Compton Scattering from Low to High Energies

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HUGS 2004 @ JLab, June 1-18 2004

Outline

- Lecture 1 : Real Compton scattering on the nucleon and sum rules
- Lecture 2 : Forward virtual Compton scattering & nucleon structure functions
- Lecture 3 : Deeply virtual Compton scattering & generalized parton distributions
- Lecture 4 : Two-photon exchange physics in elastic electron-nucleon scattering

…if you want to read more details

in preparing these lectures,

I have primarily used some review papers :

- Lecture 1, 2 :

  Drechsel, Pasquini, Vdh : Physics Reports 378 (2003) 99 - 205

- Lecture 3 :

  Guichon, Vdh : Prog. Part. Nucl. Phys. 41 (1998) 125 – 190

  Goeke, Polyakov, Vdh : Prog. Part. Nucl. Phys. 47 (2001) 401 - 515

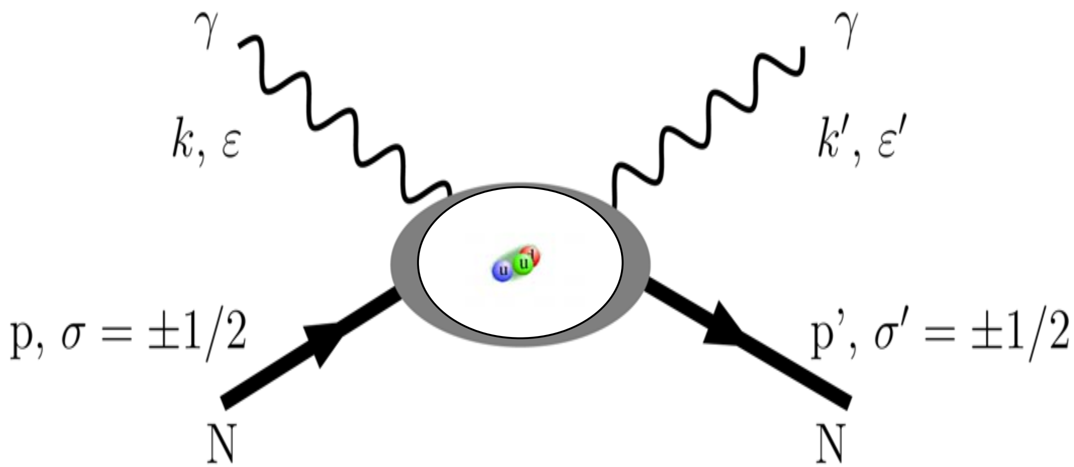
- Lecture 4 : research papers , field in rapid development since 2002

1st lecture :

Real Compton scattering on the nucleon & sum rules

Introduction :

the real Compton scattering (RCS) process



ε, ε' : photon polarization vectors

σ, σ' : nucleon spin projections

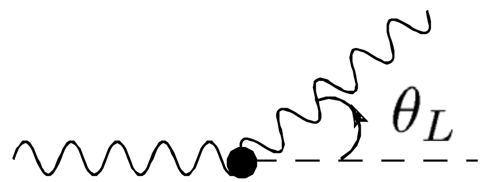
Kinematics in LAB system :

$$k(\nu, \vec{k}) \quad \text{with} \quad |\vec{\mathrm{k}}| = \nu$$

$$k'(\nu', \vec{k}') \quad \text{with} \quad |\vec{\mathrm{k}}'| = \nu'$$

$$p(M_N, 0)$$

$$p' = k + p - k'$$



$$\boxed{\nu' = \frac{\nu}{1 + \frac{\nu}{M_N}\left[1 - \cos\theta_L\right]}}$$

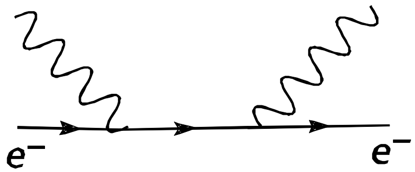
shift in wavelength of scattered photon

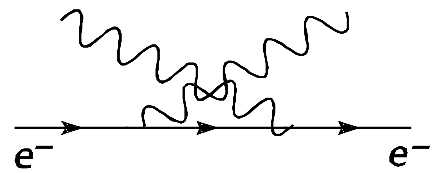
Compton (1923)

Compton scattering on point particles

➡ Compton scattering on spin 1/2 point particle (Dirac)

e.g. e^-





$$\left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{KN}}_{\mathrm{Lab}} = \left(\frac{e^2}{4\pi}\right)^2 \frac{1}{2\,M^2} \left(\frac{\nu'}{\nu}\right)^2 \left[\frac{\nu'}{\nu} + \frac{\nu}{\nu'} - \sin^2\theta_L\right]$$

Klein-Nishina (1929)

$$\xrightarrow[\Theta_L = 0]{} \left(\frac{e^2}{4\pi M}\right)^2$$

: Thomson term

➡ Compton scattering on spin 1/2 particle with anomalous magnetic moment

$$\boxed{\begin{array}{l} \Gamma^\mu = \gamma^\mu + \kappa_N i\sigma^{\mu\nu}\dfrac{k_\nu}{2M_N} \\ \kappa_p = 1.793 \qquad \kappa_n = -1.913 \end{array}}$$

