Few-Nucleon Forces and Systems in Chiral Effective Field Theory

Evgeny Epelbaum

Research Centre Jülich & University Bonn, Germany

Lecture 1: Chiral Perturbation Theory: the basics

Lecture 2: Inclusion of nucleon(s)

- Single-nucleon sector
- Few nucleons
  - a) A toy model
  - b) Warm-up exercise: π-less EFT
  - c) EFT with perturbative π's
  - d) EFT with nonperturbative π's a la Weinberg
- Summary

Lecture 3: Chiral EFT & nuclear forces

Lecture 4: Applications

Inclusion of the nucleons

Recall from lecture 1: π´s can be introduced via

$$U = \frac{1}{F}(I\sqrt{1-\vec{\pi}^{\,2}} + i\vec{\pi}\cdot\vec{\tau}) \quad \text{or} \quad U = e^{i/F\vec{\pi}\cdot\vec{\tau}} \quad \text{such that} \quad U \xrightarrow{G} U' = R\,U\,L^{-1} \equiv e^{-i/2\,\vec{\theta}_R\cdot\vec{\tau}}\,U\,e^{i/2\,\vec{\theta}_L\cdot\vec{\tau}}$$

Remember further: isospin rotations, $H \in G$, correspond to: $\vec{\theta}_R = \vec{\theta}_L \equiv \vec{\theta}_V$

It is convenient to switch to $u = \sqrt{U} = e^{i/(2F)\,\vec{\pi}\cdot\vec{\tau}} \xrightarrow{G} huL^{-1} = Ruh^{-1}$ with $h = e^{-i/2\,\vec{\theta}\cdot\vec{\tau}}$

<u>Notice</u>: $\vec{\theta}$ is a complicated nonlinear function of $\vec{\theta}_R, \vec{\theta}_L, \vec{\pi}$ that reduces to $\vec{\theta} = \vec{\theta}_V$ for $H \in G$.

How do nucleons transform under chiral rotations?

- nucleons should transform linearly under isospin group H: $N \xrightarrow{H\in G} N' = e^{-i/2\,\vec{\theta}_V\cdot\vec{\tau}}N$
- they can transform nonlinearly under the chiral group G

Coleman-Callan-Wess-Zumino (CCWZ) realization: $\boxed{N \xrightarrow{G} N' = hN \equiv e^{-i/2\,\vec{\theta}(\vec{\theta}_L,\vec{\theta}_R,\vec{\pi})\cdot\vec{\tau}}N}$

One can show that (see ***Coleman, Callan, Wess & Zumino, PR 177 (69) 2239; 2247***):

- this indeed defines a nonlinear realization of G;
- other realizations may be reduced to the CCWZ one via nonlinear field redefinitions.

How to construct the most general χ–invariant $\mathcal{L}_{\rm eff}$ with nucleons?

<u>Idea</u>: use covariant quantities O, $O \xrightarrow{G} O' = hOh^{-1}$, as building blocks.

$\Longrightarrow$ terms like $\bar{N}O_1\ldots O_nN, \quad \bar{N}O_1\ldots Tr(O_iO_{i+1}\ldots)\ldots O_nN, \quad \ldots$ are χ–invariant

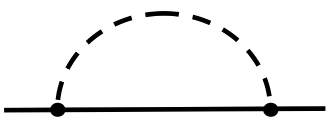
- covariant derivative of the pion field: $u_\mu \equiv i u^\dagger (\partial_\mu U) u^\dagger = -iu(\partial_\mu U^\dagger) u \xrightarrow{G} h u_\mu h^{-1}$
- covariant derivative of N: $D_\mu N = [\partial_\mu + \Gamma_\mu] N \xrightarrow{G} h D_\mu N$ where $\Gamma_\mu \equiv \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger)$
- covariant derivative of O: $D_\mu O = \partial_\mu O + [\Gamma_\mu,\, O] \xrightarrow{G} h\,(D_\mu O)\, h^{-1}$

Lowest-order Lagrangian: $\mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\gamma^\mu D_\mu - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu \right) N$

(notice: no $\bar{N}\gamma^\mu\gamma_5 Tr(u_\mu) N$-term since $Tr(u_\mu) = 0$)

Problem: new hard mass scale m $\Longrightarrow$ power counting ??

