Phenomenology at low Q^2

Lecture II

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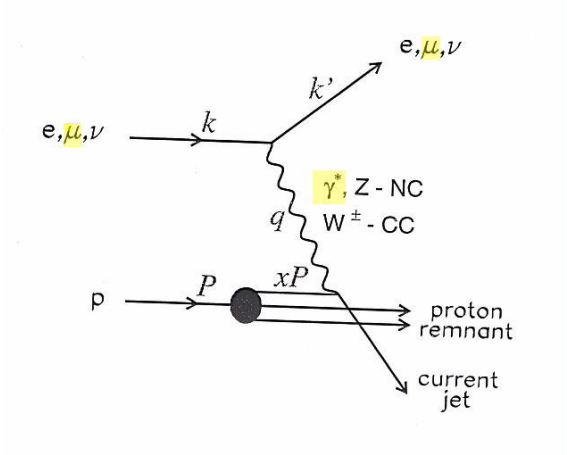
Outline

1. Important practical example: radiative corrections
2. What do the data show around $Q^2 = 1$ GeV^2?
3. Parametrizations of F_2^p in the low Q^2, low x region
   - Goal
   - DL
   - GRV
   - JKBB
   - Martin-Ryskin-Stasto
   - ALLM97
   - ZEUS Regge fit
4. Summary

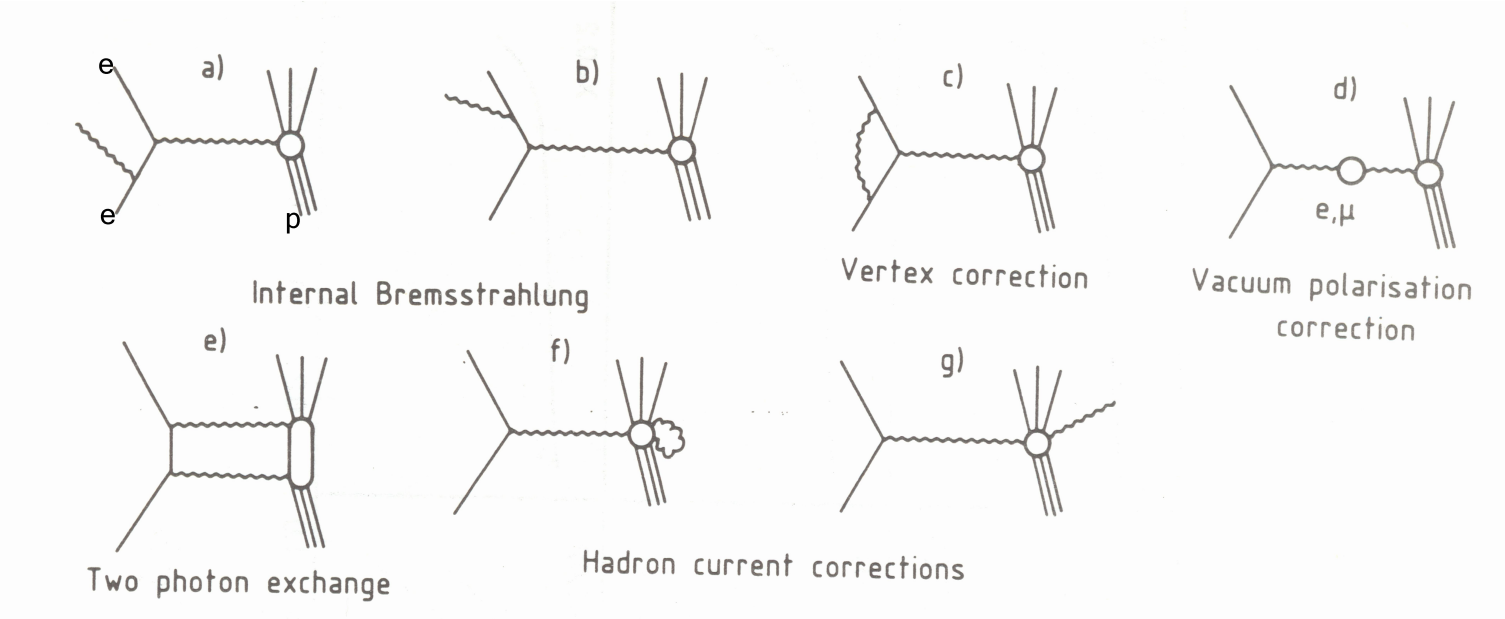
Outline

Important practical example: radiative corrections

Extraction of the Born events from measurements demands removing the QED background
$\longrightarrow$ radiative corrections procedure.



The following processes contribute up to α^3 to the Born cross section:

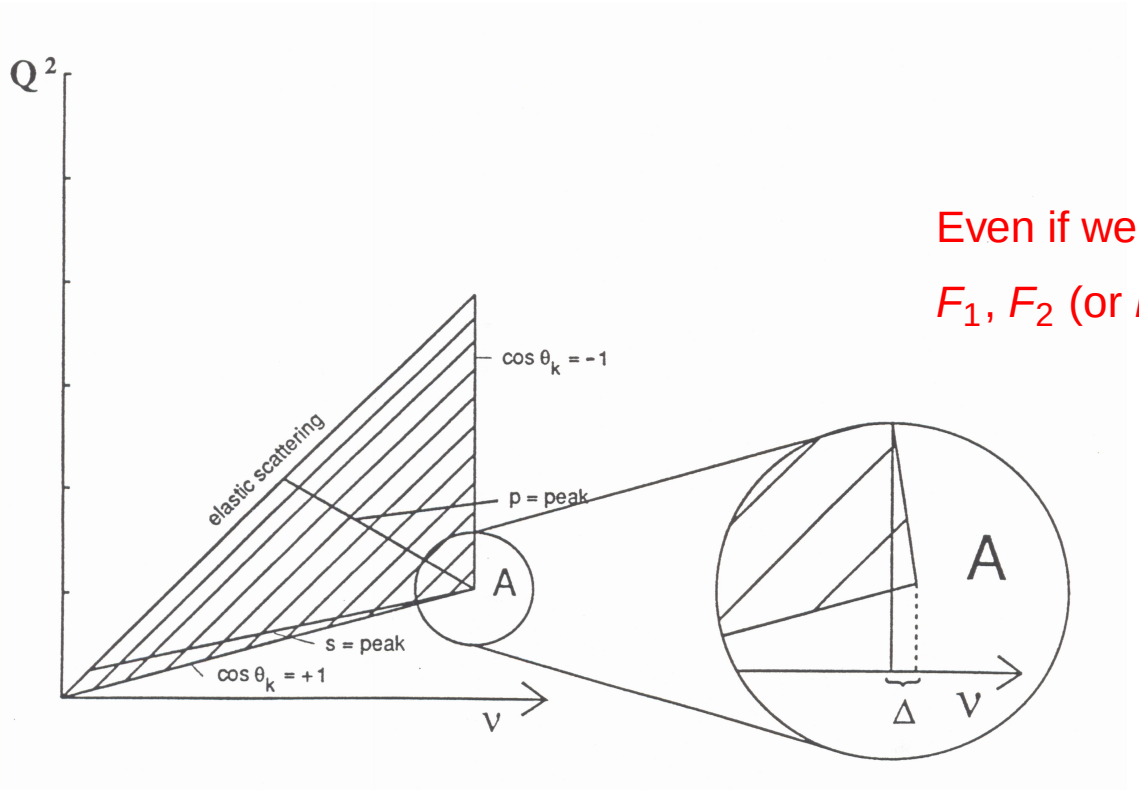


Practical example: radiative corrections...cont'd

$$\sigma_{obs} = v\sigma_{1\gamma} + \sigma_{tail} = v\sigma_{1\gamma} + \sigma_{inel} + \sigma_{el} + \sigma_{qel} \quad (1)$$

v – virtual corrections + many soft photon emission. Same for the polarised cross sections.

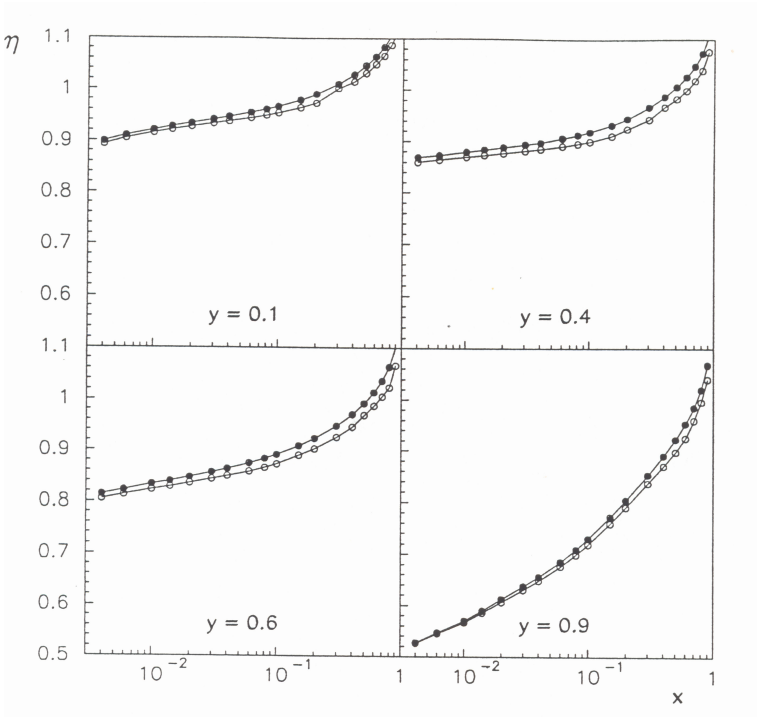
Range of kinematical variables from which the radiative tails contribute to the cross section measured at the point $A(Q^2, \nu)$; parallel lines: W = const (Badelek, Bardin, Kurek, Scholz, Z. Phys. C66 (1995) 591).



Even if we measure at DIS, information on F_1, F_2 (or R, F_2) needed down to Q^2 =0!

Practical example: radiative corrections...cont'd

Define the radiative correction: $\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{meas}}$. Example of its magnitude, calculated for μp at 280 GeV in two approaches: FERRAD35 (open symbols) and TERAD86 (closed symbols):



Important! $\eta(x,y)$ may be both $>$ and $<$ than 1.
Strongest departure from 1 at low x and high$y$
(i.e. low x and low Q^2) due to (quasi)elastic tails.
$\longrightarrow$ Hadron method removing the elastic tail!

Hadron method used by SMC for x<0.02.
Monte Carlo: few percent DIS lost,
otherwise no biases (at high x too many DIS lost).