Stern (1933)

$$\left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{Powell}}_{\mathrm{Lab}} = \left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{KN}}_{\mathrm{Lab}} + \kappa_N \left(\frac{e^2}{4\pi M_N}\right)^2 \left(\frac{\nu'}{\nu}\right)^2 \frac{\nu\nu'}{M_N^2}$$
$$\times \left[(1-\cos\theta_L)^2 + \frac{\kappa_N}{2}\left(\frac{9}{2} - 5\cos\theta_L + \frac{1}{2}\cos^2\theta_L\right) + \kappa_N^2\left(1 - \cos\theta_L + \frac{1}{2}\sin^2\theta_L\right) + \frac{\kappa_N^3}{4}\left(1 + \frac{1}{2}\sin^2\theta_L\right)\right]$$

Powell (1949)

Low energy expansion of RCS process

➡ Spin-independent RCS amplitude

note : transverse photons : $\vec{\varepsilon} \cdot \vec{k} \quad \vec{\varepsilon}\,' \cdot \vec{k}\,' = 0$

$$\boxed{T_{\rm Lab} = \delta_{\sigma\sigma'} \left\{ A_1 \, \vec{\varepsilon}\,'^* \cdot \vec{\varepsilon} + A_2 \, (\vec{\varepsilon}\,'^* \cdot \hat{k}) \, (\vec{\varepsilon} \cdot \hat{k}') \right\}}$$

➡ Low energy expansion of RCS amplitude : ν , ν' small

$$A_1 = -\frac{e^2}{4\pi M_N} + \nu \nu' (\alpha + \beta \cos\theta_L) + \mathcal{O}(\nu^3)$$

$$A_2 = -\nu \nu' \beta + \mathcal{O}(\nu^3)$$

Low energy theorem (LET) :
based on gauge invariance, Lorentz covariance, crossing and discrete symmetries

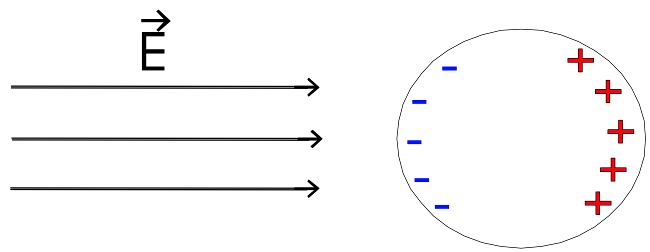
Low, Gell-Mann, Goldberger (1954)

terms parametrizing the internal structure of particle :

Electric (α) and Magnetic (β) dipole polarizabilities of nucleon

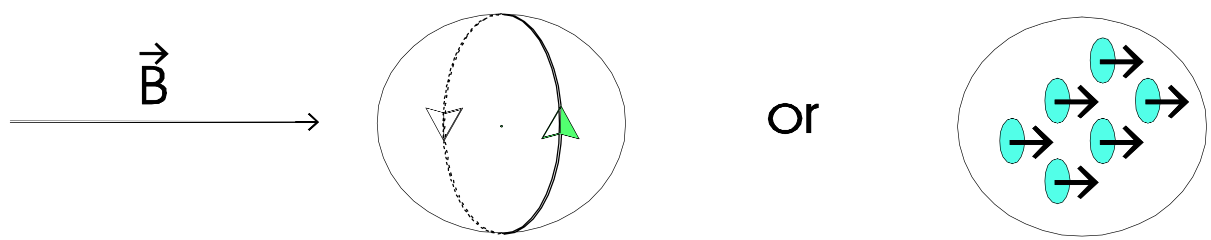
Electric and Magnetic polarizabilities of a composite system

Electric polarizability



$$\vec{P} = 4\pi\alpha\,\vec{E}$$

Magnetic polarizability



Diamagnetism

$\beta_{dia} < 0$

Paramagnetism

$\beta_{para} > 0$

the polarizability is a measure for the rigidity (stiffness) of a system

Low energy expansion of RCS cross section in terms of polarizabilities

$$
\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Lab}} &= \left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{Powell}}_{\mathrm{Lab}} \\
&- \frac{e^2}{4\pi M_N}\left(\frac{\nu'}{\nu}\right)^2 \nu\,\nu' \left\{ \frac{\alpha+\beta}{2}(1+\cos\theta_L)^2 + \frac{\alpha-\beta}{2}(1-\cos\theta_L)^2 \right\} \\
&+ \mathcal{O}(\nu^3)
\end{aligned}
$$

