Four lectures on the Color Glass Condensate

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Outline of lectures

- Lecture I: *General introduction, the DIS paradigm, QCD evolution, saturation.*
- Lecture II: *The IMF wave function, the MV model.*
- Lecture III: *Quantum evolution in the CGC, Wilson RG, analytic and numerical solutions.*
- Lecture IV: *DIS and hadronic scattering at high energies; Heavy Ion collisions at RHIC.*

Recent Reviews on the CGC:

- L. McLerran, hep-ph/0311028
- E. Iancu & R. Venugopalan, hep-ph/0303204
- A. H. Mueller, hep-ph/9911289

Outstanding phenomenological issues in QCD at high energies (I)

- *Perturbative QCD is very successful-but- describes only a small part of the cross-section:* $\sigma_{\mathrm{Rutherford}} \propto \frac{1}{Q^2}$

- *Lattice QCD-first principles approach-computes static quantities well-especially difficult to apply at high energies*

- *Outstanding problem-to understand the bulk of hadronic cross-sections in QCD*

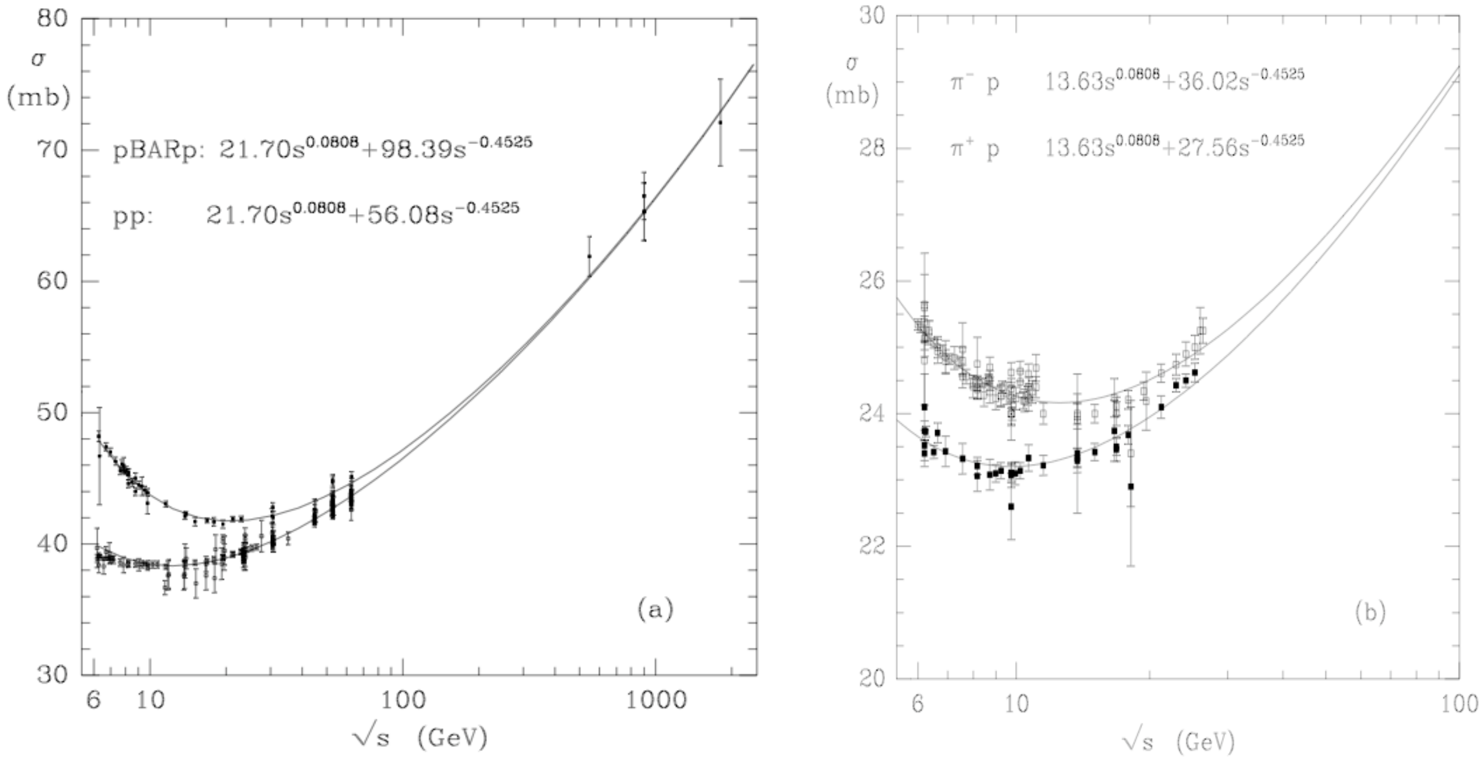
Outstanding phenomenological issues In QCD at high energies (II)

- *Can one compute the behavior of hadronic cross-sections and multiplicities at high energies? Are they intrinsically non-perturbative?*

- *Are hadronic cross-sections universal at high energies? Do the strong interactions have a fixed point at high energies?*

- *Can one compute the initial conditions for the formation of a quark gluon plasma? What are its properties?*

