Compact Stars with Exotic States of Matter
A basic (but hopefully interesting) introduction
to matter under extreme conditions

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Outline (I)

• Lect 1 – Neutron stars
  – Introduction to compact stars
    • relevant physics – theorists’ playground
    • relevant scales – get some numbers in your head
    • formation – “Little” Bangs
    • unraveling the “onion” – strange pasta
    • observational data on compact stars
  – Basic equations of structure
    • Newtonian stars – white dwarves
    • Fermi gas equation of state
    • mass vs. radius curve
    • general relativistic equations
    • neutron stars – next lecture
Outline (II)

• Lect 2 – The layers of the “onion” – Exotic states of matter
• EoS of nuclear matter
  – realistic potentials
  – solving the Schrodinger equation variationally
  – cold catalyzed nucleon matter
• Exotic states of matter
  – unpaired quark matter
  – CFL
• Building a “realistic” star
  – equations of state
  – phase transitions in nuclear and quark matter
  – maximum mass limits
References

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- S. Weinberg,  
  ‘Gravitation and cosmology’
- M. Prakash and J. Lattimer,  
- J. Lattimer,  
  ‘Stars’, SUNY Stonybrook grad course,  
  http://www.ess.sunysb.edu/lattimer/PHY521/index.html
- S. Reddy,  
  `Novel phases at high density and their roles in the structure and evolution of neutron stars’ (Zakopane Summer School),  
  nucl-th/0211045
- M. Alford,  
  `Color superconducting quark matter’, hep-ph/0102047
The Theorist’s Playground & Astrophysical Laboratory

- Relevant Theories
  - general relativity
  - classical electrodynamics
  - quantum field theory
    - Electroweak
    - QCD
    - EFT
  - statistical physics
  - transport phenomena
  - collective phenomena

- Overlapping disciplines
  - nuclear physics
  - particle physics
  - astrophysics
  - condensed matter

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Scales – Mass & Length

- Fundamental constants
  
  \( \hbar c \approx 197 \text{ MeV fm} \), \( k_B = 8.62 \times 10^{-11} \text{ MeV/K} \)

- Solar scales
  
  \( M_\odot = 1.99 \times 10^{30} \text{ kg} \)
  \( = 1.79 \times 10^{54} \text{ erg} \)
  \( R_\odot = 6.96 \times 10^5 \text{ km} \)
  \( \rho_c \approx 160 \text{ g/cc} \approx 20 \rho_{Fe} \)
  \( T_c \approx 1.6 \times 10^7 \text{ K} \)
  \( H \approx 50 \text{ G} \)

- NS scales
  
  \( M \approx 1.4 M_\odot \)
  \( R \approx 10 - 20 \text{ km} \)
  \( \rho_c \approx 10^{15} \text{ g/cc} \approx 5 - 10n_0, n_0 = 0.16 \text{ fm}^{-3} \)
  \( T_c \approx 10^6 \text{ K} \approx 0 \)
  \( H \approx 10^{12} \text{ G} \)
Formation of neutron stars: supernovae

Main stages:
(I) core collapse
(II) proto-neutron star
(III) $\nu$ heating
(IV) $\nu$ transparency
(V) photon cooling

Prakash and Lattimer
Formation of proto-neutron stars: Type II Supernova

- **Type II supernova explosion**
  - gravitational collapse massive star’s white dwarf core > 8M-
  - collapse halts @ core density » n₀
  - shock wave dynamics
    - shock wave forms @ outer core radius
    - energy loss ν and nuclear dissociation stalls shock wave 100-200 km
    - shock resuscitation from core ν’s +rotation, convection, magnetic fields, etc.
    - ν-driven explosion expels stellar mantle
  - gravitational binding energy released
    \[ \text{BE}_{\text{grav}} = \rho_{\text{avg}} \int d^3r V(r) = \frac{3}{5} (GM^2/R) \quad \frac{1}{4} \times 3 \times 10^{53} \text{ erg} \quad \frac{1}{4} \times 0.1 \text{ M}_{\text{star}} \]
  - kinetic energy mass blow-off 1!2£10^{51} erg
  - Supernova (SN) 1987A in Large Magellanic Cloud
    confirmed release of \( E_\nu = 3 \times 10^{53} \text{ erg} \)
Proto-neutron stars