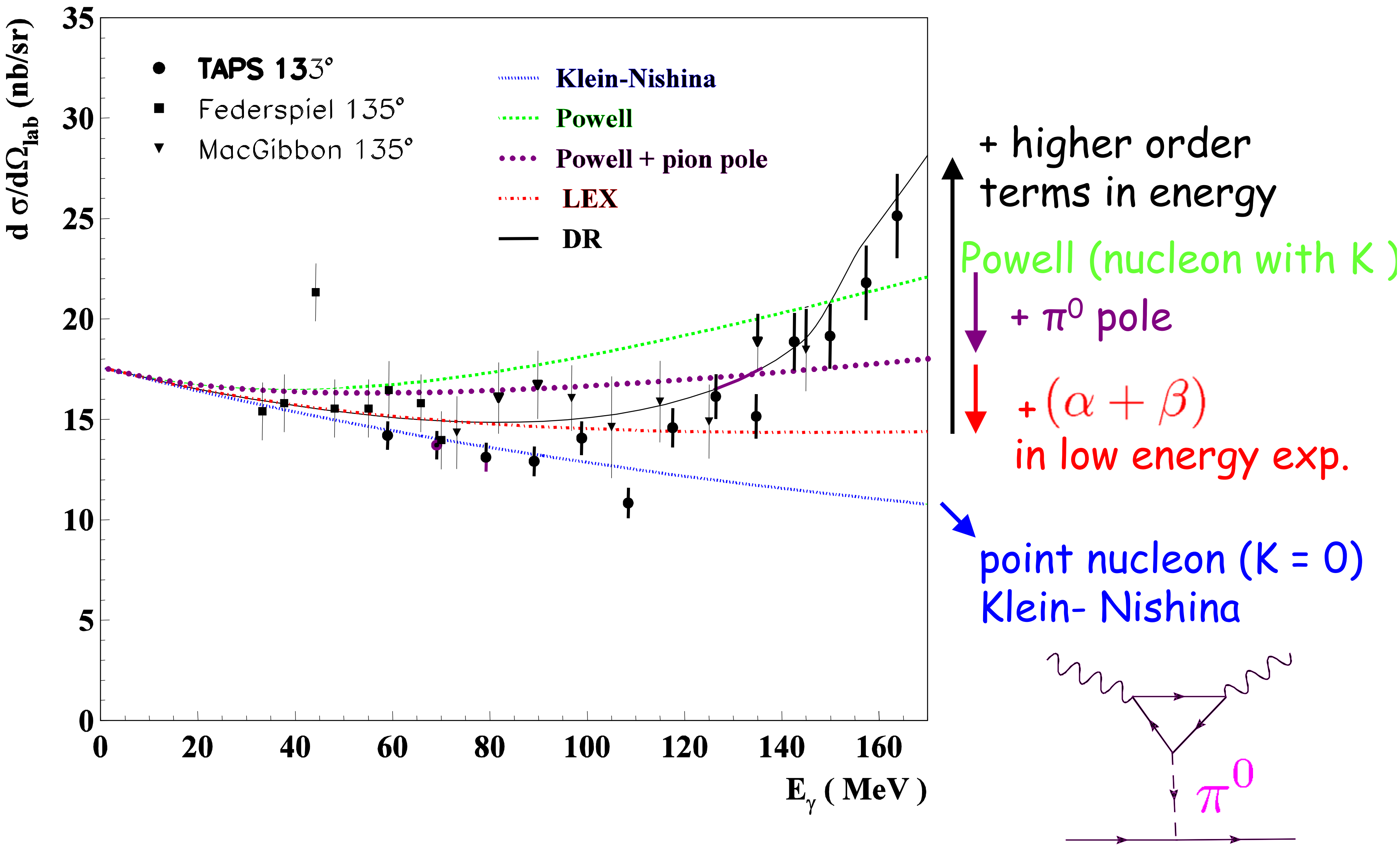
➡ Polarizability term : quadratic in the photon energy

➡ Angular dependence : disentangle α and β

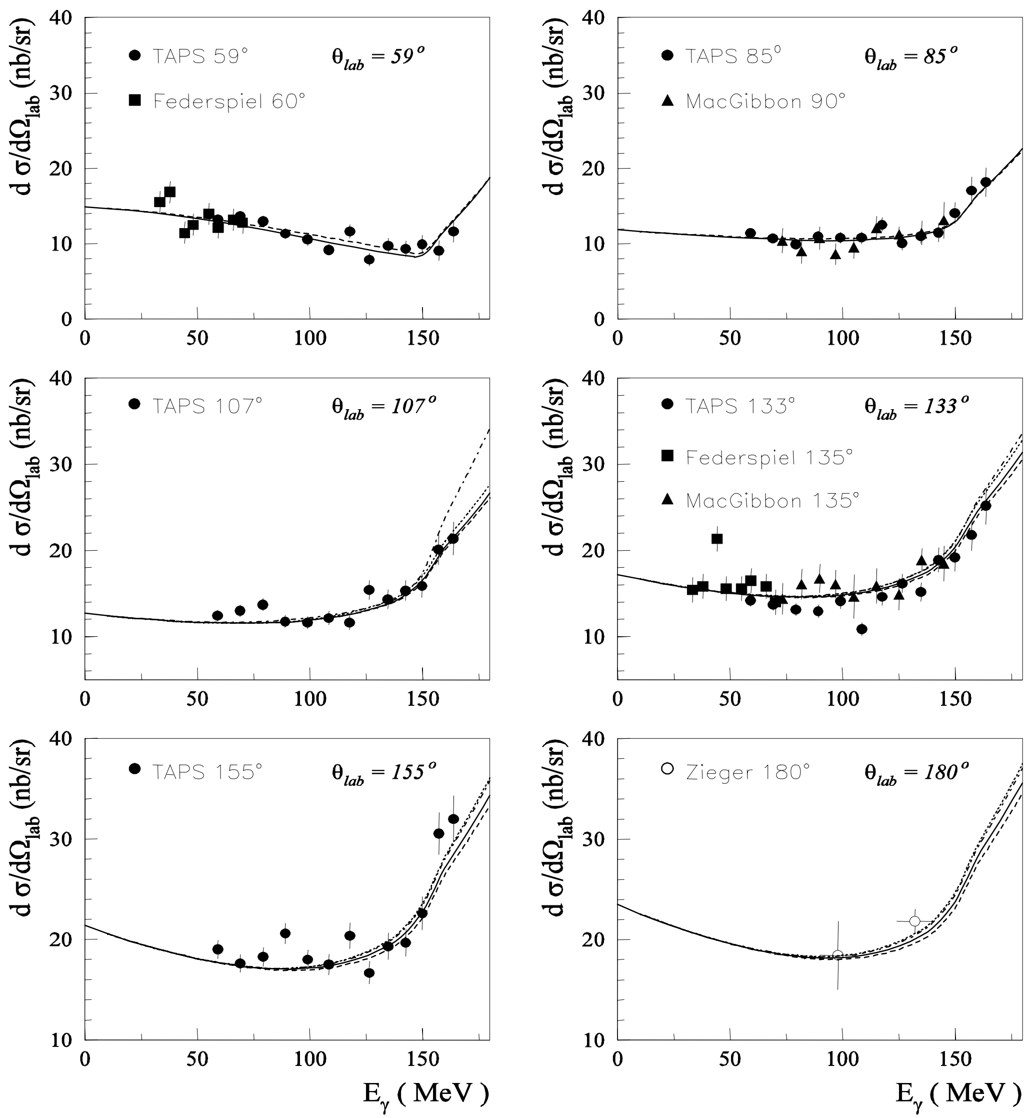
$$
\begin{aligned}
\theta_L = 0^o \quad &: \quad d\sigma \sim \alpha+\beta \\
\theta_L = 180^o \quad &: \quad d\sigma \sim \alpha-\beta
\end{aligned}
$$