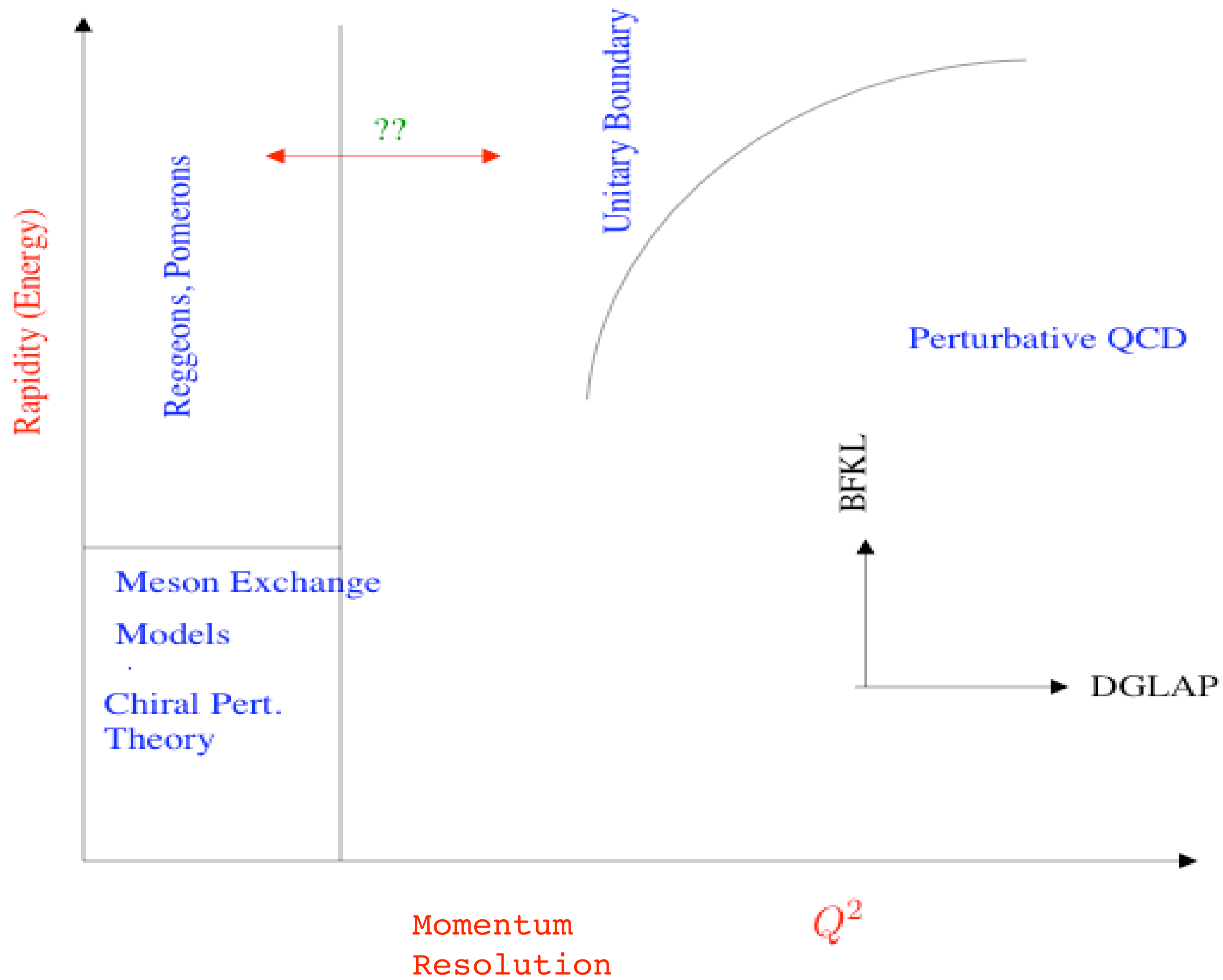
Total Cross-sections



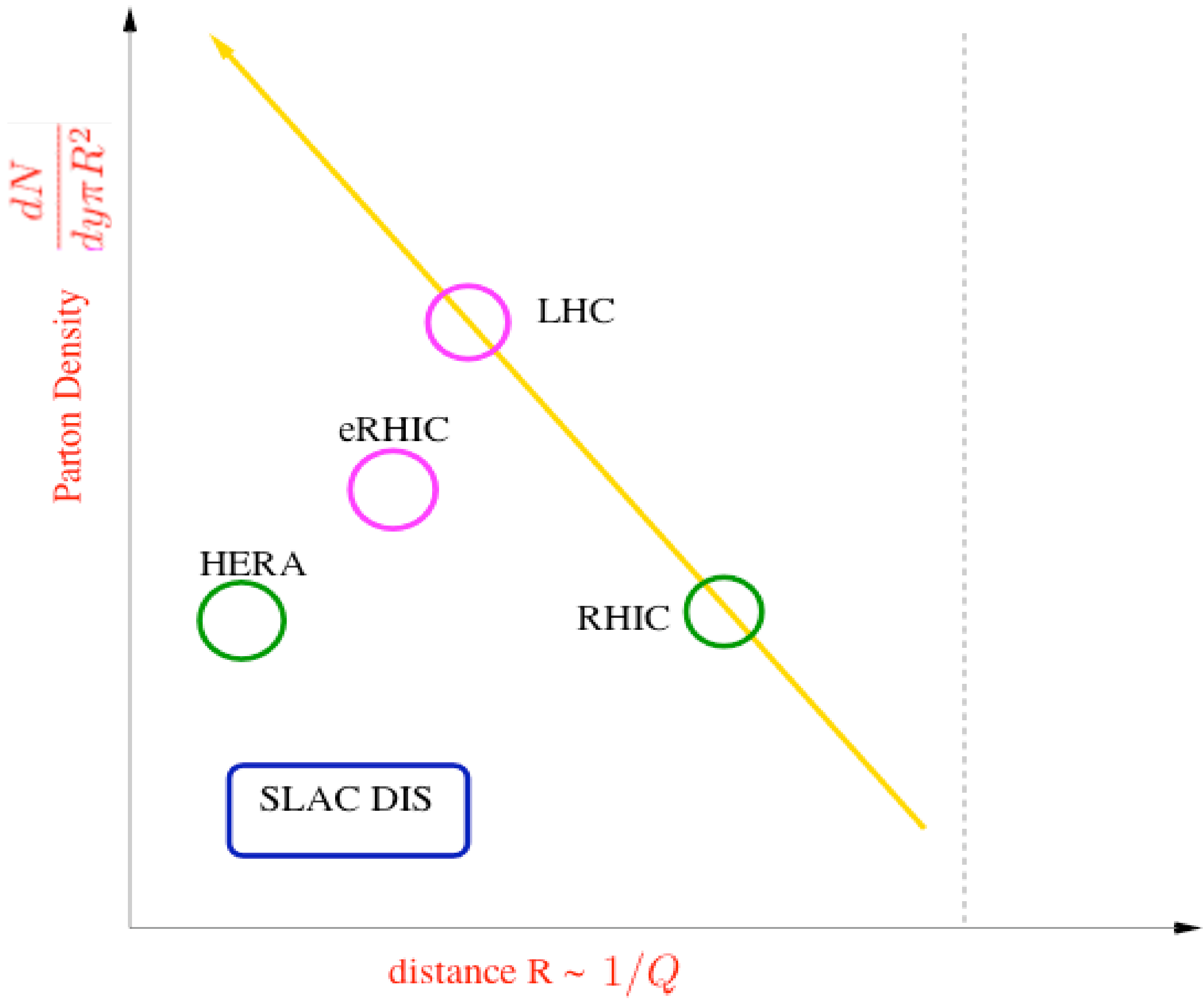
Donnachie & Landshoff

$$\sigma(s) = As^{0.0808} + Bs^{-0.4525}$$

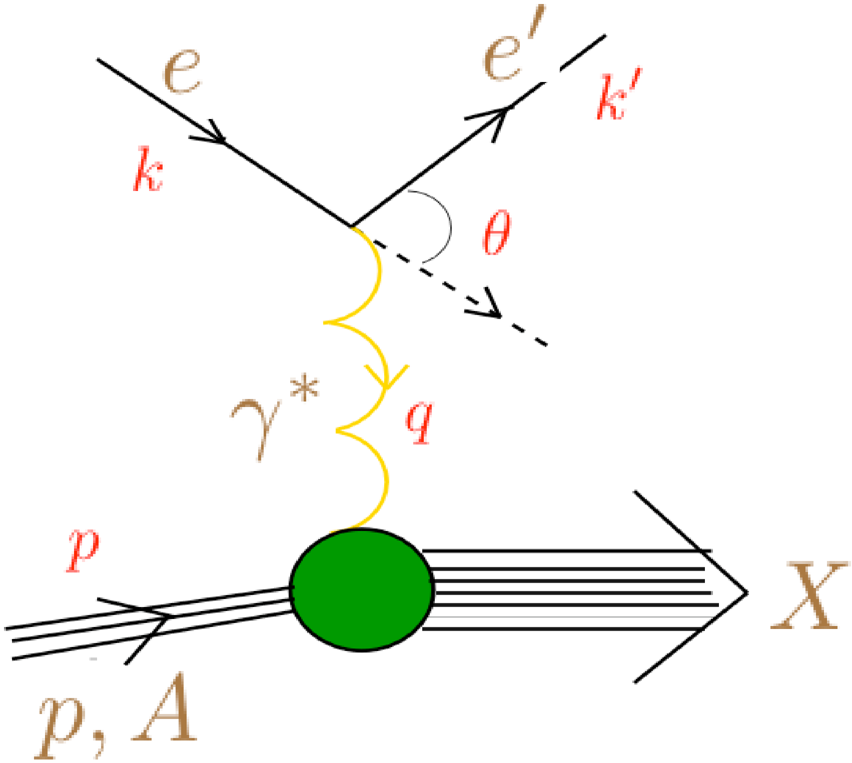
Road map of the strong interactions



Current and future colliders



The DIS Paradigm



Kinematics:

$$s = (p+k)^2$$

$$Q^2 = -q^2 = 4E_e E_{e'} \sin^2\left(\frac{\theta}{2}\right)$$

$$x_{\mathrm{Bj}} = \frac{Q^2}{2M_p(E_e - E_{e'})} \equiv \frac{Q^2}{2p\cdot q}$$

$$y = \left(1 - \frac{E_{e'}}{E_e}\right) = \frac{p\cdot q}{p\cdot k}$$

$$\boxed{x\,y \approx \frac{Q^2}{s}}$$

Cross-section:

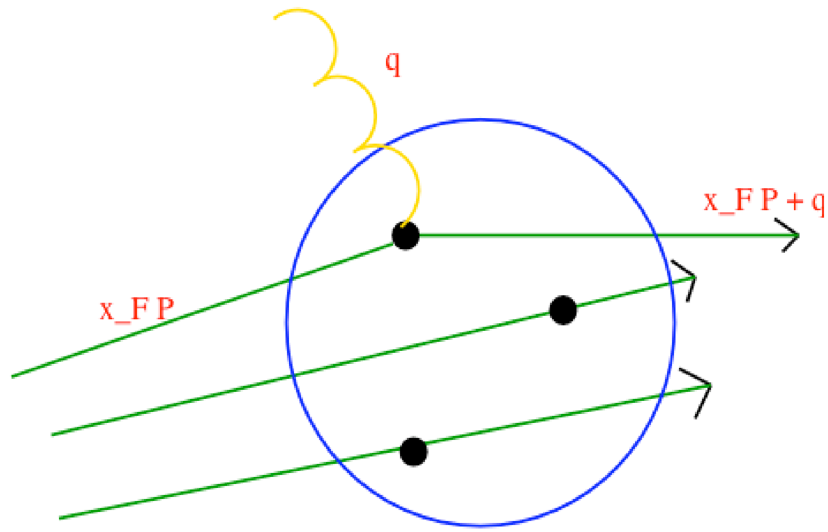
$$\frac{d^2\sigma^{ep\to eX}}{dx\,dQ^2} = \frac{4\pi\alpha_{\mathrm{em.}}}{xQ^4}\left[y^2 x F_1(x,Q^2) + (1-y)F_2(x,Q^2)\right]$$

- In the leading logarithmic approximation (or DIS scheme)

$$F_2(x,Q^2) = \sum_{\substack{q=u,c,t \\ d,s,b}} e_q^2 \left(xq(x,Q^2) + x\bar{q}(x,Q^2) \right)$$

