Few-Nucleon Forces and Systems in Chiral Effective Field Theory

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Lecture 1: **Chiral Perturbation Theory: the basics**

Lecture 2: **Inclusion of nucleon(s)**

Lecture 3: **Chiral EFT & nuclear forces**

- How to derive nuclear forces from $\mathcal{L}_{\rm eff}$
- The structure of nuclear forces upto N^{3}LO
- Summary

Lecture 4: **Applications**

Derivation of the nuclear force

We are interested in nuclear forces rather than in the scattering amplitude

$\Longrightarrow$ can, in general, not use Feynman diagrams. How to derive nuclear forces??

Nuclear potentials from chiral EFT

Energy-dependent potentials
(using old-fashioned pert. theory)
Weinberg '90,'91;
van Kolck et al. '92,'94,'96

Friar & Coon '94 →

Energy-independent potentials

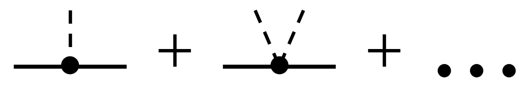
S-matrix based
Robilotta, da Rocha '97;
Kaiser et al. '97 - '01;
Higa et al. '03,'04

Unitary transformation
E.E. et al. '98,'00,'05

Notice:

- nuclear potentials are, generally, not unique
- all schemes rely on perturb. theory (expansion in low momenta & about the χ–limit)
- most methods developed in 1950s: *Brueckner & Watson '53; Taketani et al. '52; Okubo '54, ...*

Nuclear potentials from field theory: general remarques

Consider mesons interacting with non-relativistic nucleons: $H = H_0 + H_I\,, \quad H_I =$ 

It is convenient do decompose the full Fock space as $|\Psi\rangle = |\phi\rangle + |\psi\rangle$, where

$$|\phi\rangle \equiv |N\rangle + |NN\rangle + |NNN\rangle + \ldots \qquad \longleftarrow \text{ no mesons}$$

$$|\psi\rangle \equiv |N\pi\rangle + |N\pi\pi\rangle + \ldots + |NN\pi\rangle + \ldots \qquad \longleftarrow \text{ at least one meson}$$

Schrödinger equation:

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix}$$

projection operators

infinite-dimensional equation due to πN-coupling $\Longrightarrow$ can not solve…

How to reduce this equation to an effective one for $|\phi\rangle$, which can be solved by standard methods?

Method 1:

Tamm '45, Dancoff '50

Idea: use the Schrödinger equation to project out the unwanted Fock-space component $|\psi\rangle$.

$$\begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} = E \begin{pmatrix} |\phi\rangle \\ |\psi\rangle \end{pmatrix} \Longrightarrow |\psi\rangle = \frac{1}{E - \lambda H \lambda} H |\phi\rangle$$

$$\Longrightarrow \left(H_0 + V_{\rm eff}^{\rm TD}(E)\right)|\phi\rangle = E|\phi\rangle$$

where the effective potential is given by $V_{\rm eff}^{\rm TD}(E) = \eta H_I \eta + \eta H_I \lambda \frac{1}{E - \lambda H \lambda} \lambda H_I \eta$

Notice:

- $V_{\rm eff}^{\rm TD}$ depends on E
- $|\phi\rangle$ not orthonormal: $\langle \phi_i | \phi_j \rangle = \langle \Psi_i | \Psi_j \rangle - \langle \psi_i | \psi_j \rangle = \delta_{ij} - \langle \phi_i | H_I \left(\frac{1}{E - \lambda H \lambda} \right)^2 H_I | \phi_j \rangle$
- reduces to old-fashioned time-ordered perturbation theory:

$$V_{\rm eff}^{\rm TD} = \eta H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \eta + \eta H_I \frac{\lambda}{E - H_0} H_I \frac{\lambda}{E - H_0} H_I \eta + \ldots$$

Method 2: unitary transformation

Okubo '54, Fukuda et al. '54

Idea: use unitary transformation U in order to decouple the $|\phi\rangle$ and $|\psi\rangle$ components!

$$H = \begin{pmatrix} \eta H \eta & \eta H \lambda \\ \lambda H \eta & \lambda H \lambda \end{pmatrix} \Longrightarrow \tilde{H} \equiv U^\dagger H U = \begin{pmatrix} \eta \tilde{H} \eta & 0 \\ 0 & \lambda \tilde{H} \lambda \end{pmatrix}$$

Advantages:

- no dependence on energy (per construction),
- unitary transformation preserves the norm of $|\phi\rangle$

How to compute U?