➡ Higher terms in photon energy : can be treated in a dispersion relation formalism (see later)

Low energy RCS on proton



Low energy RCS on proton : global fit



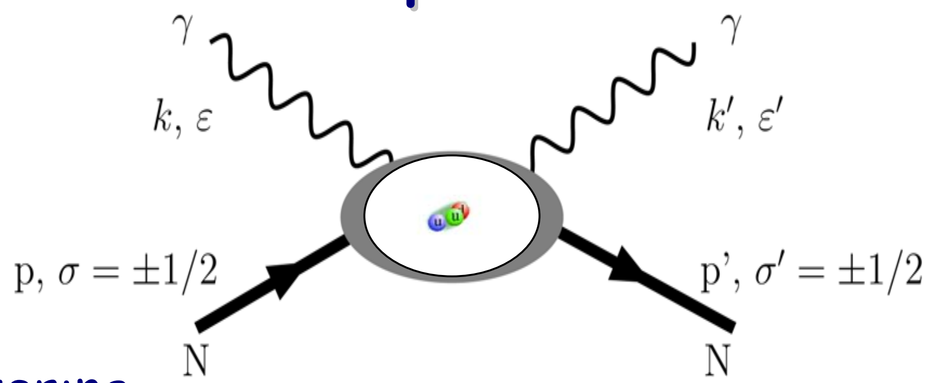
in units 10^{-4} fm^3 !

$$\alpha = 12.1 \pm 0.3(\mathrm{stat}) \mp 0.4(\mathrm{syst}) \pm 0.3(\mathrm{mod})$$

$$\beta = 1.6 \pm 0.4(\mathrm{stat}) \pm 0.4(\mathrm{syst}) \pm 0.4(\mathrm{mod})$$

Olmos de Leon (2001)

Forward real Compton scattering (RCS)



- forward scattering
  k = k', p = p'

$$\boxed{T(\nu,\,\theta=0) \quad = \quad \vec{\varepsilon}'^{*}\cdot\vec{\varepsilon}\,f(\nu) \;+\; i\vec{\sigma}\cdot(\vec{\varepsilon}'^{*}\times\vec{\varepsilon})\,g(\nu)}$$

- photon crossing: $f(\nu) \;=\; f(-\nu) \qquad g(\nu) \;=\; -g(-\nu)$

- low energy expansion: ν ! 0

$$f(\nu) \;=\; \frac{-e^2}{4\pi M_N} \;+\; (\alpha+\beta)\,\nu^2 \;+\; \mathcal{O}(\nu^4)$$

$$g(\nu) \;=\; \frac{-e^2\,\kappa_N^2}{8\pi M_N^2}\,\nu \;+\; \gamma_0\,\nu^3 \;+\; \mathcal{O}(\nu^5)$$

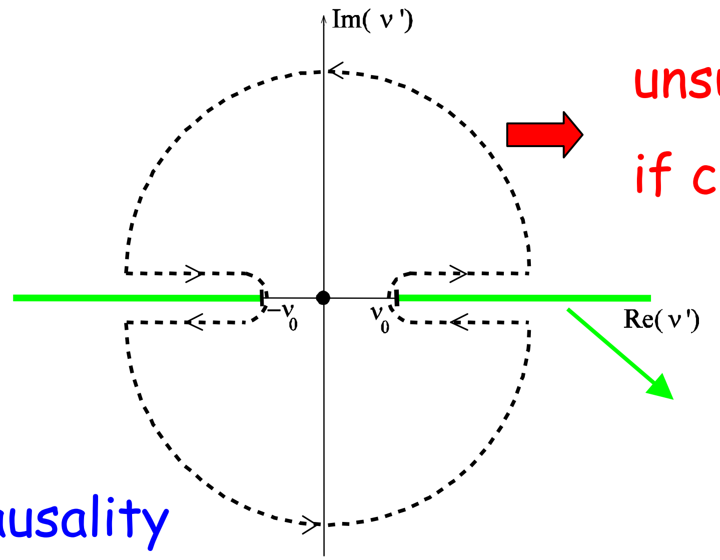
Low Energy Theorem

Low, Gell-Mann, Goldberger (1954)

dipole polarizabilities

higher order polarizab.

