Few-Nucleon Forces and Systems in Chiral Effective Field Theory

Evgeny Epelbaum

Research Centre Jülich & University Bonn, Germany

Lecture 1: Chiral Perturbation Theory: the basics

- Effective (field) theories: basic principles
- Introduction to Chiral Perturbation Theory
  - a) Chiral symmetry of QCD
  - b) Effective chiral Lagrangian
  - c) Power counting
  - d) Example: pion scattering
- Summary

Lecture 2: Inclusion of nucleon(s)

Lecture 3: Chiral EFT & nuclear forces

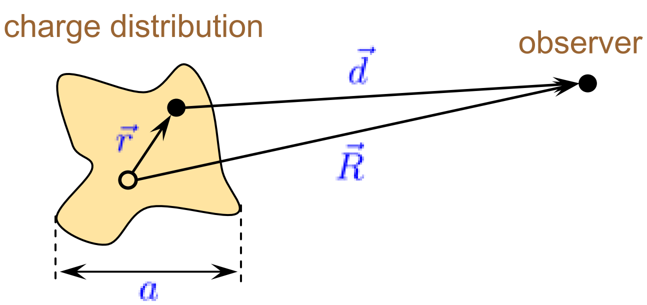
Lecture 4: Applications

Effective (Field) Theories

An effective (field) theory is an approximate theory whose scope is to describe phenomena which occur at a chosen length (or energy) range.

Examples:

1) Multipole expansion for electric potentials



$$
\begin{aligned}
V &\propto \int \frac{\rho(\vec{r}\,)}{d}\, d^3r \\
&= \int \frac{\rho(\vec{r}\,)}{\sqrt{R^2 + 2rR\cos\theta + r^2}}\, d^3r \\
&= \sum_{n=0}^{\infty} \frac{1}{R^{n+1}} \int r^n\, P_n(\cos\theta)\rho(\vec{r}\,)\, d^3r \\
&= q\frac{1}{R} + P\frac{1}{R^2} + Q\frac{1}{R^3} + \dots
\end{aligned}
$$

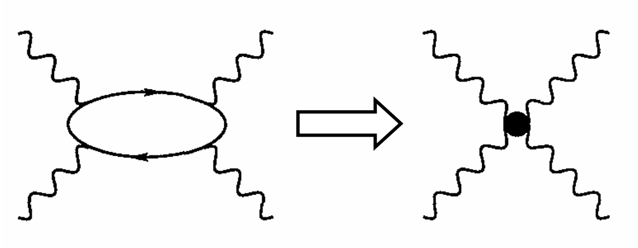
⟵ The sum converges rapidly for $a \ll R$

2) Photon-photon scattering at low energy

Euler & Kockel '35; Heisenberg & Euler '36

At low energy, $E \ll m_e$, one cannot probe details of γe interactions

$\Longrightarrow$ e's can be integrated out.



This leads to an effective Lagrangian which includes all possible photon interactions consistent with Lorentz & gauge symmetry.

Building blocks: $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu\,, \quad \tilde{F}^{\mu\nu} = -1/2\,\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2}{m_e^4}\left[a_1 \left(F_{\mu\nu} F^{\mu\nu}\right)^2 + a_2 \left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)^2\right] \underbrace{+ \dots}_{\text{higher-order (in } 1/m_e \text{ and } \alpha \text{) terms}}$$

constants

Each $F^{\mu\nu}$ generates photon momentum $\Longrightarrow$ $\boxed{\text{Amplitude } \simeq E_\gamma^4}$

For 1-loop analysis based on $\mathcal{L}_{\text{eff}}$ see J.Halter, PLB 316 (1993) 155.

"Weinberg's Theorem"

"if one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetry principles, and then calculates S-matrix elements with this Lagrangian to any order in perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles"

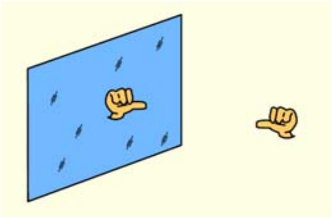
S.Weinberg, Physica A96 (79) 327

- identify the symmetries of the underlying theory,
- construct the most general $\mathcal{L}_{\rm eff}$ in terms of relevant d.o.f. and consistent with the symmetries,
- do standard quantum field theory with the effective Lagrangian.