COMPASS: hadrons in the trigger

Practical example: radiative corrections...cont'd

Example of systematic errors on F_2^p due to QED RC for μp at 90 GeV

(NMC, Nucl. Phys. B483 (1997) 3).

x	Q^2 (GeV^2)	y	$d^2\sigma_p^{meas}/dxdQ^2$ ($\mathrm{b \cdot GeV^{-2}}$)	E [%]	E' [%]	AC [%]	RC [%]	RE [%]	R	$F_2^p \pm \Delta F_2^{stat} \pm \Delta F_2^{syst}$
0.0078	0.80	0.635	8.600e-06	0.5	-0.1	2.3	1.7	0.3	0.337	0.2720 ± 0.0034 ± 0.0080
0.0092	1.09	0.709	4.125e-06	0.4	0.1	2.1	2.1	0.5	0.337	0.2943 ± 0.0082 ± 0.0091
0.0120	0.90	0.462	5.290e-06	0.7	-0.3	1.6	0.8	0.2	0.246	0.2763 ± 0.0064 ± 0.0054
0.0125	1.22	0.596	2.707e-06	0.6	-0.1	1.6	1.4	0.3	0.246	0.3022 ± 0.0037 ± 0.0067
0.0139	1.62	0.703	1.387e-06	0.4	0.0	1.5	1.8	0.5	0.246	0.3204 ± 0.0101 ± 0.0078
0.0173	1.27	0.448	2.013e-06	0.7	-0.2	1.1	0.7	0.2	0.190	0.2988 ± 0.0048 ± 0.0045
0.0176	1.72	0.593	1.078e-06	0.6	-0.1	1.2	1.2	0.3	0.190	0.3316 ± 0.0054 ± 0.0059
0.0185	2.16	0.703	6.251e-07	0.5	0.0	1.1	1.5	0.4	0.190	0.3354 ± 0.0118 ± 0.0065
0.0246	1.28	0.320	1.601e-06	1.0	-0.4	0.4	0.6	0.1	0.099	0.3053 ± 0.0056 ± 0.0039
0.0247	1.75	0.437	8.198e-07	0.7	-0.2	0.7	0.6	0.2	0.099	0.3268 ± 0.0044 ± 0.0038
0.0254	2.35	0.565	4.349e-07	0.6	-0.1	0.8	0.8	0.3	0.099	0.3504 ± 0.0046 ± 0.0046
0.0276	3.24	0.707	1.950e-07	0.5	0.1	1.0	1.7	0.5	0.099	0.3415 ± 0.0195 ± 0.0070
0.0348	1.30	0.231	1.182e-06	1.2	-0.7	0.3	0.2	0.1	0.108	0.3082 ± 0.0085 ± 0.0045
0.0348	1.76	0.309	6.482e-07	1.0	-0.4	0.2	0.2	0.1	0.108	0.3306 ± 0.0059 ± 0.0037
0.0349	2.44	0.427	3.239e-07	0.8	-0.2	0.4	0.4	0.2	0.108	0.3541 ± 0.0046 ± 0.0035
0.0355	3.34	0.570	1.622e-07	0.6	0.0	0.6	1.1	0.3	0.108	0.3732 ± 0.0092 ± 0.0053
0.0364	4.26	0.705	9.547e-08	0.5	0.1	0.8	2.4	0.4	0.108	0.3878 ± 0.0317 ± 0.0101

What do the data show around $Q^2 = 1$ GeV^2?

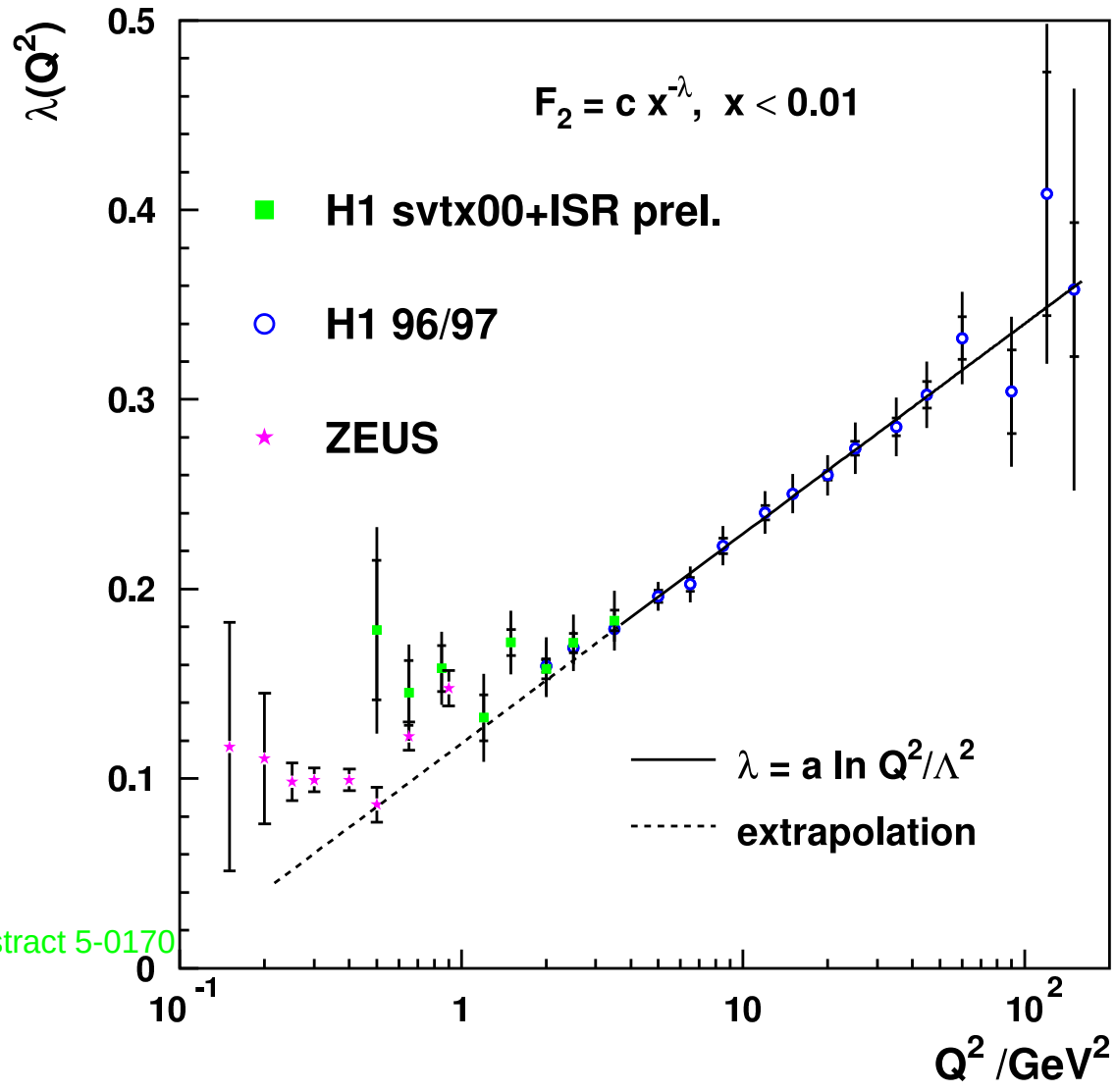
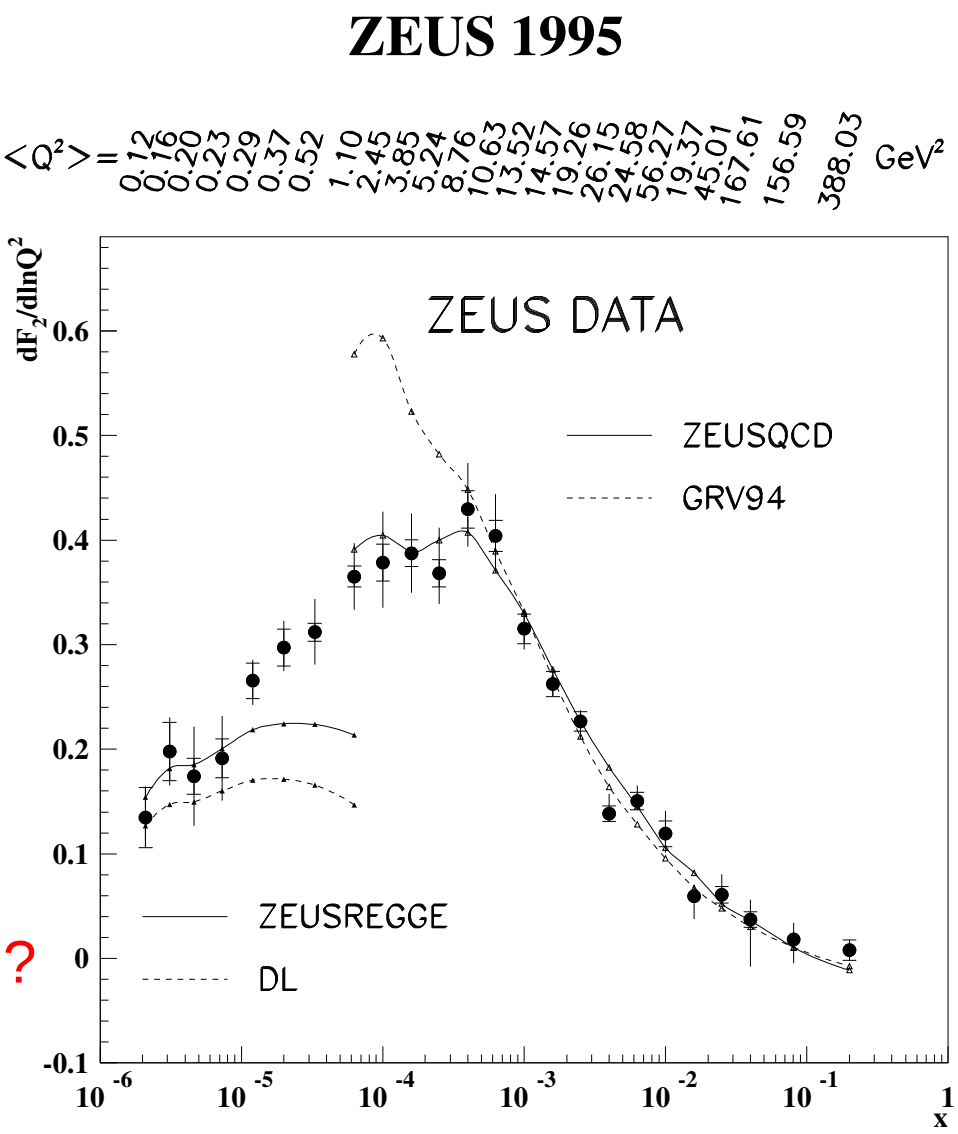


Figure from ICHEP2004, abstract 5-0170

What do the data show around Q^2 = 1 GeV^2?...cont'd



Gluons softer x ↘ ?