Dispersion relations for forward RCS



unsubtracted DR :

if circle at ∞ vanishes

branch cuts :

πN, ππN, ... thresholds

- analyticity in ν, causality

➡ Cauchy integral formula

$$f(\nu + i\varepsilon) \;=\; \frac{1}{2\pi i} \oint_C d\nu' \, \frac{f(\nu')}{\nu' - \nu - i\varepsilon}$$

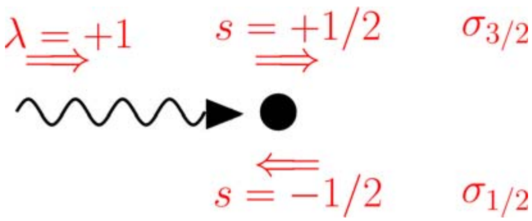
- crossing symmetry

$$f(-\nu) \;=\; f(\nu) \Longrightarrow \mathrm{Re}\, f(\nu) \;=\; \frac{2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\nu' \, \mathrm{Im} f(\nu')}{\nu'^2 - \nu^2}$$

$$g(-\nu) \;=\; -g(\nu) \Longrightarrow \mathrm{Re}\, g(\nu) \;=\; \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\mathrm{Im} g(\nu')}{\nu'^2 - \nu^2}$$

Sum rules for forward RCS

- Unitarity ➡ Optical Theorem



$$\mathrm{Im}\left\{ \begin{array}{c} f \\ g \end{array} \right\} = \frac{\nu}{8\pi}\left(\sigma_{1/2} \pm \sigma_{3/2}\right)$$

$$\mathrm{Re}\, f(\nu) = \frac{1}{4\pi^2}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^2 \left(\sigma_{1/2} + \sigma_{3/2}\right)}{\nu'^2 - \nu^2}$$

$$\mathrm{Re}\, g(\nu) = \frac{\nu}{4\pi^2}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \left(\sigma_{1/2} - \sigma_{3/2}\right)}{\nu'^2 - \nu^2}$$

- make a low energy expansion of both left and right sides of DRs
  ➡ SUM RULES

Spin independent sum rules for RCS

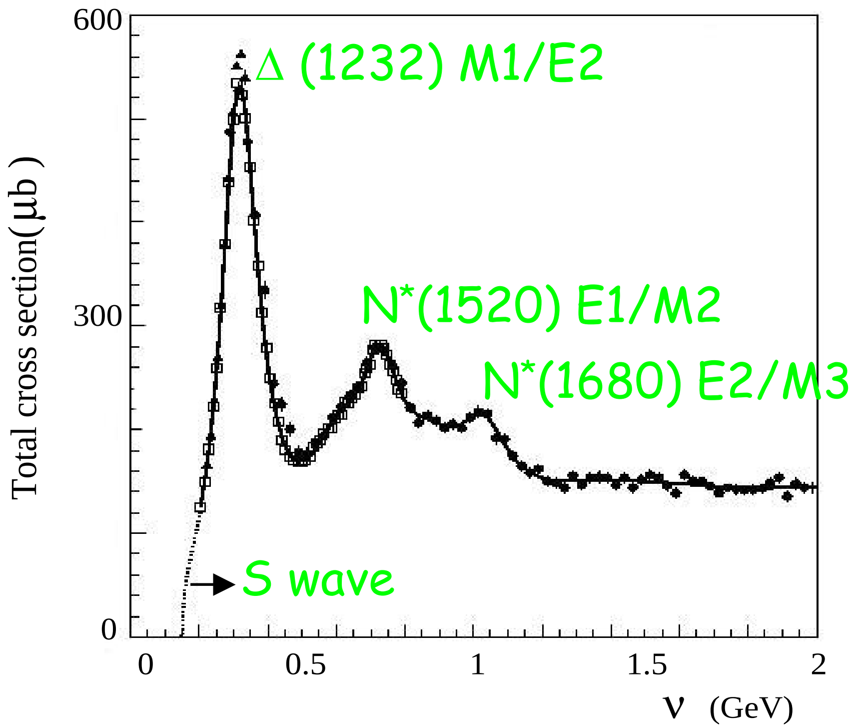
$$\sum_{n=0} \left(\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{(\sigma_{1/2} + \sigma_{3/2})}{\nu'^{2n}} \right) \nu^{2n} = -\frac{e^2}{4\pi M_N} + (\alpha + \beta)\, \nu^2 + \mathcal{O}(\nu^4)$$