For example: 1-loop correction to m *(Gasser, Sainio & Svarc '88)*



$\Longrightarrow$ $\delta m \xrightarrow{M\to 0} -\frac{3g_A^2 m^3}{(4\pi F)^2}\left[\log\frac{m}{\mu} + \mu^{d-4}\left(\frac{1}{d-4} + const\right)\right]$

Solution: heavy-baryon approach *(Jenkins & Manohar '91; Bernard et al. '92)*

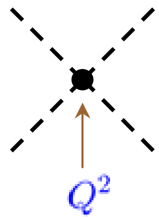
$$\mathcal{L}_{\pi N}^{(1)} = N'^\dagger \left(iD_0 + \frac{g_A}{2}\vec{\sigma}\cdot\vec{u} \right) N' + \mathcal{O}(1/m)$$

m disappeared from $\mathcal{L}_{\pi N}^{(1)}$ $\Longrightarrow$ power counting manifest! For example: $(\delta m)^{\rm HB} = -\frac{3g_A^2 M_\pi^3}{32\pi F^2}$

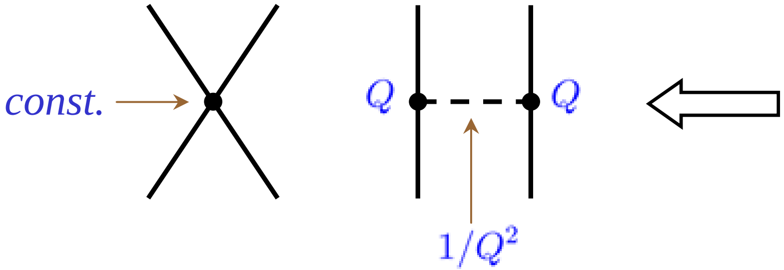
Notice: Lorentz invariant formulations preferable in certain cases!

Ellis & Tang '98; Becher & Leutwyler '99; Gegelia et al. '03; ...

In the $\pi\pi$, πN sectors, S-matrix can be evaluated in perturbation theory (Goldstone bosons do not interact at $E = 0,\ m_q = 0$)



But this is not the case for 2 and more nucleons:



do <u>not</u> vanish for $E = 0,\ m_q = 0$

The presence of shallow bound states (^{2}H, ^{3}H, ^{3}He, ^{4}He, …) indicates breakdown of perturbation theory even at very low energy!

Can one do EFT for strongly interacting nucleons?

Reminder: analytic properties of S-matrix

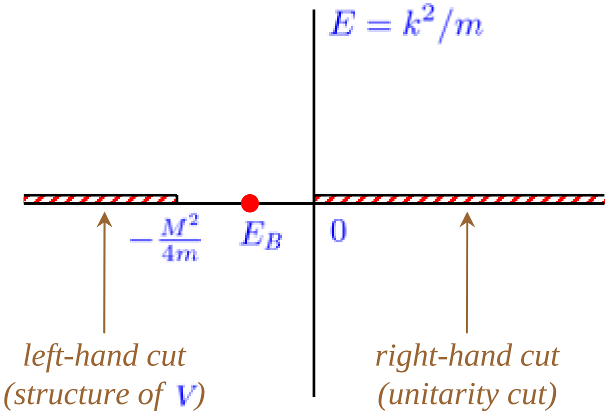
NN scattering (non-relativistic):

$$S(k) = e^{2i\delta(k)} = 1 - 2\pi i \underbrace{T(k)}_{\text{on-shell T-matrix}}\,, \qquad \underbrace{T(p,k) = V(p,k) + \int q^2 dq\, V(p,q) \frac{m}{k^2 - q^2 + i\epsilon} T(q,k)}_{\text{Lippmann-Schwinger equation for half-off-shell T-matrix}}$$

$$\propto \frac{1}{F(k) - ik}$$

effective range function

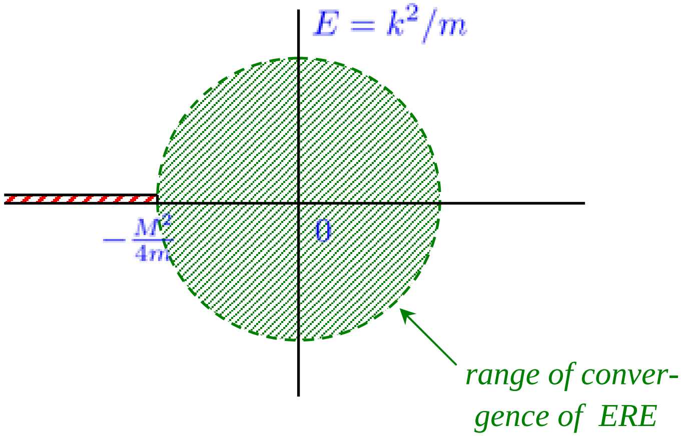
Analytic structure of S-matrix



$$\frac{1}{(\vec{p}\,' - \vec{p})^2 + M^2} \xrightarrow{p = p' = k} \frac{1}{2k^2(1 - \cos\theta) + M^2}$$