Parton model:

Impulse approximation



$$(x_F p + q)^2 = m_q^2 \approx 0 \rightarrow 2x_F p \cdot q = -q^2$$

$$\boxed{=> x_F \approx \frac{Q^2}{2p \cdot q} \equiv x_{\mathrm{Bj}}}$$

$x_{\mathrm{Bj}} =$ *Fraction of hadron momentum carried by a parton...*

In the parton model, $F_2(x_{\rm Bj}, Q^2) \to F_2(x_{\rm Bj})$

Bjorken scaling - the theory is scale invariant

Parton model $\neq$ QCD

QCD has scaling violations which are only logarithmic in the Bjorken limit:

$$\boxed{Q^2 \to \infty; s \to \infty; x_{\rm Bj} \sim \frac{Q^2}{s} = fixed}$$

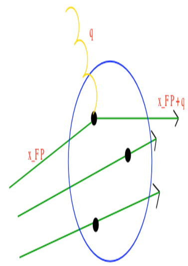
In QCD, for $\boxed{Q^2 >> \Lambda^2_{\rm QCD} (\Lambda_{\rm QCD} \approx 200 {\rm MeV})}$

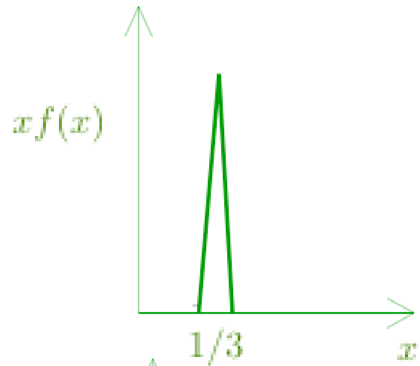
the OPE enables **F_2** *to be expressed in terms of local operators*

The Renormalization Group describes evolution of moments of operators with change of scale