➡ term in ν^0

$$-\frac{e^2}{4\pi M_N} \neq \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, (\sigma_{1/2} + \sigma_{3/2})$$

for ν!1, $\sigma_{TOT} = (\sigma_{1/2}+\sigma_{3/2})/2$ » $\nu^{0.08}$

) NO convergence



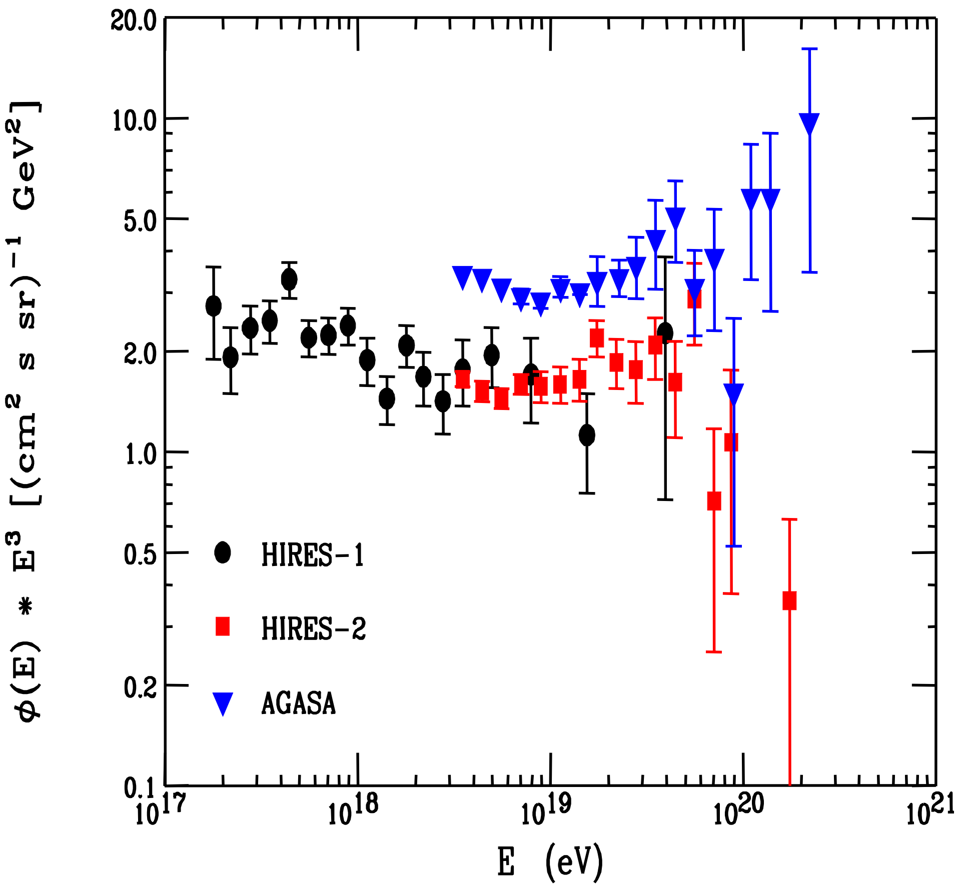
➡ term in ν^2

$$\boxed{\alpha + \beta = \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{(\sigma_{1/2} + \sigma_{3/2})}{\nu'^2}}$$

Baldin sum rule (1960)

$$\alpha_p + \beta_p = (13.69 \pm 0.14) \times 10^{-4} \mathrm{fm}^3$$

Digression : ultra-high energy cosmic rays and the total photoabsorption cross section on the proton



energy spectrum of ultra-high energy cosmic rays (protons)

P. Lipari

(ISVHECRI 2002)

ultra-high energy cosmic rays : GZK cut-off and the total photoabsorption cross section on the proton

➡ Protons scatter from Cosmic Microwave Background (CMB)

$$\gamma_{CMB} = 2.7^o \text{ K} \qquad \langle\omega\rangle \simeq 10^{-3}\,\text{eV}$$

dominant processes :

$$p + \gamma_{CMB} \longrightarrow n + \pi^+$$
$$p + \gamma_{CMB} \longrightarrow p + \pi^0$$

$$\begin{aligned} W^2 &= (k+p)^2 = M_N^2 + 2\,\omega\,E_p\,(1-\cos\theta) \\ W_{\rm thr}^2 &= (M_N + m_\pi)^2 \end{aligned}$$

$$\Downarrow$$

$$E_p = \frac{M_N m_\pi + m_\pi^2/2}{\langle\omega\rangle} \simeq \frac{0.14\,\text{GeV}^2}{10^{-3}\,\text{eV}} \simeq 10^{20}\text{eV}$$

GZK cut-off

Greisen (1966)

Zatsepin & Kuzmin (1966)

➡ Puzzle to see protons with E_p > GZK cut-off

"exotic" sources near our galaxy ?

decay of very large mass particles $M_X \sim M_{GUT} \sim 10^{24}$ eV ?

Spin dependent sum rules for RCS

$$\boxed{\mathrm{Re}\, g(\nu) \;=\; \frac{\nu}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \left(\sigma_{1/2} - \sigma_{3/2}\right)}{\nu'^2 - \nu^2}}$$

$$-\frac{e^2 \kappa_N^2}{8\pi M_N^2}\, \nu \;+\; \gamma_0 \; \nu^3 + \mathcal{O}(\nu^5) \;=\; \sum_{n=1} \left(\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\left(\sigma_{1/2} - \sigma_{3/2}\right)}{\nu'^{2n-1}} \, \nu^{2n-1} \right)$$

➡ term in ν

$$\boxed{\frac{e^2 \pi \kappa_N^2}{2\, M_N^2} \;=\; \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\left(\sigma_{3/2} - \sigma_{1/2}\right)}{\nu'}}$$

Gerasimov-Drell-Hearn (GDH) sum rule (1966)

consequence of :

gauge invariance,

analyticity (DR), unitarity,

+ convergence assumption (no-subtraction)

GDH sum rule in QED

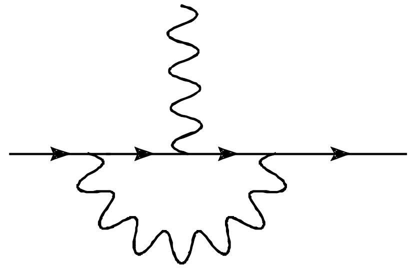
$$\boxed{\frac{e^2 \pi\, \kappa^2}{2\, M^2} = \int_0^\infty d\nu\, \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu}}$$

➡ Compute both sides of the sum rule in perturbation theory.
Is the GDH sum rule verified?