Figure from hep-ex/9809005

Figure 9: $dF_2/d\ln Q^2$ as a function of x calculated by fitting ZEUS F_2 data in bins of x to the form $a + b\ln Q^2$. The inner error bar shows the statistical error and the

What do the data show around Q^2 = 1 GeV^2?...cont'd

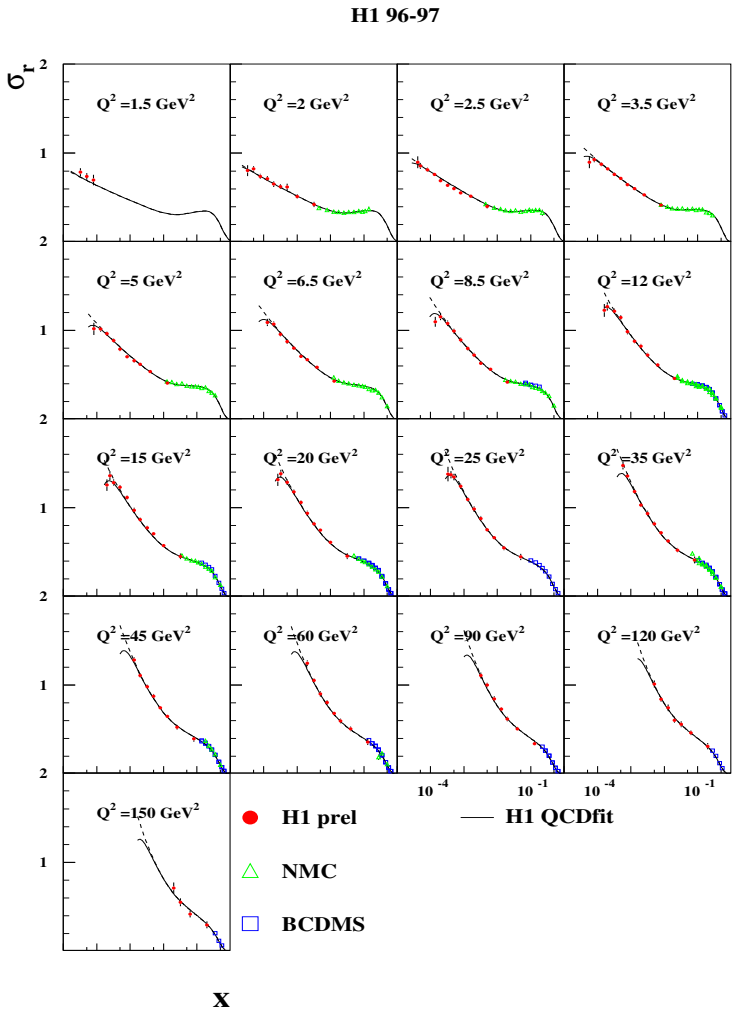


Figure from hep-ex/0008069

Figure 3: *The preliminary H1 data on the reduced cross section (proportional to F_2) from the 1996-97 data-taking period. Also shown are points from the fixed-target experiments NMC (triangles) and BDCMS (squares). The solid curve shows the NLO QCD fit carried out by H1, while the dotted curve visible at the lowest x corresponds to the expectation for $F_L = 0$, as discussed in the text.*

What do the data show around Q^2 = 1 GeV^2?...cont'd

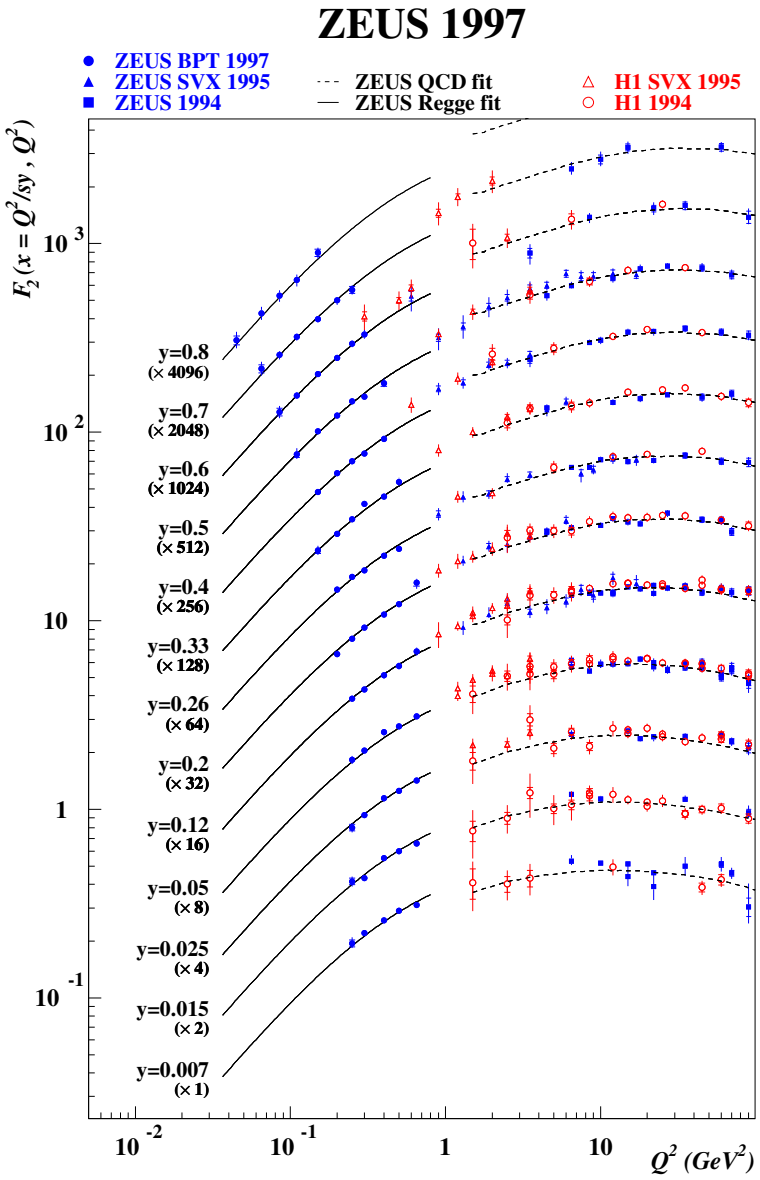


Figure from: ZEUS, Eur. Phys. J. C7 (1999)609

Outline

F_2^p in the low Q^2, low x region

Goal

Excellent compendium of nucleon structure functions: Cooper-Sarkar, Devenish, De Roeck, Int. J. Mod. Phys. A13 (1998) 3385

Measurements of F_2 and R are described by different parametrizations, depending on underlying physics ideas.
They are

- either physics motivated fits or models of dynamic origin and
- have to have a proper asymptotic behaviour: at $Q^2 \to 0$ fulfilling the conditions

$$F_2 = O(Q^2), \qquad \frac{F_1}{M} + \frac{F_2}{M}\frac{pq}{q^2} = O(Q^2).$$

  given in Lecture I, eq.(3) and at $Q^2 \to \infty$ joining the QCD improved parton model expressions, valid in the DIS region.

Parametrizations are usually valid in limited kinematic intervals.
Below follows the review of most important physics ideas at low Q^2 and the resulting parametrizations.

Outline

F_2^p in the low Q^2, low x region

DL (see e.g. Donnachie, Landshoff, Z. Phys. C61 (1994) 139)

- Based of the Regge model which describes well the energy dependence of the photoproduction cross section $\Longrightarrow$ low Q^2 data as well ?
- $$F_2 = (F_2)_P + (F_2)_R$$

  each of the above terms (soft pomeron, reggeon) is

  $$\left(\frac{Q^2}{Q^2 + M_{R,P}^2}\right)^{\alpha_{R,P}} x^{(1-\alpha_{R,P})}(1-x)^{\beta_{R,P}}$$
- By construction the W dependence of $\sigma^{\gamma^* p}$ is weak (as for hadron-hadron).

Result: low W data well reproduced but not the high W (i.e. HERA) data; for $Q^2 \gtrsim 0.4$ GeV^2 the DL model has too weak energy dependence.