$\Longrightarrow$ run into the pole for $k^2 \leq -M^2/4$

Effective range function $F = k^{2l+1} \cot\delta$



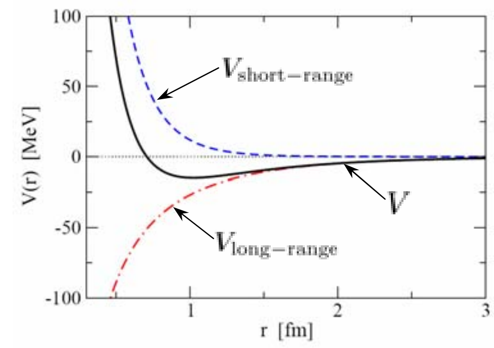
Effective range expansion (ERE):

$$F(k^2) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots$$

Can one do EFT for strongly interacting nucleons?

Toy model

$$V(\vec{q}) = \underbrace{\frac{\alpha_l}{\vec{q}^{\,2} + M_l^2}}_{M_l = 200\text{ MeV}} + \underbrace{\frac{\alpha_h}{\vec{q}^{\,2} + M_h^2}}_{M_h = 750\text{ MeV}} \quad \longrightarrow \quad V(\vec{r}) = \underbrace{\frac{\alpha_l}{4\pi r} e^{-M_l r}}_{\textit{long-range}} + \underbrace{\frac{\alpha_h}{4\pi r} e^{-M_h r}}_{\textit{short-range}}$$



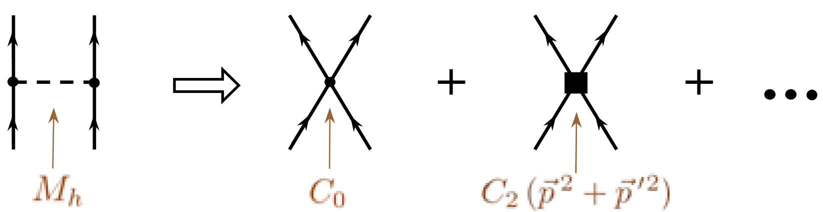
$$\left.\begin{aligned} \alpha_l &= -1.50 \\ \alpha_h &= 10.81 \end{aligned}\right\} \Longrightarrow$$ S-wave bound state with: $E_B = 2.2229$ MeV

Effective theory

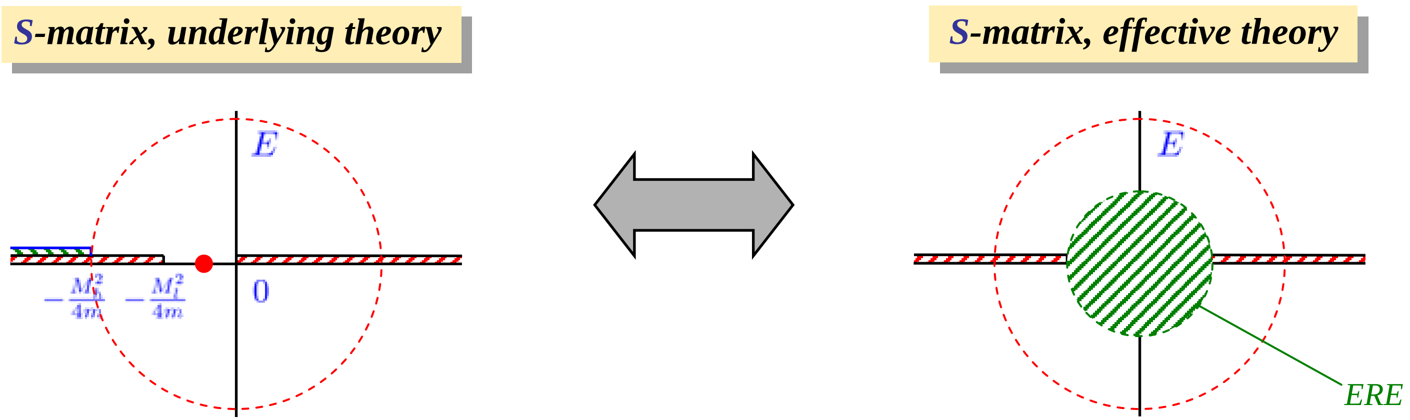
At low energy, $q \sim M_l \ll M_h$, the precise structure of $V_{\text{short-range}}$ is irrelevant

$\Longrightarrow$ mimic $V_{\text{short-range}}$ by a generic set of point-like interactions

$$\boxed{V \rightarrow V_{\text{eff}} = V_{\text{long-range}} + C_0 + C_2\,(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + C_4\,\vec{p}^{\,2}\vec{p}^{\,\prime 2} + \ldots}$$



What to expect?