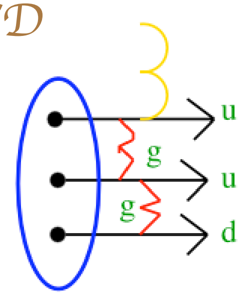
The proton at high energies

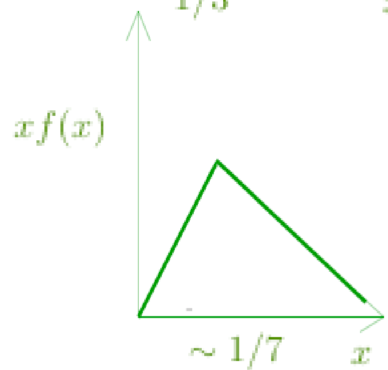
Parton Model





QCD

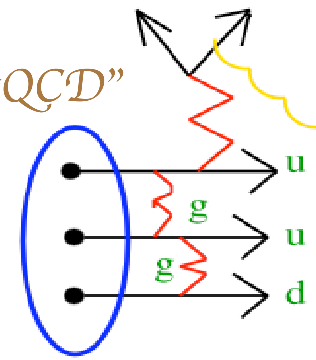


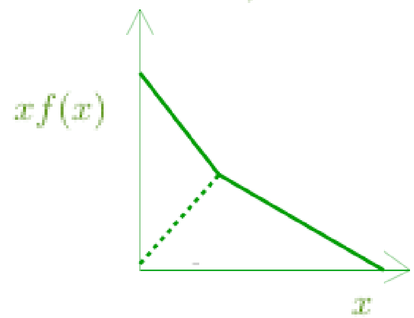


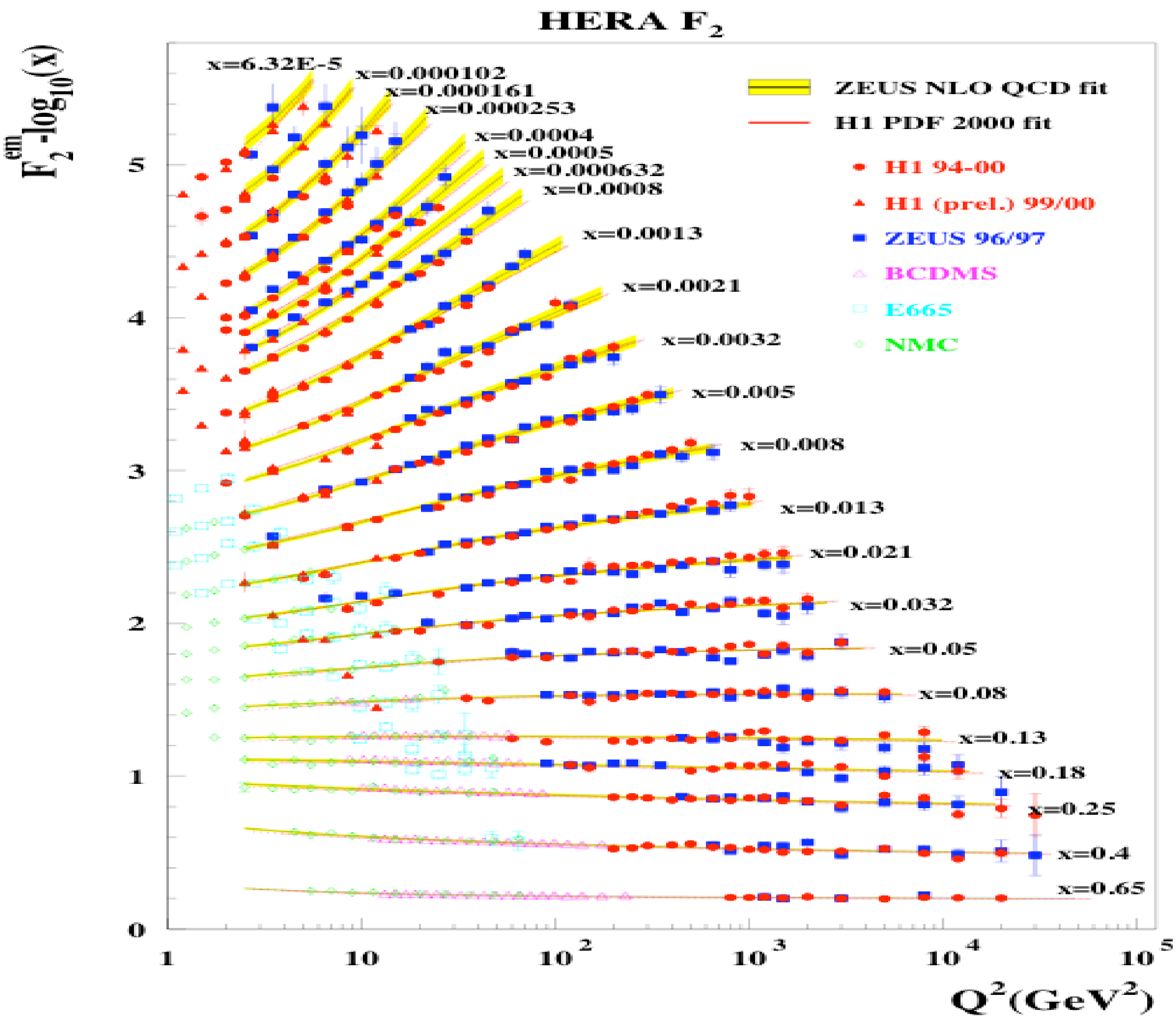
$$\int_0^1 \frac{dx}{x}\,(xq(x) - x\bar{q}(x)) = 3$$