F_2^p in the low Q^2, low x region

DL...cont'd

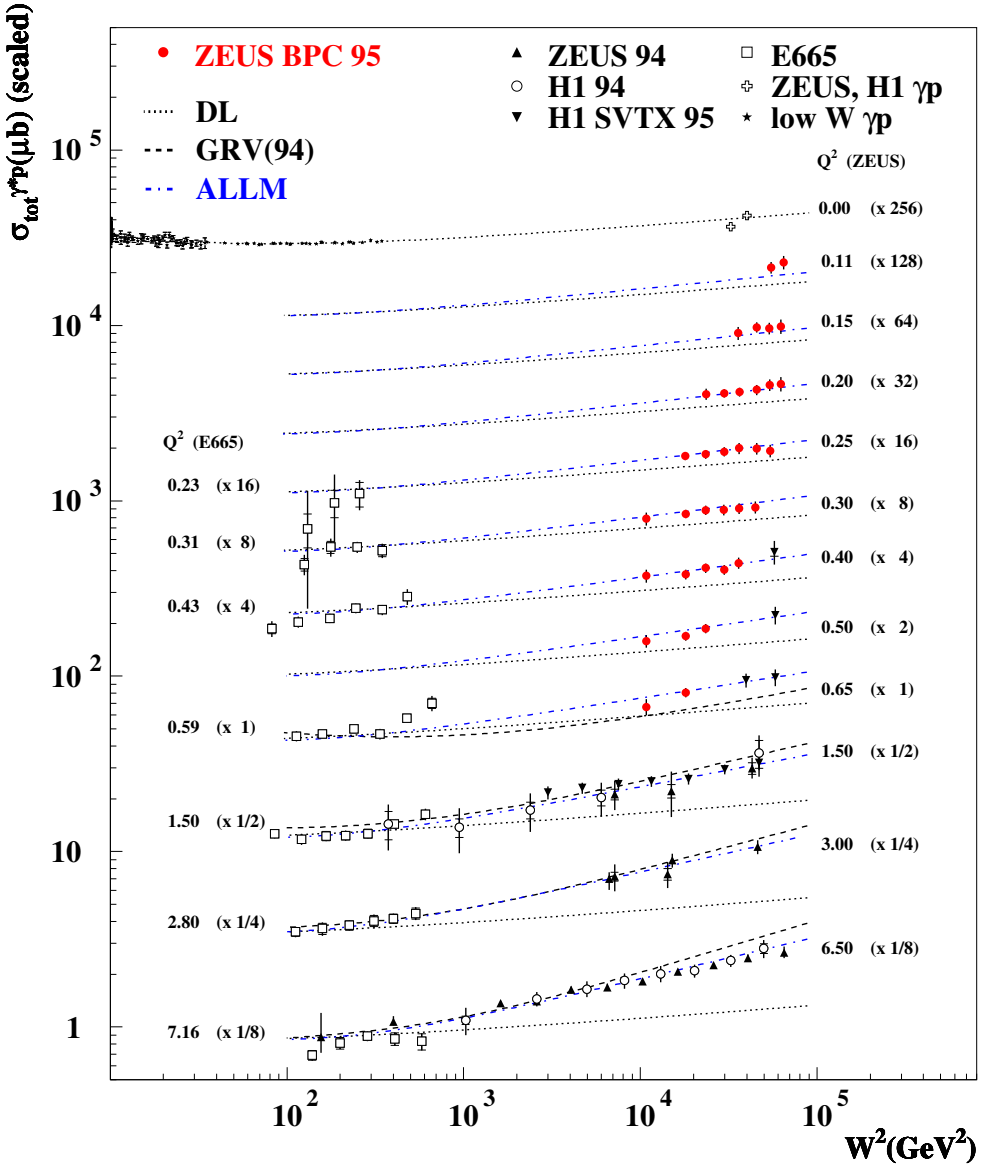


Figure from Abramowicz, Levy, hep-ph/9712415

Outline

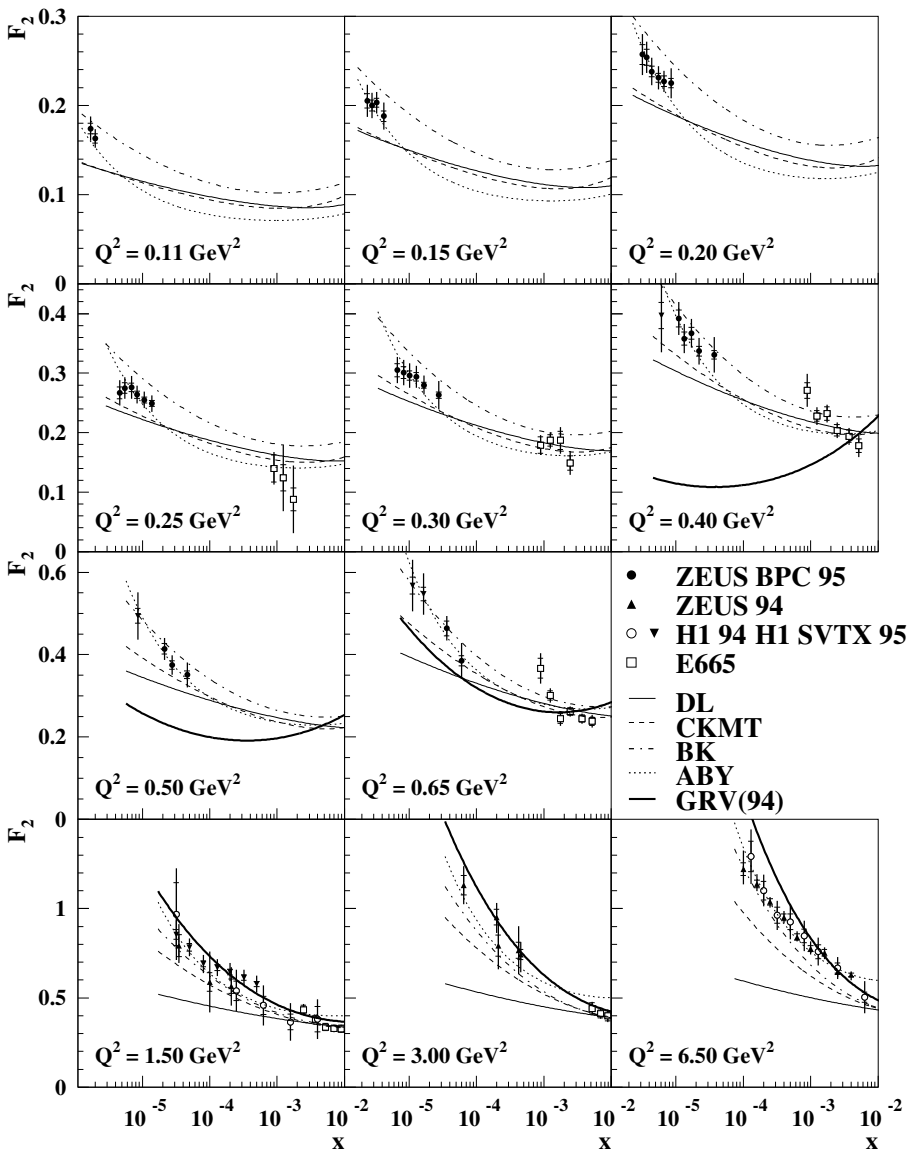
F_2^p in the low Q^2, low x region

GRV (Glück, Reya, Vogt, Eur. Phys.J. C5 (1998) 461)

- One of the global fits of parton distributions.
- Contrary to other approaches (MRS, CTEQ), results do not strongly depend on the (nonperturbative) input parametrization at Q_0^2.
- Original idea: at low scale $Q^2 = \mu^2$ the nucleon contains only valence quarks.
- Other partons (gluons, sea) are generated dynamically through DGLAP as $Q^2 \nearrow$ e.g. in processes: $q \to qg, \;\; g \to q\bar{q}$.
- However data required both valence gluons and valence sea, of valence-like (small at low x) input distributions $\Longrightarrow$ a structure of constituent quarks ?
- Valence distributions taken at Q_0^2 =4 GeV^2 and evolved backwards to μ^2. Valence-like sea and gluons at μ^2 parametrized in a simple way.
- Scale μ^2 set so that gluons there are as hard as valence (e.g. u_v) distribution; observe that the evolution softens gluons.
- Finally normal evolution to high Q^2 and fit of the distribution parameters.
- Currently μ^2 = 0.3 GeV^2; calculations are perturbatively stable for observables like F_2. However they are applicable only to the leading twists (LT) of QCD while at low Q^2 a contribution of HT may be nonnegligible. Thus the calculations describe the reality only for $Q^2 \gtrsim$ 0.6 GeV^2. Indeed they agree with data for $Q^2 \gtrsim$ 0.8 GeV^2.

F_2^p in the low Q^2, low x region

GRV...cont'd



Outline

Parametrizations of F_2 in the low Q^2, low x region

JKBB (Kwieciński, Badełek, Z. Phys. C43 (1989) 43; Phys. Lett. B295 (1992) 263)

The starting point is the Generalised Vector Meson Dominance (GVMD) representation of the structure function $F_2(x, Q^2)$:

$$\begin{aligned} F_2[x = Q^2/(s + Q^2 - M^2), Q^2] &= \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2} + Q^2 \int_{Q_0^2}^{\infty} dQ'^2 \frac{\Phi(Q'^2, s)}{(Q'^2 + Q^2)^2} \\ &\equiv F_2^{(v)}(x, Q^2) + F_2^{(p)}(x, Q^2) \end{aligned} \tag{2}$$

The function $\Phi(Q^2, s)$ is expressed as follows:

$$\Phi(Q'^2, s) = -\frac{1}{\pi} Im \int^{-Q'^2} \frac{dQ''^2}{Q''^2} F_2^{AS}(x', Q''^2) \tag{3}$$

- Asymptotic structure function $F_2^{AS}(x, Q^2)$ assumed to be given.
- By construction, $F_2(x, Q^2) \to F_2^{AS}(x, Q^2)$ for large Q^2.
- The first term in (2) corresponds to the low mass vector meson dominance.
- Contribution of vector mesons heavier then Q_0 is included in the integral in (2).
- This integral can be looked upon as the extrapolation of the (QCD improved) parton model for arbitrary Q^2 (including Q^2 =0).
- The representation (2) is written for fixed s and is expected to be valid at $s \gg Q^2$, i.e. at low x but for arbitrary Q^2 – and above the resonances.

F_2^p in the low Q^2, low x region...cont'd

JKBB...cont'd

- Choosing the parameter $Q_0^2 > (M_v^2)_{max}$ where $(M_v)_{max}$ is the mass of the heaviest vector meson included in the sum one explicitly avoids double counting when adding two separate contributions to F_2.
- Q_0 should be smaller than the mass of the lightest vector meson not included in the sum.
- Representation (2) for the partonic part $F_2^{(p)}(x, Q^2)$ may be simplified as follows:

$$F_2^{(p)}(x, Q^2) = \frac{Q^2}{(Q^2 + Q_0^2)} F_2^{AS}(\bar{x}, Q^2 + Q_0^2) \tag{4}$$

where

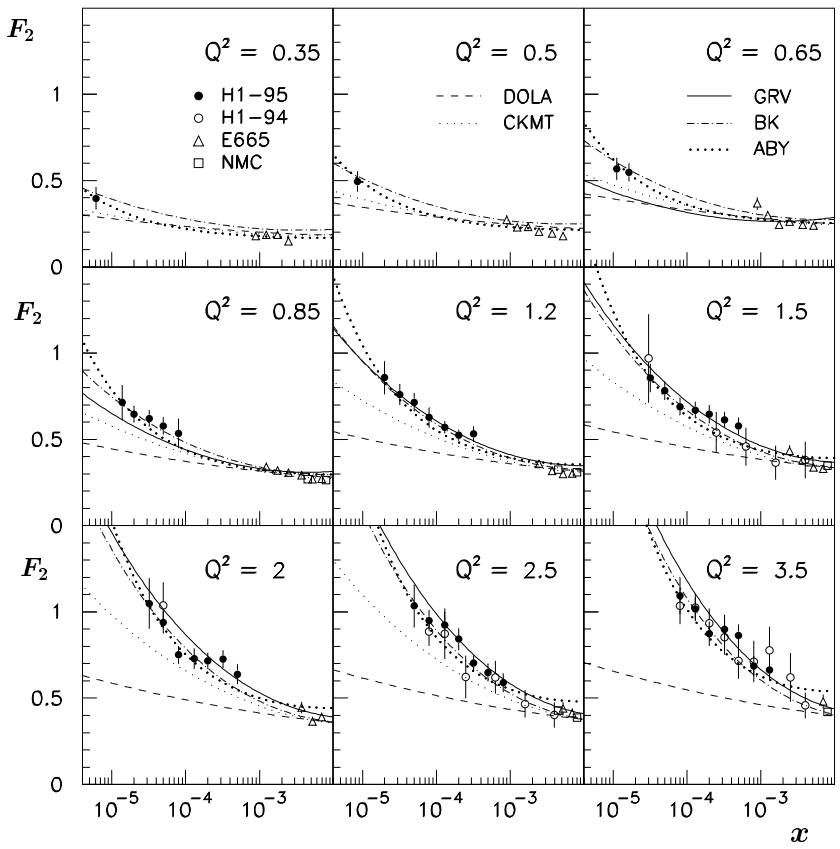
$$\bar{x} = \frac{Q^2 + Q_0^2}{s + Q^2 - M^2 + Q_0^2} \equiv \frac{Q^2 + Q_0^2}{2M\nu + Q_0^2} \tag{5}$$

