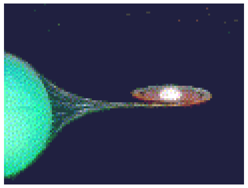

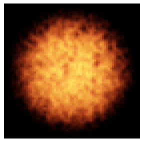

Compact Stars with Exotic States of Matter

A basic (but hopefully interesting) introduction
to matter under extreme conditions

Hampton University Graduate Studies
Thomas Jefferson National Accelerator Facility
2004 June 3

Mark Paris (JLab)

Outline (I)

- Lect 1 – Neutron stars
  - Introduction to compact stars
    - relevant physics – theorists' playground
    - relevant scales – get some numbers in your head
    - formation – "Little" Bangs
    - unraveling the "onion" – strange pasta
    - observational data on compact stars
  - Basic equations of structure
    - Newtonian stars – white dwarves
    - Fermi gas equation of state
    - mass vs. radius curve
    - general relativistic equations
    - neutron stars – next lecture

Outline (II)

- Lect 2 – The layers of the “onion” – Exotic states of matter
- EoS of nuclear matter
  - realistic potentials
  - solving the Schrodinger equation variationally
  - cold catalyzed nucleon matter
- Exotic states of matter
  - unpaired quark matter
  - CFL
- Building a “realistic” star
  - equations of state
  - phase transitions in nuclear and quark matter
  - maximum mass limits

References

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- S. Weinberg,
  'Gravitation and cosmology'
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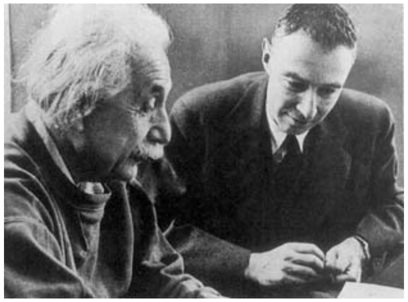
  'The physics of neutron stars', astro-ph/0405262
- J. Lattimer,
  'Stars', SUNY Stonybrook grad course,
  http://www.ess.sunysb.edu/lattimer/PHY521/index.html
- S. Reddy,
  \`Novel phases at high density and their roles in the structure and evolution of neutron stars' (Zakopane Summer School),
  nucl-th/0211045
- M. Alford,
  \`Color superconducting quark matter', hep-ph/0102047

The Theorist's Playground & Astrophysical Laboratory

- Relevant Theories
  - general relativity
  - classical electrodynamics
  - quantum field theory
    - Electroweak
    - QCD
    - EFT
  - statistical physics
  - transport phenomena
  - collective phenomena



- Overlapping disciplines
  - nuclear physics
  - particle physics
  - astrophysics
  - condensed matter

Scales – Mass & Length

- Fundamental constants

$$\hbar c \approx 197 \text{ MeV fm}, k_B = 8.62 \times 10^{-11} \text{ MeV/K}$$

- Solar scales

$$\begin{aligned} M_\odot &= 1.99 \times 10^{30} \text{ kg} \\ &= 1.79 \times 10^{54} \text{ erg} \\ R_\odot &= 6.96 \times 10^{5} \text{ km} \\ \rho_c &\approx 160 \text{ g/cc} \approx 20\rho_{Fe} \\ T_c &\approx 1.6 \times 10^{7} \text{ K} \\ H &\approx 50 \text{ G} \end{aligned}$$

- NS scales

$$\begin{aligned} M &\approx 1.4 M_\odot \\ R &\approx 10 - 20 \text{ km} \\ \rho_c &\approx 10^{15} \text{ g/cc} \approx 5 - 10 n_0, n_0 = 0.16 \text{ fm}^{-3} \\ T_c &\approx 10^{6} \text{ K} \approx 0 \\ H &\approx 10^{12} \text{ G} \end{aligned}$$

thimbleful of NM»10^9 tons

Formation of neutron stars: supernovae

Main stages:

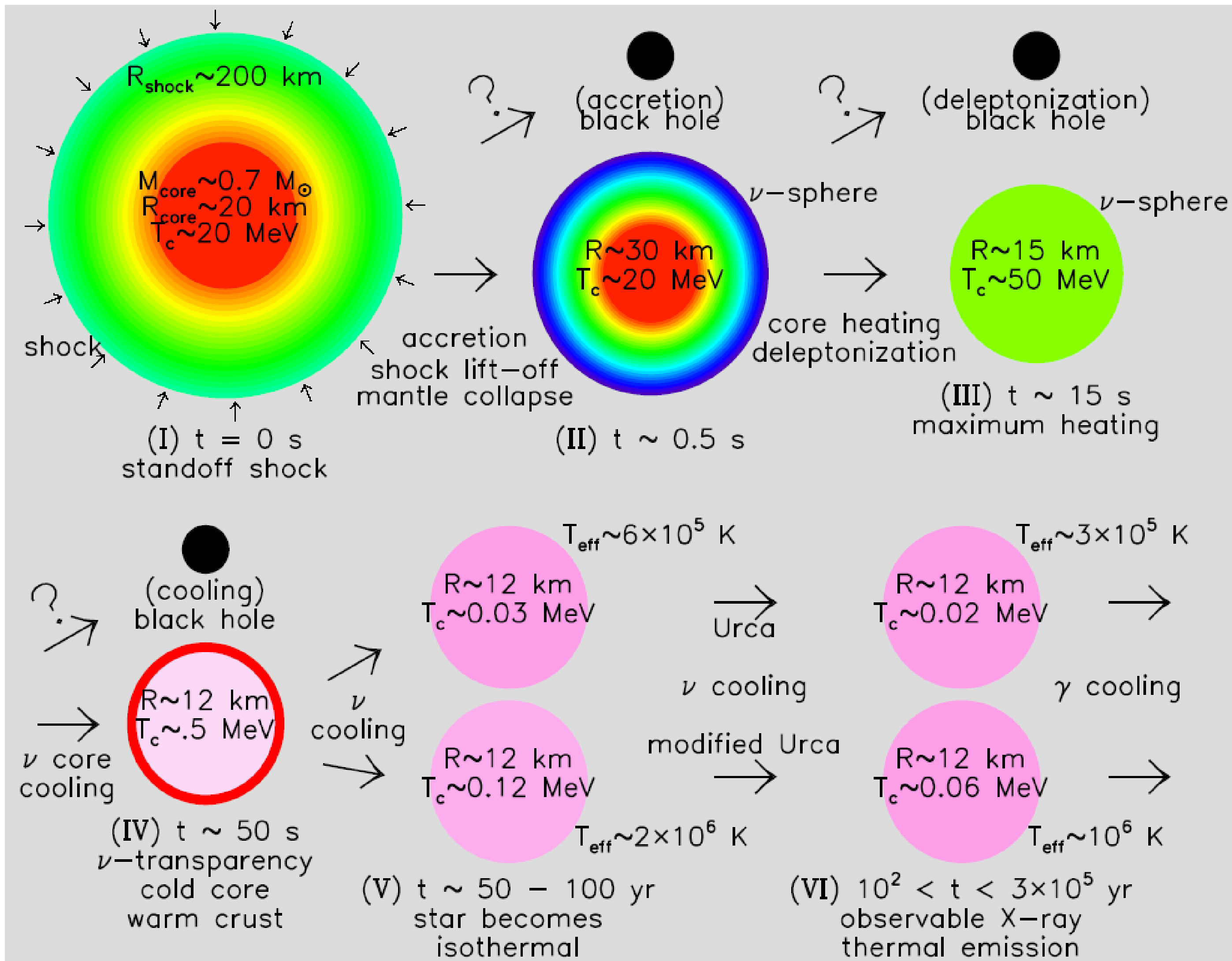
(I) core collapse

(II) proto-neutron star

(III) ν heating

(IV) ν transparency

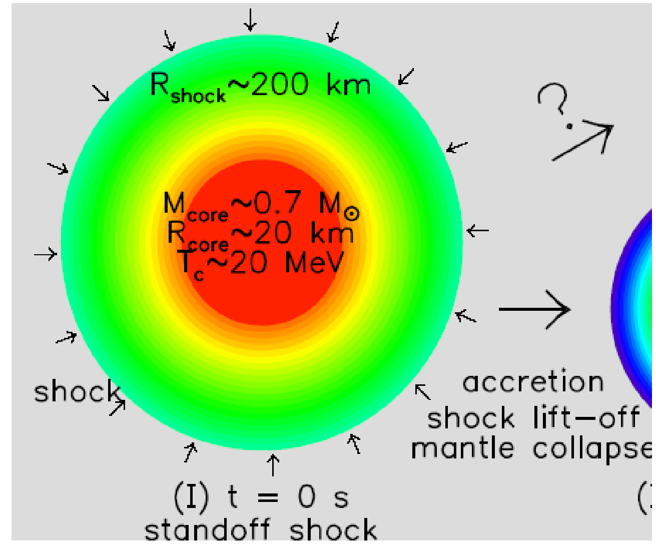
(V) photon cooling



Prakash and Lattimer

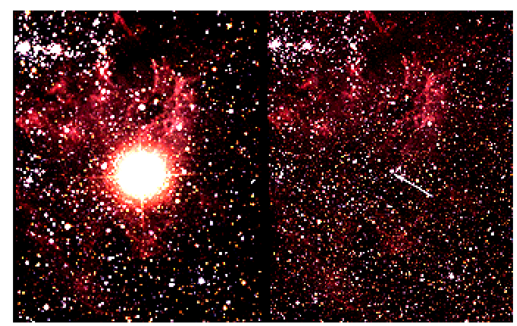
Formation of proto-neutron stars: Type II Supernova

- Type II supernova explosion
  - gravitational collapse massive star's white dwarf core > 8M–
  - collapse halts @ core density » n_0
  - shock wave dynamics
    - shock wave forms @ outer core radius
