_\mathrm{s}}{r}\,\frac{\alpha_\mathrm{s}^3}{\pi}\,\frac{C_A^3}{12}\left(\ln\frac{C_A\alpha_\mathrm{s}}{2r}\frac{1}{\mu}+\frac{1}{2\epsilon}\right)$$

1) $\langle r\cdot E(t)\, r\cdot E(0)\rangle = \frac{r^2}{3}\langle E(t)\cdot E(0)\rangle$

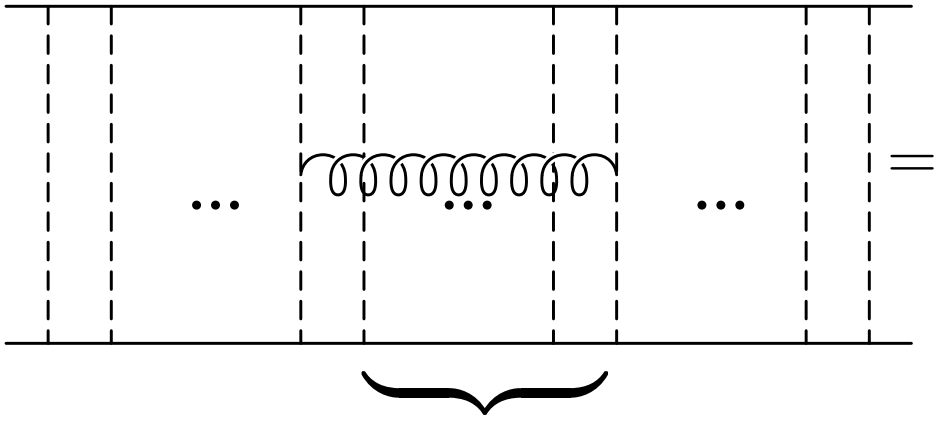
2) for $t>0$ at LO

$$\langle \mathrm{Tr}(E(t)\cdot E(0))\rangle = \mathrm{Tr}\{T^aT^a\}\,\mu^{4-D}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\,k\,e^{-i\,k\,t} \qquad \mathrm{Tr}\{T^aT^a\} = N_c\,C_F$$

3) $\int_0^\infty dt\, e^{-it(V_o-V_s+k)} = \frac{-i}{k+V_o-V_s}$ $\qquad V_o-V_s = \frac{C_A}{2}\frac{\alpha_\mathrm{s}}{r}$

4) $\mu^{4-D}\int\frac{d^{D-1}k}{(2\pi)^{D-1}}\frac{k}{k+V_o-V_s} = \frac{(V_o-V_s)^3}{4\pi^2}\left(\frac{1}{\epsilon}+2\ln\frac{V_o-V_s}{\mu}\right)+\ldots$