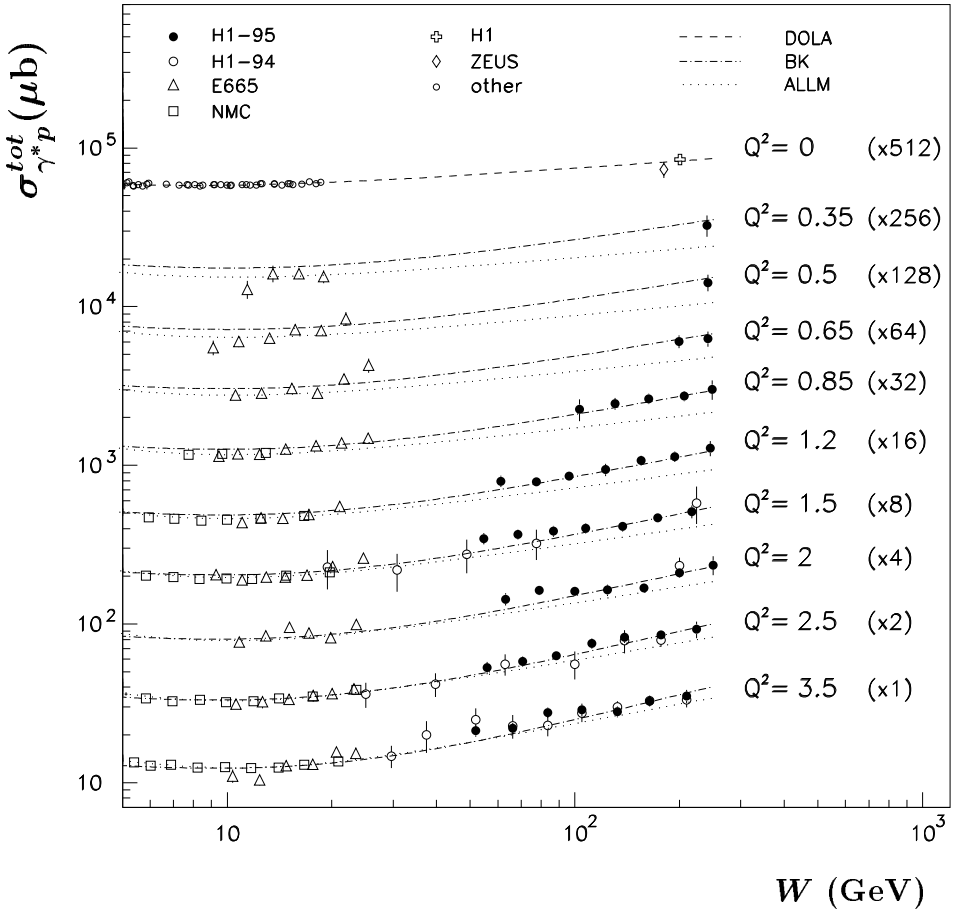
- Simplified parametrization (4) connecting $F_2^{(p)}(x, Q^2)$ to F_2^{AS} by an appropriate change of the arguments posseses all the main properties of the second term in (2).

Apart from Q_0^2, constrained by physical requirements, the representation (2) does not contain any other free parameters except those which are implicitly present in F_2^{AS}.

F_2^p in the low Q^2, low x region...cont'd

JKBB...cont'd

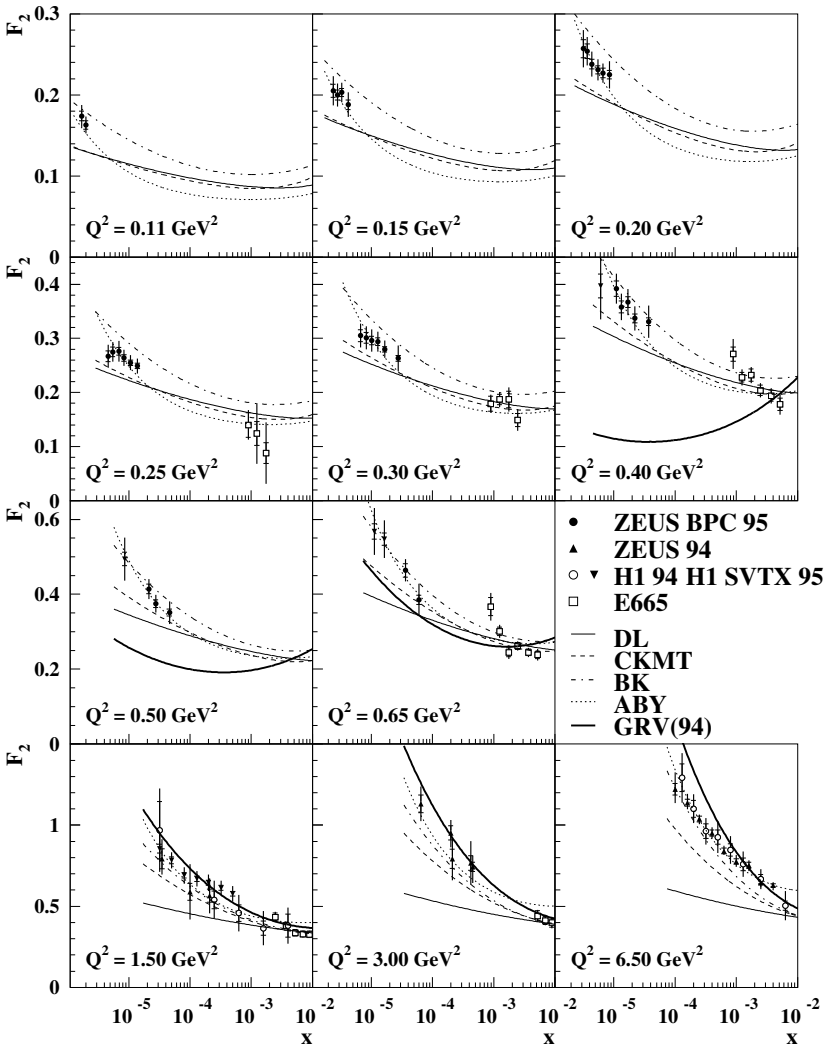


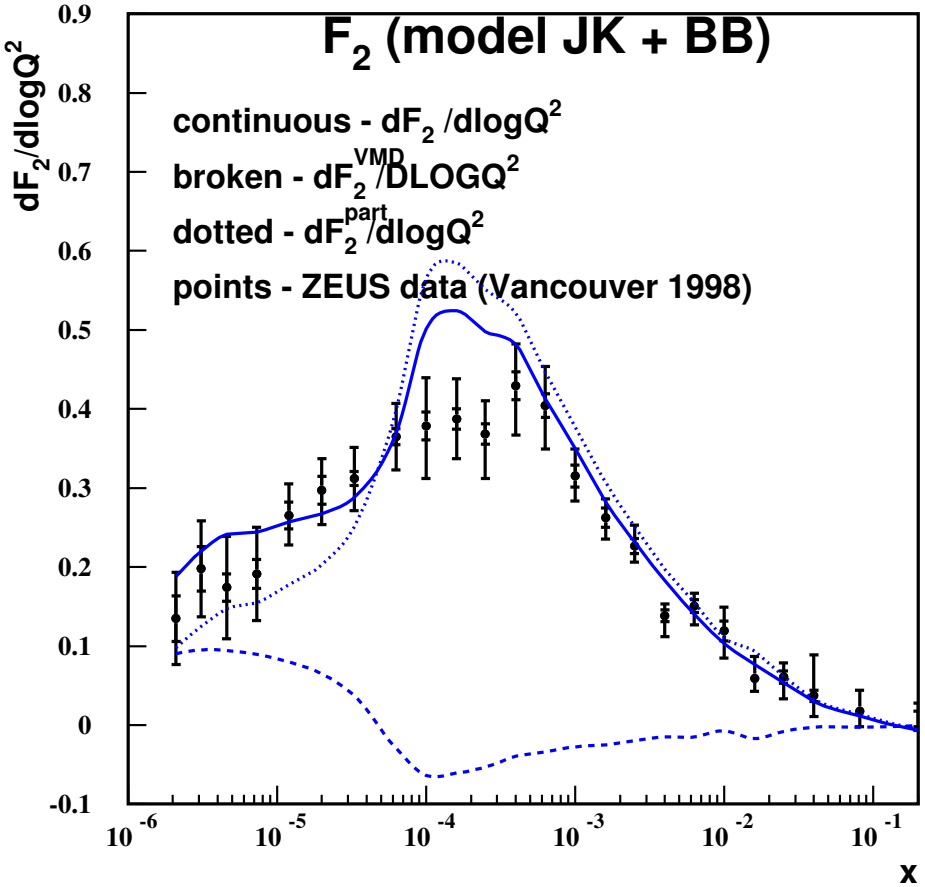


H1 Collaboration, DESY 97-042

F_2^p in the low Q^2, low x region...cont'd

JKBB...cont'd





Outline

F_2^p in the low Q^2, low x region

Martin-Ryskin-Stasto (Martin, Ryskin, Stasto, Eur. Phys. J. C7 (1999) 643)

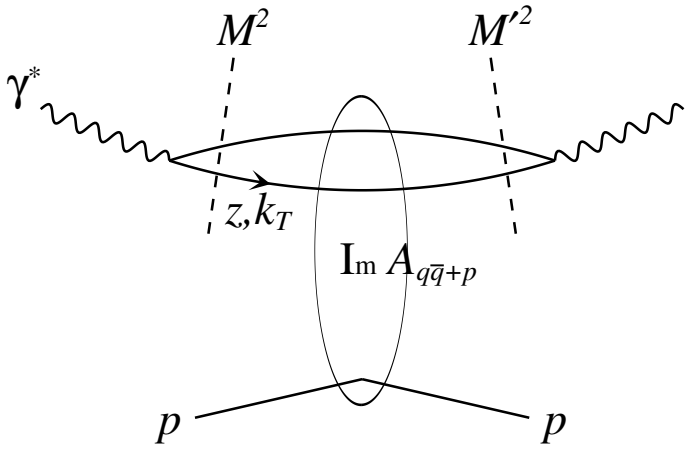
Exploits further the idea of BBJK.