    - energy loss ν and nuclear dissociation stalls shock wave 100-200 km
    - shock resuscitation from core ν's +rotation, convection, magnetic fields, etc.
    - ν-driven explosion expels stellar mantle
  - gravitational binding energy released

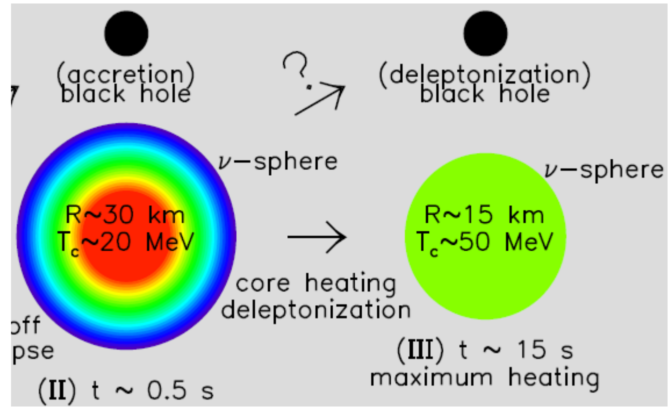


$BE_{grav} = \rho_{av}$s d^3r $V(r) = 3/5(GM^2/R)$ ¼ 3£ 10^{53} erg ¼ 0.1 M_{star}

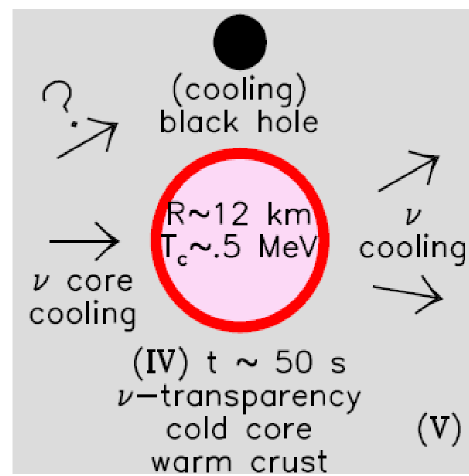
  - kinetic energy mass blow-off 1!2£10^{51} erg
  - Supernova (SN) 1987A in Large Magellanic Cloud confirmed release of E_ν=3£10^{53} erg