$$\int_0^1 \frac{dx}{x}\,(xq(x) + x\bar{q}(x)) \to \infty$$

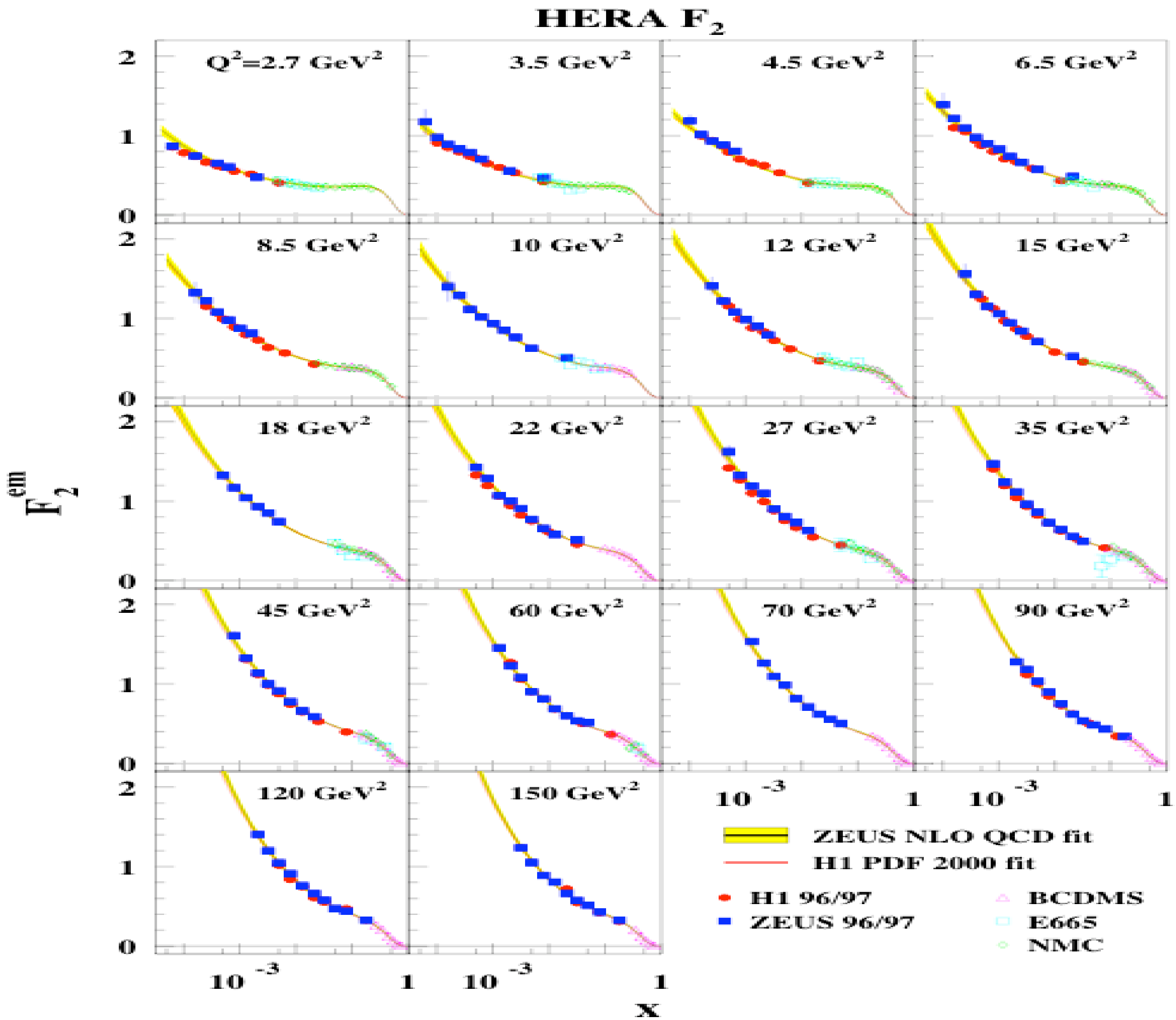
"xQCD"



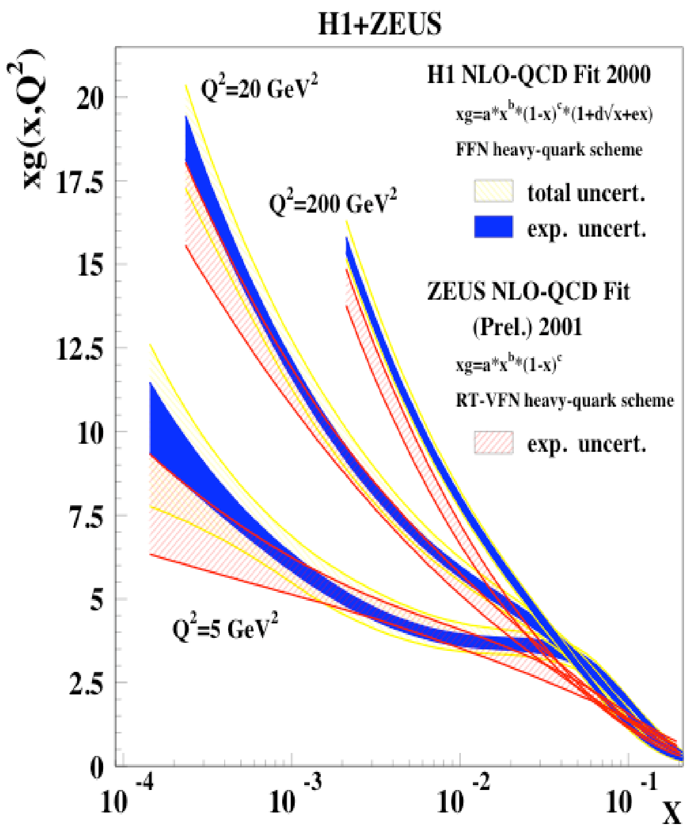




HERA F2
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 94-00
H1 (prel.) 99/00
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
F_2^{em}-$\log_{10}(x)$
$Q^2(GeV^2)$



HERA F2
Q²=2.7 GeV²
3.5 GeV²
4.5 GeV²
6.5 GeV²
8.5 GeV²
10 GeV²
12 GeV²
15 GeV²
18 GeV²
22 GeV²
27 GeV²
35 GeV²
45 GeV²
60 GeV²
70 GeV²
90 GeV²
120 GeV²
150 GeV²
F2 em
x
ZEUS NLO QCD fit
H1 PDF 2000 fit
H1 96/97
ZEUS 96/97
BCDMS
E665
NMC



From the QCD evolution Equations (DGLAP eqns.)

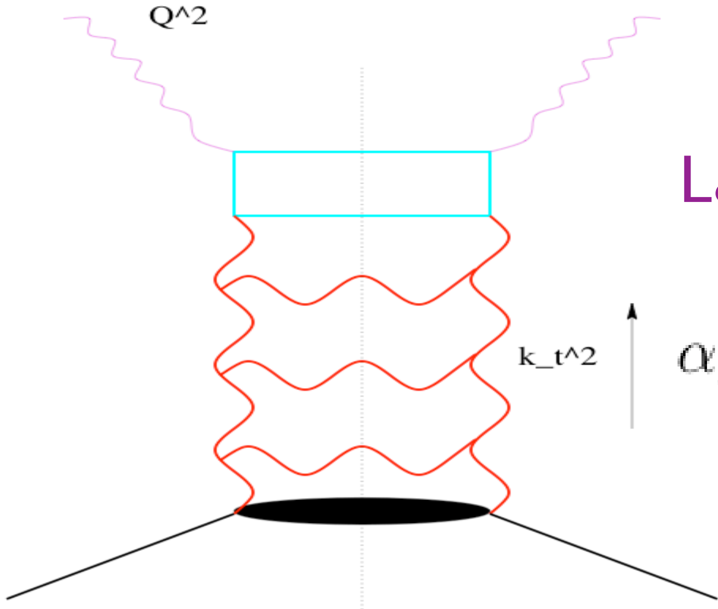
$$xG(x, Q^2) \propto \frac{\partial F_2}{\partial \ln Q^2}$$

QCD evolution equations at small x

a) The DGLAP equation (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

If x_Bj << 1, gluon bremsstrahlung is dominant in QCD evolution:

P_gg > P_qg > P_qq



Large Logs from Bremsstrahlung

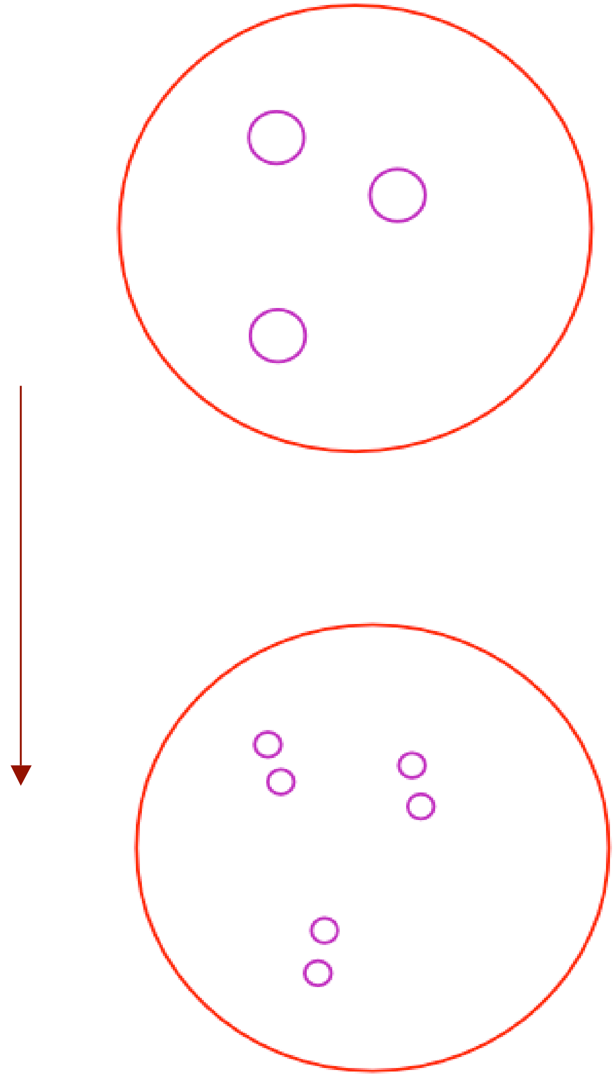
$$\alpha_S \int \frac{dx}{x} \frac{d^2 k_\perp}{k_\perp^2} \longrightarrow \alpha_S^p \ln^m(1/x) \ln^n(Q^2)$$

For $Q^2 >> \Lambda^2_{\mathrm{QCD}}$ and x~1, resum $\alpha_S \ln(Q^2)$
At small x, sum double logs

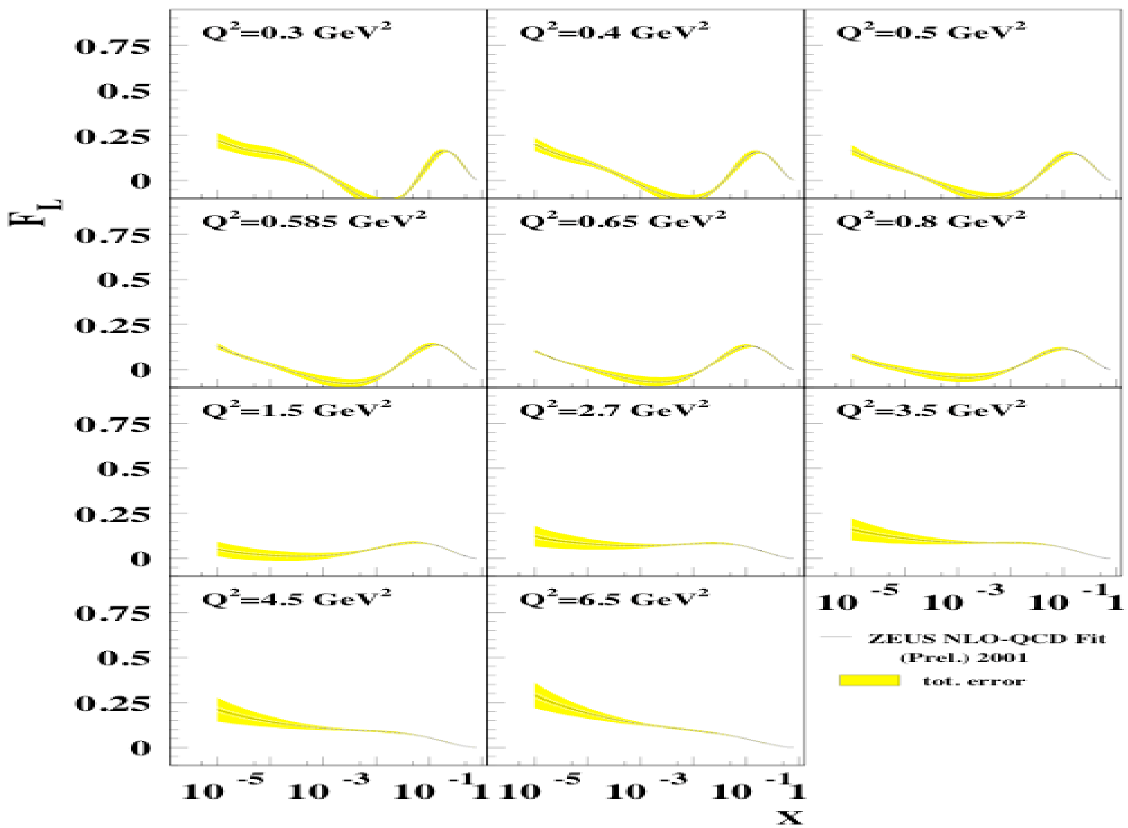
# of gluons grows rapidly...

increasing

Q^2



But… the phase space density decreases
-the proton becomes more dilute



$$F_L \propto \alpha_S x G(x, Q^2)$$

F_L is a positive definite quantity-result hints at problem with leading twist NLO pQCD at low x and moderate Q^2

Thus far, our discussion has focused on the Bjorken limit in QCD:

$$\boxed{Q^2 \rightarrow \infty\,;\, s \rightarrow \infty\,;\, x_{\mathrm{Bj}} \approx \frac{Q^2}{s} = \text{fixed}}$$

Asymptotic freedom, Factorization Theorems, machinery of precision physics in QCD...

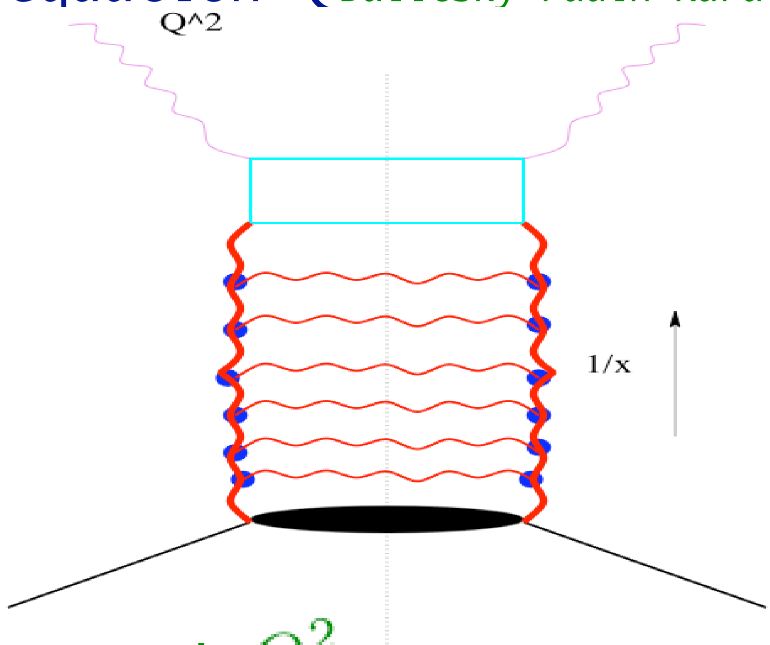
Other interesting limit-is the Regge limit of QCD:

$$\boxed{x_{\mathrm{Bj}} \rightarrow 0\,;\, s \rightarrow \infty\,;\, Q^2(>> \Lambda^2_{\mathrm{QCD}}) = \text{fixed}}$$