- Perturbative and non-perturbative QCD contributions separated by the distance configurations of the $q\bar{q}$ pair in the $\gamma^* \to q\bar{q}$:
- small distance configurations ($k_T^2 > k_0^2$) given by pQCD (unified equations, DGLAP + BFKL, unintegrated gluon distribution);
- large distance configurations ($k_T^2 < k_0^2$) given by VMD (for low $q\bar{q}$ fluctuation masses, $M^2 < Q_0^2$), and additive quark model (for high $q\bar{q}$ masses, $M^2 > Q_0^2$).
- Excellent description of the data throughout the whole Q^2 region, including Q^2 =0.
- Fitted (at x <0.05) are 3 parameters of the gluon distribution; scales k_0^2 and Q_0^2 chosen as: k_0^2 =0.2 GeV^2 (crucial) and Q_0^2 =1.5 gV^2. Choice of k_0^2 yields physically sensible g and F_L.
- Interference between states of different $q\bar{q}$ masses is crucial for description of the data.

- Importance of the perturbative contribution in the non-perturbative domain.

F_2^p in the low Q^2, low x region

Martin-Ryskin-Stasto...cont'd



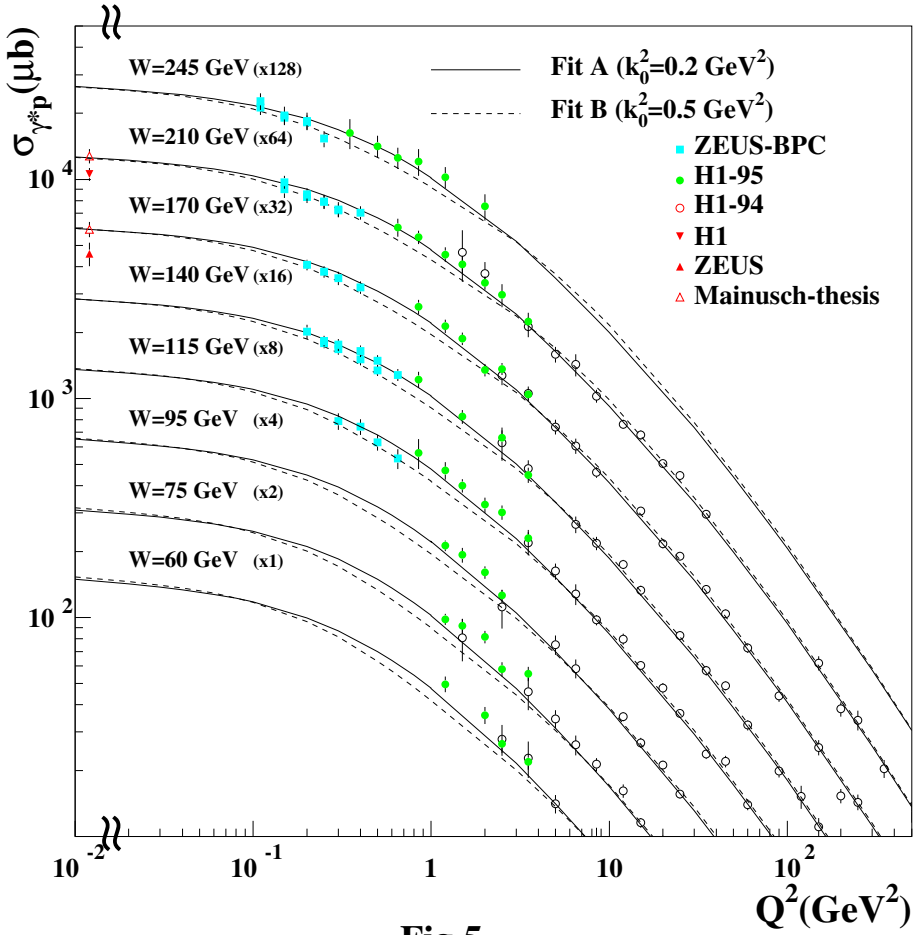


Fig.5

F_2^p in the low Q^2, low x region

Martin-Ryskin-Stasto...cont'd

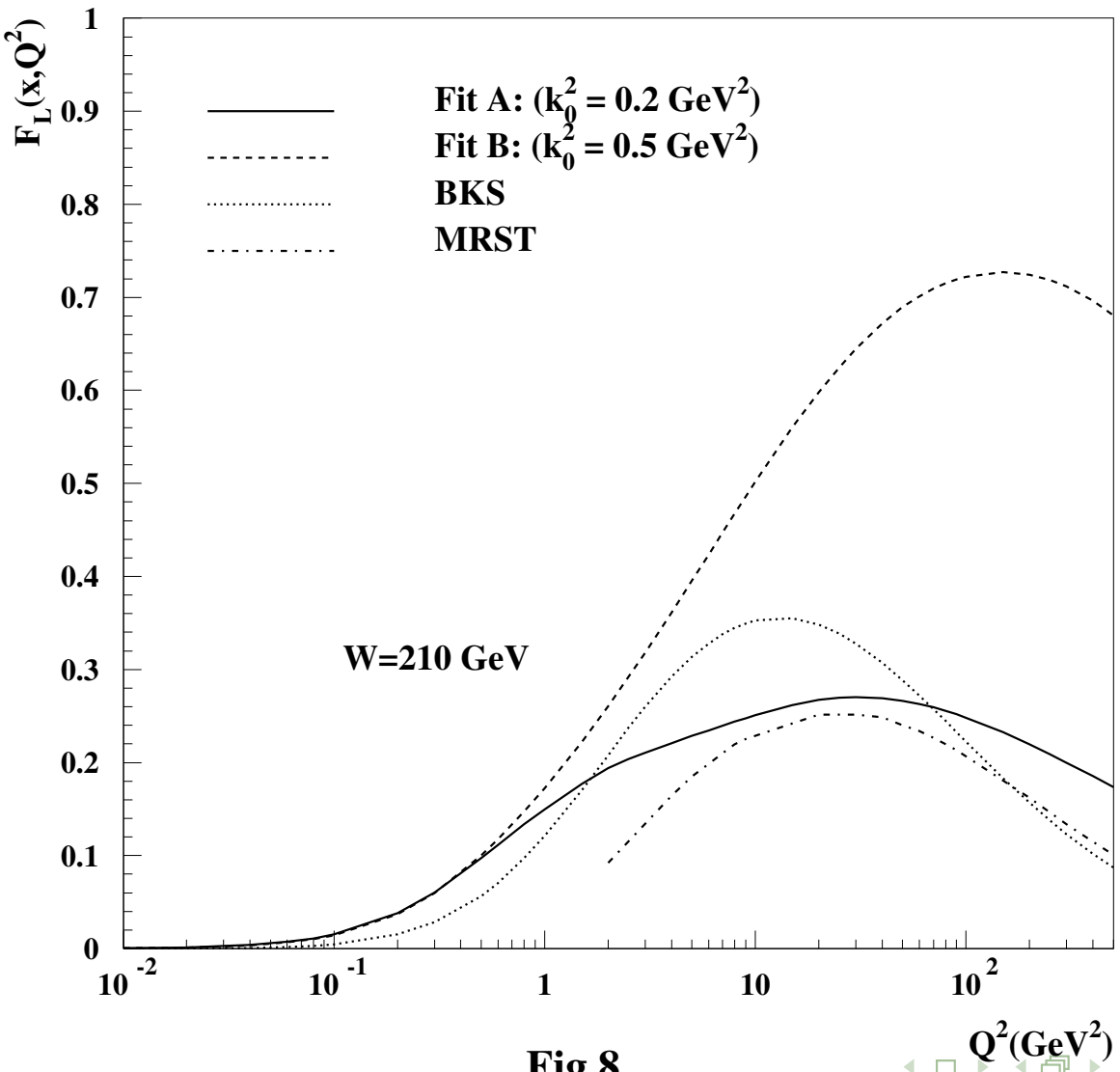


Fig.8

Outline

F_2^p in the low Q^2, low x region

ALLM97 (Abramowicz, Levy, hep-ph/9712415)

- Parametrization of the $\sigma_{tot}(\gamma^* p)$ at $W^2 \gtrsim$ 3 GeV^2 (above resonances).
- Valid everywhere in x and Q^2 (including photoproduction).
- Based on Regge–type approach; extention to large Q^2 compatible with QCD.
- Observe that it is a fit of 23 parameters to all the data
- Fit contains contributions of the pomeron (P) and reggeon (R):

$$F_2(x,Q^2) = \frac{Q^2}{Q^2+m_0^2}\left[F_2^P(x,Q^2)+F_2^R(x,Q^2)\right] \tag{6}$$

of the form

$$F_2^P(x,Q^2) = c_P(t)x_P^{\alpha(t)}(1-x)^{b_P(t)}, \qquad F_2^R(x,Q^2) = c_R(t)x_R^{\alpha(t)}(1-x)^{b_R(t)} \tag{7}$$

where

$$t = \ln\left(\frac{\ln Q^2+Q_0^2}{\Lambda^2} \;/\; \ln\frac{Q_0^2}{\Lambda^2}\right) \tag{8}$$

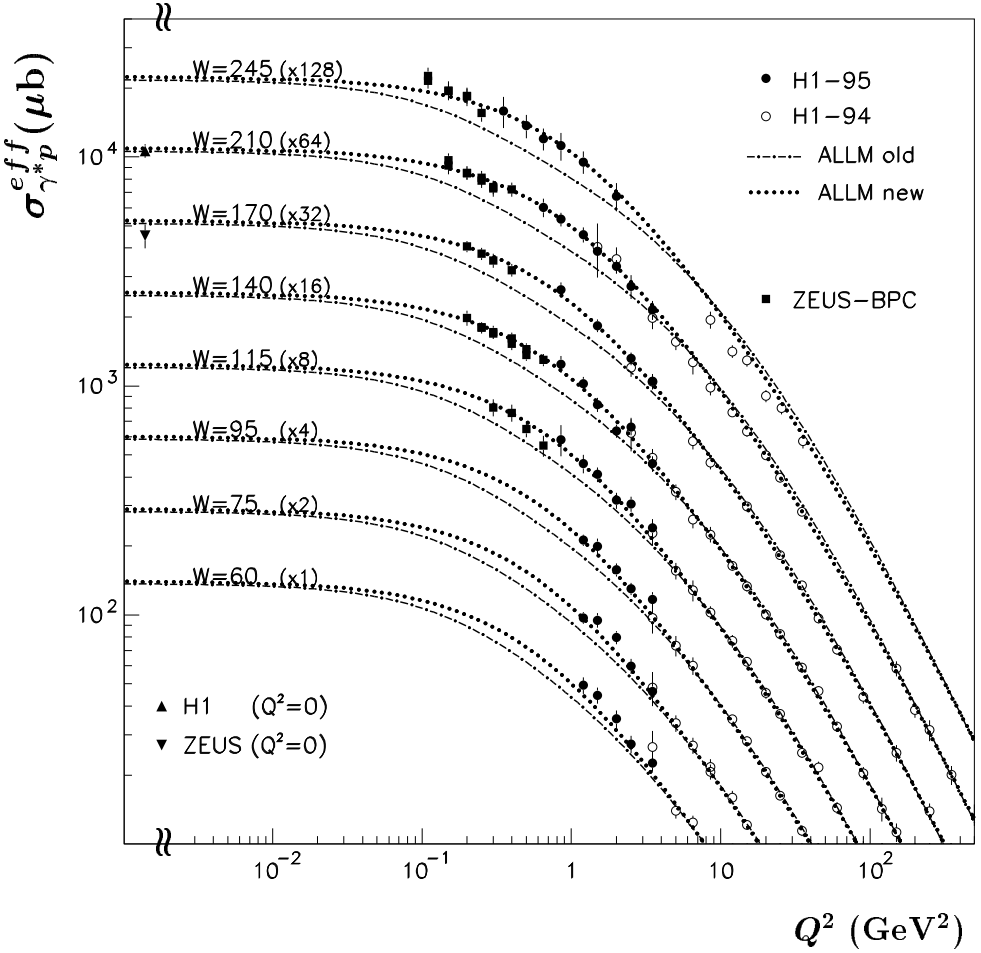
and

$$\frac{1}{x_P} = 1+\frac{W^2-M^2}{Q^2+m_P^2}, \qquad \frac{1}{x_R} = 1+\frac{W^2-M^2}{Q^2+m_R^2} \tag{9}$$