Proto-neutron stars

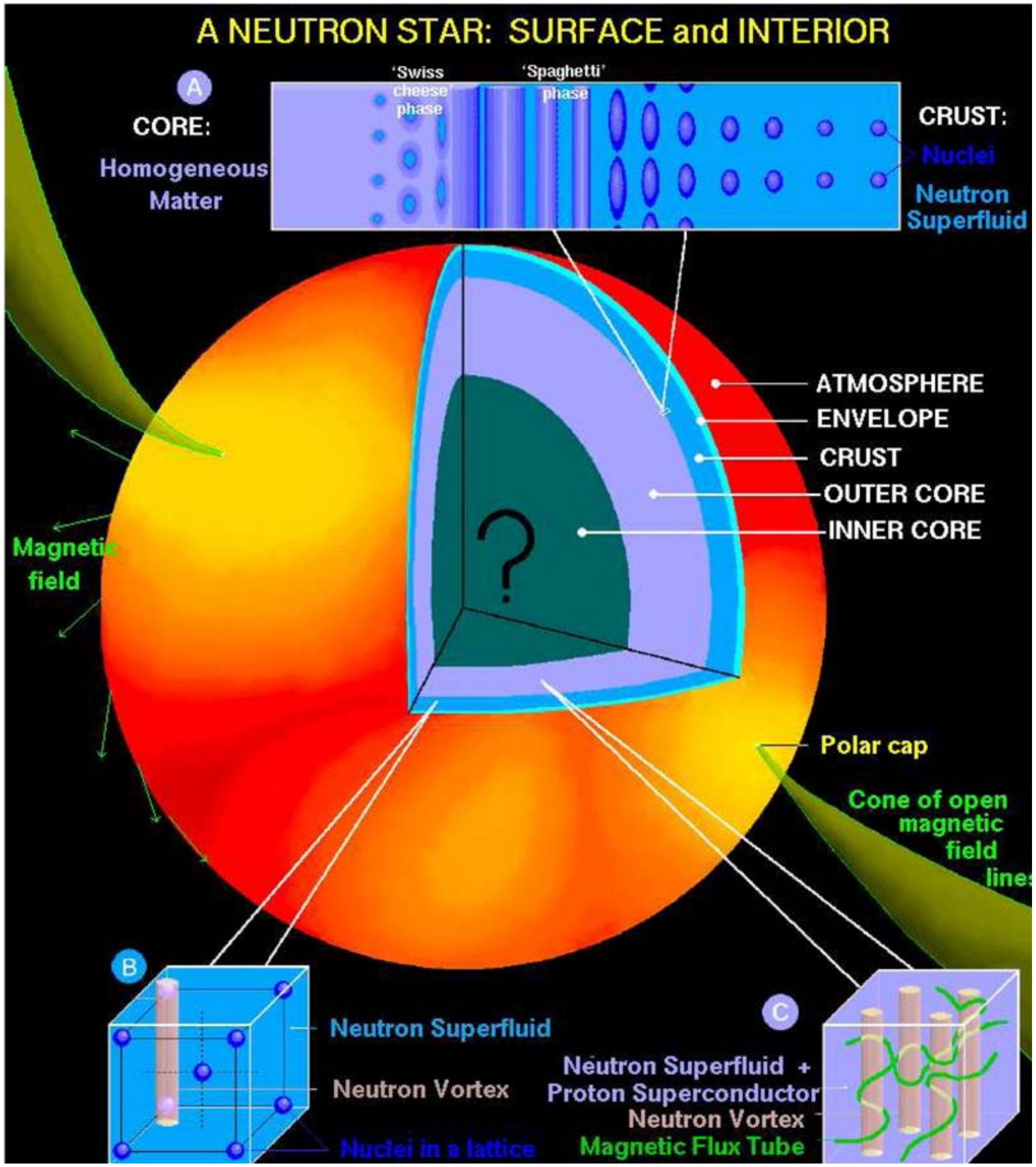


- proto-neutron star R ¼ 20 km
  - lepton rich - e^- and ν_e
  - baryon number density n=2!3n_0
  - trapped neutrinos $\sigma_{\nu A}$ ¼ 10^{-40} cm^2
    - λ ¼ $(\sigma\ n)^{-1}$ ¼ 10 cm
    - compare $\sigma_{\nu A}$ to σ_{eA}¼ 10^{-24} cm^2
  - shrinks due to pressure loss from ν emission at surface
  - escape of ν from interior on diffusion time scale
    τ ¼ $3R^2/\lambda c$ ¼ 10 s
  - ν loss) e^- capture on protons and initially warms the interior as the ν's make their way out; mostly neutrons
  - core temperature T_c ¼ 50 MeV (6£ 10^{11} K)
  - cooling starts
  - σ / λ^{-1} / h E_ν^2 i) λ > R after » 50s