- should work for momenta up to $|k| \leq \frac{M_h}{2} = 375 \text{ MeV}$ or, equivalently, up to energy $E_{\text{lab}} \leq \frac{M_h^2}{2m} \sim 300 \text{ MeV}$

- should be able to go beyond the effective range expansion which converges for $|k| \leq \frac{M_l}{2} = 100 \text{ MeV}$ or $E_{\text{lab}} \leq \frac{M_l^2}{2m} \sim 20 \text{ MeV}$

Effective theory: $V \to V_{\rm eff} = V_{\rm long-range} + C_0 + C_2\,(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + C_4\,\vec{p}^{\,2}\vec{p}^{\,\prime 2} + \ldots$

T-matrix:

- weak interaction, $|\alpha_{l,h}| \ll 1$: $\langle f|T|i\rangle \simeq \langle f|V_{\rm eff}|i\rangle$
- strong interaction, $|\alpha_{l,h}| \geq 1$: $\langle f|T|i\rangle = \langle f|V_{\rm eff}|i\rangle + \sum_n \frac{\langle f|V_{\rm eff}|n\rangle\langle n|V_{\rm eff}|i\rangle}{E_i - E_n + i\epsilon} + \ldots$

probe high-momentum physics, integral diverges…

Solution: introduce ultraviolet cutoff Λ: $M_l \ll \Lambda \sim M_h$

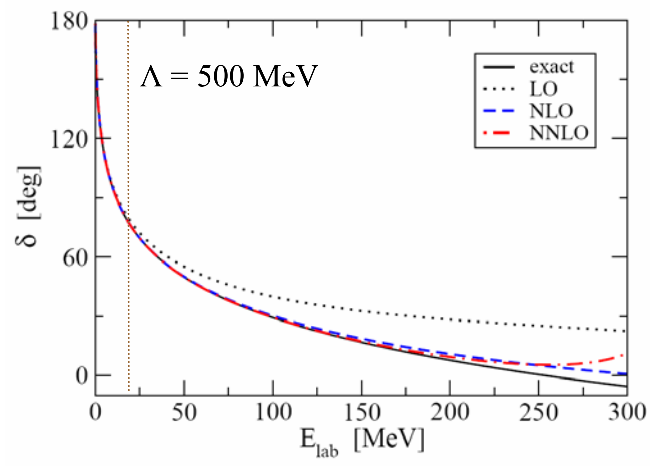
Fix $C_{2i}(\Lambda)$ from some low-energy data and make predictions!

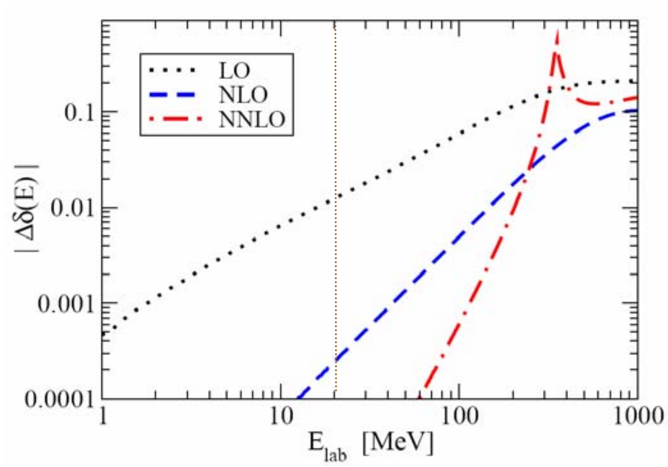
For example, the coefficients in the ERE: $k\cot\delta = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + \ldots$

LO: $V_{\rm eff} = V_{\rm long-range} + C_0\, f_\Lambda(p,p')$, *where:* $\underbrace{f_\Lambda(p,\,p')}_{\textit{cutoff function}} = e^{-\frac{\vec{p}^{\,2}+\vec{p}^{\,\prime 2}}{\Lambda^2}}$ $\longleftarrow$ C_0 *from* a

NLO: $V_{\rm eff} = V_{\rm long-range} + \left[C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2})\right] f_\Lambda(p,p')$ $\longleftarrow$ $C_{0,2}$ *from* a, r