Here M is the proton mass; $m_0^2, m_P^2, m_R^2, Q_0^2$ allow a smooth transition to photoproduction.
For $Q^2 \gg m_P^2, Q^2 \gg m_R^2$, $x_P \to x$, $x_R \to x$;
$c_R, a_R, b_R, b_P \nearrow Q^2 \nearrow$; $c_P, a_P \searrow Q^2 \nearrow$.

F_2^p in the low Q^2, low x region

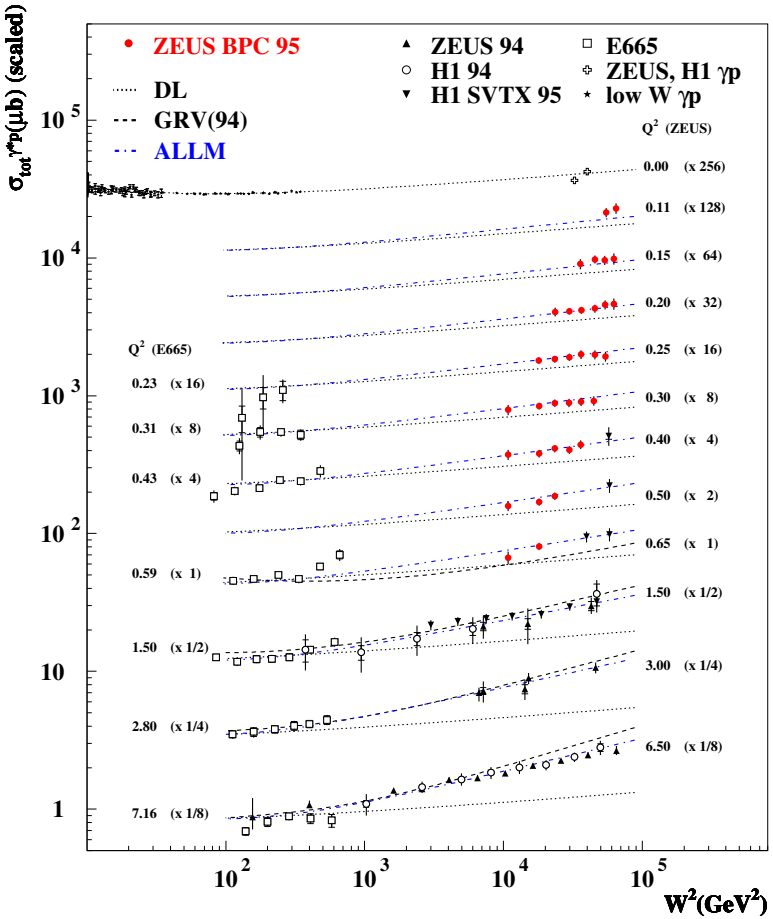
ALLM97...cont'd

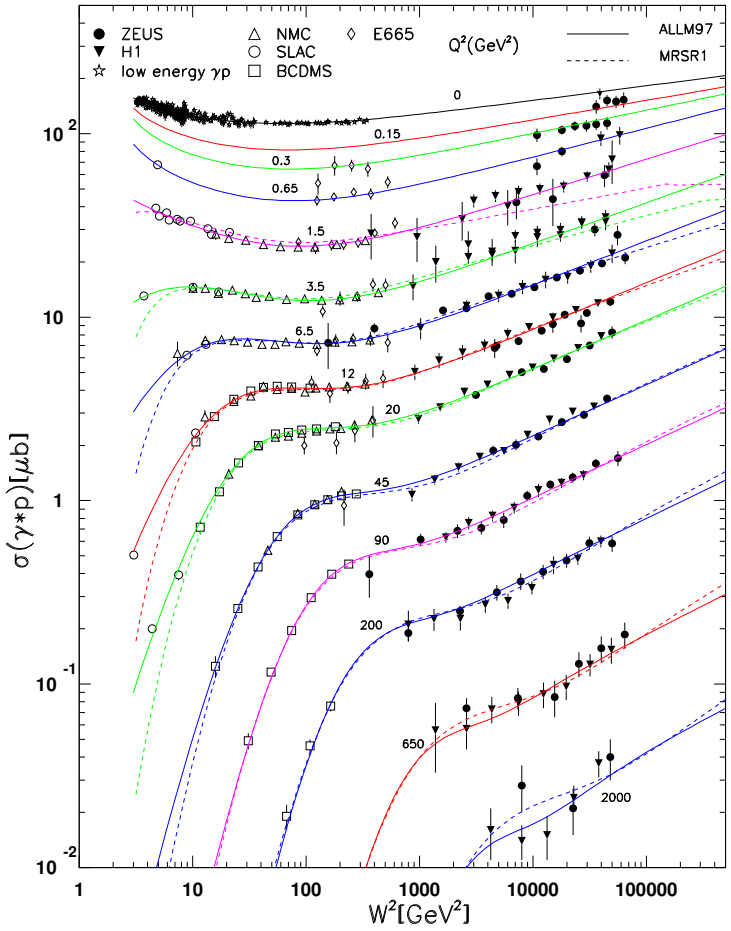


($\sigma^{eff}_{\gamma^* p} \approx \sigma^{tot}_{\gamma^* p}$ at HERA energies)

F_2^p in the low Q^2, low x region

ALLM97...cont'd

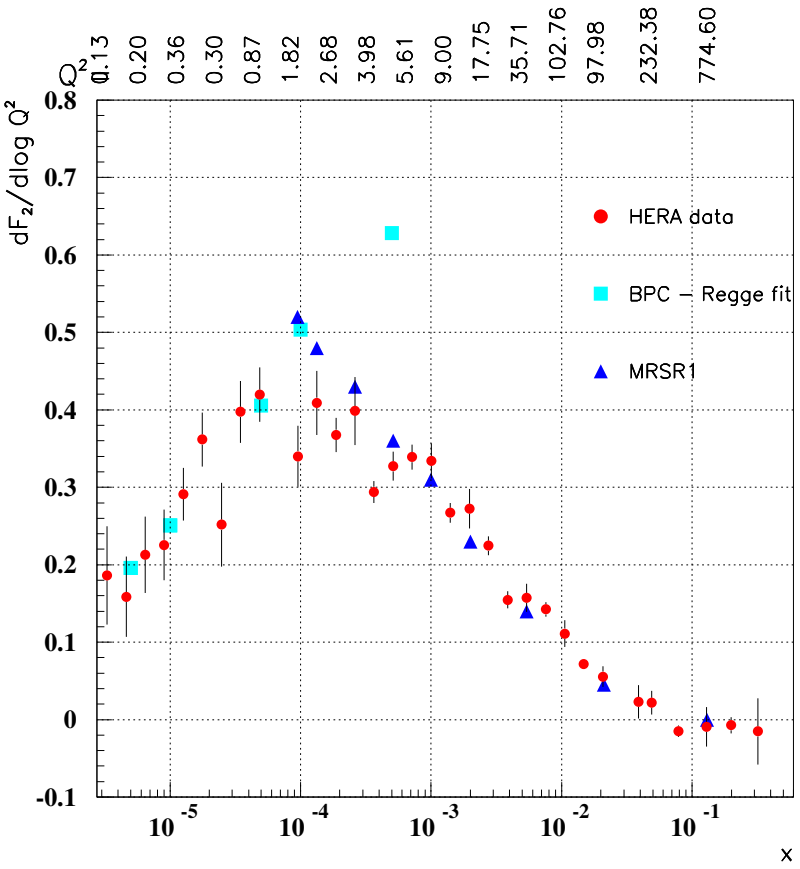


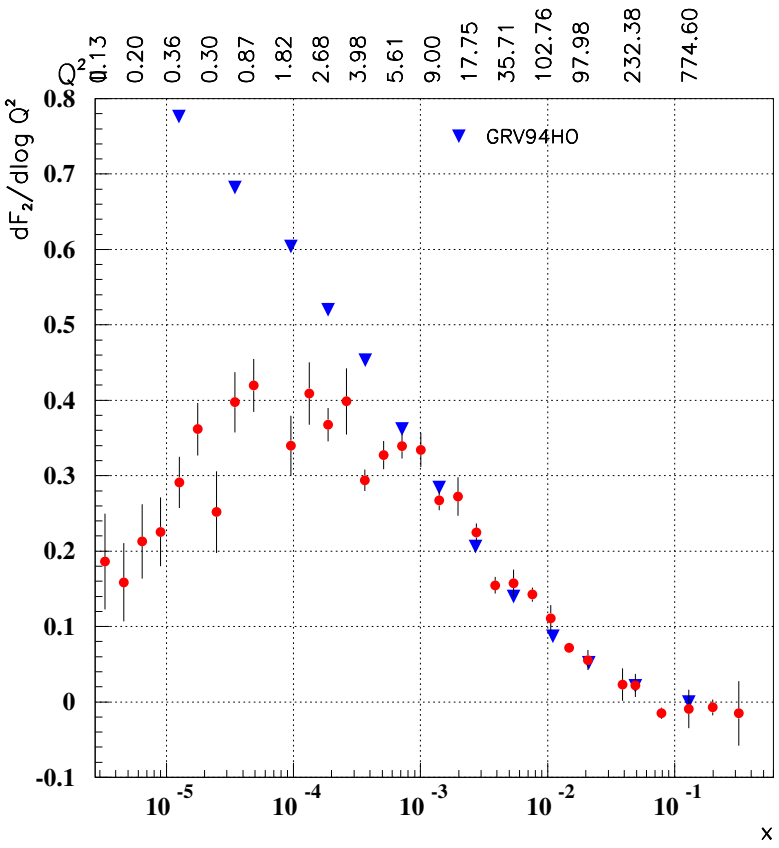


F_2^p in the low Q^2, low x region

ALLM97 – the transition region

$$\sigma_{tot}(\gamma^* p) = \frac{4\pi^2\alpha}{Q^2(1-x)}\frac{Q^2+4M^2x^2}{Q^2}F_2(W^2,Q^2); \quad \sigma_{tot}(\gamma p) = \lim_{Q^2\to 0} 4\pi^2\alpha\frac{F_2}{Q^2} \quad (\textit{limit at fixed } \nu)$$