➡ for Dirac particle : $\kappa = 0 \Rightarrow g \equiv 2(1+\kappa) = 2$

➡ electron anomalous magnetic moment (loop effect)

to 1-loop accuracy :



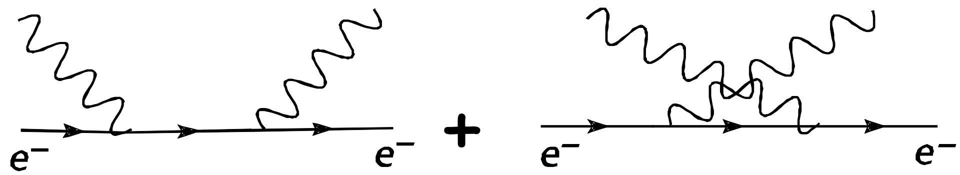
$$\kappa = \left(\frac{e^2}{4\pi}\right) \frac{1}{2\pi} \Rightarrow g = 2.0023$$

Schwinger

experiment electron : g = 2.002319304374 ± 8 . 10^{-12} !

GDH sum rule in QED : O(e^4)

$$\sigma_{3/2} - \sigma_{1/2} \quad \sim \quad \Big| \quad + \quad \Big|^2$$



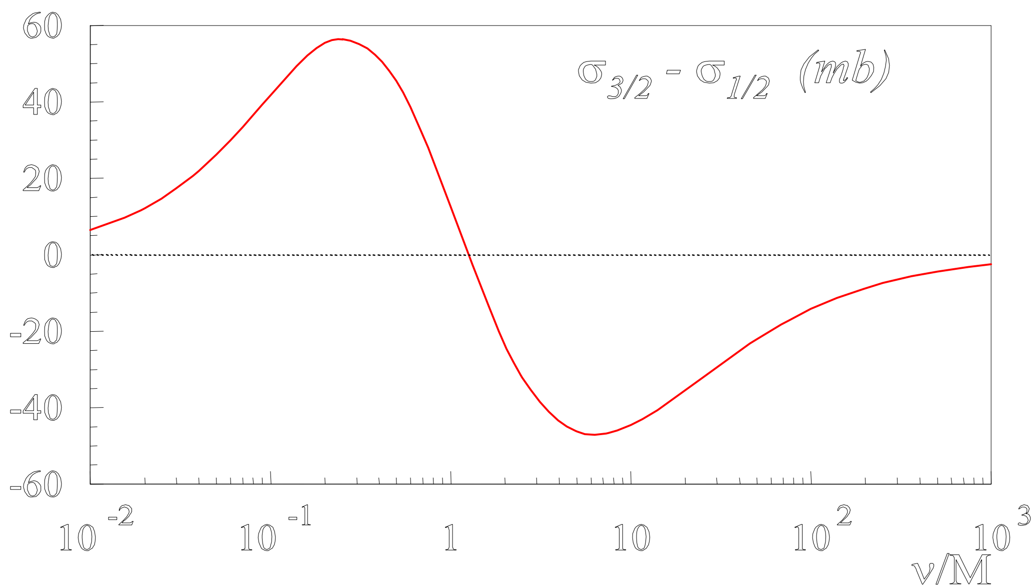
$$\boxed{\sigma_{3/2} - \sigma_{1/2} \;=\; -\frac{e^4}{8\,\pi\,M^2}\,\frac{1}{x}\left[\left(1+\frac{1}{x}\right)\ln\left(1+2\,x\right) - 2\left(1+\frac{x^2}{(1+2x)^2}\right)\right]}$$

$$x \,=\, \nu\,/\,M$$

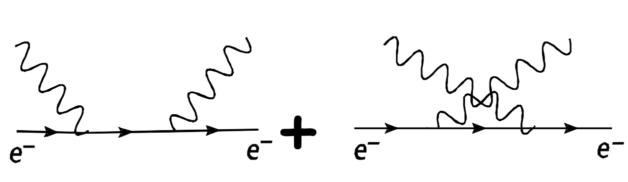
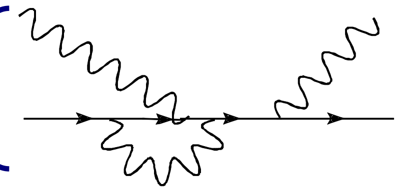
at O(e4) :

$$0 \overset{☺}{=} \int_0^\infty \frac{\mathrm{d}\nu}{\nu}\left[\sigma_{3/2} - \sigma_{1/2}\right]$$

Altarelli, Cabibbo, Maiani (1972)