It is convenient to parameterize U in terms of the operator $A = \lambda A \eta$ (Okubo '54):

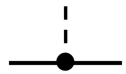
$$U = \begin{pmatrix} \eta(1 + A^\dagger A)^{-1/2} & -A^\dagger(1 + A A^\dagger)^{-1/2} \\ A(1 + A^\dagger A)^{-1/2} & \lambda(1 + A A^\dagger)^{-1/2} \end{pmatrix}$$

Require that $\eta \tilde{H} \lambda = \lambda \tilde{H} \eta = 0 \Longrightarrow \boxed{\lambda \left(H - [A,\ H] - A H A\right) \eta = 0}$

(decoupling equation)

The major problem is to solve the nonlinear decoupling equation.

<u>Example</u>: expansion in powers of the coupling constant

$H_I =$  $\propto g \quad \Longrightarrow$ ansatz: $A = A^{(1)} + A^{(2)} + A^{(3)} + \ldots$

Recursive solution of the decoupling equation $\lambda \left(H - [A,\ H] - AHA \right) \eta = 0$

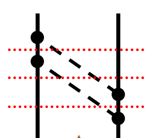
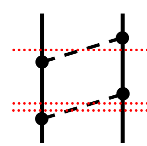
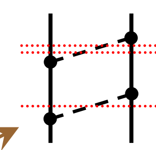
$$g^1: \quad \lambda(H_I - [A^{(1)},\ H_0])\eta = 0 \qquad \Longrightarrow \qquad A^{(1)} = -\lambda \frac{H_I}{E_\eta - E_\lambda} \eta$$

$$g^2: \quad \lambda(H_I\, A^{(1)} - [A^{(2)},\ H_0])\eta = 0 \qquad \Longrightarrow \qquad A^{(2)} = -\lambda \frac{H_I\, A^{(1)}}{E_\eta - E_\lambda} \eta$$

$$\ldots$$

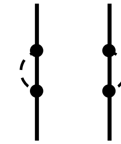
In the static approximation, i.e. in the limit $m \to \infty$, one has: $E_\eta - E_\lambda \sim E_\pi$. One obtains:

$$\begin{aligned} V_{\rm eff} \;=\; & - \eta H_I \frac{\lambda}{E_\pi} H_I \eta \\ & \underbrace{- \eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta}_{} + \underbrace{\frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta}_{} + \underbrace{\frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta}_{} + \ldots \end{aligned}$$

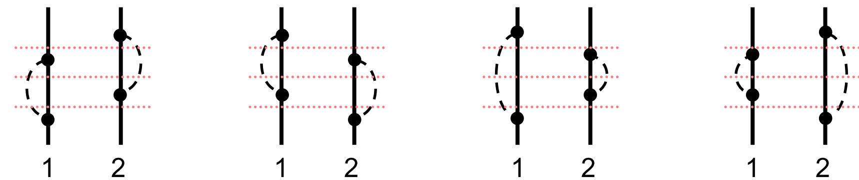
same as in old-fashioned perturbation theory

wave-function renormalization (missing in old-fashioned perturbation theory)

Consider contributions due to one self-energy insertion at each nucleon: 

Nucleon self-energies lead to wave-function and mass renormalization (i.e. contribute to the 1N Hamilton operator). Expect no contributions to the 2N Hamilton operator!