The Appelquist–Dine–Muzinich diagrams

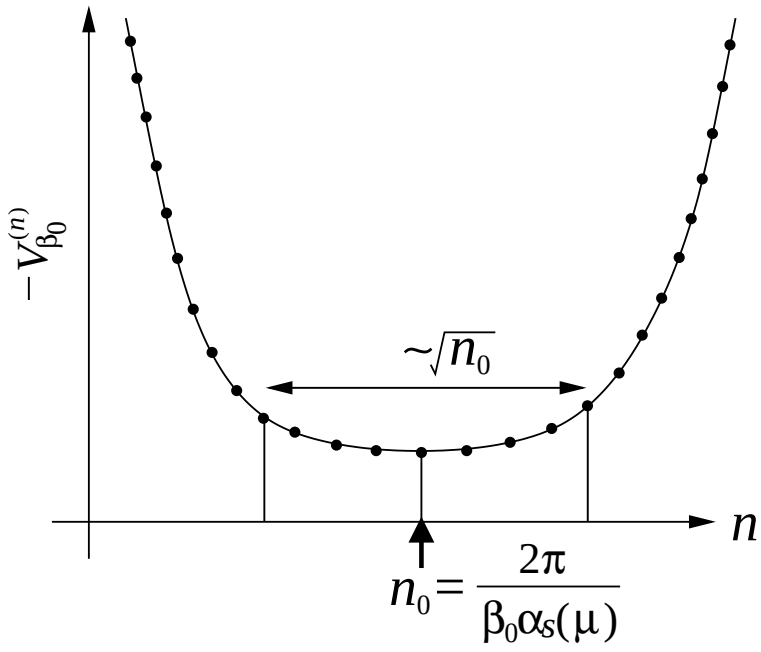


$$= -\frac{C_F C_A^3}{12}\frac{\alpha_s}{r}\frac{\alpha_s^3}{\pi}\ln\left[\frac{C_A\alpha_s}{2r}\times r\right]$$

$$\sim \exp(-i(V_o - V_s)\,T)$$

Appelquist Dine Muzinich 78, Brambilla Pineda Soto Vairo 99

The asymptotic behaviour of $V_{\beta_0}^{(n)}$



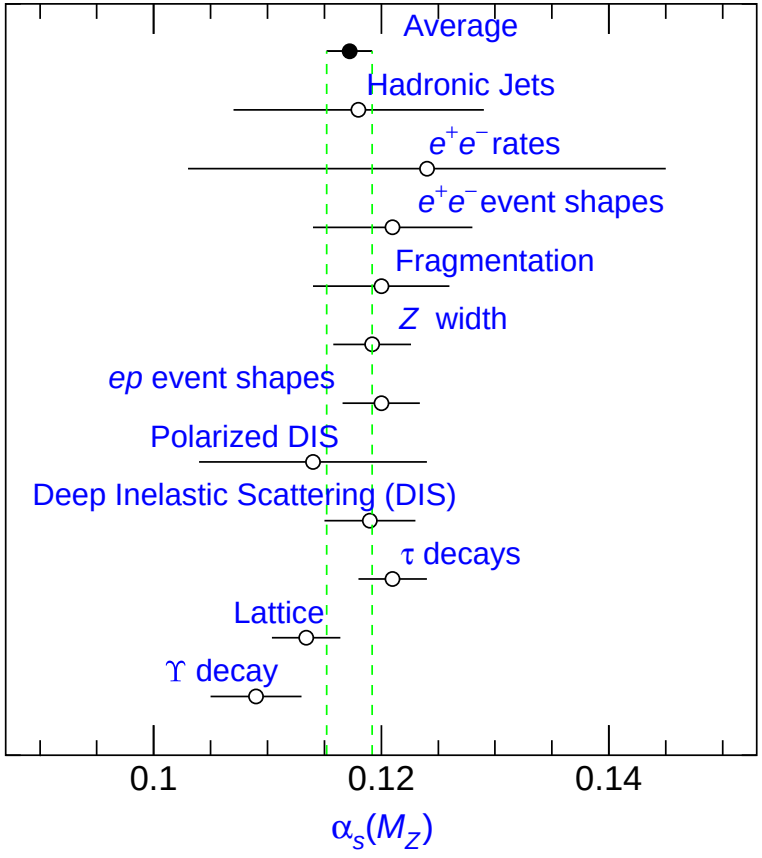
$$\delta V_{\beta_0} \sim \sum_{n=n_0-\sqrt{n_0}}^{n_0+\sqrt{n_0}} |V_{\beta_0}^{(n)}| \sim \Lambda_{\rm QCD} \qquad \Lambda_{\rm QCD} = \mu \exp\left\{-\frac{2\pi}{\beta_0 \alpha_{\rm s}(\mu)}\right\}$$

$$C_0 = \sum_{n=1}^{\infty} N_m \, \mu \left(\frac{\beta_0}{2\pi}\right)^n \alpha_{\rm s}^{n+1}(\mu) \sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}$$

Only b *and* $c_{0,1,2}$ *are known. For* N_m *only an approximate calculation is possible:* $N_m \simeq 0.622036$ *(*$N_f = 0$*).*

Beneke 95, 99, Lee 97, 99, Pineda 02

$$\alpha_{\rm s}(M_{\rm Z}) = 0.117(2)$$



Hinchliffe Manohar 00, PDG 02