Introduction to Chiral Perturbation Theory

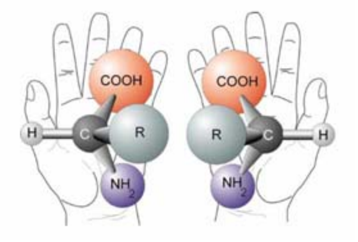
What is chiral?

Lord Kelvin, 1904: *"I call any geometrical figure, or group of points, chiral, and say that it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself."*



mirror image of object ≠ object

$\Longrightarrow$ *object is* ***chiral***



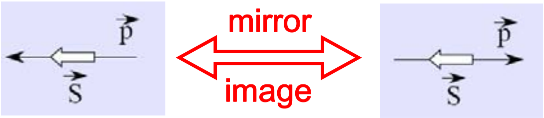
$\Longleftarrow$ Chirality is an important concept in chemistry, biology, etc. E.g. all biological polymers have definite chirality!

Some objects in real world occur with both chiralities…

Chiral symmetry of QCD

$$\mathcal{L}_{\rm QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$

Left- and right-handed quark fields: $q_{L,R} = \frac{1}{2}(1 \pm \gamma_5)q.$



(for m = 0: chirality = helicity)

Chiral group is a group of independent rotations of $q_{L,\,R}$ in the flavor space.

For 2 flavors: $\mathrm{G} = \mathrm{SU(2)_L \times SU(2)_R}$ and $\begin{cases} q_L \xrightarrow{G} q'_L = g_L\, q_L \\ q_R \xrightarrow{G} q'_R = g_R\, q_R \end{cases}$ with $g_{L,R} \in \mathrm{SU}(2)_{L,R}$

Chiral $\mathrm{SU(2)}$ Lie algebra:

$$[\Gamma_i^L,\ \Gamma_j^L] = i\epsilon_{ijk}\Gamma_k^L$$
$$[\Gamma_i^R,\ \Gamma_j^R] = i\,\epsilon_{ijk}\,\Gamma_k^R$$
$$[\Gamma_i^L,\ \Gamma_j^R] = 0$$

generators of $\mathrm{SU(2)_{L,R}}$

Or, equivalently:

$$[V_i,\ V_j] = i\,\epsilon_{ijk}\,V_k$$
$$[A_i,\ A_j] = i\,\epsilon_{ijk}\,V_k$$
$$[V_i,\ A_j] = i\,\epsilon_{ijk}\,A_k$$

where

$V_i = \Gamma_i^R + \Gamma_i^L$ ← vector (isospin) generators

$A_i = \Gamma_i^R - \Gamma_i^L$ ← axial generators

Transformation properties of $\mathcal{L}_{\rm QCD}$: $\bar q(i\not{D}-m)q = \underbrace{\bar q_L i\not{D} q_L + \bar q_R i\not{D} q_R}_{\textit{chiral invariant}} - \underbrace{m(\bar q_L q_R + \bar q_R q_L)}_{\textit{not chiral invariant}}$

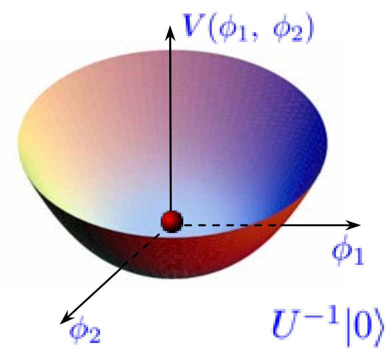
$m_{u,d} \sim 5\ \mathrm{MeV} \ll M_\rho \sim 770\ \mathrm{MeV}$ $\Longrightarrow$ $\mathcal{L}_{\rm QCD}$ is approximately chiral invariant

Notice: $\mathrm{SU}(2)$ chiral symmetry is an accurate symmetry of QCD, i.e. $M_\pi^2/M_\rho^2 \sim 0.03$.