Peeling the astrophysical onion

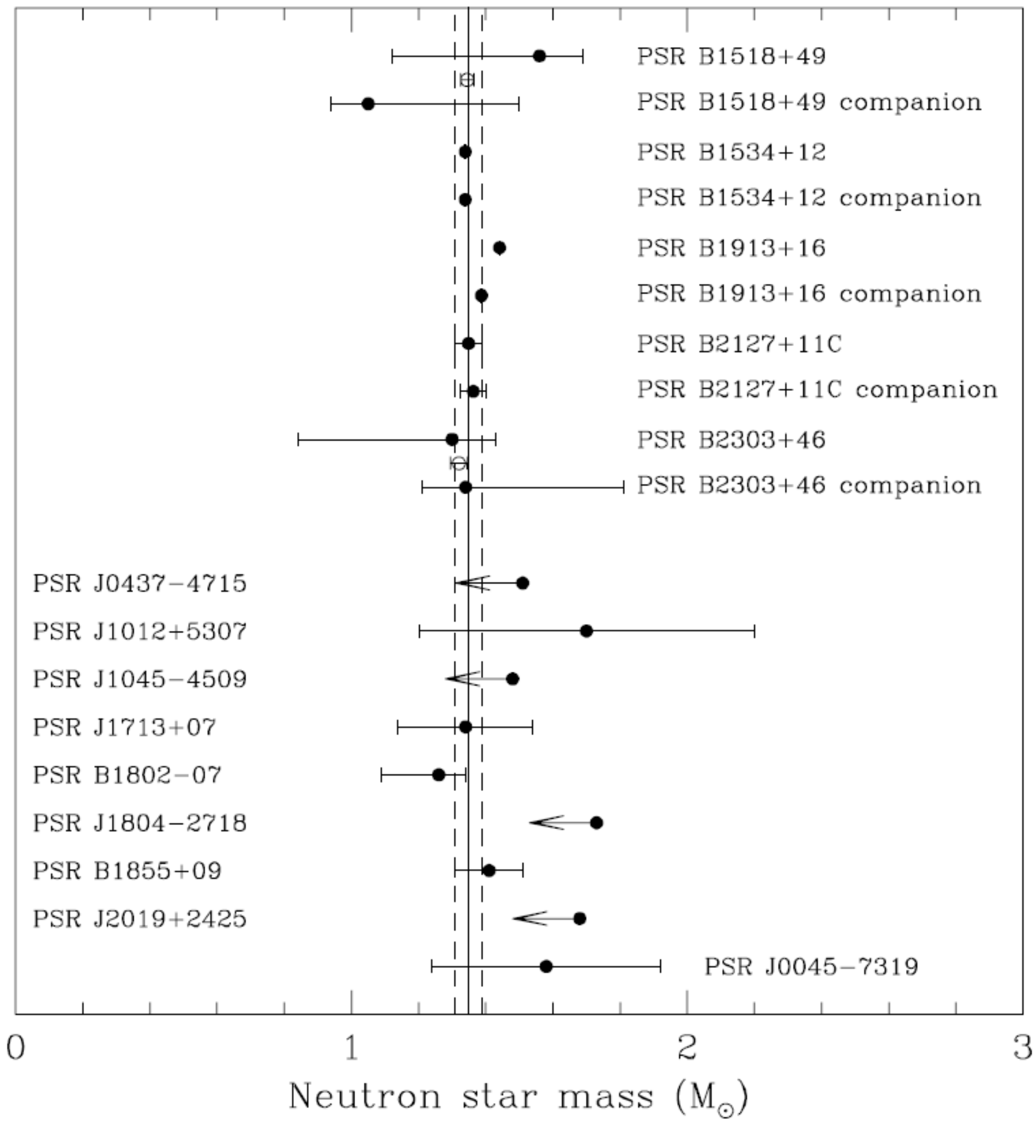
- Atmosphere } neglibile mass
- Envelope
- Crust
  - nuclear lattice
  - neutron superfluid
- Transition region
  - inhomogeneous "pasta" phases
- Outer core
  - pion condensation
  - hyperonic matter
- Inner core
  - quark matter
  - color superconductors
    - CFL
    - 2SC



Observation of astrophysical objects

- Varieties of astrophysical objects
  - main sequence stars
  - white dwarves
  - pulsars
  - binary systems
  - quasars
- Observational techniques
  - radio astronomy
    - Very Large Array – Socorro, NM
    - Arecibo – Puerto Rico
  - optical telescopes
    - low earth orbit
      - Hubble Space Telescope
    - land based
      - Mauna Kea, Chile, Arizona, etc.
  - x-ray observatories
    - Chandra
    - XMM

Radio binary pulsar data



S.E. Thorsett, D. Chakrabarty, Ap. J. 512, 288 (1999)

