Compton Scattering from Low to High Energies

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Outline

- Lecture 1: Real Compton scattering on the nucleon and sum rules
- Lecture 2: Forward virtual Compton scattering & nucleon structure functions
- Lecture 3: Deeply virtual Compton scattering & generalized parton distributions
- Lecture 4: Two-photon exchange physics in elastic electron-nucleon scattering
...if you want to read more details

in preparing these lectures,

I have primarily used some review papers:

- **Lecture 1, 2:**
  

- **Lecture 3:**
  
  

- **Lecture 4:** research papers, field in rapid development since 2002
1\textsuperscript{st} lecture:

Real Compton scattering on the nucleon & sum rules
Introduction: the real Compton scattering (RCS) process

Kinematics in LAB system:

\[ k(\nu, \vec{k}) \quad \text{with} \quad |\vec{k}| = \nu \]
\[ k'(\nu', \vec{k}') \quad \text{with} \quad |\vec{k}'| = \nu' \]
\[ p(M_N, 0) \]
\[ p' = k + p - k' \]

\[ \nu' = \frac{\nu}{1 + \frac{\nu}{M_N} \left[ 1 - \cos \theta_L \right]} \]

\( \varepsilon, \varepsilon' \): photon polarization vectors
\( \sigma, \sigma' \): nucleon spin projections

shift in wavelength of scattered photon
Compton (1923)
Compton scattering on point particles

- Compton scattering on spin 1/2 point particle (Dirac)
  
  e.g. $e^-$

  $\left( \frac{d\sigma}{d\Omega} \right)_{\text{Lab}}^{\text{KN}} = \left( \frac{e^2}{4\pi} \right)^2 \frac{1}{2M^2} \left( \frac{\nu'}{\nu} \right)^2 \left[ \frac{\nu'}{\nu} + \frac{\nu}{\nu'} - \sin^2 \theta_L \right]$  
  
  $\Theta_L = 0$ : Thomson term

- Compton scattering on spin 1/2 particle with anomalous magnetic moment

  $\Gamma^\mu = \gamma^\mu + \kappa_N i\sigma^{\mu\nu} \frac{k_\nu}{2M_N}$

  $\kappa_p = 1.793$  $\kappa_n = -1.913$

  Powell (1949)
Low energy expansion of RCS process

Spin-independent RCS amplitude

note: transverse photons: \[ \vec{\epsilon} \cdot \vec{k} \quad \vec{\epsilon}' \cdot \vec{k}' = 0 \]

\[ T_{\text{Lab}} = \delta_{\sigma\sigma'} \left\{ A_1 \vec{\epsilon}'^* \cdot \vec{\epsilon} + A_2 (\vec{\epsilon}'^* \cdot \hat{k}) (\vec{\epsilon} \cdot \hat{k}') \right\} \]

Low energy expansion of RCS amplitude: \( \nu, \nu' \) small

\[
A_1 = -\frac{e^2}{4\pi M_N} + \nu \nu' (\alpha + \beta \cos \theta_L) + \mathcal{O}(\nu^3) \\
A_2 = -\nu \nu' \beta + \mathcal{O}(\nu^3)
\]

Low energy theorem (LET): based on gauge invariance, Lorentz covariance, crossing and discrete symmetries

Low, Gell-Mann, Goldberger (1954)

terms parametrizing the internal structure of particle:

Electric (\( \alpha \)) and Magnetic (\( \beta \)) dipole polarizabilities of nucleon
Electric and Magnetic polarizabilities of a composite system

Electric polarizability

\[ \vec{P} = 4\pi \varepsilon_0 \vec{E} \]

Magnetic polarizability

Diamagnetism
\[ \beta_{\text{dia}} < 0 \]

Paramagnetism
\[ \beta_{\text{para}} > 0 \]

the polarizability is a measure for the rigidity (stiffness) of a system
Low energy expansion of RCS cross section in terms of polarizabilities

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Lab}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Lab}}^{\text{Powell}} - \frac{e^2}{4\pi M_N} \left( \frac{\nu'}{\nu} \right)^2 \nu \nu' \left\{ \frac{\alpha + \beta}{2} (1 + \cos \theta_L)^2 + \frac{\alpha - \beta}{2} (1 - \cos \theta_L)^2 \right\} + \mathcal{O}(\nu^3)
\]

- Polarizability term: quadratic in the photon energy
- Angular dependence: disentangle $\alpha$ and $\beta$

\[
\theta_L = 0^\circ : \quad d\sigma \sim \alpha + \beta \\
\theta_L = 180^\circ : \quad d\sigma \sim \alpha - \beta
\]

- Higher terms in photon energy: can be treated in a dispersion relation formalism (see later)
Low energy RCS on proton

- Powell (nucleon with $\kappa$) + $\pi^0$ pole + $(\alpha + \beta)$ terms in energy