- proto-neutron star $R \approx 20$ km
  - lepton rich - $e^-$ and $\nu_e$
  - baryon number density $n=2!3n_0$
  - trapped neutrinos $\sigma_{\nu A} \approx 10^{-40}$ cm$^2$
    - $\lambda \approx (\sigma n)^{-1} \approx 10$ cm
    - compare $\sigma_{\nu A}$ to $\sigma_{e A} \approx 10^{-24}$ cm$^2$
  - shrinks due to pressure loss from $\nu$ emission at surface
  - escape of $\nu$ from interior on diffusion time scale
    $\tau \approx 3R^2/\lambda c \approx 10$ s
  - $\nu$ loss $\to e^-$ capture on protons and initially warms the interior as the $\nu$'s make their way out; mostly neutrons
  - core temperature $T_c \approx 50$ MeV ($6\times 10^{11}$ K)
  - cooling starts
    - $\sigma / \lambda^{-1} / h E_{\nu}^2 i \approx \lambda > R$ after $\approx 50$s
Peeling the astrophysical onion

- **Atmosphere**
  - neglibile mass

- **Envelope**

- **Crust**
  - nuclear lattice
  - neutron superfluid

- **Transition region**
  - inhomogeneous “pasta” phases

- **Outer core**
  - pion condensation
  - hyperonic matter

- **Inner core**
  - quark matter
  - color superconductors
    - CFL
    - 2SC
Observation of astrophysical objects

- Varieties of astrophysical objects
  - main sequence stars
  - white dwarves
  - pulsars
  - binary systems
  - quasars

- Observational techniques
  - radio astronomy
    - Very Large Array – Socorro, NM
    - Arecibo – Puerto Rico
  - optical telescopes
    - low earth orbit
      - Hubble Space Telescope
    - land based
      - Mauna Kea, Chile, Arizona, etc.
  - x-ray observatories
    - Chandra
    - XMM
Radio binary pulsar data

- Timing observations
  - orbital sizes and periods gives total mass
  - relativistic effects give mass of each component
- NS-NS binaries
  precision $\leq 0.0003M_{\odot}$
- NS-white dwarf binaries
  precision $\leq 0.1M_{\odot}$
- x-ray binaries
  larger errors
- An aside:
  radio observation of ms pulsars and extrasolar planets by A. Wolszczan

Newtonian stars: warm up on white dwarves

• Assume:
  – spherically symmetric, static star
  – uniform (entropy & chemical composition)
  – $E/V \approx \frac{1}{4} m_N N/V$ – neglect general relativistic effects

• Newtonian equation of motion – hydrostatics
  – gravity – $F_g$
  – degeneracy pressure of electron gas – $F_{\text{deg}}$

\[
\hat{r} \cdot \mathbf{F} = 0
\]

\[-F_g + F_{\text{deg}} = 0\]

\[F_{\text{deg}} = P(r + dr)dA(r + dr) - P(r)dA(r)\]

\[F_g = \frac{GM(r)}{r^2} \rho(r)dV\]

\[\frac{dP(r)}{dr} = -\frac{GM(r) \rho(r)}{r^2}\]

\[\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)\]

\[\frac{d}{dr} \frac{r^2}{\rho(r)} \frac{dp(r)}{dr} = -4\pi G r^2 \rho(r)\]
Stellar structure equation

- **Structure equation**
  - obtain \( p(r), \rho(r) \)
  \[
  \frac{d}{dr} \frac{r^2}{\rho(r)} \frac{dp(r)}{dr} = -4\pi Gr^2 \rho(r)
  \]

- **Boundary condition**
  - \( p_c = p(0), \rho_c = \rho(0) \)

- **Integrate out from central values to** \( p(R) = \rho(R) = 0 \)

- **Requires Equation of State (EoS):** \( p(\rho) \)
  - EoS depends on species present
  - interactions

- **Properties of EoS**
  - “stiffness” \( \frac{1}{4} \) adiabatic compressibility
    - \( \kappa_s = \frac{1}{\rho} \left( \frac{dp}{d\rho} \right)_s \sim \frac{1}{\rho_c^2} \)
      - smaller slope (at fixed \( \rho \)) “harder” EoS
      - “hard” EoS) large wave propagation speed
        - all EoS’s are limited by superluminal wave speed

- **Mass vs. radius** \( M(R) \) curve
  - scan over central density/pressure
  - obtain total mass, \( M \) and radius, \( R \)
  - maximum mass