Physics of strong fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

QCD evolution equations at small x

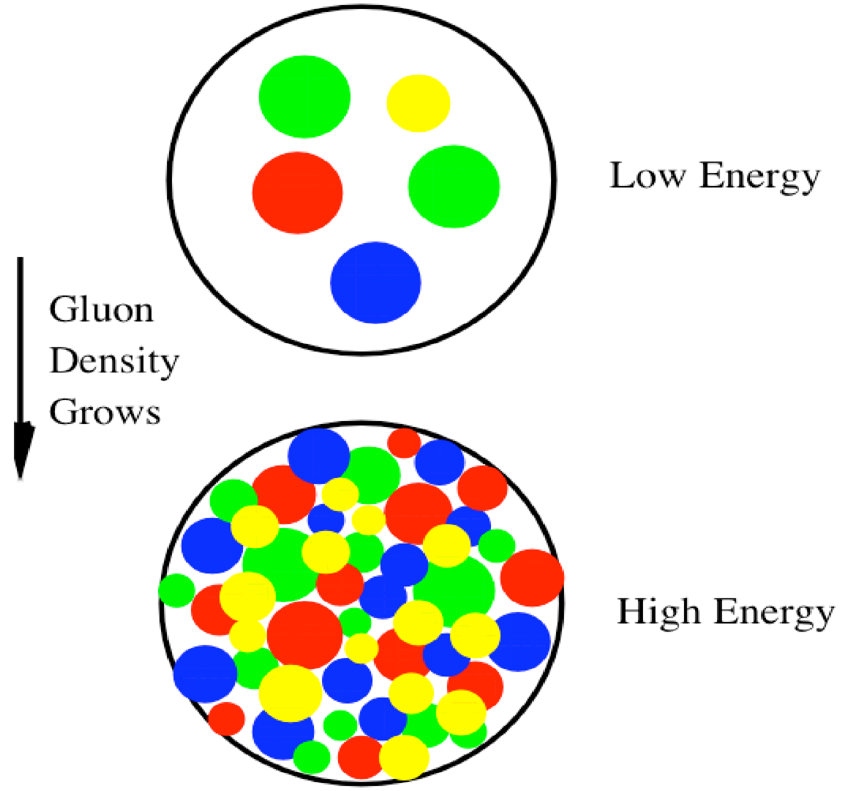
b) The BFKL equation (Balitsky-Fadin-Kuraev-Lipatov)



Evolution in x, not Q^2
Re-sums $\alpha_S \ln(1/x)$

# of gluons grows even more rapidly

$$xG(x, Q^2) \propto \frac{1}{x^{\lambda}}\,;\ \lambda \sim 0.5$$



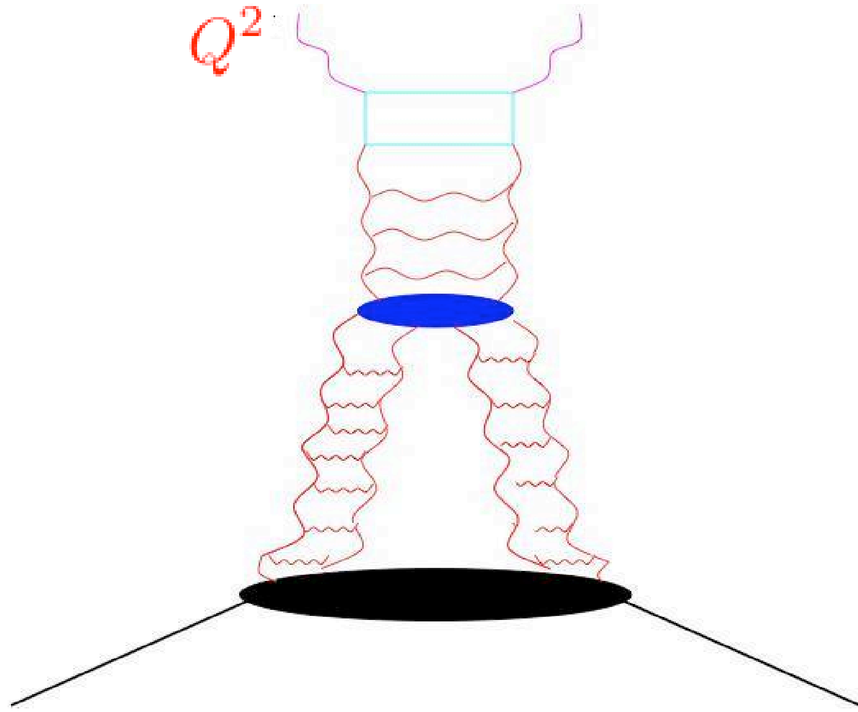
-Large x

-Small x

Phase space density grows rapidly-BFKL evolution breaks down when phase space density f~ 1

