2nd lecture :
Forward virtual Compton scattering & nucleon structure functions

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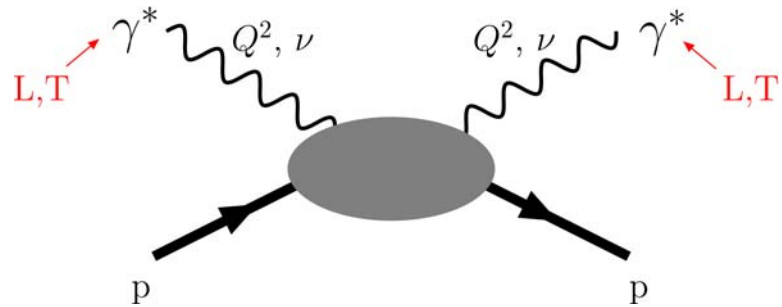
Forward double virtual Compton scattering (VVCS)



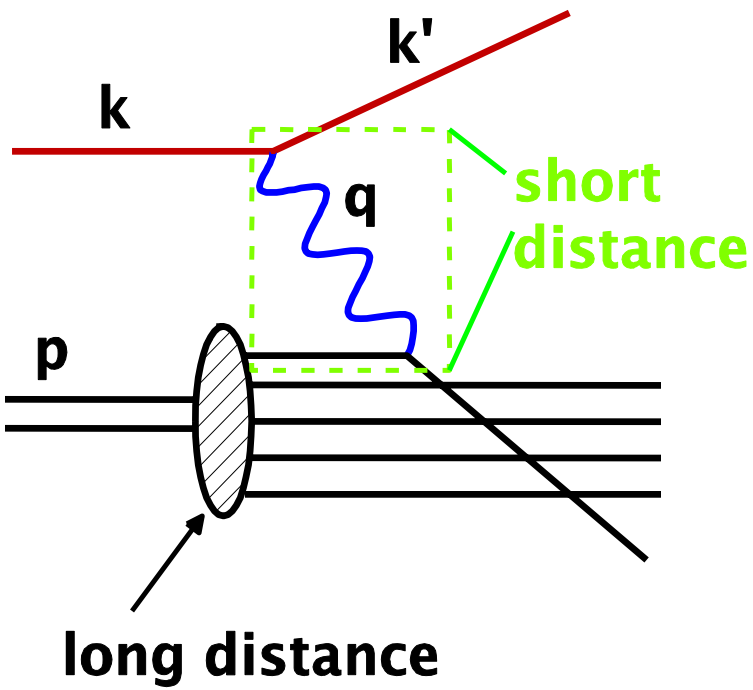
$$T(\nu, Q^2, \theta = 0) = \vec{\varepsilon}'^* \cdot \vec{\varepsilon}\, f_T(\nu, Q^2) + f_L(\nu, Q^2)$$
$$+ \; i\vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon})\, g_{TT}(\nu, Q^2) - i\vec{\sigma} \cdot [(\vec{\varepsilon}'^* - \vec{\varepsilon}) \times \hat{q}]\, g_{LT}(\nu, Q^2)$$

→ optical theorem

$$\mathrm{Im} f_T(\nu, Q^2) = \frac{K}{4\pi}\sigma_T(\nu, Q^2) \qquad \mathrm{Im}\, g_{TT}(\nu, Q^2) = \frac{K}{4\pi}\sigma_{TT}(\nu, Q^2)$$
$$\mathrm{Im}\, f_L(\nu, Q^2) = \frac{K}{4\pi}\sigma_L(\nu, Q^2) \qquad \mathrm{Im}\, g_{LT}(\nu, Q^2) = \frac{K}{4\pi}\sigma_{LT}(\nu, Q^2)$$

nucleon parton distribution

Deep-inelastic lepton nucleon scattering



$q(\nu, \vec{q})$ $\quad$ $p(M, \vec{0})$

$$Q^2 = \vec{q}^{\,2} - \nu^2$$

$$x_B = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

$$d\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

leptonic tensor

hadronic tensor

Hadronic tensor in DIS

➡ $$W^{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \, \delta^4(q+p-p_X) \, \langle N(p,s) \,|\, J^\mu(0) \,|\, X \rangle \, \langle X \,|\, J^\nu(0) \,|\, N(p,s) \rangle$$

$$= W^{\mu\nu\,(S)} + W^{\mu\nu\,(A)}$$

➡ 4 structure functions of nucleon

$$W^{\mu\nu\,(S)} = 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{p\cdot q} \left(p^\mu - \frac{p\cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p\cdot q}{q^2} q^\nu \right) F_2(x_B, Q^2) \right\}$$

$$W^{\mu\nu\,(A)} = \frac{2 M_N}{p\cdot q} \, i \, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \left\{ S_\beta \, g_1(x_B, Q^2) + \left[S_\beta - \frac{S\cdot q}{p\cdot q} p_\beta \right] g_2(x_B, Q^2) \right\}$$

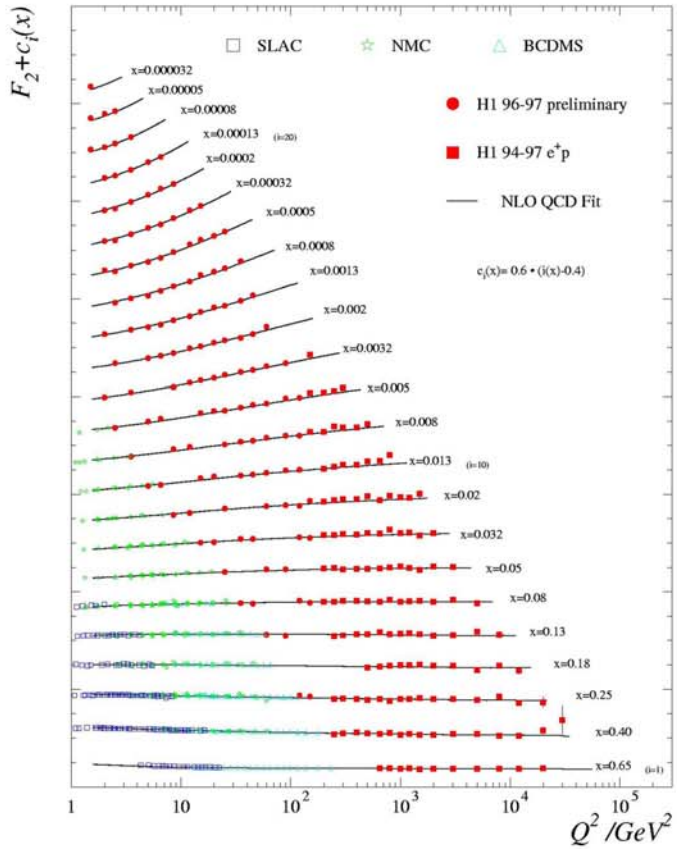
➡ Bjorken scaling : $Q^2 \to \infty$, x_B fixed

$$F_1(x_B, Q^2) \longrightarrow F_1(x_B) \qquad g_1(x_B, Q^2) \longrightarrow g_1(x_B)$$