- Timing observations
  - orbital sizes and periods gives total mass
  - relativistic effects give mass of each component
- NS-NS binaries

  precision&0.0003M–
- NS-white dwarf binaries

  precision&0.1M–
- x-ray binaries

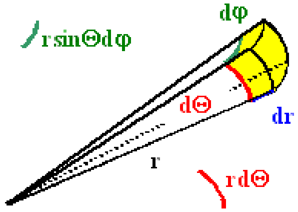
  larger errors
- An aside:

  radio observation of ms pulsars and extrasolar planets by A. Wolszczan

Newtonian stars: warm up on white dwarves

- Assume:
  - spherically symmetric, static star
  - uniform (entropy & chemical composition)
  - E/V ¼ m_N N/V – neglect general relativisitic effects
- Newtonian equation of motion – hydrostatics
  - gravity – F_g
  - degeneracy pressure of electron gas – F_{deg}

$$\begin{aligned}
\hat{\mathbf{r}} \cdot \sum \mathbf{F} &= 0 \\
-F_g + F_{deg} &= 0 \\
F_{deg} &= P(r+dr)dA(r+dr) - P(r)dA(r) \\
F_g &= \frac{GM(r)}{r^2}\rho(r)dV \\
\frac{dP(r)}{dr} &= -\frac{GM(r)\rho(r)}{r^2} \\
\frac{dM(r)}{dr} &= 4\pi r^2 \rho(r)
\end{aligned}$$



$dV = r^2 \cdot \sin\Theta d\varphi \cdot d\Theta \cdot dr$

$$\frac{d}{dr}\frac{r^2}{\rho(r)}\frac{dp(r)}{dr} = -4\pi G r^2 \rho(r)$$

Stellar structure equation

- Structure equation
  - obtain p(r), ρ(r)

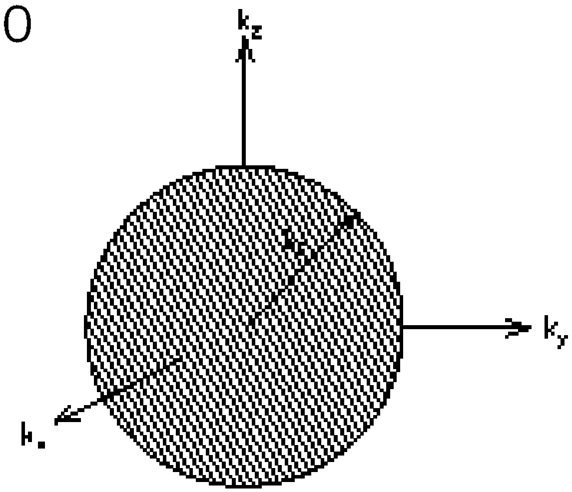
$$\frac{d}{dr}\frac{r^2}{\rho(r)}\frac{dp(r)}{dr} = -4\pi G r^2 \rho(r)$$

- Boundary condition
  - p_c=p(0), ρ_c=ρ(0)
- Integrate out from central values to p(R)=ρ(R)=0
- Requires Equation of State (EoS): p(ρ)
  - EoS depends on species present
  - interactions
- Properties of EoS
  - “stiffness” ¼ adiabatic compressibility – $\kappa_S = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right)_S \sim \frac{1}{\rho c^2}$
    - smaller slope (at fixed ρ)) “harder” EoS
    - “hard” EoS) large wave propagation speed
      - all EoS’s are limited by superluminal wave speed
- Mass vs. radius M(R) curve
  - scan over central density/pressure
  - obtain total mass, M and radius, R
  - maximum mass

Fermi gas model EoS

- Assume cold, relativistic Fermi gas of electrons – all momenta filled to Fermi level, k_F

$$n(k) = \left[e^{(\epsilon(k)-\mu)/T} + 1\right]^{-1} \to \theta(k_F - k), T \to 0$$



$$\begin{aligned} n = \frac{N}{V} &= g\int \frac{d^3k}{(2\pi)^3}\theta(k_F - k) \\ &= \frac{g}{2\pi^2}\int_0^\infty dk k^2 \theta(k_F - k) \\ &= \frac{g}{6\pi^2}k_F^3 \end{aligned}$$

$$\begin{aligned} \epsilon(k_F) = \frac{E}{V} &= g\int \frac{d^3k}{(2\pi)^3}\theta(k_F - k)\sqrt{k^2 + m^2} \\ &= \frac{g}{2\pi^2}\frac{m^4}{\pi^2}\int_0^{k_F/m} du u^2 (1 + u^2)^{1/2} \\ &= g\frac{\epsilon_0}{2}\left[(2x^3 + x)(1 + x^2)^{1/2} - \sinh^{-1} x\right] \end{aligned}$$

$$\epsilon_0 = \frac{m^4}{\pi^2}, x = \frac{k_F}{m}$$

Fermi gas EoS (II)