NNLO: $V_{\rm eff} = V_{\rm long-range} + \left[C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + C_4\,\vec{p}^{\,2}\vec{p}^{\,\prime 2}\right] f_\Lambda(p,p')$ $\longleftarrow$ $C_{0,2,4}$ *from* a, r, v_2





Error at order ν : $\Delta\delta(k) \sim (k/\bar{\Lambda})^{2\nu}$, $\bar{\Lambda} \sim 400$ MeV ⟵ agrees with $\bar{\Lambda} \sim \frac{M_h}{2}$

Results for the bound state: $E_B = \underbrace{2.1594}_{LO} + \underbrace{0.0638}_{NLO} - \underbrace{0.0003}_{NNLO} = 2.2229$ MeV

Lesson learned:

- Incorporate the correct long-range force.
- Add local correction terms to V_{eff}. Respect symmetries.
- Introduce an ultraviolet cutoff Λ (large enough but not necessarily ∞).
- Fix unknown constants from some date and make predictions.

⟹ At low energy model independent and systematically improvable approach!

For more details see: G.P.Lepage, "How to renormalize the Schrödinger equation", nucl-th/9706029

Warm-up exercise: few nucleons at very low energy

Effective Lagrangian: for $Q \ll M_\pi$ only point-like interactions

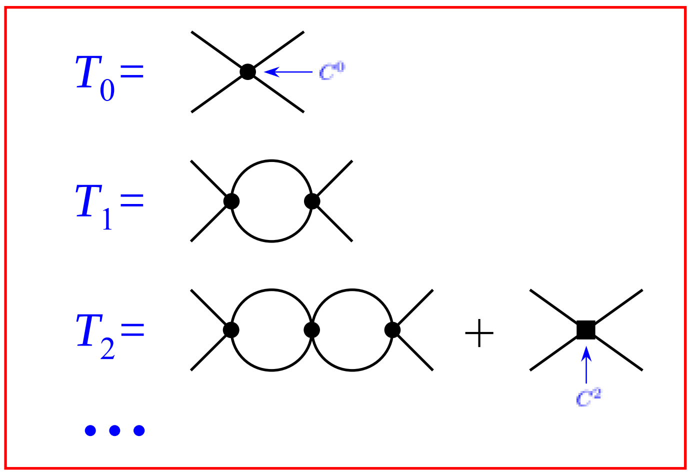
$$\mathcal{L}_{\rm eff} = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m} \right) N - \frac{1}{2} C_1^0 (N^\dagger N)^2 - \frac{1}{2} C_2^0 (N^\dagger \vec{\sigma} N)^2 - \frac{1}{4} C_1^2 (N^\dagger \vec{\nabla}^2 N)(N^\dagger N) + \text{h.c.} + \ldots$$

Scattering amplitude (S-waves):

$$S = e^{2i\delta} = 1 - i \left(\frac{km}{2\pi} \right) T \,, \qquad T = -\frac{4\pi}{m} \frac{1}{k \cot\delta - ik} = -\frac{4\pi}{m} \frac{1}{\left(-\frac{1}{a} + \frac{1}{2} r_0 k^2 + v_2 k^4 + v_3 k^6 + \ldots\right) - ik}$$

Natural case

$$|a| \sim M_\pi^{-1} \,, \; |r| \sim M_\pi^{-1} \,, \; \ldots \quad \Longrightarrow \quad T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m} \Big[\underset{\sim Q^0}{1} - \underset{\sim Q^1}{iak} + \underbrace{\left(\frac{a r_0}{2} - a^2 \right) k^2}_{\sim Q^2} + \ldots \Big]$$



The EFT expansion can be arranged to match the above expansion for T.

Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for T;
- scaling of the LECs: $C^i \sim Q^0$

<u>In reality</u>: $a_{^1S_0} = -23.741 \text{ fm} = -16.6\, M_\pi^{-1}$ $\quad a_{^3S_1} = -5.42 \text{ fm} = 3.8\, M_\pi^{-1}$