old-fashioned perturbation theory



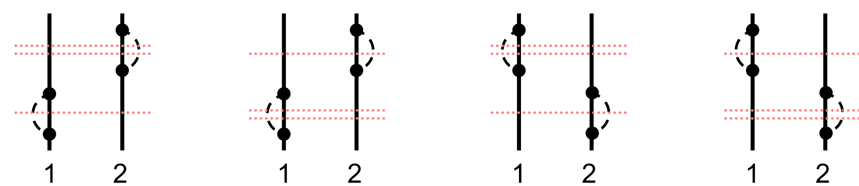
$$
\begin{aligned}
V_{\rm eff}^{\rm TD} &= -\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \\
&= \mathcal{M} \left(-\frac{2}{\omega_1 \omega_2 (\omega_1 + \omega_2)} - \frac{1}{\omega_1^2 (\omega_1 + \omega_2)} - \frac{1}{\omega_2^2 (\omega_1 + \omega_2)} \right) \\
&= \mathcal{M} \left(-\frac{1}{\omega_1^2 \omega_2} - \frac{1}{\omega_1 \omega_2^2} \right) \quad \text{where} \quad \omega_i \equiv \sqrt{\vec{p}_i^{\,2} + M_\pi^2}
\end{aligned}
$$

common isospin, spin & momentum structure (depends on the form of H_I)

What is wrong ??

method of unitary transformation

Additional contributions
(wave-function renormalization)

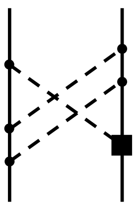


$$
V_{\rm eff} = V_{\rm eff}^{\rm TD} + \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta + \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta = V_{\rm eff}^{\rm TD} + \mathcal{M} \left(\frac{1}{\omega_1^2 \omega_2} + \frac{1}{\omega_1 \omega_2^2} \right) = 0
$$

Application to chiral Lagrangians (E.E. et al., '98)

expansion in g ⇒ chiral expansion

Power counting



$\sim \left(\frac{Q}{\Lambda}\right)^{\nu}$ ← count powers of Q using dim. analysis

an alternative: count powers of Λ!

The scale Λ may only arise through coupling constants ⇒ $\boxed{\nu = -2 + \sum_i V_i \kappa_i}$

$$\mathcal{L}_i = c_i \, (N^\dagger(\ldots)N)^{\frac{n_i}{2}} \, \pi^{p_i} \, (\partial_\mu, M_\pi)^{d_i} \quad \Rightarrow \quad [c_i] = (mass)^{-\kappa_i} \quad \textit{with} \quad \kappa_i = d_i + \tfrac{3}{2} n_i + p_i - 4$$

Remember:

- $\kappa_i < 0$ – *relevant (superrenorm.)*
- $\kappa_i = 0$ – *marginal (renorm.)*
- $\kappa_i > 0$ – *irrelevant (nonrenorm.)*

Examples:

$N^\dagger \boldsymbol{\tau} \vec{\sigma} N \cdot \vec{\nabla} \boldsymbol{\pi} \longrightarrow \kappa_i = 1$

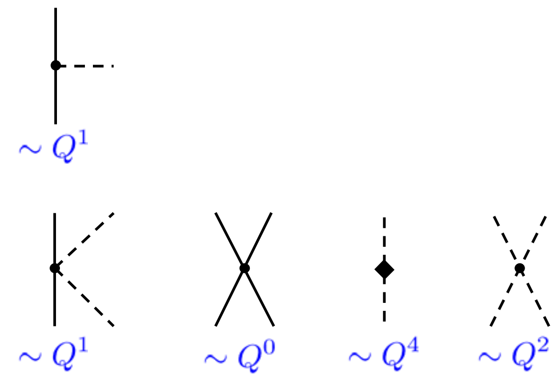
$(N^\dagger N)\,(N^\dagger N) \longrightarrow \kappa_i = 2$

- expansion in coupling constant ($H_i \sim g^{n_i}$) ⟷ chiral expansion ($H_i \sim (Q/\Lambda)^{\kappa_i}$)
- perturbation theory works since all $\kappa_i > 0$ (as a consequence of χ–symmetry)

Vertices in $\mathcal{L}_{\rm eff}$:

$\kappa = 1$

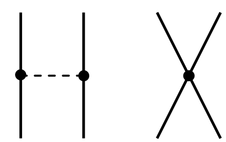
$\kappa = 2$



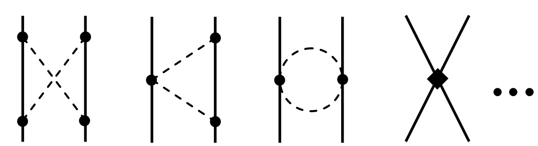
$\vdots$

Power counting: $\boxed{\nu = -2 + \sum_i V_i \kappa_i}$

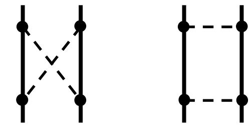
- the leading 2N potential arises at order $\nu = 0$ from graphs with two $\kappa = 1$-vertices or with one $\kappa = 2$-vertex



- no contributions at order $\nu = 1$
- corrections at order $\nu = 2$ from graphs with
  - a) four $\kappa = 1$-vertices,
  - b) two $\kappa = 1$-vertices and one $\kappa = 2$-vertex,
  - c) two $\kappa = 2$-vertices,
  - d) one $\kappa = 4$-vertex



Example: chiral 2π-exchange potential proportional to $g_A{}^4$:



$$V_{2\pi}^{(2)}(q) = -\,\eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \;+\; \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta \;+\; \frac{1}{2}\eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta$$

$$= -\frac{g_A^4}{2(2F_\pi)^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{\omega_+^3 \omega_-^3 (\omega_+ + \omega_-)} \left\{ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\vec{l}^{\,2} - \vec{q}^{\,2} \right)^2 + 6(\vec{\sigma}_2 \cdot [\vec{q} \times \vec{l}\,])(\vec{\sigma}_1 \cdot [\vec{q} \times \vec{l}\,]) \right\}$$

where $\quad \omega_\pm = \sqrt{(\vec{q} \pm \vec{l}\,) + 4M_\pi^2}$

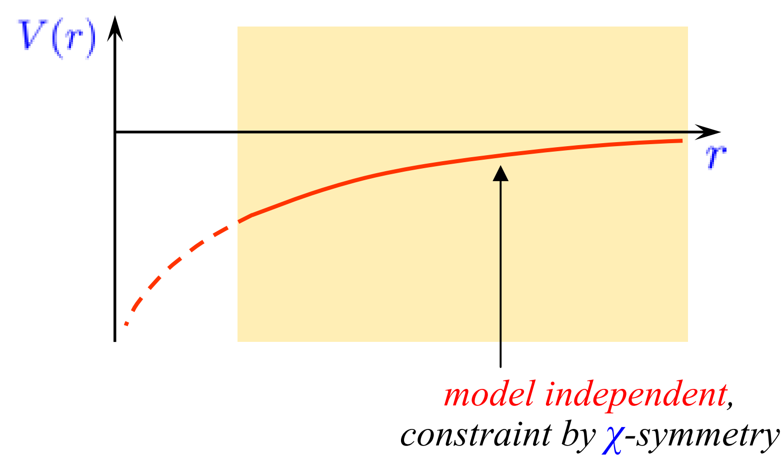
The integral has logarithmic and quadratic divergences, which can be removed by the short-range counter terms:

$$V_{\rm cont} = (\alpha_1 + \alpha_2\, q^2)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \alpha_3 (\vec{\sigma}_1 \cdot \vec{q}\,)(\vec{\sigma}_2 \cdot \vec{q}\,) + \alpha_4 (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\, q^2$$