H invariant under G

$H \xrightarrow{G} H' = U^{-1} H U = H$

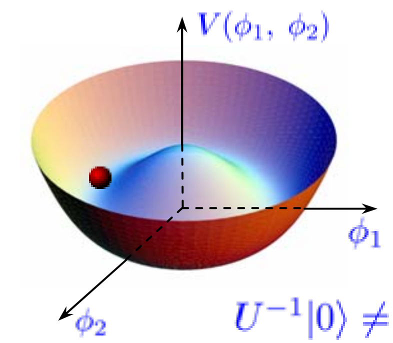
Wigner-Weyl mode



$U^{-1}|0\rangle = |0\rangle$

$\Longrightarrow$ degenerate multiplets according to irred. representations of G

Nambu-Goldstone mode

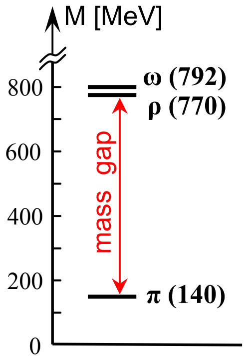


$U^{-1}|0\rangle \neq |0\rangle$

$\Longrightarrow$ spontaneous symmetry breaking, massless Goldstone bosons

There is a strong evidence that chiral symmetry is spontaneously broken down to the isospin group:

- Only isospin but not chiral multiplets are observed in the particle spectrum (axial charges would lead to parity doublets)



- Triplet of unnaturally light pseudoscalar mesons (pions) — natural candidates for Goldstone bosons

- Scalar quark condensate:

$$\langle 0|\bar{q}q|0\rangle\Big|_{\overline{MS},\,2\,GeV} = -(262 \pm 12\ \mathrm{MeV})^3$$ (Lattice, quenched, Hasenfratz et al. '02)

- Further theoretical arguments given by:
  Vafa & Witten '84; 't Hooft '80; Coleman & Witten '80

Chiral perturbation theory

Goldstone bosons + matter fields

$$e^{iZ[J]} = \int [Dq][D\bar{q}][DG]\, e^{\int id^4x\, \mathcal{L}_{\rm QCD}[q,\bar{q},G;\, J]} \Leftrightarrow \int [D\Phi]\, e^{\int id^4x\, \mathcal{L}_{\rm eff}[\Phi;\, J]}$$

(for details see Leutwyler, Ann. Phys. 235 (1994) 165)

Cannot derive $\mathcal{L}_{eff}$ $\Longrightarrow$ write down the <u>most general expression</u> consistent with the chiral symmetry of QCD, i.e.:

- Include all possible χ-invariant terms,
- Include all terms that break χ-symmetry in the same way as $\bar{q}mq$ in $\mathcal{L}_{\rm QCD}$ does.

Consider the pure Goldstone boson sector and neglect the term $\bar{q}mq$.

How to write down most general χ-invariant $\mathcal{L}_{eff}$?

How do π's transform under G?

- Isospin subgroup $H \in G$ realized linearly (π's build an isospin triplet).
- Chiral group necessarily realized nonlinearly:

  $SU(2)_L \times SU(2)_R$ is isomorphic to $SO(4)$ $\Longrightarrow$ need at least 4 dimensions to construct a nontrivial linear realization

Infinitesimal $\mathrm{SO}(4)$ rotation of the 4-vector $(\pi_1,\, \pi_2,\, \pi_3,\, \sigma)$:

$$\begin{pmatrix} \vec{\pi} \\ \sigma \end{pmatrix} \to \begin{pmatrix} \vec{\pi}\,' \\ \sigma' \end{pmatrix} = \left[1 + \vec{\theta}^{\,V} \cdot \vec{V} + \vec{\theta}^{\,A} \cdot \vec{A}\right] \begin{pmatrix} \vec{\pi} \\ \sigma \end{pmatrix}$$

where: $\vec{\theta}^{\,V} \cdot \vec{V} = \begin{pmatrix} 0 & -\theta_3^V & \theta_2^V & 0 \\ \theta_3^V & 0 & -\theta_1^V & 0 \\ -\theta_2^V & \theta_1^V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\vec{\theta}^{\,A} \cdot \vec{A} = \begin{pmatrix} 0 & 0 & 0 & \theta_1^A \\ 0 & 0 & 0 & \theta_2^A \\ 0 & 0 & 0 & \theta_3^A \\ -\theta_1^A & -\theta_2^A & -\theta_3^A & 0 \end{pmatrix}$