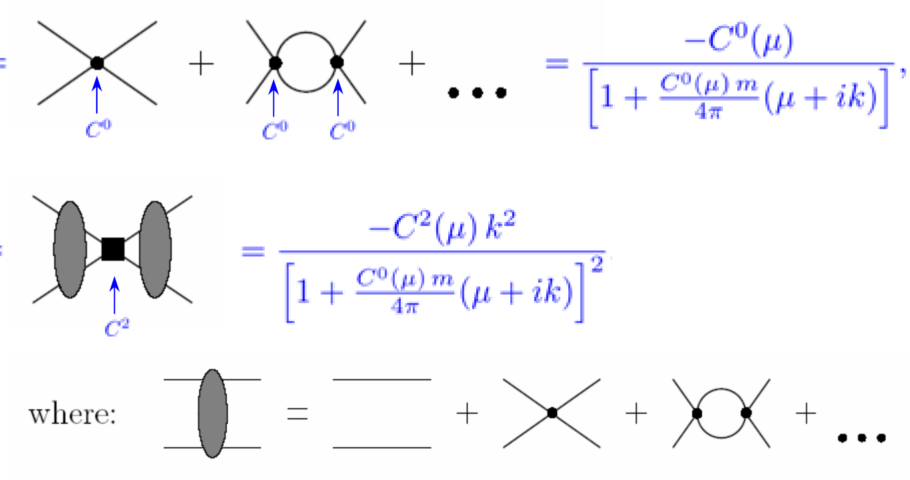
Unnatural case: $|a| \gg M_\pi^{-1}$ (Kaplan, Savage & Wise '97)

Keep ak fixed, i.e. count $a \sim Q^{-1}$:

$$T = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a}+\frac{1}{2}r_0k^2+v_2k^4+v_3k^6+\ldots\right)-ik} = \frac{4\pi}{m}\frac{1}{(1+iak)}\Bigg[\underset{\sim Q^{-1}}{a} + \underbrace{\frac{ar_0}{2(a^{-1}+ik)}}_{\sim Q^0}k^2 + \underset{\sim Q^{1}}{\ldots}\Bigg]\,.$$

<u>Notice</u>: perturbation theory breaks down (T has a pole at $|k| \sim |a|^{-1} \ll M_\pi$)

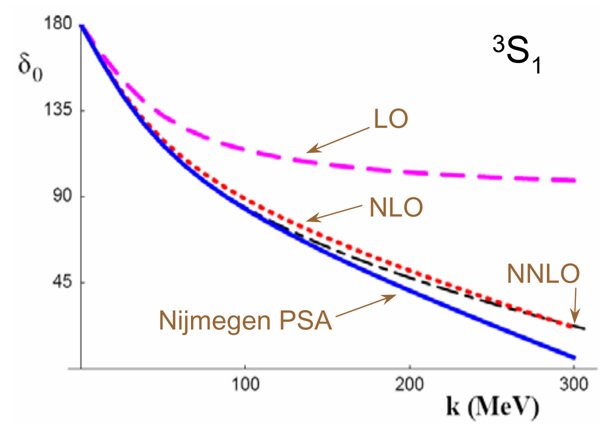
KSW expansion for the amplitude (using DR & Power Divergence Subtraction)



$$T^{(-1)} = \ldots = \frac{-C^0(\mu)}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]},$$

$$T^{(0)} = \frac{-C^2(\mu)\,k^2}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]^2}$$

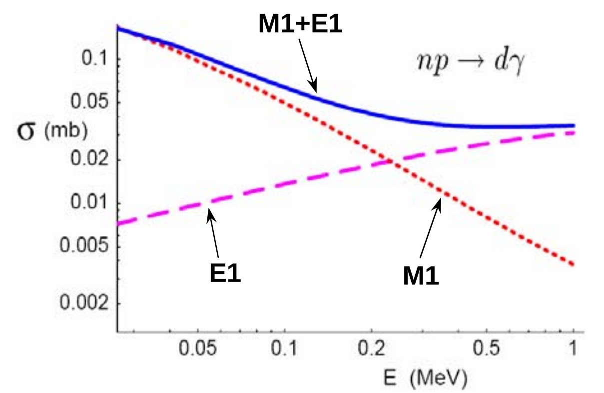
...



from: *Chen, Rupak & Savage NPA653 (1999)*

<u>Notice</u>: LECs scale differently compared to the natural case: $C^0 \sim 1/Q, \quad C^2 \sim 1/Q^2, \quad \ldots$

Applications and extensions



(from: Chen & Savage '99)

$np \to d\gamma$: *Chen, Rupak & Savage '99; Chen & Savage '99; Rupak '00*
(at N^{4}LO accurate to 1% for $E \lesssim 1\,\mathrm{MeV}$)

$pp \to de^+\nu_e$: *Kong & Ravndal '99, '01; Butler & Chen '01*

$ed \to ed$: *Chen, Rupak & Savage '99*

$\nu(\bar{\nu})d$: *Butler & Chen '00; Butler, Chen & Kong '01; Chen '01*

$3N$: *Bedaque, Hammer, van Kolck '98; Gabbiani, Bedaque, Grießhammer '00; Blankleider, Gegelia '01; Grießhammer '05; ...*

$4N$: *Platter, Hammer & Meißner '04*

halo-nuclei: *Bertulani, Hammer & van Kolck '02; Bedaque, Hammer & van Kolck '03*

. . .