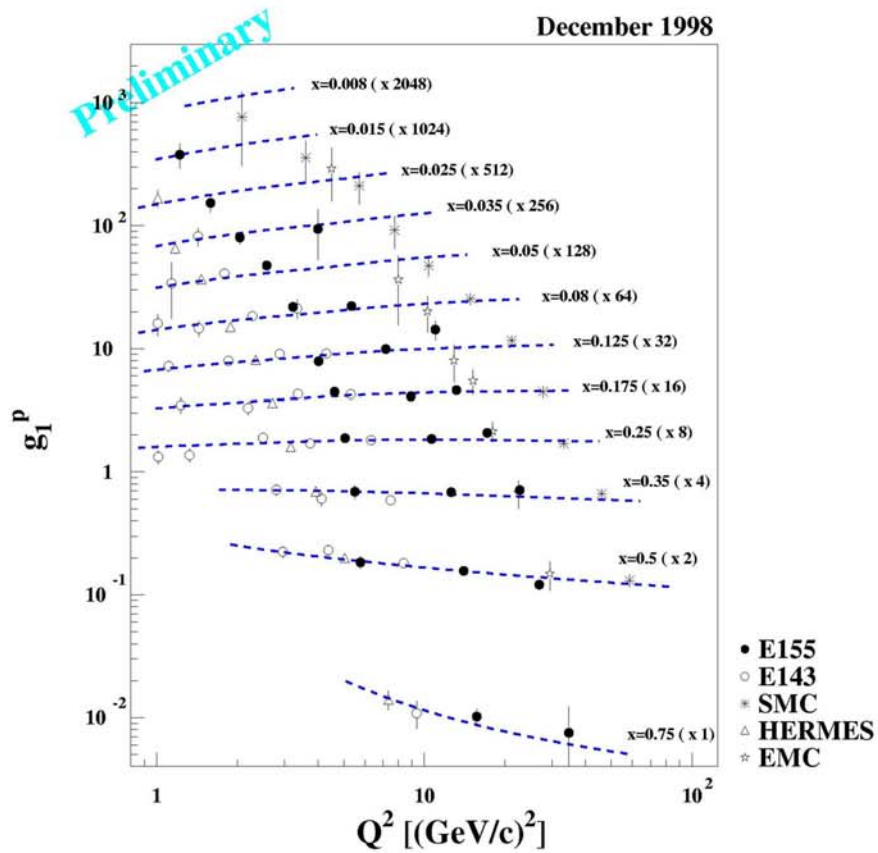
$$F_2(x_B, Q^2) \longrightarrow F_2(x_B) \qquad g_2(x_B, Q^2) \longrightarrow g_2(x_B)$$

Proton structure functions

World data on F_1^p



World data on g_1^p



Dispersion relation for f_T

➡ subtracted dispersion relation

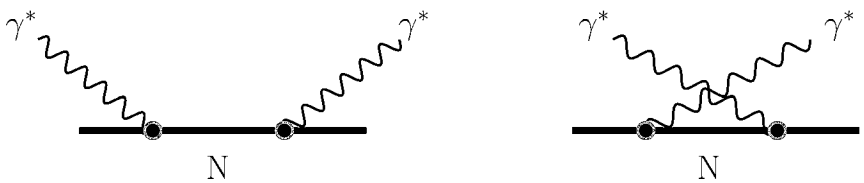
$$\mathrm{Re}\, f_T(\nu, Q^2) = \mathrm{Re}\, f_T(0, Q^2) + \frac{2\nu^2}{\pi}\mathcal{P}\int_0^\infty \mathrm{d}\nu' \, \frac{\mathrm{Im} f_T(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

subtraction

➡ elastic contribution

singularities at : $\nu = \S\, \nu_B$ with $\nu_B = Q^2/2M_N$

($ x_B=1)



$$\mathrm{Re} f_T^{\mathrm{Pole}}(\nu, Q^2) = -\frac{e^2}{4\pi\, M_N}\frac{\nu_B^2}{\nu^2 - \nu_B^2}\, G_M^2(Q^2)$$

$$\mathrm{Re} f_T(\nu, Q^2) - \mathrm{Re} f_T^{\mathrm{Pole}}(\nu, Q^2) = \mathrm{Re} f_T(0, Q^2) - \mathrm{Re} f_T^{\mathrm{Pole}}(0, Q^2)$$

$$+ \frac{\nu^2}{2\pi^2}\mathcal{P}\int_{\nu_0}^\infty \mathrm{d}\nu' \, \frac{K(\nu', Q^2)\sigma_T(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

ν_0: π threshold

➡ low energy expansion for the non-pole part

$$\mathrm{Re} f_T(\nu, Q^2) - \mathrm{Re} f_T^{\mathrm{Pole}}(\nu, Q^2) = \mathrm{Re} f_T(0, Q^2) - \mathrm{Re} f_T^{\mathrm{Pole}}(0, Q^2)$$

$$+ \nu^2(\alpha(Q^2) + \beta(Q^2)) + \mathcal{O}(\nu^4)$$

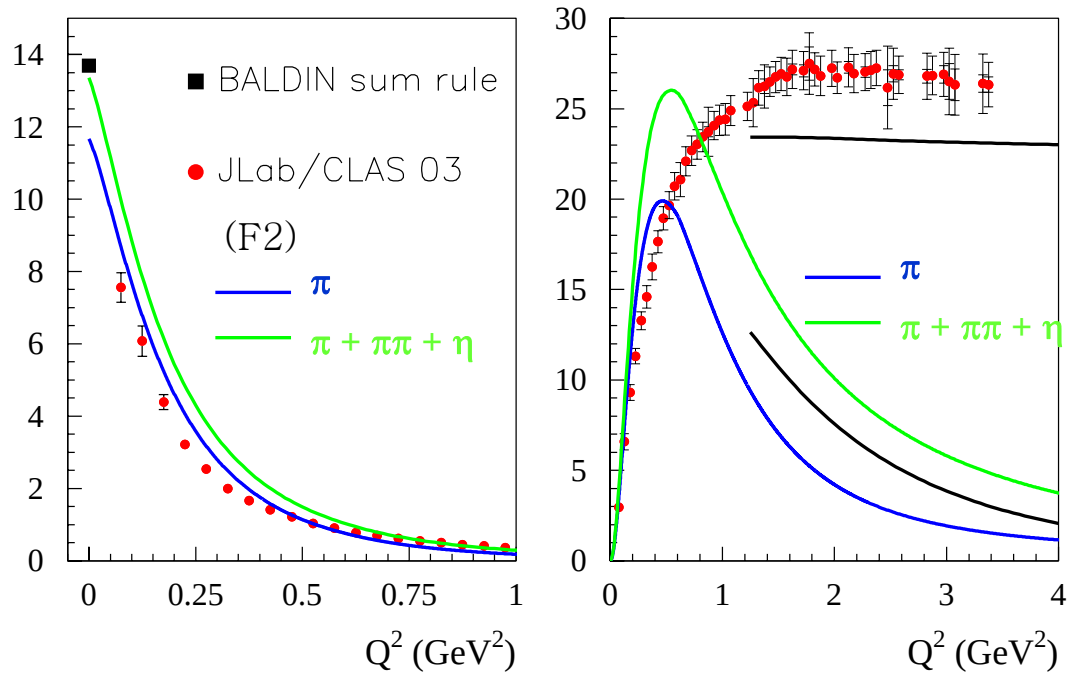
Generalized Baldin sum rule

$$\alpha(Q^2)+\beta(Q^2) = \frac{1}{2\pi^2}\int_{\nu_0}^{\infty}\frac{K(\nu,Q^2)}{\nu}\,\frac{\sigma_T(\nu,Q^2)}{\nu^2}\,d\nu\,,$$

$$= \frac{e^2 M_N}{\pi Q^4}\int_0^{x_0} 2x\,F_1(x,Q^2)\,dx \longrightarrow \frac{e^2 M_N}{\pi Q^4}\int_0^{x_0} F_2(x,Q^2)\,dx$$

$Q^2 >>$

$\alpha + \beta$ $(10^{-4}\ fm^3)$ $\qquad$ $(\alpha + \beta)\cdot Q^4/(2\,M)$ (10^{-4})



→ **DIS (MRST 01)**

→ **"DIS"**
(W < 2 GeV)

resonance estimate (W < 2 GeV, MAID π + ππ + η)
Drechsel, Pasquini, Vdh (2003)

CLAS data: Osipenko et al. (2003)

Spin dependent forward double VCS

$$T(\nu, Q^2, \theta = 0) = i\vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon})\, g_{TT}(\nu, Q^2) - i\vec{\sigma} \cdot [(\vec{\varepsilon}'^* - \vec{\varepsilon}) \times \hat{q}]\, g_{LT}(\nu, Q^2)$$

$$= \frac{i}{M_N} \epsilon_{\mu\nu\alpha\beta}\, q^\alpha s^\beta\, S_1(\nu, Q^2) + \frac{i}{M_N^3} \epsilon_{\mu\nu\alpha\beta}\, q^\alpha (p \cdot q\; s^\beta - s \cdot q\; p^\beta)\, S_2(\nu, Q^2)$$

- link between S_1, S_2 & g_{TT}, g_{LT}

$$S_1(\nu, Q^2) = \frac{\nu M_N}{\nu^2 + Q^2} \left[g_{TT}(\nu, Q^2) + \frac{Q}{\nu} g_{LT}(\nu, Q^2) \right]$$

$$S_2(\nu, Q^2) = -\frac{M_N^2}{\nu^2 + Q^2} \left[g_{TT}(\nu, Q^2) - \frac{\nu}{Q} g_{LT}(\nu, Q^2) \right]$$