- Klein-Nishina

- TAPS 135°
- Federspiel 135°
- MacGibbon 135°

- Powell
- $\kappa = 0$

- LEX
- DR

Point nucleon ($K = 0$)
Klein-Nishina

$\pi^0$
Low energy RCS on proton: global fit

\[ \alpha = 12.1 \pm 0.3(\text{stat}) \pm 0.4(\text{syst}) \pm 0.3(\text{mod}) \]
\[ \beta = 1.6 \pm 0.4(\text{stat}) \pm 0.4(\text{syst}) \pm 0.4(\text{mod}) \]

Olmos de Leon (2001)
Forward real Compton scattering (RCS)

\[ T(\nu, \theta = 0) = \tilde{\varepsilon}'* \cdot \tilde{\varepsilon} f(\nu) + i\tilde{\sigma} \cdot (\tilde{\varepsilon}'* \times \tilde{\varepsilon}) g(\nu) \]

\[ f(\nu) = \frac{-e^2}{4\pi M_N} + (\alpha + \beta) \nu^2 + \mathcal{O}(\nu^4) \]

\[ g(\nu) = \frac{-e^2 \kappa^2}{8\pi M_N^2} \nu + \gamma_0 \nu^3 + \mathcal{O}(\nu^5) \]

forward scattering: \( k = k', \ p = p' \)

photon crossing: \( f(\nu) = f(-\nu) \quad g(\nu) = -g(-\nu) \)

low energy expansion: \( \nu \rightarrow 0 \)

Low Energy Theorem
Low, Gell-Mann, Goldberger (1954)

higher order polarizabilities
Dispersion relations for forward RCS

- analyticity in $\nu$, causality
  - Cauchy integral formula
    \[ f(\nu + i\varepsilon) = \frac{1}{2\pi i} \oint_C d\nu' \frac{f(\nu')}{\nu' - \nu - i\varepsilon} \]

- crossing symmetry
  \[ f(-\nu) = f(\nu) \implies \text{Re } f(\nu) = \frac{2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\nu' \text{Im } f(\nu')}{\nu'^2 - \nu^2} \]
  \[ g(-\nu) = -g(\nu) \implies \text{Re } g(\nu) = \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } g(\nu')}{\nu'^2 - \nu^2} \]

unsubtracted DR:
if circle at $\infty$ vanishes
branch cuts:
$\pi N, \pi \pi N, \ldots$ thresholds
Sum rules for forward RCS

- Unitarity $\implies$ Optical Theorem

\[ \lambda = +1 \quad s = +1/2 \quad \sigma_{3/2} \]
\[ s = -1/2 \quad \sigma_{1/2} \]

\[ \text{Im} \left\{ \begin{array}{c} f \\ g \end{array} \right\} = \frac{\nu}{8\pi} (\sigma_{1/2} \pm \sigma_{3/2}) \]

- make a low energy expansion of both left and right sides of DRs $\implies$ SUM RULES

\[
\begin{align*}
\text{Re} \ f(\nu) &= \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \text{d}\nu' \frac{\nu'^2 (\sigma_{1/2} + \sigma_{3/2})}{\nu'^2 - \nu^2} \\
\text{Re} \ g(\nu) &= \frac{\nu}{4\pi^2} \int_{\nu_0}^{\infty} \text{d}\nu' \frac{\nu' (\sigma_{1/2} - \sigma_{3/2})}{\nu'^2 - \nu^2}
\end{align*}
\]
Spin independent sum rules for RCS

\[ \sum_{n=0} \left( \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{1/2} + \sigma_{3/2})}{\nu'^{2n}} \right) \nu^{2n} = -\frac{e^2}{4\pi M_N} + (\alpha + \beta) \nu^2 + O(\nu^4) \]

\[ \rightarrow \text{ term in } \nu^0 \]

\[ -\frac{e^2}{4\pi M_N} \neq \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \left( \sigma_{1/2} + \sigma_{3/2} \right) \]

for \( \nu \neq 1, \sigma_{\text{TOT}} = (\sigma_{1/2} + \sigma_{3/2})/2 \gg \nu^{0.08} 

\rightarrow \text{ term in } \nu^2 \]

\[ \alpha + \beta = \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{1/2} + \sigma_{3/2})}{\nu'^2} \]

Baldin sum rule (1960)

\[ \alpha_p + \beta_p = (13.69 \pm 0.14) \times 10^{-4} \text{fm}^3 \]
Digression: ultra-high energy cosmic rays and the total photoabsorption cross section on the proton

Energy spectrum of ultra-high energy cosmic rays (protons)

P. Lipari
(ISVHECRI 2002)
ultra-high energy cosmic rays: GZK cut-off
and the total photoabsorption cross section on
the proton

Protons scatter from Cosmic Microwave Background (CMB)

\[ \gamma_{\text{CMB}} = 2.7^\circ \text{ K} \]

\[ \langle \omega \rangle \approx 10^{-3} \text{ eV} \]

dominant processes:

\[ p + \gamma_{\text{CMB}} \rightarrow n + \pi^+ \]

\[ p + \gamma_{\text{CMB}} \rightarrow p + \pi^0 \]

\[ W^2 = (k + p)^2 = M_N^2 + 2 \omega E_p (1 - \cos \theta) \]

\[ W_{\text{thr}}^2 = (M_N + m_\pi)^2 \]

\[ E_p = \frac{M_N m_\pi + m_\pi^2 / 2}{\langle \omega \rangle} \approx \frac{0.14 \text{ GeV}^2}{10^{-3} \text{ eV}} \approx 10^{20} \text{ eV} \]