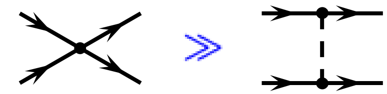
Notice:

- π-less EFT is equivalent to ERE in the 2N case,
- it becomes nontrivial for >2N and/or external probes

Need to include pions in order to go to higher energies and/or beyond the ERE!

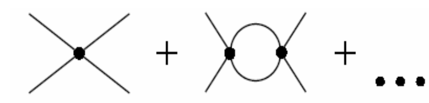
Chiral EFT for few nucleons: are pions perturbative?

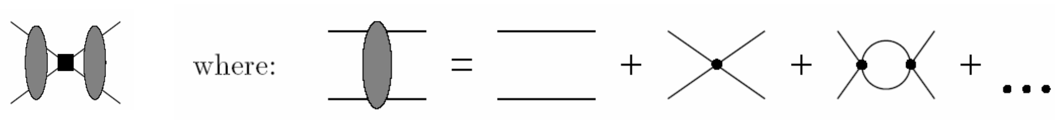
It is straightforward to generalize the KSW power counting assuming that π-exchanges can be treated in perturbation theory, i.e.:



$$T(k) = T^{(-1)}(k) + T^{(0)}(k) + T^{(1)}(k) + \ldots$$

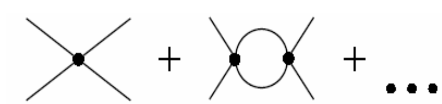
- **EFT without pions**

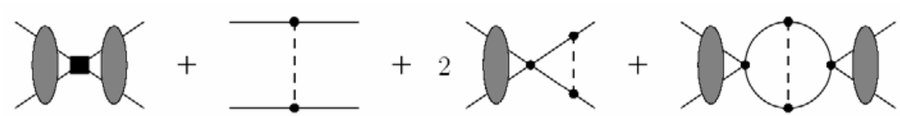
$T^{(-1)} =$ 

$T^{(0)} =$ 

...

- **EFT with perturbative pions**

$T^{(-1)} =$ 

$T^{(0)} =$ 

...

Does the expansion based on perturbative pions converge?

- **"Low-energy theorems"** **(Cohen & Hansen '99)**

<u>Idea:</u> if pions are properly incorporated, one should be able to go beyond the effective range expansion, i.e. to predict the shape parameters.

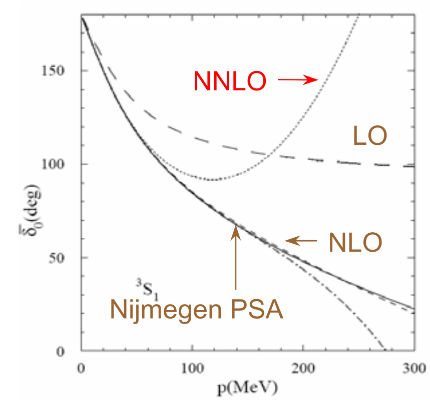
$^1S_0$ at NLO: $k\cot\delta = -a^{-1} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$

$$v_2 = \boxed{\frac{g_A^2 m}{16\pi F_\pi^2}\left(-\frac{16}{3a^2 M_\pi^4} + \frac{32}{5a M_\pi^3} - \frac{2}{M_\pi^2}\right)} \qquad v_3 = \boxed{\frac{g_A^2 m}{16\pi F_\pi^2}\left(\frac{16}{a^2 M_\pi^6} - \frac{128}{7a M_\pi^5} + \frac{16}{3M_\pi^4}\right)}$$

	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
theory	-3.3	17.8	-108.	-0.95	4.6	-25.
data	-0.5	3.8	-17.	0.04	0.7	-4.0
	spin-singlet			spin-triplet		

- **Higher-order calculation** **(Mehen & Stewart '00)**

NNLO results obtained by Mehen & Stewart show no signs of convergence in spin-triplet channels →