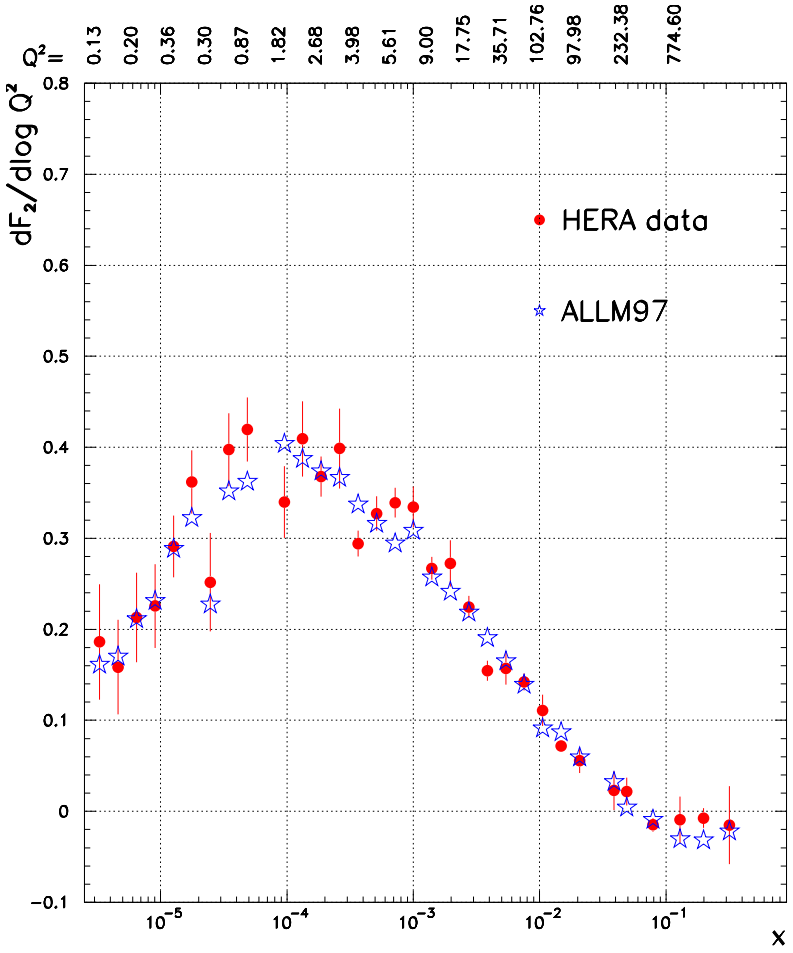




Transition at Q^2: 1 – 3 GeV^2 ?

F_2^p in the low Q^2, low x region

ALLM97 – the transition region



Outline

F_2^p in the low Q^2, low x region

ZEUS Regge fit (ZEUS, Eur. Phys. J. C7 (1999) 609)

Combines the Q^2 dependence of the VMD with the energy dependence from the Regge model:

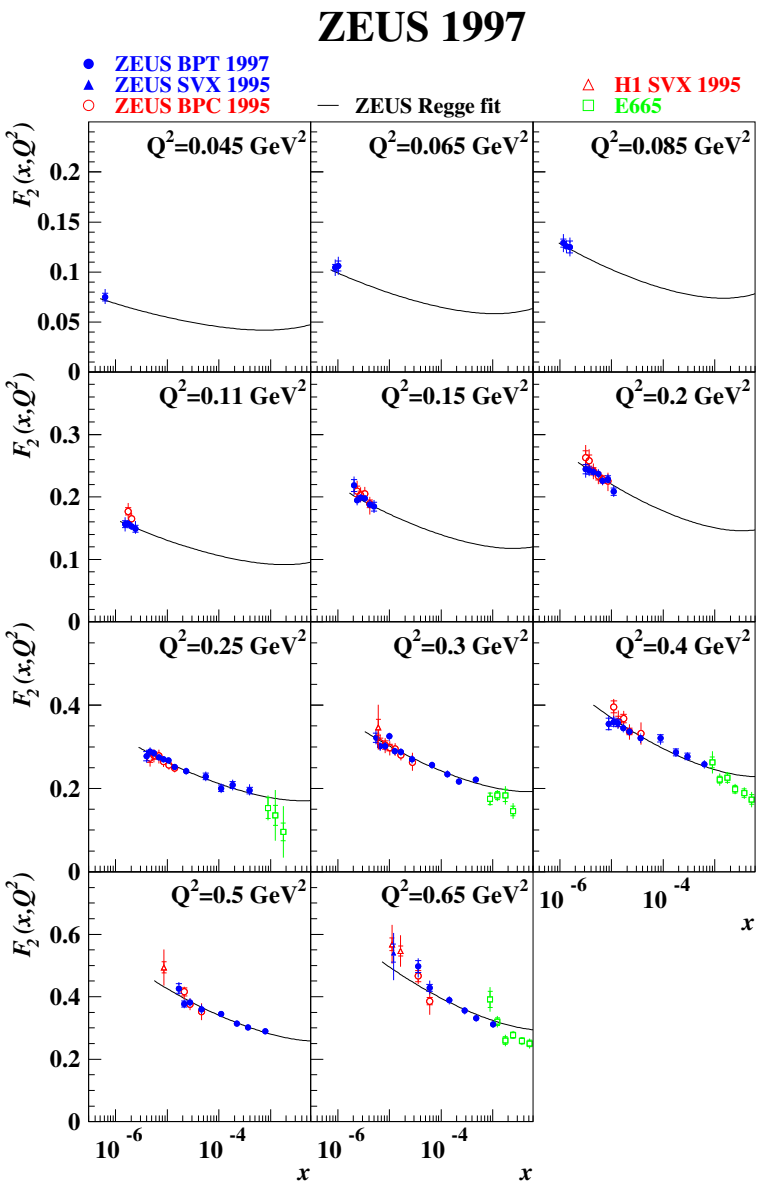
$$F_2(x,Q^2) = \left(\frac{Q^2}{4\pi^2\alpha}\right)\cdot\left(\frac{M_0^2}{M^2+Q^2}\right)\cdot\left[A_R\cdot(W^2)^{\alpha_R-1}+A_P\cdot(W^2)^{\alpha_P-1}\right]$$

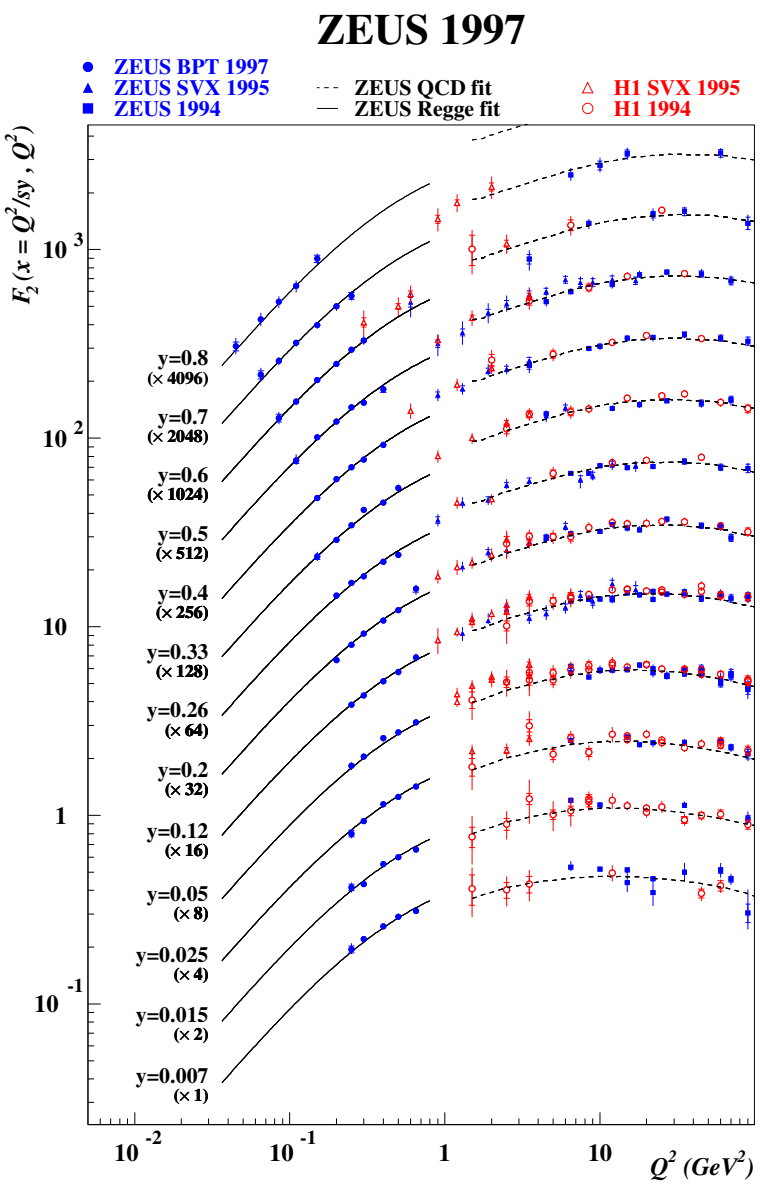
where A_R, A_P, M_0 are constants; α_R, α_P are reggeon and pomeron intercepts. Fixed: M_0^2 = 0.53 GeV^2, α_R = 0.53.
Remaining 3 parameters fitted to Q^2 =0 data at W^2 >3 GeV^2.

Result: α_P = 1.097 $\pm$ 0.002.

F_2^p in the low Q^2, low x region

ZEUS Regge fit...cont's





Outline

Summary

- Apart of physics interest, knowledge of structure functions also outside the DIS region is necessary, e.g. due to radiative corrections.
- Several approaches determining F_2 at low Q^2 exist: physics motivated fits and fully dynamical models.
- Observe that the resulting F_2 parametrizations are valid in limited Q^2 (and/or x) intervals.