- pressure

$$
\begin{aligned}
p &= -\frac{\partial E}{\partial V}\Big)_{T=0} = n\mu - \epsilon \\
&= g\int \frac{d^3k}{(2\pi)^3}\left(\mu - \sqrt{k^2+m^2}\right)\theta(k_F - k) \\
&= g\frac{1}{6\pi^2}\int_0^{k_F} dk k^4 (k^2+m^2)^{-1/2}, \quad \text{I.B.P.}, \quad \mu = \sqrt{k_F^2+m^2} \\
p &= g\frac{\epsilon_0}{48}\left[(2x^3-3x)(1+x^2)^{1/2} + 3\sinh^{-1}x\right] \\
\epsilon &= nm_N A/Z + \epsilon(k_F), \quad \epsilon(k_F) \ll nm_N \\
\epsilon &\approx \rho
\end{aligned}
$$

- eliminate x to obtain p(ρ)

Fermi gas EoS (III)

- Relativistic

$$
\begin{aligned}
p(k_F) &= g\frac{1}{24\pi^2}k_F^4 \\
&= K_{rel}g^{-1/3}\epsilon^{4/3}, \quad K_{rel} = \frac{1}{24\pi^2}\left(\frac{6\pi^2 Z}{m_N A}\right)^{4/3}
\end{aligned}
$$

- Non-relativistic

$$
\begin{aligned}
p(k_F) &= g\frac{1}{2\pi^2}\frac{k_F^5}{15m_e} \\
&= K_{nr}g^{-2/3}\epsilon^{5/3}, \quad K_{nr} = \frac{1}{30\pi^2 m_e}\left(\frac{6\pi^2 Z}{m_N A}\right)^{5/3}
\end{aligned}
$$

- Polytropic EoS

$$p = K\epsilon^{\gamma}$$

Mass vs. radius

- Polytropic EoS – exactly soluble

$$M = 4\pi R^{(3\gamma-4)/(\gamma-2)} \left(\frac{K\gamma}{4\pi G(\gamma-1)}\right)^{-1/(\gamma-2)} \xi_1^{-(3\gamma-4)/(\gamma-2)} \xi_1^2 |\theta'(\xi_1)|$$

  - Lane-Emden function $\theta(\xi)$
  - $\gamma>6/5$
  - $\gamma=4/3$) M independent of R, $\rho(0)$
- Relativistic EoS and Chandrasekhar limit
  - $\gamma=4/3$

$$M = 5.87 \left(\frac{Z}{A}\right)^2 M_\odot \approx 1.26 M_\odot, \quad Z/A = 26/56$$

Neutron stars: General relativistic equation

- Tolman-Oppenheimer-Volkov Equation
  - gravitational and special relativistic corrections increase the strength of gravity relative to Newtonian case
  - neglects rotation

$$
\begin{aligned}
\frac{dP(r)}{dr} &= -\frac{G\rho(r)m(r)}{r^2} \\
&\times \left[1+\frac{P}{\rho(r)c^2}\right] \\
&\times \left[1+\frac{4\pi r^3 P}{m(r)c^2}\right] \\
&\times \left[1-\frac{2Gm(r)}{r}\right]^{-1}
\end{aligned}
$$

special relativistic mass-energy corrections

gravitational length contraction

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$$

Neutron stars

- For masses, $M>((\sim c)^{3/2}/m_N^2 G^{3/2})$¼ $2M_{–}$ (Chandrasekhar mass)
  - electron degeneracy can't support gravity
  - white dwarf collapses
    - possibly to a black hole
    - or a neutron star
- Similar to white dwarf – now neutron degeneracy
  - reaction: $p+e^-$! $n+\nu$
  - mostly neutrons, some protons – enough to prevent neutron decay
  - central density > white dwarf's » $(m_N/m_e)^3$ » 10^9
  - radius < white dwarf's » $m_N/2m_e$ » 10^3

- Next lecture – neutron stars from "realistic" equations of state; and
- A "realistic" compact object taking into account "exotic matter"