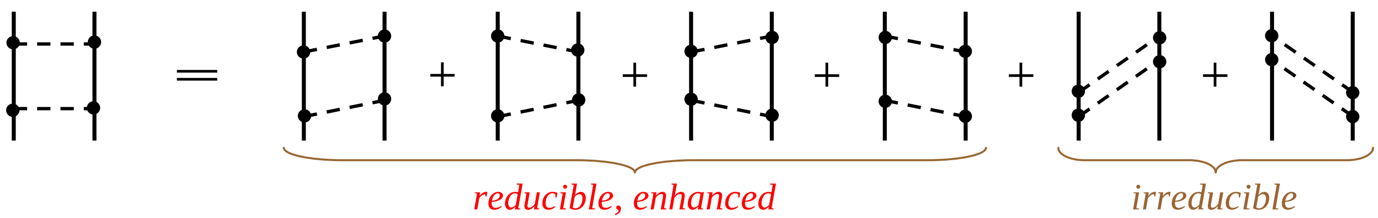


⇨ $\boxed{\pi\text{'s must be treated non-perturbatively}}$

Chiral EFT for few nucleons á la Weinberg

Perturbation theory fails due to enhancement caused by reducible (i.e. infrared divergent in the limit $m_N \to \infty$) diagrams.

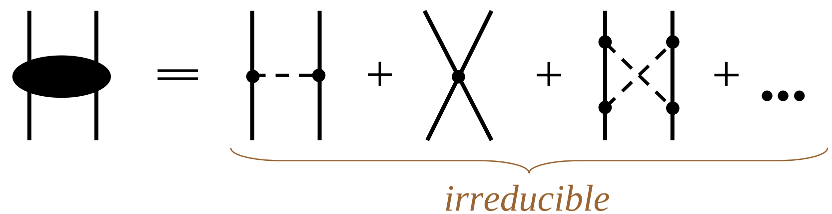
Switch to time-ordered theory: $\text{Amp} = \langle NN|H^I|NN\rangle + \sum_{\Psi} \frac{\langle NN|H^I|\Psi\rangle\langle\Psi|H^I|NN\rangle}{E_{NN} - E_\Psi + i\epsilon} + \ldots$



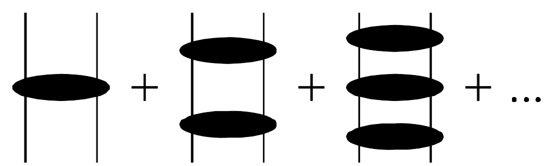
$$\frac{1}{E_{NN} - E_\Psi} = \frac{m_N}{\vec{p}^{\,2} - \vec{q}^{\,2}} \sim \frac{m_N}{Q^2} \gg \frac{1}{Q} \qquad \frac{1}{E_{NN} - E_\Psi} \sim \frac{1}{M_\pi} \sim \frac{1}{Q}$$

Weinberg's proposal:

- Use ChPT to calculate $V_{\text{eff}} =$



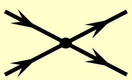
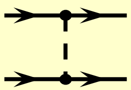
- Solve the Schrödinger eq. $T =$



Kaplan-Savage-Wise *versus* Weinberg

Kaplan-Savage-Wise

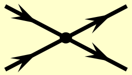
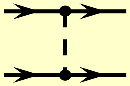
- failure of perturbation theory attributed to large C^0
- naive dimensional analysis doesn't apply to contact terms

 $\sim Q^{-1} \gg$  $\sim Q^0$

- only LO contact terms need to be summed up to infinite order
- analytical calculations possible

Weinberg

- failure of perturbation theory attributed to enhancement in reducible graphs (large m)
- naive dimensional analysis for contact & pion terms

 $\sim$  $\sim Q^0$

- 1π-exchange and contact terms need to be summed up to infinite order
- numerical calculations required

Toy Model *versus* Weinberg

- long-range part of the potential is known exactly
- Short-range part is represented by a series of contact terms

$$V_{\rm eff} = \sum_\nu \Big[\underbrace{V_{\rm long-range}}_{\text{Yukawa}} + \underbrace{V^{(\nu)}_{\rm short-range}}_{\text{contact terms}} \Big]$$

- long-range part of the potential is calculated in ChPT
- Short-range part is represented by a series of contact terms

$$V_{\rm eff} = \sum_\nu \Big[\underbrace{V^{(\nu)}_{\rm long-range}}_{1\pi,\ 2\pi,\ \ldots,\ \text{exchange}} + \underbrace{V^{(\nu)}_{\rm short-range}}_{\text{contact terms}} \Big]$$

Summary

- Chiral perturbation theory can be extended to the one-nucleon sector in a straightforward way.
- Two- and more-nucleon systems have to be treated nonperturbatively. EFT is a useful tool to deal with nonperturbative problems.
- π-less EFT provides a simple and systematic framework to study few-nucleon systems at very low energy.
- Pions need to be included nonperturbatively! Weinberg's program: use chiral expansion to derive nuclear forces which can be applied as input in few-nucleon calculations.
- How to derive nuclear forces??

Derivation and structure of nuclear forces will be discussed in lecture 3 …