Puzzle to see protons with \( E_p > \) GZK cut-off

“exotic” sources near our galaxy?

decay of very large mass particles \( M_X \sim M_{\text{GUT}} \sim 10^{24} \text{ eV} \)?
Spin dependent sum rules for RCS

\[
\text{Reg}(\nu) = \frac{\nu}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{\nu'(\sigma_{1/2} - \sigma_{3/2})}{\nu'^2 - \nu^2}
\]

\[-\frac{e^2 \kappa_N^2}{8\pi M_N^2} \nu + \gamma_0 \nu^3 + O(\nu^5) = \sum_{n=1}^{\infty} \left( \frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{1/2} - \sigma_{3/2})}{\nu'^{2n-1}} \nu^{2n-1} \right)\]

\[\text{term in } \nu\]

\[\frac{e^2 \pi \kappa_N^2}{2 M_N^2} = \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'}\]

Gerasimov-Drell-Hearn (GDH) sum rule (1966)

consequence of:
- gauge invariance,
- analyticity (DR), unitarity,
- + convergence assumption (no-subtraction)
Compute both sides of the sum rule in perturbation theory. Is the GDH sum rule verified?

for Dirac particle: \( \kappa = 0 \) \( \Rightarrow \) \( g \equiv 2(1 + \kappa) = 2 \)

electron anomalous magnetic moment (loop effect) 
 to 1-loop accuracy:

\[
\frac{e^2 \pi \kappa^2}{2 M^2} = \int_0^\infty d\nu \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu}
\]

\[\kappa = \left(\frac{e^2}{4\pi}\right) \frac{1}{2\pi} \Rightarrow g = 2.0023 \]

Schwinger

experiment electron: \( g = 2.002319304374 \pm 8 \cdot 10^{-12} \) !
GDH sum rule in QED: $O(e^4)$

$$\sigma_{3/2} - \sigma_{1/2} \sim -\frac{e^4}{8\pi M^2} \frac{1}{x} \left[ \left(1 + \frac{1}{x}\right) \ln (1 + 2x) - 2 \left(1 + \frac{x^2}{(1 + 2x)^2}\right) \right]$$

$$x = \nu / M$$

at $O(e^4)$:

$$0 = \int_0^\infty \frac{d\nu}{\nu} \left[ \sigma_{3/2} - \sigma_{1/2} \right]$$

Altarelli, Cabibbo, Maiani (1972)
GDH sum rule in QED: $O(e^6)$

\[ \sigma_{3/2} - \sigma_{1/2} \sim \left( \begin{array}{c}
\begin{array}{c}
\text{e}^- \rightarrow \text{e}^-
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\text{e}^- \rightarrow \text{e}^-
\end{array}
\end{array} \right) \begin{array}{c}
\begin{array}{c}
\text{tree level}
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\text{1-loop diagrams}
\end{array}
\end{array} \end{array} + \ldots \]

\[ \frac{e^2 \pi}{2 M^2} \left[ \left( \frac{e^2}{4 \pi} \right) \frac{1}{2 \pi} \right]^2 = \int_0^\infty d\nu \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu} \quad \text{Dicus, Vega (2001)} \]

GDH sum rule is satisfied in QED to order $O(e^6)$
Spin dependent sum rules for RCS on proton

GDH sum rule (1966):

\[
\frac{e^2 \pi k^2_N}{2 M^2_N} = \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'}
\]

sum rule for proton:

lsh = 205 \, \mu b
Spin dependent sum rules for RCS on proton

\[ \frac{e^{-2\pi R^2}}{2 M_N^2} = \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'} \]  
GDH sum rule

\[ \gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{(\sigma_{3/2} - \sigma_{1/2})}{\nu'^3} \]  
Forward Spin Polarizability (FSP)

<table>
<thead>
<tr>
<th>PROTON results</th>
<th>E[GeV]</th>
<th>GDH [(\mu)b]</th>
<th>FSP [10(^{-4}) fm(^4)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAID2000 estimate</td>
<td>&lt; 0.2</td>
<td>-28.5 ± 2</td>
<td>0.95 ± 0.05</td>
</tr>
<tr>
<td>MAMI experiment</td>
<td>0.2 - 0.8</td>
<td>226 ± 5 ± 12</td>
<td>-1.87 ± 0.08 ± 0.10</td>
</tr>
<tr>
<td>ELSA experiment</td>
<td>0.8 - 2.9</td>
<td>27.5 ± 2.0 ± 1.2</td>
<td>-0.03</td>
</tr>
<tr>
<td>Bianchi-Thomas / Simula et al. : estimate</td>
<td>&gt; 2.9</td>
<td>-14 ± 2</td>
<td>+0.01</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>211 ± 15</td>
<td>-0.94 ± 0.15</td>
</tr>
<tr>
<td><strong>GDH sum rule</strong></td>
<td></td>
<td>205</td>
<td></td>
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