One reads off:
$$\vec{\pi}\,' = \vec{\pi} + \vec{\theta}^{\,V} \times \vec{\pi} + \vec{\theta}^{\,A}\sigma$$
$$\sigma' = \sigma - \vec{\theta}^{\,A} \cdot \vec{\pi}$$

Switch to the nonlinear realization of $\mathrm{SO}(4)$:
only 3 out of 4 components of the vector $(\vec{\pi},\, \sigma)$ are independent, i.e. $\vec{\pi}^2 + \sigma^2 = F^2$

$$\sigma = \sqrt{F^2 - \vec{\pi}^2} \quad \Longrightarrow \quad \left\{ \begin{array}{ll} \vec{\pi} \xrightarrow{\vec{\theta}^{\,V}} \vec{\pi}' = \vec{\pi} + \vec{\theta}^{\,V} \times \vec{\pi} & \longleftarrow \textit{linear under } \vec{\theta}^{\,V} \\ \vec{\pi} \xrightarrow{\vec{\theta}^{\,A}} \vec{\pi}' = \vec{\pi} + \vec{\theta}^{\,A}\sqrt{F^2 - \vec{\pi}^2} & \longleftarrow \textit{nonlinear under } \vec{\theta}^{\,A} \end{array} \right.$$

<u>Notice:</u> it is more convenient to use a 2 x 2 matrix notation:

$$U = \tfrac{1}{F}\left(\sigma\, I + i\vec{\pi} \cdot \vec{\tau}\right) \xrightarrow[\textit{realization}]{\textit{nonlinear}} U = \tfrac{1}{F}\left(I\sqrt{1 - \vec{\pi}^{\,2}} + i\vec{\pi} \cdot \vec{\tau}\right)$$

$\mathrm{SO}(4)$ or chiral rotations:

$U \longrightarrow U' = L U R^\dagger$ with $L = e^{-i/2(\vec{\theta}^{\,V} - \vec{\theta}^{\,A})\cdot\vec{\tau}}$ and $R = e^{-i/2(\vec{\theta}^{\,V} + \vec{\theta}^{\,A})\cdot\vec{\tau}}$

The above realization of G is not unique. How does this non-uniqueness affect S-matrix?

- All realizations of G are equivalent to each other by means of nonlinear field redefinitions $\vec{\pi} \to \vec{\pi}' = \vec{\pi}\, F[\vec{\pi}\,],\; F[0] = 1$ (Coleman, Callan, Wess & Zumino '69)
- Such field redefinitions do not affect S-matrix (Haag '58)

Construction of the χ- and Lorentz-invariant ($\Longrightarrow$ no terms with odd # of ∂_μ) $\mathcal{L}_{eff}$:

- 0 derivatives: $UU^\dagger = U^\dagger U = 1$ — plays no role
- 2 derivatives: $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \xrightarrow{g\in G} \mathrm{Tr}(L\partial_\mu U R^\dagger R\, \partial^\mu U^\dagger L^\dagger) = \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$
- 4 derivatives: $[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2\,; \quad \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\,\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$

…

<u>Notice</u>: terms with $\partial_\mu\partial_\nu U,\; \partial_\mu\partial_\nu\partial_\rho U,\; \partial_\mu\partial_\nu\partial_\rho\partial_\sigma U$ can be eliminated via e.o.m. & partial integration

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

where:

$$\begin{aligned} \mathcal{L}_\pi^{(2)} &= \frac{F^2}{4}\,\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \\ \mathcal{L}_\pi^{(4)} &= l_1[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2 + l_2 \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\,\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger) \\ \ldots \end{aligned}$$

low-energy constants

<u>Notice</u>: only derivative couplings allowed (Goldstone bosons do not interact at $E = 0$)

What is the meaning of F ?