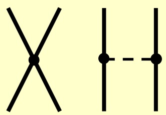
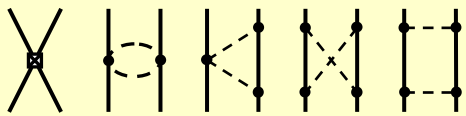
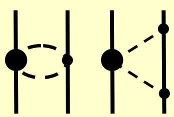
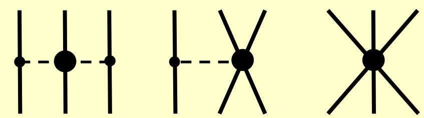
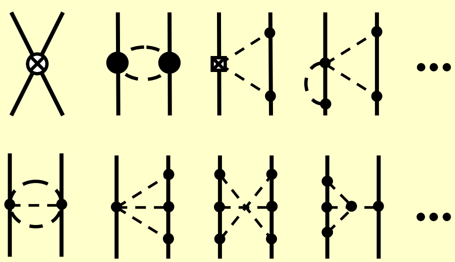
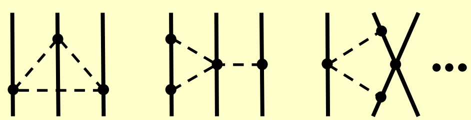
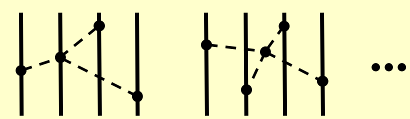
Coordinate space representation:

$$V_{2\pi}^{(2)}(q) \longrightarrow V_{2\pi}^{(2)}(r)$$

The large-r behavior (i.e. the long-range part) of the potential is uniquely determined and does not depend on regularization.



Few-nucleon forces in chiral EFT (Weinberg´s counting)

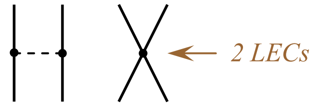
	Two-nucleon force	Three-nucleon force	Four-nucleon force
Q^0		—	—
Q^2		—	—
Q^3			—
Q^4		 work in progress…	

2 nucleon force $\gg$ 3 nucleon force $\gg$ 4 nucleon force . . .

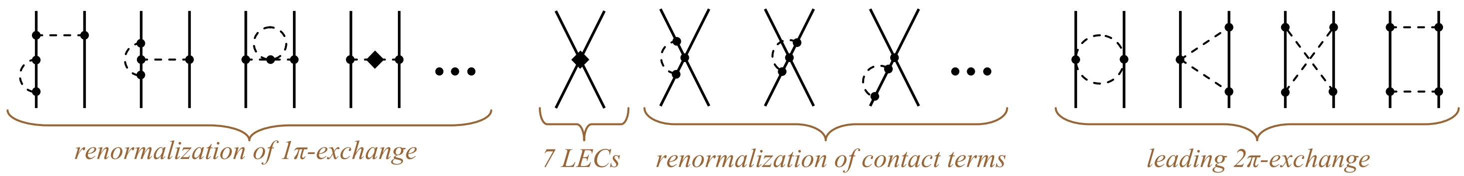
Two-nucleon force

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97;
E.E. et al. '98, '03; Kaiser '99,'00,'01; Higa et al. '03