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Fermi gas model EoS

- Assume cold, relativistic Fermi gas of electrons – all momenta filled to Fermi level, $k_F$

\[ n(k) = \left[ e^{(\epsilon(k)-\mu)/T} + 1 \right]^{-1} \rightarrow \theta(k_F - k), T \to 0 \]

\[ n = \frac{N}{V} = g \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k) \]
\[ = \frac{g}{2\pi^2} \int_0^{\infty} dk k^2 \theta(k_F - k) \]
\[ = \frac{g}{6\pi^2} k_F^3 \]

\[ \epsilon(k_F) = \frac{E}{V} = g \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k) \sqrt{k^2 + m^2} \]
\[ = \frac{g}{2\pi^2} \frac{m^4}{\pi^2} \int_0^{k_F/m} du u^2 (1 + u^2)^{1/2} \]
\[ = g \frac{\epsilon_0}{2} \left[ (2x^3 + x)(1 + x^2)^{1/2} - \sinh^{-1} x \right] \]

\[ \epsilon_0 = \frac{m^4}{\pi^2}, \quad x = \frac{k_F}{m} \]
Fermi gas EoS (II)

- pressure

\[
p = - \left( \frac{\partial E}{\partial V} \right)_{T=0} = n\mu - \epsilon
\]

\[
= g \int \frac{d^3k}{(2\pi)^3} \left( \mu - \sqrt{k^2 + m^2} \right) \theta(k_F - k)
\]

\[
= g \frac{1}{6\pi^2} \int_0^{k_F} dk k^4 (k^2 + m^2)^{-1/2}, \quad \text{I.B.P.,} \quad \mu = \sqrt{k_F^2 + m^2}
\]

\[
p = g \frac{\epsilon_0}{48} \left[ (2x^3 - 3x)(1 + x^2)^{1/2} + 3 \sinh^{-1} x \right]
\]

\[
\epsilon = nm_N A/Z + \epsilon(k_F), \quad \epsilon(k_F) \ll nm_N
\]

\[
\epsilon \approx \rho
\]

- eliminate x to obtain p(\rho)

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Fermi gas EoS (III)

- Relativistic

\[ p(k_F) = g \frac{1}{24\pi^2} k_F^4 \]

\[ = K_{rel} g^{-1/3} \epsilon^{4/3}, \quad K_{rel} = \frac{1}{24\pi^2} \left( \frac{6\pi^2 Z}{m_N A} \right)^{4/3} \]

- Non-relativistic

\[ p(k_F) = g \frac{1}{2\pi^2} \frac{k_F^5}{15 m_e} \]

\[ = K_{nr} g^{-2/3} \epsilon^{5/3}, \quad K_{nr} = \frac{1}{30\pi^2 m_e} \left( \frac{6\pi^2 Z}{m_N A} \right)^{5/3} \]

- Polytropic EoS

\[ p = K \epsilon^\gamma \]
Mass vs. radius

- Polytropic EoS – exactly soluble

\[ M = 4\pi R^{(3\gamma-4)/(\gamma-2)} \left( \frac{K\gamma}{4\pi G(\gamma - 1)} \right)^{-1/(\gamma-2)} \xi_1^{-(3\gamma-4)/(\gamma-2)} \xi_1^2 |\theta'(\xi_1)| \]

- Lane-Emden function \( \theta(\xi) \)
- \( \gamma > 6/5 \)
- \( \gamma = 4/3 \) \( M \) independent of \( R, \rho(0) \)

- Relativistic EoS and Chandrasekhar limit
  - \( \gamma = 4/3 \)

\[ M = 5.87 \left( \frac{Z}{A} \right)^2 M_\odot \approx 1.26 M_\odot, \quad Z/A = 26/56 \]
Neutron stars: General relativistic equation

- Tolman-Oppenheimer-Volkov Equation
  - gravitational and special relativistic corrections increase the strength of gravity relative to Newtonian case
  - neglects rotation

\[
\frac{dP(r)}{dr} = -\frac{G\rho(r)m(r)}{r^2} \times \left[ 1 + \frac{P}{\rho(r)c^2} \right] \times \left[ 1 + \frac{4\pi r^3 P}{m(r)c^2} \right] \times \left[ 1 - \frac{2Gm(r)}{r} \right]^{-1}
\]

\[
\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)
\]
Neutron stars

• For masses, $M > (\sim c^{3/2}/m_N^2G^{3/2})^{1/4} \ 2M_\odot$ (Chandrasekhar mass)
  - electron degeneracy can’t support gravity
  - white dwarf collapses
    • possibly to a black hole
    • or a neutron star

• Similar to white dwarf – now neutron degeneracy
  - reaction: $p+e^- \rightarrow n+\nu$
  - mostly neutrons, some protons – enough to prevent neutron decay
  - central density > white dwarf’s $\sim (m_N/m_e)^3 \sim 10^9$
  - radius < white dwarf’s $\sim m_N/2m_e \sim 10^3$

• Next lecture – neutron stars from “realistic” equations of state; and
• A “realistic” compact object taking into account “exotic matter”