- optical theorem

$$\mathrm{Im}\, S_1 = \frac{e^2}{4M_N} \frac{M_N}{\nu}\, g_1 \qquad \mathrm{Im}\, g_{TT} = \frac{K}{4\pi}\, \sigma_{TT}$$

$$\mathrm{Im}\, S_2 = \frac{e^2}{4M_N} \frac{M_N^2}{\nu^2}\, g_2 \qquad \mathrm{Im}\, g_{LT} = \frac{K}{4\pi}\, \sigma_{LT}$$

Dispersion relations for g_{TT} and S_1

- UNsubtracted dispersion relation

$$\mathrm{Re}\, g_{TT}(\nu, Q^2) \;=\; \mathrm{Re}\, g_{TT}^{\mathrm{Pole}}(0, Q^2) \;+\; \frac{\nu}{2\pi^2}\mathcal{P}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{K(\nu', Q^2)\, \sigma_{TT}(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

$$\mathrm{Re}\, S_1(\nu, Q^2) \;=\; \mathrm{Re}\, S_1^{\mathrm{Pole}}(0, Q^2) \;+\; \frac{2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\nu'\, \mathrm{Im}\, S_1(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

- low energy expansion for inelastic part

$$\mathrm{Re}\, g_{TT}(\nu, Q^2) - \mathrm{Re}\, g_{TT}^{\mathrm{Pole}}(\nu, Q^2) = \frac{e^2}{2\pi\, M_N^2}\, I_A(Q^2)\, \nu \;+\; \gamma_0(Q^2)\, \nu^3 + \mathcal{O}(\nu^5)$$

$$\mathrm{Re}\, S_1(\nu, Q^2) - \mathrm{Re}\, S_1^{\mathrm{Pole}}(\nu, Q^2) = \frac{e^2}{2\pi\, M_N}\, I_1(Q^2) + \left[\frac{e^2}{2\pi\, M_N}\, \frac{1}{Q^2}\, \big(I_A(Q^2) \;-\; I_1(Q^2)\big) \;+\; M_N\, \delta_{LT}(Q^2)\right] \nu^2 + \mathcal{O}(\nu^4)$$

$$\Rightarrow\; I_A(Q^2) \;\equiv\; \frac{M_N^2}{\pi\, e^2}\int_{\nu_0}^{\infty} d\nu\, \frac{K(\nu, Q^2)}{\nu}\, \frac{\sigma_{TT}\,(\nu,\, Q^2)}{\nu} \;\rightarrow\; -\frac{\kappa_N^2}{4}\,, \quad Q^2 \to 0 \;:\; \text{GDH sum rule}$$

$$\Rightarrow\; I_1(Q^2) \;\equiv\; \frac{2M_N^2}{Q^2}\int_0^{x_0} dx\; g_1(x,\, Q^2) \rightarrow \begin{cases} -\frac{\kappa_N^2}{4}, & Q^2 \to 0 : \quad \text{GDH sum rule} \\ \frac{2M_N^2}{Q^2}\, \Gamma_1(Q^2), & Q^2 \to \infty : \quad \text{Bjorken sum rule} \end{cases}$$

Generalized GDH sum rule for proton

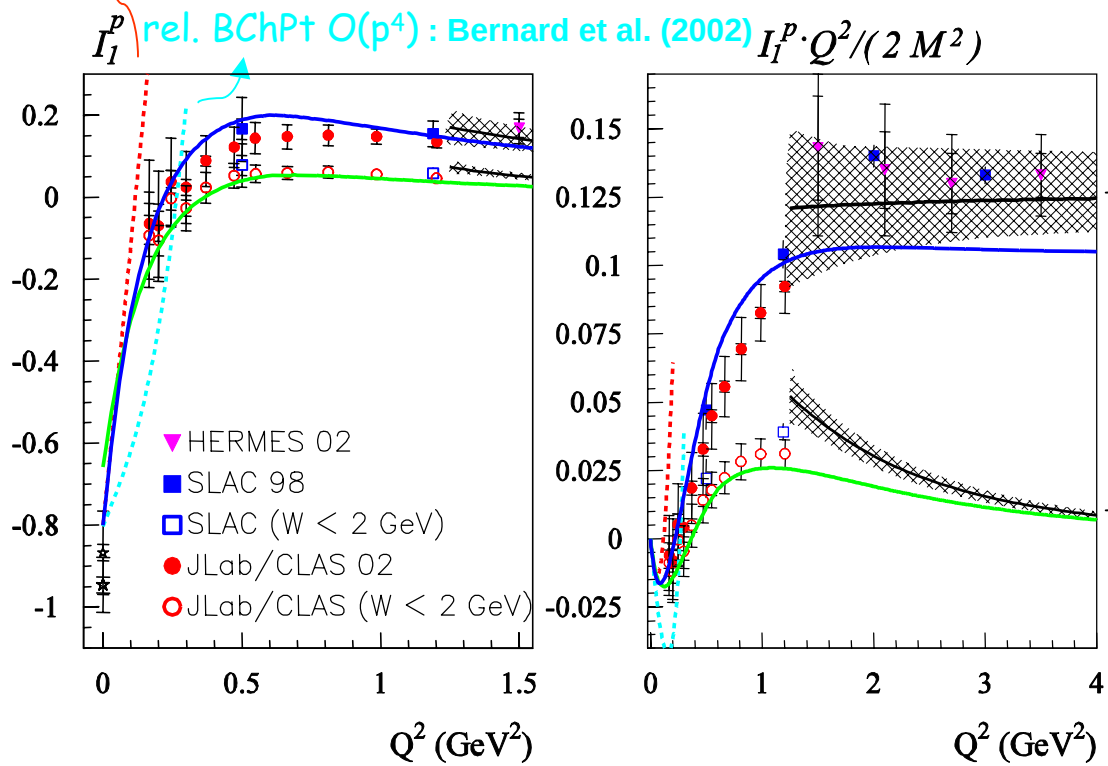
$$I_1(Q^2) \equiv \frac{2M_N^2}{Q^2}\int_0^{x_0} dx\, g_1(x,Q^2)$$

$$= \frac{M_N^2}{\pi e^2}\int_{\nu_0}^{\infty} d\nu \frac{K(\nu,Q^2)}{(\nu^2+Q^2)}\left\{\sigma_{TT}(\nu,Q^2) + \frac{Q}{\nu}\sigma_{LT}(\nu,Q^2)\right\}$$

$$\rightarrow \begin{cases} -\frac{\kappa_p^2}{4}, & Q^2 \rightarrow 0: \quad \text{GDH sum rule} \\ \frac{2M_N^2}{Q^2}\Gamma_1^p(Q^2), & Q^2 \rightarrow \infty: \quad \text{Bjorken sum rule} \end{cases}$$

HBChPt $O(p^4)$:
Ji, Kao, Osborne (2000)

rel. BChPt $O(p^4)$: **Bernard et al. (2002)**



DIS: **Bluemlein, Boettcher (2003)**

resonance estimate + DIS (VMD)