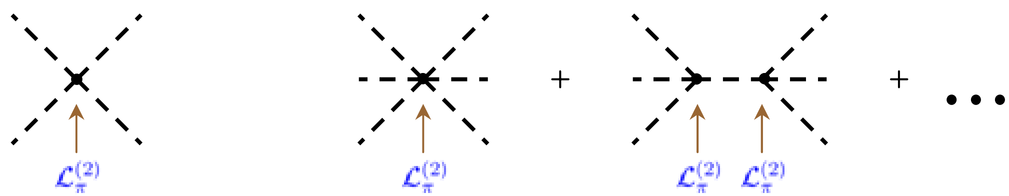
Axial current from $\mathcal{L}_{eff}$: $J^i_{A\mu} = i\mathrm{Tr}[\tau^i(U^\dagger\partial_\mu U - U\partial_\mu U^\dagger)] = -F\partial_\mu \pi^i + \ldots$ *more pion fields*

$$\langle 0 | J^i_{A\mu} | \pi^i(\vec{p})\rangle \equiv i p_\mu F_\pi \delta^{ij} \implies F = F_\pi = 92.4 \text{ MeV}$$

What are the consequences of chiral symmetry?

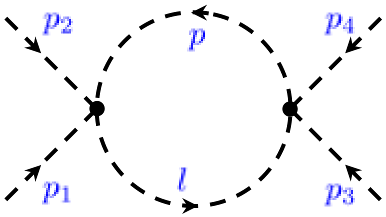
$$U = \frac{1}{F}[\sqrt{F^2 - \vec{\pi}^{\,2}} + i\vec{\tau}\cdot\vec{\pi}] = 1 + \frac{i\vec{\tau}\cdot\vec{\pi}}{F} - \frac{\vec{\pi}^2}{2F^2} + \ldots \implies \mathcal{L}_\pi^{(2)} = \frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} + \frac{1}{2F^2}(\partial_\mu\vec{\pi}\cdot\vec{\pi})^2 + \ldots$$

Leading (i.e. $\sim \mathcal{O}(E^2)$) contributions to multi-pion scattering:



no free parameters $\implies$ <u>*remarkable predictive power!*</u>

So far only tree graphs considered. What about quantum corrections (loops)?



$$p = l + p_3 + p_4$$

$$\Longrightarrow \int \frac{d^4 l}{(2\pi)^4} \frac{p_1 \cdot p_2 \; p_3 \cdot p_4}{[l^2 + i\epsilon]\,[(l + p_3 + p_4)^2 + i\epsilon]} \sim \mathcal{O}(E^4)$$

$\Longrightarrow$ suppressed compared to the L.O. contribution

UV divergences removed e.g. using DR, $\int d^4 l \to \mu^{d-4} \int d^d l$, and redefining the LECs l_i from $\mathcal{L}_\pi^{(4)}$ (need only local χ–invariant counter terms of the order $\sim E^4$)

General observation: n-loop diagrams are suppressed by the factor E^{2n} compared to the tree ones $\sim E^2$.

Power counting (Weinberg '79)

amplitude phase space factors

Consider S-matrix element: $S = \delta^4(p_1 + p_2 + \ldots + p_N)\, M\, \Pi$

The amplitude can be rewritten as: $M \equiv M(E,\, \mu,\, g^r) = E^D\, f\left(\frac{E}{\mu},\, g^r\right)$

combination of LECs

Dimensional analysis:

— pion propagators: $1/(p^2 - M_\pi^2) \sim 1/Q^2$

— momentum integrations: $d^4 l \sim Q^4$

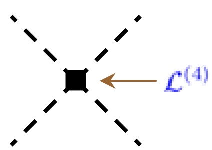
— delta functions: $\delta^4(p - p') \sim 1/Q^4$

— derivatives: $\partial_\mu \sim Q$

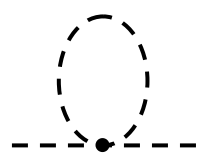
For a Feynman graph with L loops and N_d vertices with d derivatives:

$$\boxed{D = 2 + 2L + \sum_d N_d (d-2)}$$

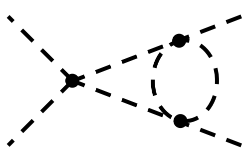
Examples:



$D = 2 + 0 + 2 = 4$



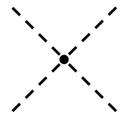
$D = 2 + 2 + 0 = 4$



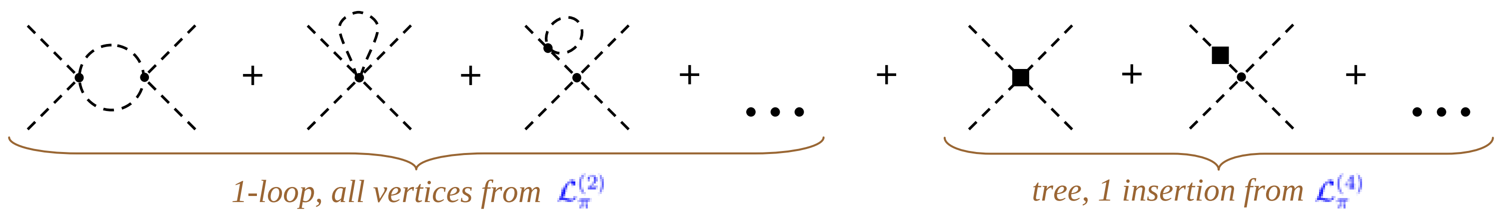
$D = 2 + 4 + 0 = 6$

Example: $\pi\pi$ scattering

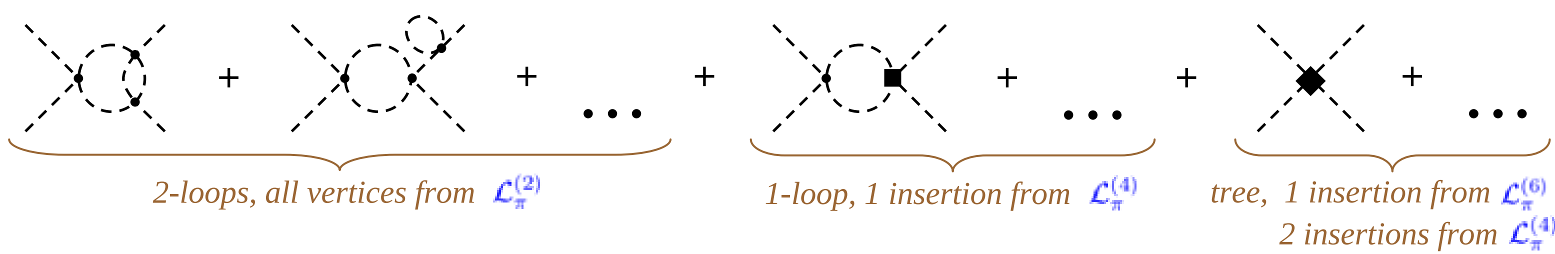
$\mathcal{O}(E^2)$



$\mathcal{O}(E^4)$



$\mathcal{O}(E^6)$



Amplitude is obtained via expansion in E/Λ_χ . What is the value of Λ_χ?

- Chiral expansion breaks down for $E \sim M_\rho \;\Longrightarrow\; \Lambda_\chi \sim M_\rho = 770\ \mathrm{MeV}$
- Consistency arguments yield: $\Lambda_\chi \leq 4\pi F_\pi = 1.2\ \mathrm{GeV}$ (Manohar & Georgi '84)

So far assumed exact χ–symmetry. How to account for explicit χ–symmetry breaking?

$$\delta\mathcal{L}_{\rm QCD} = -\bar q m q = -m_q\,\bar q(1+\epsilon\,\tau_3)q \quad \text{where} \quad m_q = \frac{1}{2}(m_u+m_d) \quad \text{and} \quad \epsilon = \frac{m_u-m_d}{m_u+m_d}$$