Gluon density saturates at $f = \dfrac{1}{\alpha_S}$

Gluon recombination-higher twist effects



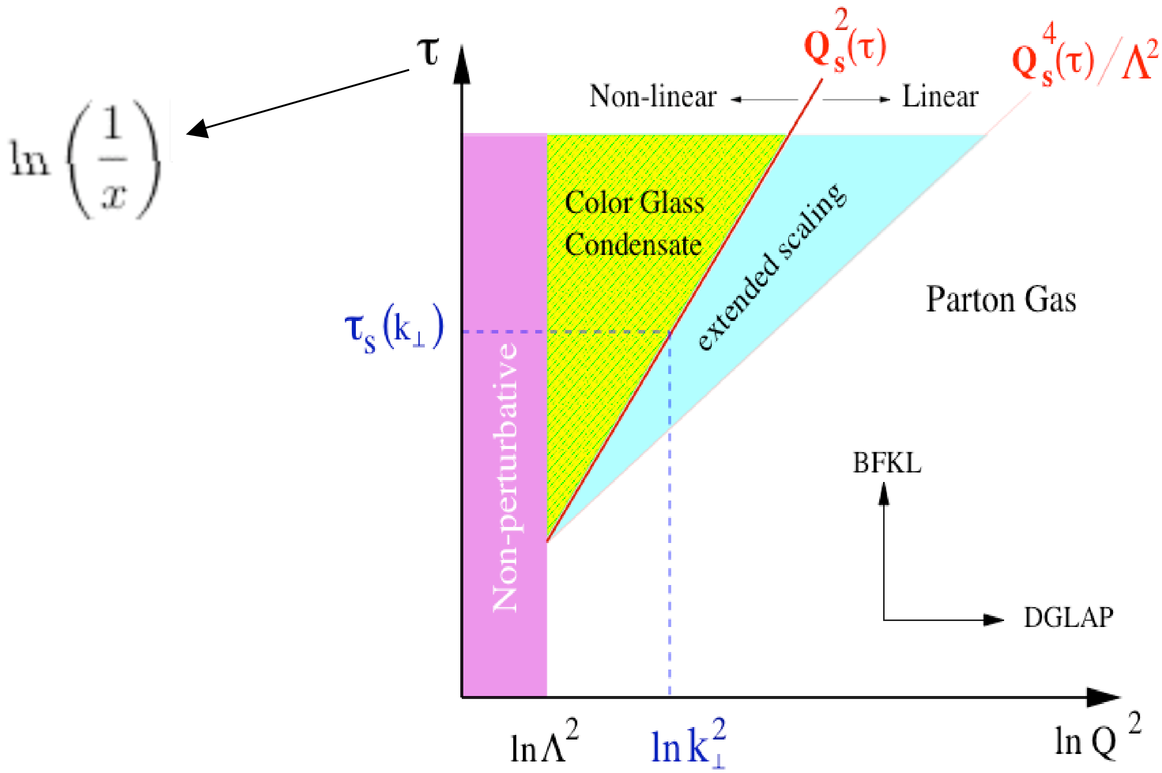
Gribov, Levin, Ryskin
Mueller, Qiu
Blaizot, Mueller

Recombination effects compete with DGLAP Bremsstrahlung effects when

$$\boxed{\alpha_S x G(x, Q^2) \sim R^2 Q^2}$$

Saturation of the gluon density for $Q \equiv Q_s(x)$

Novel regime of QCD evolution at high energies



The Color Glass Condensate

Golec-Biernat & Wusthoff's model

$$\sigma_{T,L}^{\gamma^* p} = \int d^2 r_\perp \int dz |\psi_{T,L}(r_\perp, z, Q^2)|^2 \sigma_{q\bar{q}p}(r_\perp, x)$$

$$\sigma_{q\bar{q}p}(r_\perp, x) = \sigma_0 \left[1 - \exp(-r_\perp^2 Q_s^2(x))\right]$$

with the saturation scale Q_s defined as $Q_s(x)^2 = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$

- Fit to HERA data for $x \leq 0.01$ and $Q^2 \leq 20\ \mathrm{GeV}^2$

(see also Bartels, Golec-Biernat,Kowalski)

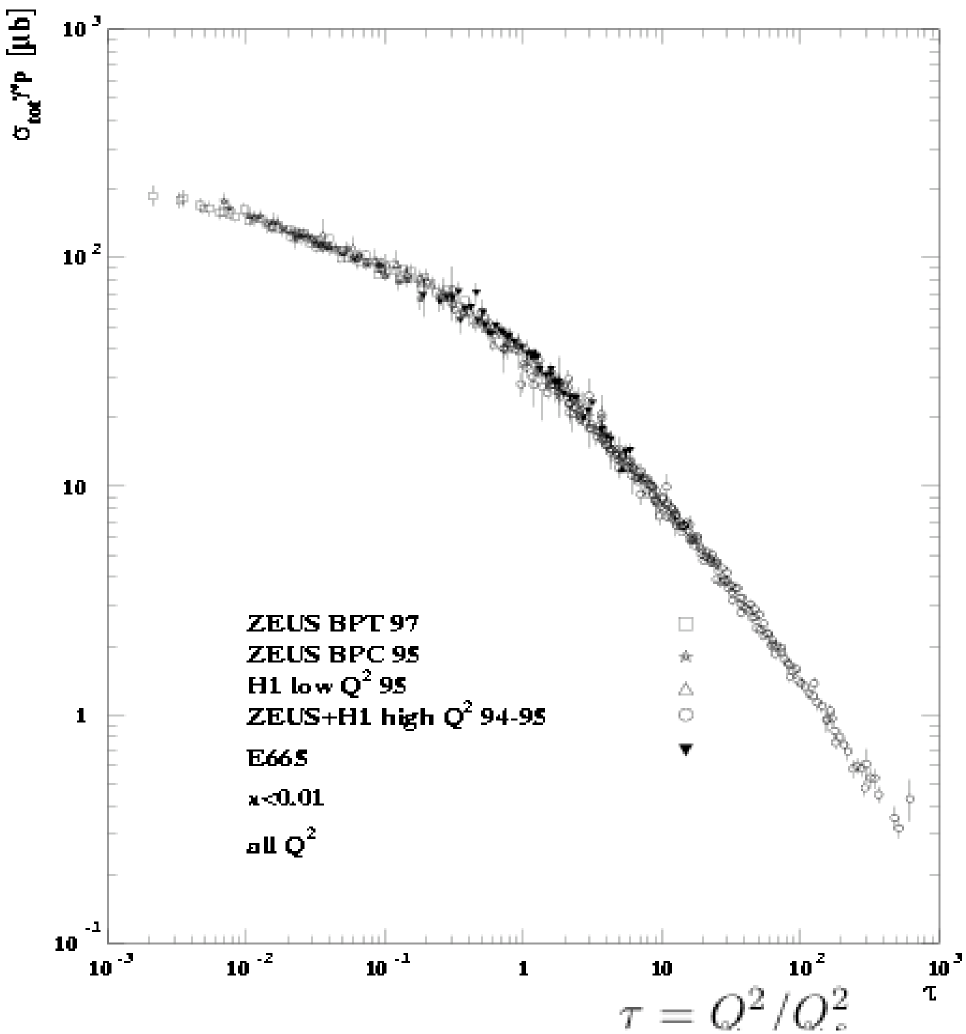
$Q_0 = 1\,\mathrm{GeV}\,;\ \lambda = 0.3\,;\ x_0 = 3 \cdot 10^{-4}\,;\ \sigma = 23\,\mathrm{mb}$

- Also good fit to HERA diffractive data

Golec-Biernat-Wusthoff; Kowalski-Teaney; Mueller,Stasto,Munier; Gonsalves, Machado

Geometrical scaling at HERA

(Golec-Biernat,Kwiecinski,Stasto)



Scaling seen for all x ‹ 0.01 and $0.045 < Q^2 < 450\,\mathrm{GeV}^2$