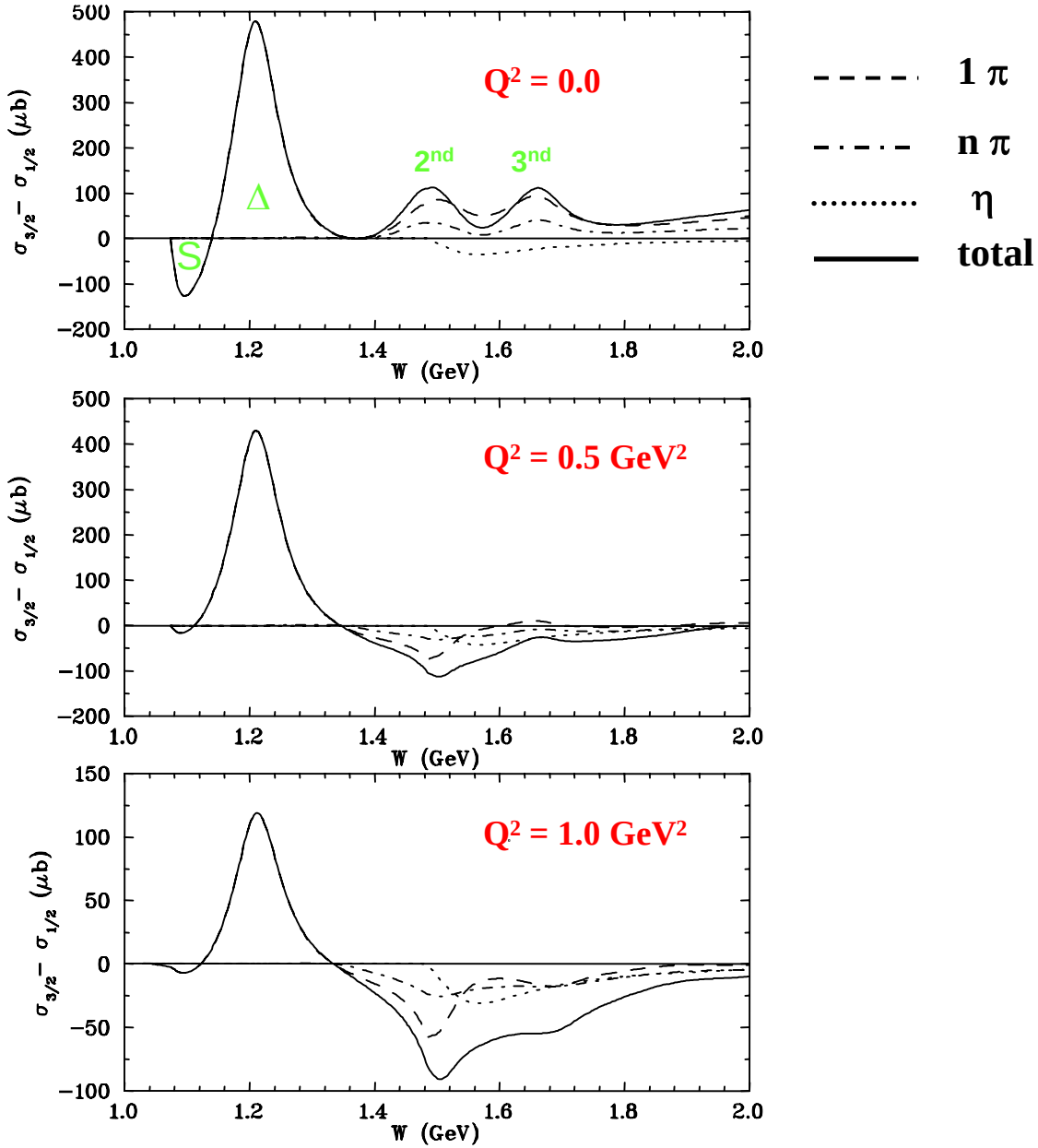
Anselmino, Ioffe, Leader / Burkert, Ioffe

"DIS": W < 2 GeV

resonance estimate W < 2 GeV:
$\pi + \pi\pi + \eta$

Drechsel, Pasquini, Vdh (2003)

Q^2 dependence of $\sigma_{3/2} - \sigma_{1/2}$

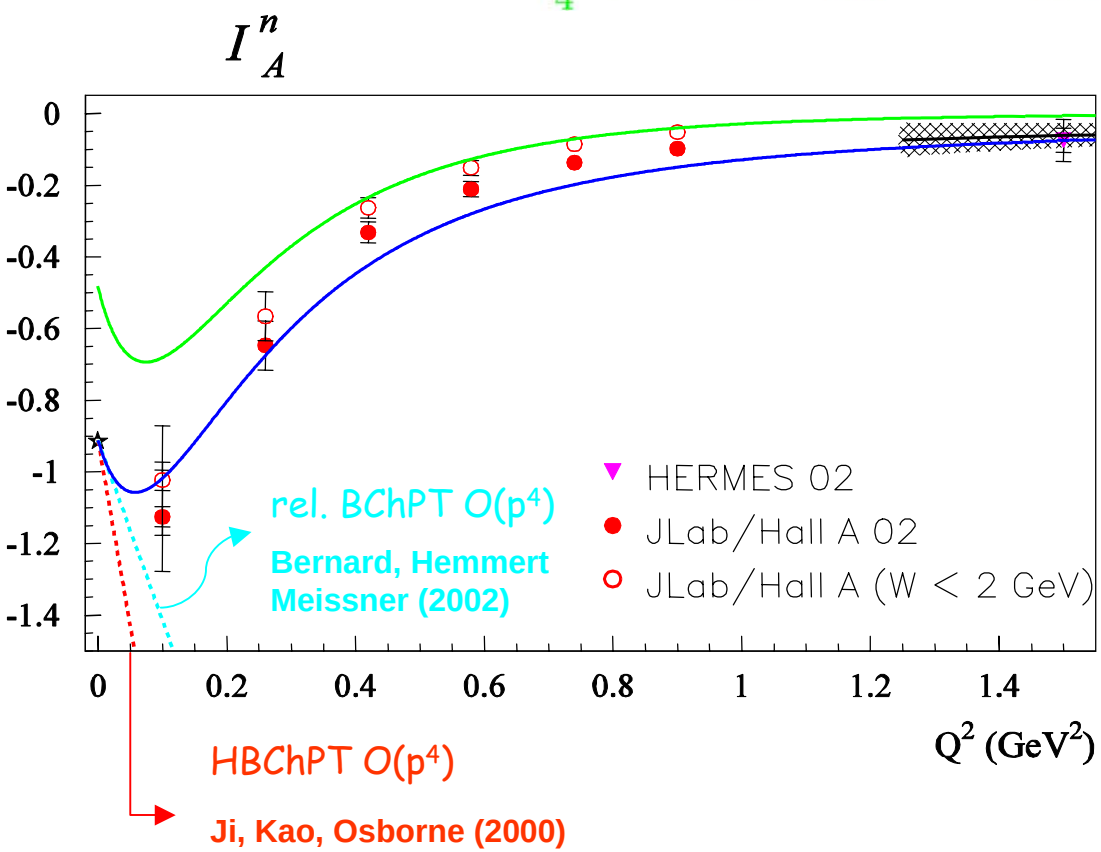


Generalized GDH sum rule for neutron

$$I_A(Q^2) \equiv \frac{2M_N^2}{Q^2}\int_0^{x_0} dx \left(g_1(x,Q^2) - \frac{4M_N^2}{Q^2}x^2 g_2(x,Q^2)\right)$$

$$= \frac{M_N^2}{\pi e^2}\int_{\nu_0}^{\infty} d\nu \frac{K(\nu,Q^2)}{\nu}\frac{\sigma_{TT}(\nu,Q^2)}{\nu}$$

$$\rightarrow -\frac{\kappa_n^2}{4}, \qquad Q^2 \rightarrow 0: \qquad \text{GDH sum rule}$$



DIS:

Bluemlein, Boettcher (2003)

resonance + DIS estimate (VMD)