4th comp. of SO(4)-vector *3rd comp. of SO(4)-vector*

$\Longrightarrow$ include in $\mathcal{L}_{eff}$ all possible 3rd and 4th components of $\rm SO(4)$ tensors of rank n multiplied by $(\epsilon m_q)^n$ and m_q^n (assuming m_q can be treated perturbatively).

external hermitian field

More convenient: method of external sources $\delta\mathcal{L}_{\rm QCD} = -\bar q \mathcal{M} q\Big|_{\mathcal{M}=m}$

The term $-\bar q\mathcal{M}q = -\bar q_L\mathcal{M}q_R - \bar q_R\mathcal{M}q_L$ is χ–invariant if: $\mathcal{M} \xrightarrow{G} \mathcal{M}' = g_R\mathcal{M}g_L^{-1} = g_L\mathcal{M}g_R^{-1}$

$\Longrightarrow$ write down all possible χ–invariant terms with $\mathcal{M}$ and then set $\mathcal{M}=m$

The leading (i.e. no ∂_μ and $\propto\mathcal{M}$) SB term in $\mathcal{L}_{eff}$:

$$\mathcal{L}_{\rm SB} = \frac{BF^2}{2}{\rm Tr}[(U+U^\dagger)\mathcal{M}]\Big|_{\mathcal{M}=m} = 2BF^2 m_q - B\,m_q\,\vec\pi^{\,2} + \mathcal{O}(\vec\pi^{\,4}) \;\Longrightarrow\; M_\pi^2 = 2m_qB + \mathcal{O}(m_q^2)$$

LEC B is related to the scalar quark condensate via $\langle 0|\bar u u|0\rangle = \langle 0|\bar d d|0\rangle = -BF^2 + \mathcal{O}(\mathcal{M})$

Notice: the generalized scenario (Stern et al. '91) in which $2m_qB \ll M_\pi^2$ is ruled out by recent data on $\pi\pi$ scatt. length.

CHPT = simultaneous expansion in energy and about the χ–limit keeping $E \sim Q \sim M_\pi$

- Write down the most general $\mathcal{L}_{eff}$ consistent with the χ-symmetry of QCD,
- Calculate S-matrix in perturbation theory (based on chiral expansion),
- Fix the unknown LECs from some data and make predictions.

Predictive power? $\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$

2 7 53 low-energy constants

Example: S-wave ππ scattering lengths $a_0^{I=0}$ and $a_0^{I=2}$

LO: $a_0^0 = 0.16$ (Weinberg '66)

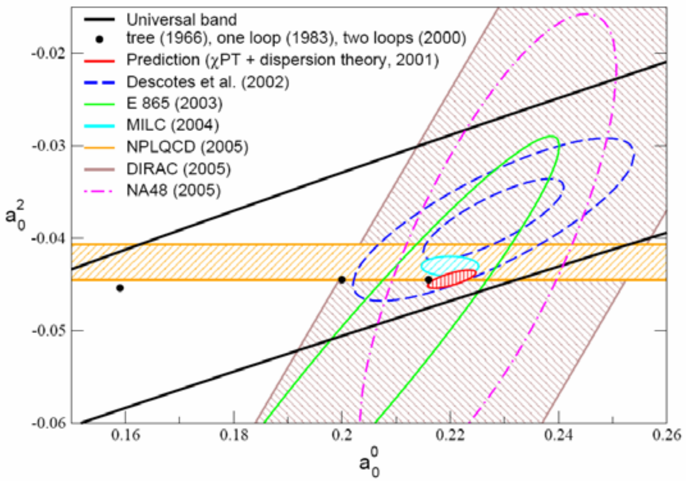
NLO: $a_0^0 = 0.20$ (GL '83)

NNLO: $a_0^0 = 0.217$ (Bijnens et al. '95)

NNLO + dispersion relations:

$a_0^0 = 0.220 \pm 0.005$

(Colangelo et al. '01)



(from Caprini et al. hep-ph/0509266)

Summary

- Effective (field) theory is a powerful method to study phenomena which occur in a certain energy range.
- The principles of Chiral Perturbation Theory, the EFT of the Standard Model, have been introduced.
- How to include nucleons ???

Some ideas will be discussed in lecture 2 …