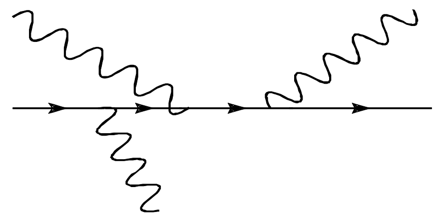


GDH sum rule in QED : O(e^6)

$\sigma_{3/2} - \sigma_{1/2} \sim$ [] [ + ...]

tree level

1-loop diagrams

+ |  + ... |^2

$$\frac{e^2 \pi}{2\, M^2} \left[\left(\frac{e^2}{4\pi} \right) \frac{1}{2\pi} \right]^2 = \int_0^\infty \mathrm{d}\nu \, \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu}$$

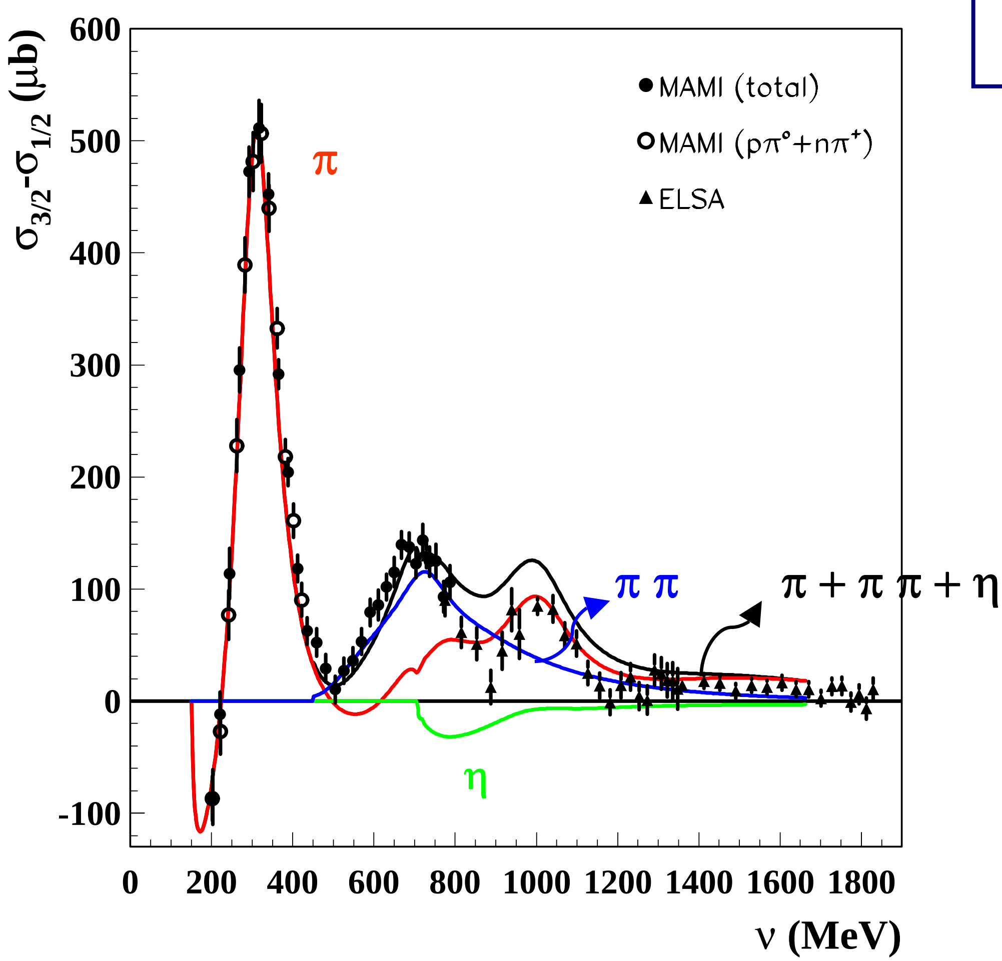
Dicus, Vega (2001)

GDH sum rule is satisfied in QED to order O(e^6)

Spin dependent sum rules for RCS on proton

➡ GDH sum rule (1966) :

$$\frac{e^2 \pi \kappa_N^2}{2\, M_N^2} = \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'}$$



sum rule for proton :

lhs = 205 μb

● MAMI [200, 800] MeV

▲ ELSA [0.7, 3.1] GeV

π MAID2000

Drechsel, Kamalov, Tiator (`00)

ππ Holvoet, Vdh (`01)

η ETA- MAID2000

π + ππ + η

Spin dependent sum rules for RCS on proton

➡ term in ν : $$\frac{e^2 \pi \kappa_N^2}{2\, M_N^2} = \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'}$$ GDH sum rule

➡ term in ν^3 : $$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'^3}$$ Forward Spin Polarizability (FSP)

PROTON results	E_γ [GeV]	GDH [μb]	FSP [10^{-4} fm^4]
MAID2000 estimate	< 0.2	-28.5 ± 2	0.95 ± 0.05
MAMI experiment	0.2 - 0.8	226 ± 5 ± 12	-1.87 ± 0.08 ± 0.10
ELSA experiment	0.8 - 2.9	27.5 ± 2.0 ± 1.2	-0.03
Bianchi-Thomas / Simula et al. : estimate	> 2.9	-14 ± 2	+0.01
Total		211 ± 15	-0.94 ± 0.15
GDH sum rule		205	