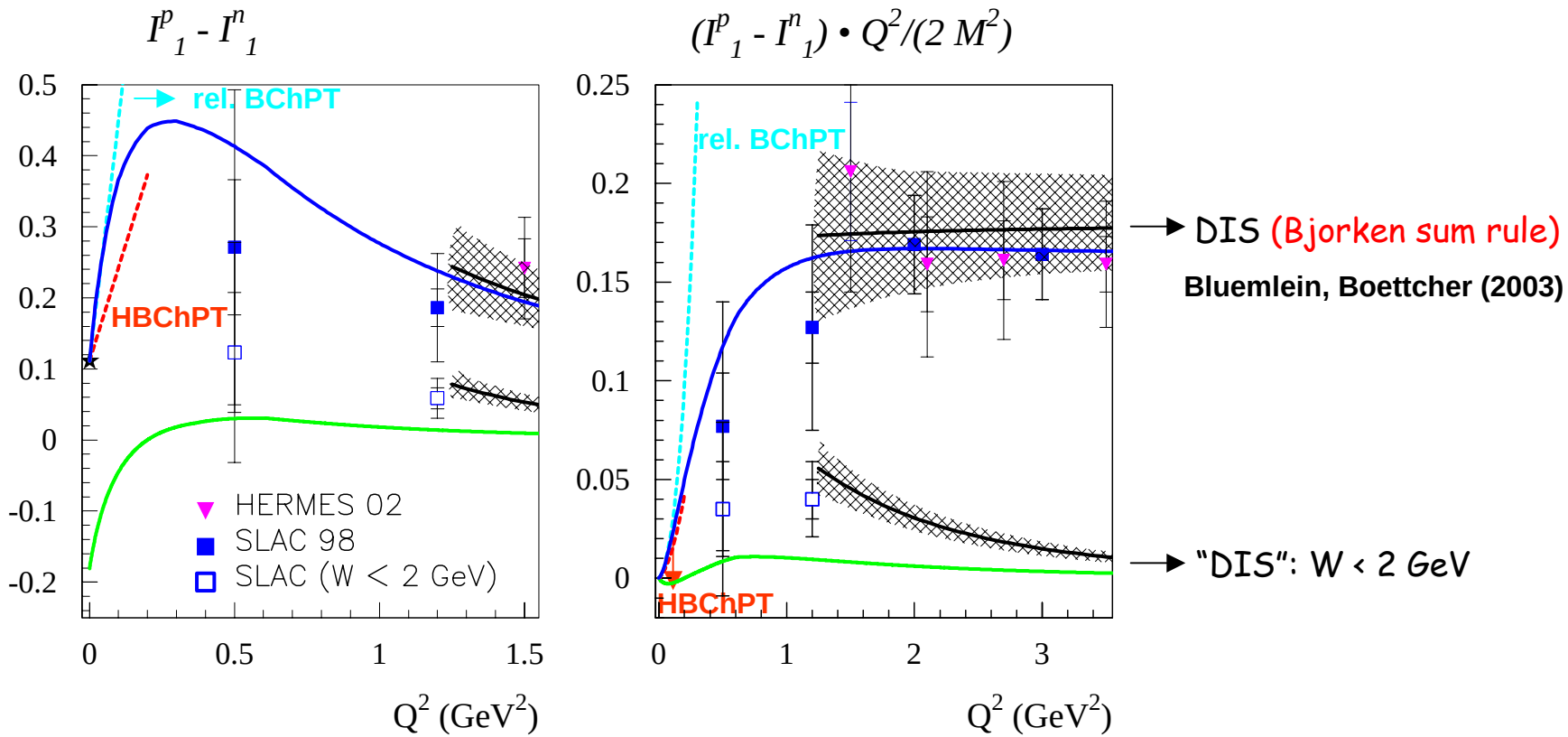
Anselmino, Ioffe, Leader / Burkert, Ioffe

resonance estimate :

π (MAID)

Drechsel, Pasquini, Vdh (2003)

GDH sum rule for p - n



HBChPT O(p4): **Ji, Kao, Osborne (2000)**

rel. BChPT O(p^4): **Bernard, Meissner, Hemmert (2002)**

resonance estimate (W < 2 GeV): π N (MAID)

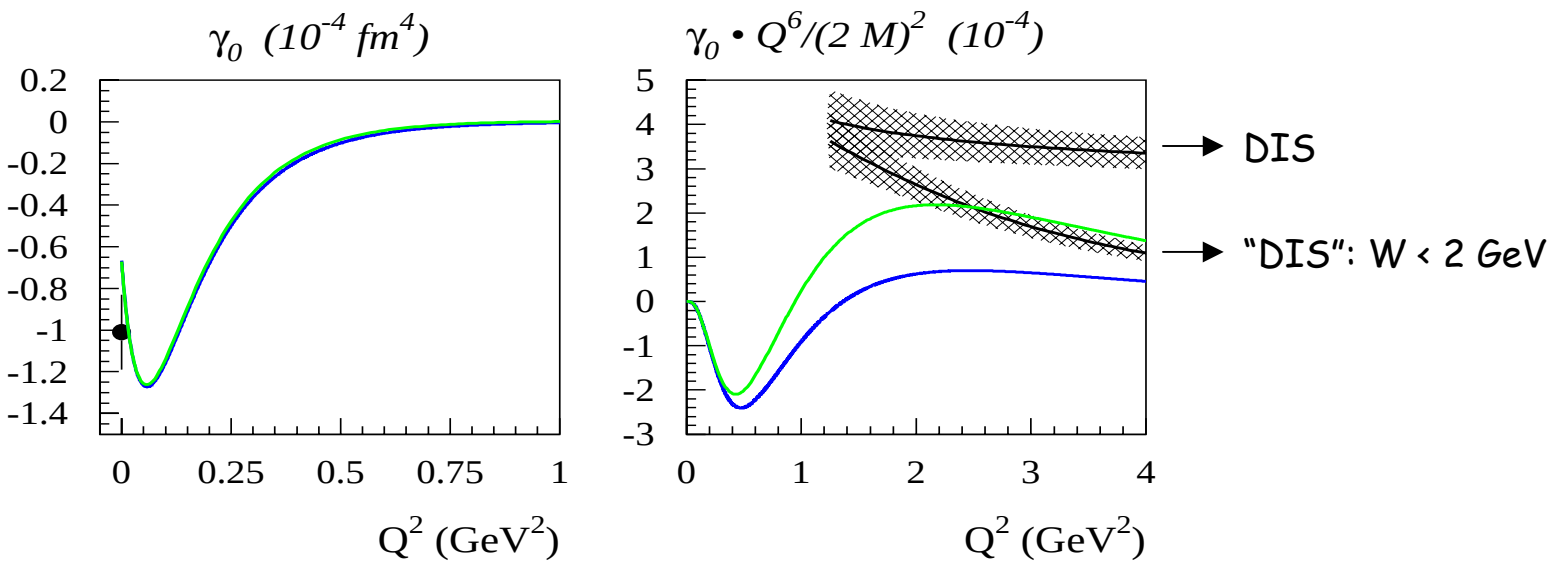
resonance + DIS estimate (VMD)

} **Drechsel, Pasquini, Vdh (2003)**

Forward spin polarizability γ^0 of proton

$$\gamma_0(Q^2) = \frac{1}{2\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{K(\nu, Q^2)}{\nu}\, \frac{\sigma_{TT}(\nu, Q^2)}{\nu^3}$$

$$= \frac{e^2\, 4\, M_N^2}{\pi\, Q^6}\int_0^{x_0} dx\, x^2 \left\{ g_1(x, Q^2) - \frac{4\, M_N^2}{Q^2}\, x^2\, g_2(x, Q^2) \right\}$$



resonance estimate (W < 2 GeV) : π + $\pi\pi$ + η

resonance estimate (W < 2 GeV) : π (MAID)

Dispersion relation for g_{LT}

- UNsubtracted dispersion relation

$$\mathrm{Re}\, g_{LT}(\nu, Q^2) \;=\; \mathrm{Re}\, g_{LT}^{\mathrm{Pole}}(0, Q^2) \;+\; \frac{1}{2\pi^2}\mathcal{P}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \, K(\nu', Q^2)\, \sigma_{LT}(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

- low energy expansion for inelastic part

$$\mathrm{Re}\, g_{LT}(\nu, Q^2) - \mathrm{Re}\, g_{LT}^{\mathrm{Pole}}(\nu, Q^2) \;=\; \frac{e^2}{2\pi\, M_N^2}\, Q\, (\, I_1(Q^2) \;+\; I_2(Q^2))$$

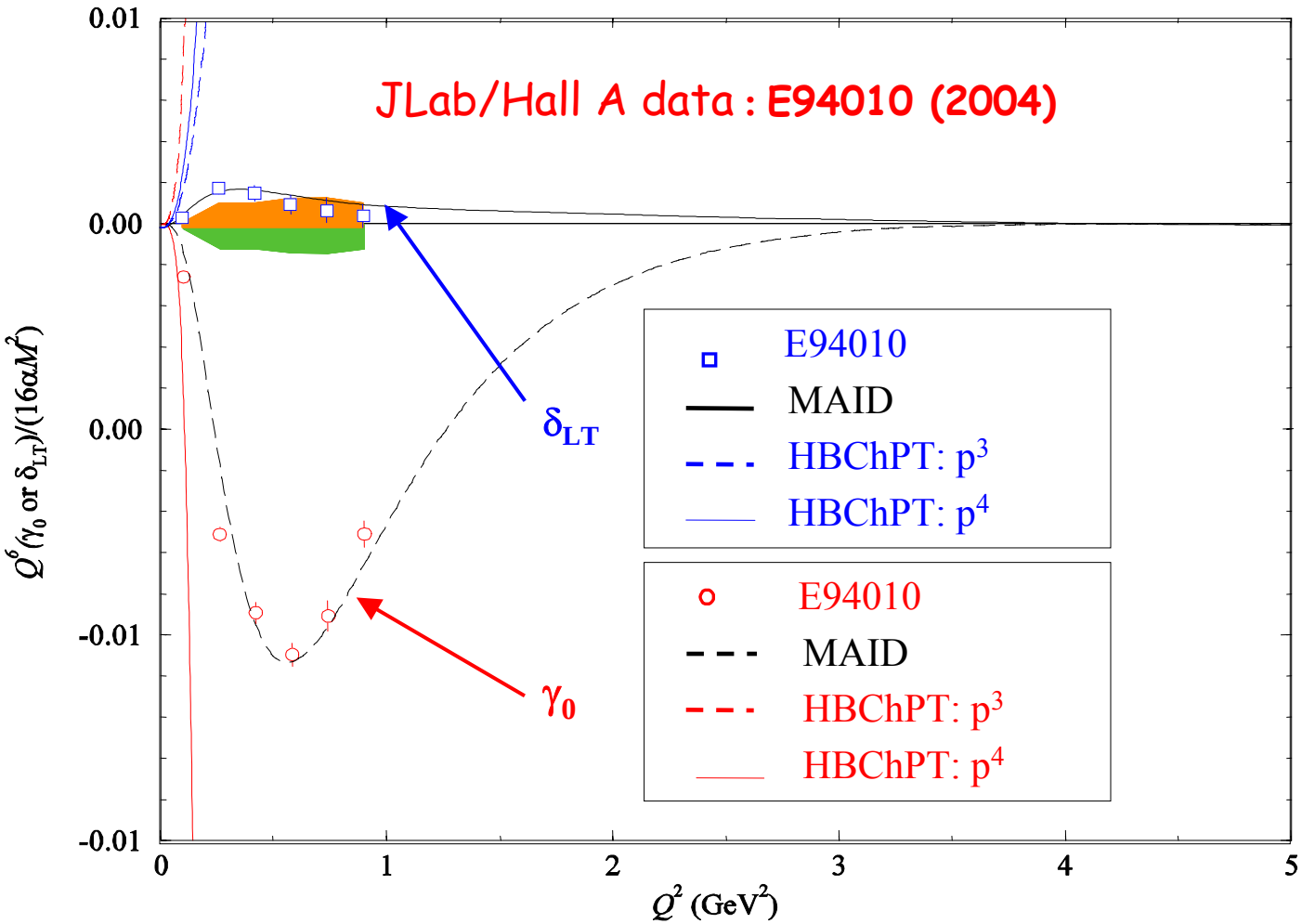
$$+ \quad \delta_{LT}(Q^2)\, \nu^2\, Q + \mathcal{O}(\nu^4)$$

➡ first moment of g_2 : $\quad I_2(Q^2) \;\equiv\; \frac{2M_N^2}{Q^2}\int_0^{x_0} \mathrm{d}x\; g_2(x, Q^2)$

➡ Longitudinal - Transverse spin polarizability

$$\delta_{LT}(Q^2) \;=\; \frac{1}{2\pi^2}\int_{\nu_0}^{\infty} d\nu\, \frac{K(\nu,\, Q^2)}{\nu}\, \frac{\sigma_{LT}(\nu,\, Q^2)}{Q\, \nu^2} \;=\; \frac{e^2\, 4\, M_N^2}{\pi\, Q^6}\int_0^{x_0} dx\, x^2\, \{g_1(x,\, Q^2) \;+\; g_2(x,\, Q^2)\}$$

Forward spin polarizability of neutron



resonance estimate (MAID) : Drechsel, Pasquini, Vdh (2003)

HBChPT: Kao, Spitzenberg, Vdh (2003)

Dispersion relations for S_2

- UNsubtracted dispersion relation

(1) $$\mathrm{Re}\, S_2(\nu, Q^2) = \frac{2\nu}{\pi} \mathcal{P} \int_0^\infty d\nu' \, \frac{\mathrm{Im}\, S_2(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

➡ assuming high energy behavior: for ν ! 1, S_2 ! 0 faster than $1/\nu$

(2) $$\mathrm{Re}\,(\nu\, S_2(\nu, Q^2)) = \frac{2}{\pi} \mathcal{P} \int_0^\infty d\nu' \, \frac{\nu'^2\, \mathrm{Im}\, S_2(\nu', Q^2)}{(\nu'^2 - \nu^2)}$$

- superconvergence relation

(1) * ν – (2) $$0 = \int_0^\infty d\nu' \, \mathrm{Im}\, S_2(\nu', Q^2)$$

➡ in terms of g_2 (x, Q^2) : $$0 = \int_0^1 dx \, g_2(x, Q^2)$$

Burkhardt – Cottingham sum rule (1970)

$$\boxed{\int_0^{x_0} dx \, g_2(x, Q^2) = \underbrace{\frac{Q^2}{8M_N^2} F_P(Q^2) \left[F_D(Q^2) + F_P(Q^2)\right]}_{\text{elastic contribution}}}$$

F_D : Dirac

F_P : Pauli

x_0: inelastic threshold

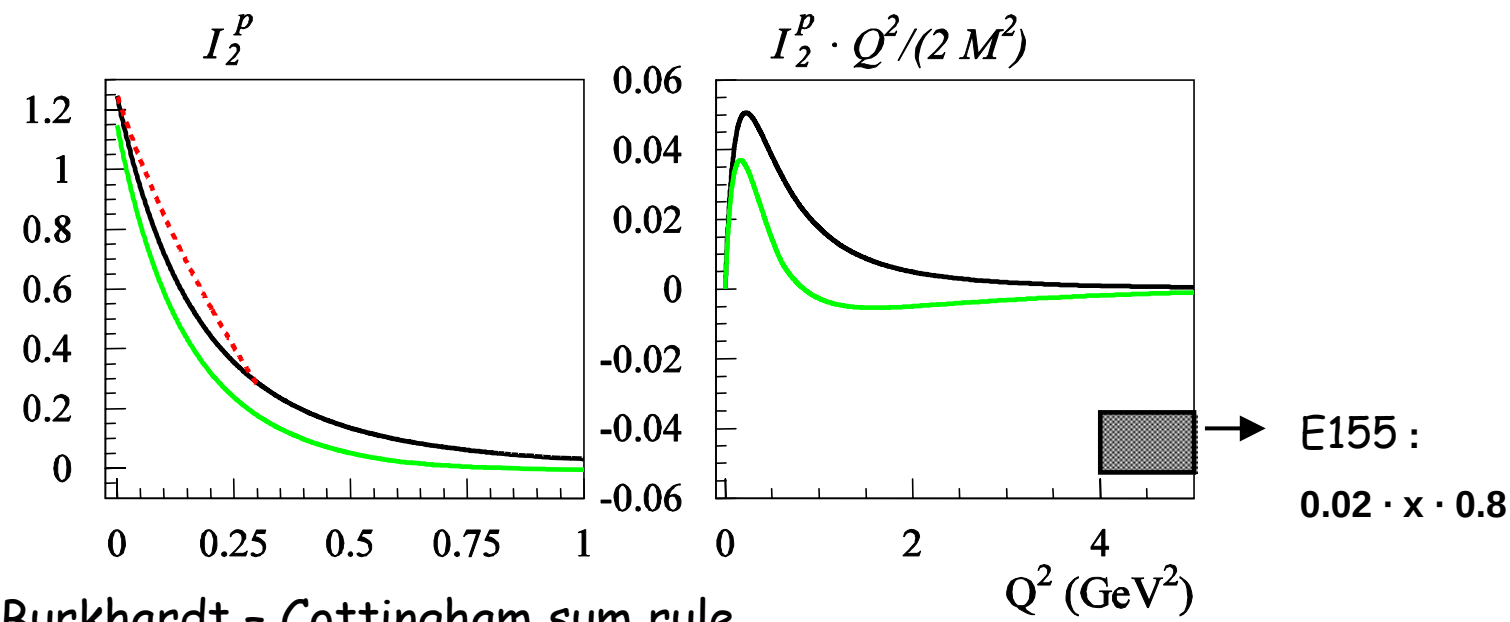
convergence condition: **g_2 ! x^α for x! 0 with α > -1**

Burkhardt-Cottingham sum rule for proton

$$I_2(Q^2) \equiv \frac{2M_N^2}{Q^2}\int_0^{x_0} \mathrm{d}x\, g_2(x,Q^2)$$

$$= \frac{M_N^2}{\pi e^2}\int_{\nu_0}^{\infty} \mathrm{d}\nu\, \frac{K(\nu,Q^2)}{\nu^2+Q^2)}\left\{-\sigma_{TT}(\nu,Q^2) + \frac{\nu}{Q}\,\sigma_{LT}(\nu,Q^2)\right\}$$

$$= \frac{1}{4}\,F_P(Q^2)\left[F_D(Q^2)+F_P(Q^2)\right], \qquad \text{BC sum rule}$$

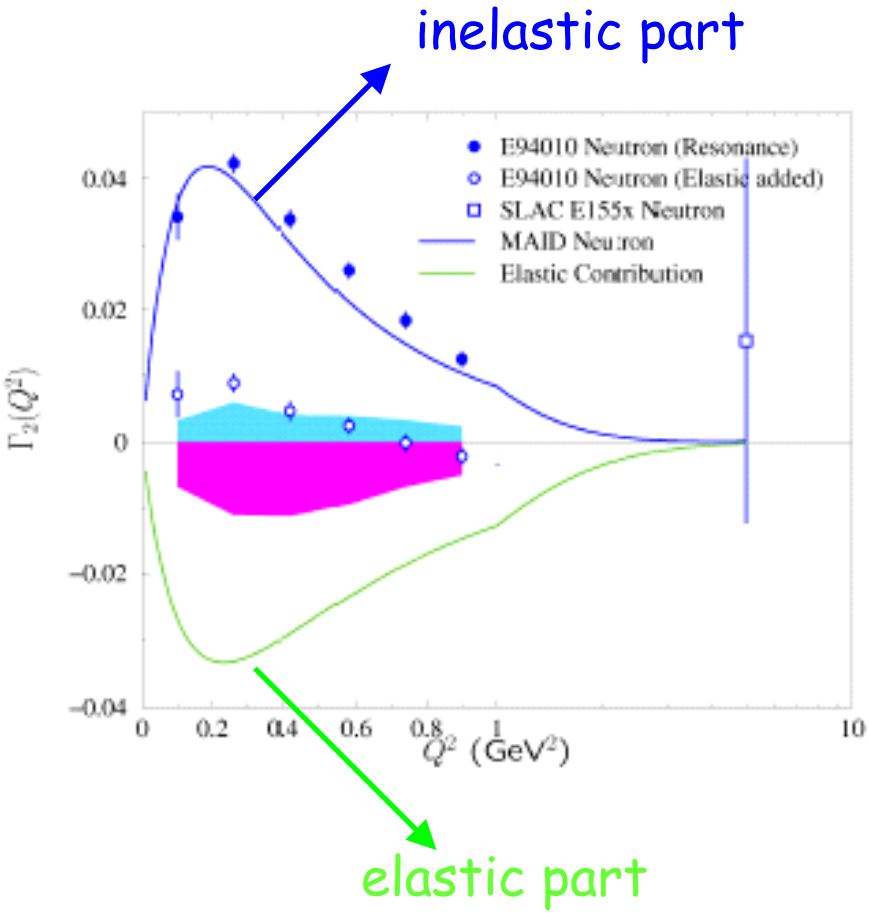


Burkhardt – Cottingham sum rule

resonance estimate (W < 2 GeV): π (MAID)

HBChPT O(p⁴) : Kao, Spitzenberg, Vdh (2002)

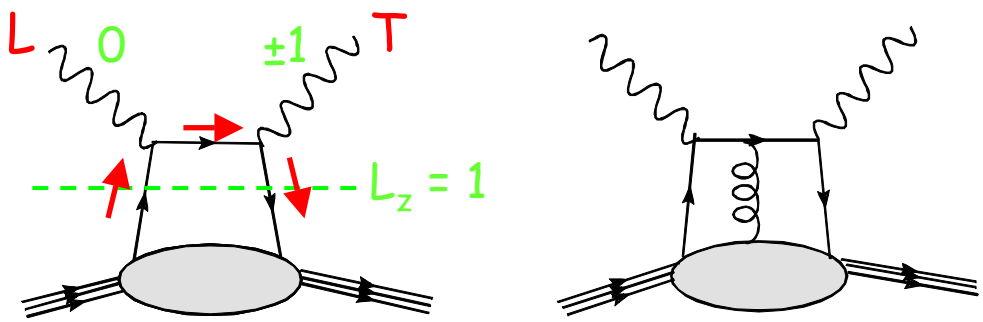
Burkhardt-Cottingham sum rule for neutron



Burhardt-Cottingham sum rule is satisfied for the neutron

Wandzura - Wilczek relation

$$g_2(x, Q^2) \quad = \quad g_2^{WW}(x, Q^2) \quad + \quad \bar{g}_2(x, Q^2)$$



Twist-2 : Wandzura-Wilczek

Twist-3 : quark-gluon correlations in nucleon

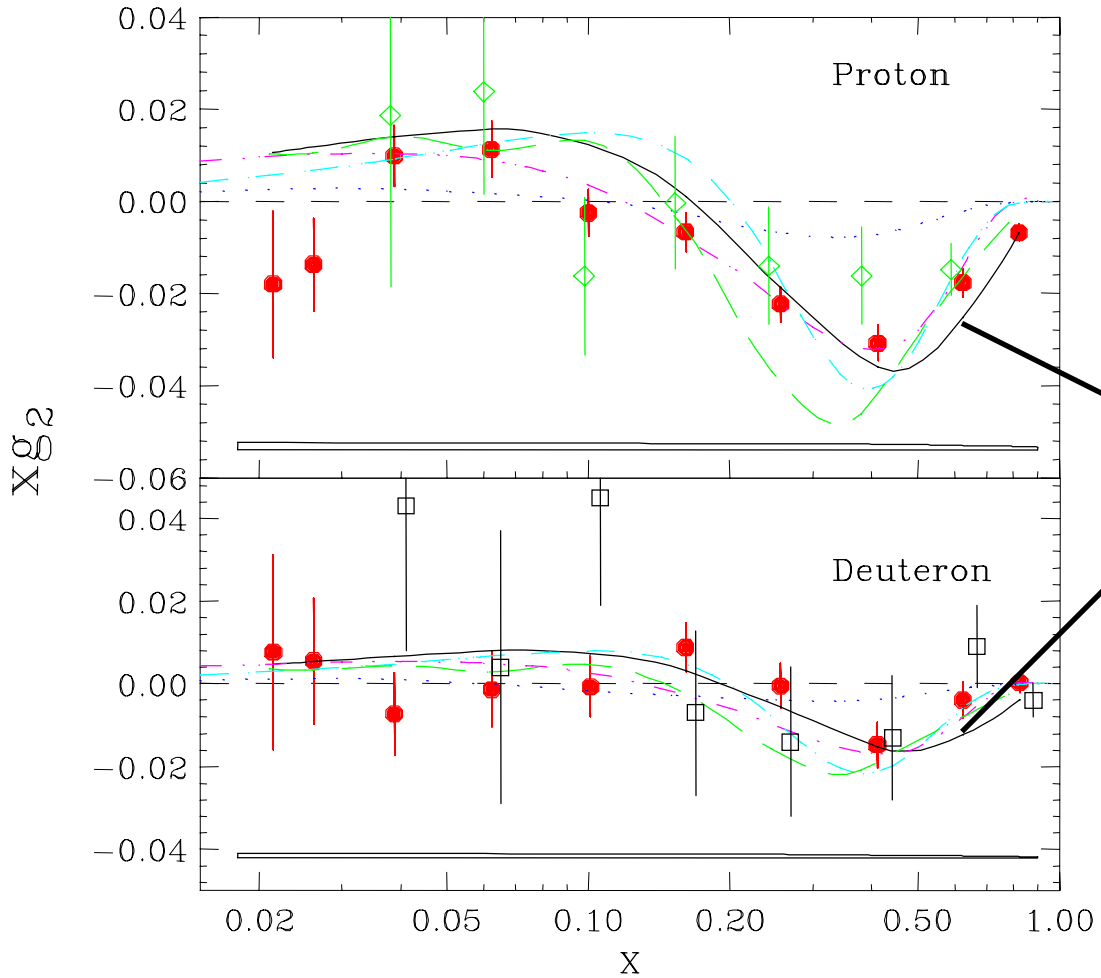
➡ for $Q^2 \to \infty$: Wandzura-Wilczek relation:

$$g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \mathrm{d}y \; \frac{g_1(y, Q^2)}{y}$$

➡ twist-3 moment d_2

$$d_2 \quad \equiv \quad 3 \int_0^1 \mathrm{d}x \; x^2 \; \bar{g}_2(x) \qquad : d_2^{WW} = 0$$

Experimental results for g_2



SLAC E155X

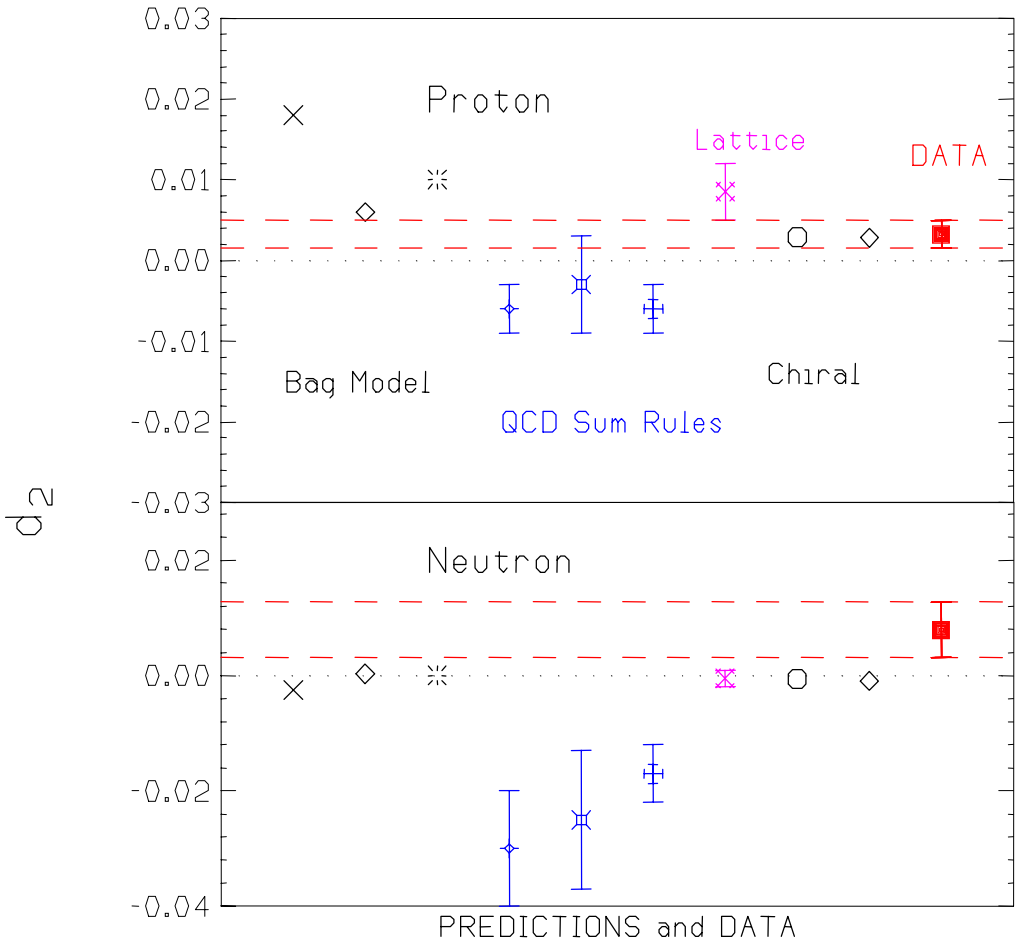
SLAC E155

SLAC E143

Wandzura-Wilczek relation

Twist-3 matrix element d_2

$$d_2 \equiv 3\int_0^1 \mathrm{d}x\, x^2\, \bar{g}_2(x) \qquad : d_2^{WW} = 0$$



data : SLAC E155X

$\langle Q^2 \rangle = 5\ \mathrm{GeV}^2$

Scale dependence of d_2

$$d_2(Q^2) \equiv \int_0^1 \mathrm{d}x\, x^2 \, \{3\, g_2(x, Q^2) + 2\, g_1(x, Q^2)\}$$

- scaling limit, $Q^2 \to \infty$: $d_2(Q^2) \longrightarrow d_2^{\mathrm{twist-3}}$
- arbitrary Q^2 :

$$d_2(Q^2) = \frac{Q^6}{8\, M_N^2} \left\{ -\frac{1}{M_N^2}\, \frac{1}{Q^2} \left(I_A(Q^2) - I_1(Q^2) \right) + \frac{4\,\pi}{e^2}\, \delta_{LT}(Q^2) \right\}$$

enter in low energy expansion of VVCS

- low Q2 : ChPT

$$d_2(Q^2) \longrightarrow \frac{Q^6}{8\, M_N^2}\, \frac{2\pi}{e^2}\, \{-\gamma_0(0) + 3\, \delta_{LT}(0)\}$$

$$= \frac{Q^6}{48\, M_N^2}\, \frac{g_A^2}{(4\pi F_\pi)^2\, m_\pi^2} \times \left\{ 1 + \frac{m_\pi}{M_N} \cdot \frac{\pi}{8}\, [(21 + 9\,\kappa_v) + (-6 + 14\,\kappa_s)\tau_3\,] \right\}$$

HBChPT : p^3 p^4

Scale dependence of d_2

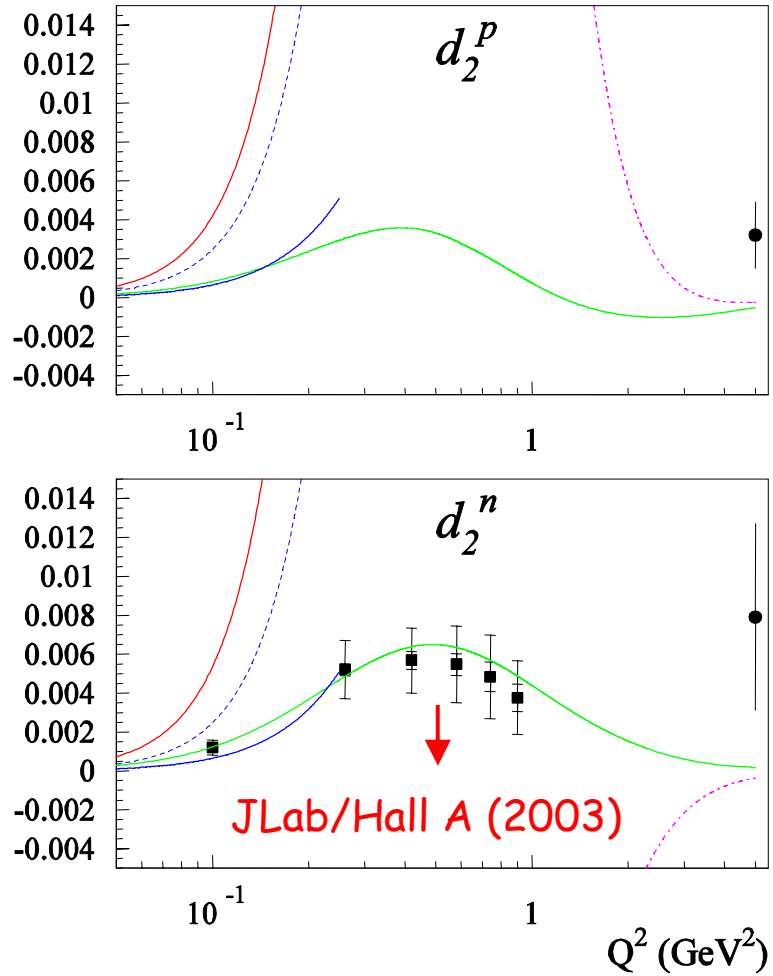
ELASTIC

HBChPT $O(p^4)$

HBChPT $O(p^3)$ incl. Δ

HBChPT $O(p^3)$

resonance estimate : π (MAID)



E155 (2002)

Kao, Drechsel, Kamalov, Vdh (2003)