- **LO:**

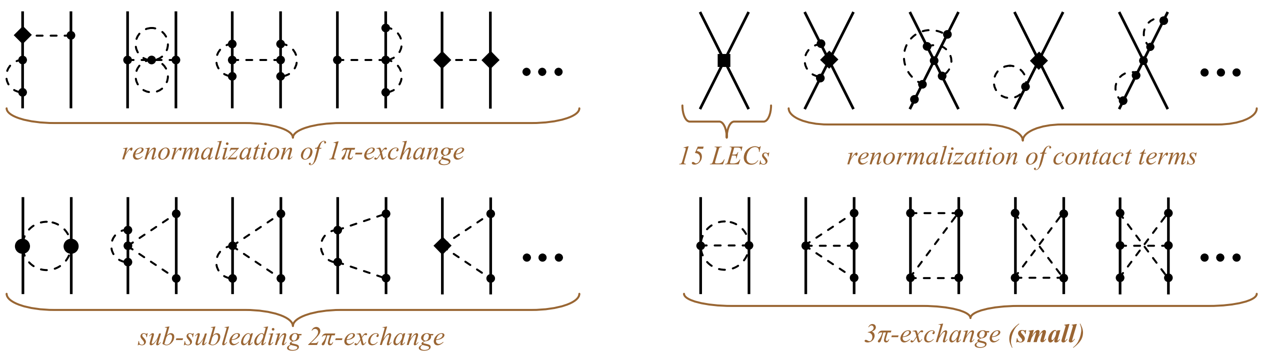


- **NLO:**



- **N^{2}LO:**

- **N^{3}LO:**



- **1/m - corrections** *(Friar '99)*

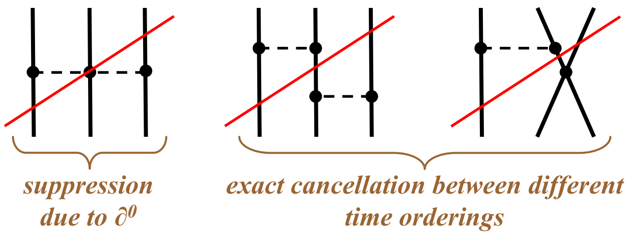
- **Isospin-breaking corrections**

  van Kolck '93, '95; van Kolck et al. '96; Friar et al. '99, '03, '04; Niskanen '02; E.E. & Meißner '05

Three-nucleon force

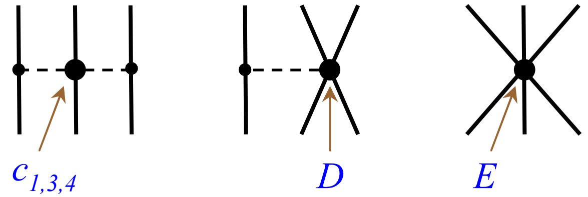
- LO: no 3NF

- NLO: 3NF vanishes
  (if one uses energy-independent formulation)
  Coon & Friar '94; Eden & Gari '96
  Weinberg '91; van Kolck '94, ...



- N^2LO: first nonvanishing 3NF
  van Kolck '94; E.E. et al. '02

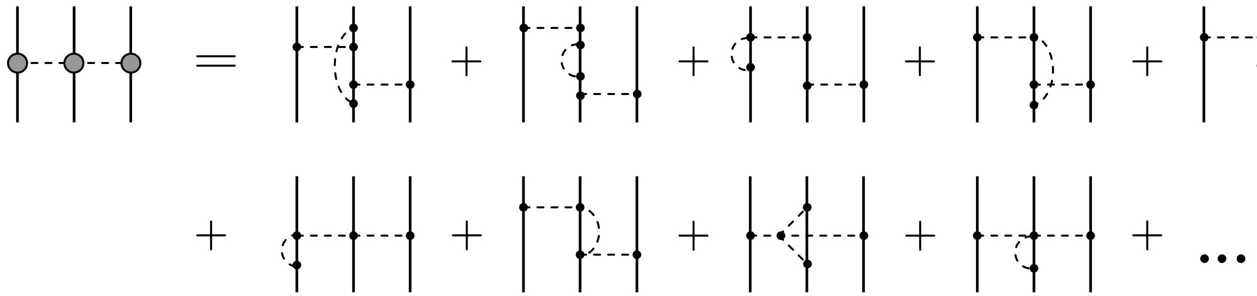
  Notice: need two 3N data points to fix the new LECs D, E



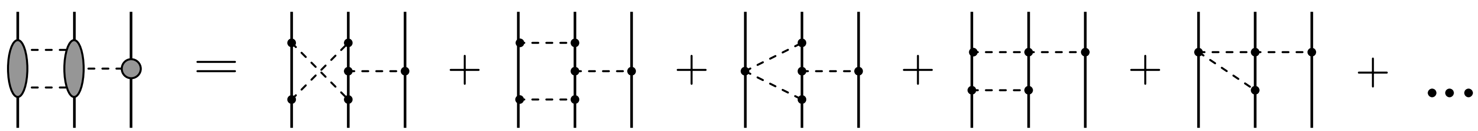
N^3LO: numerous one-loop corrections

(E.E., Veronique Bernard and Ulf-G. Meißner, work in progress)

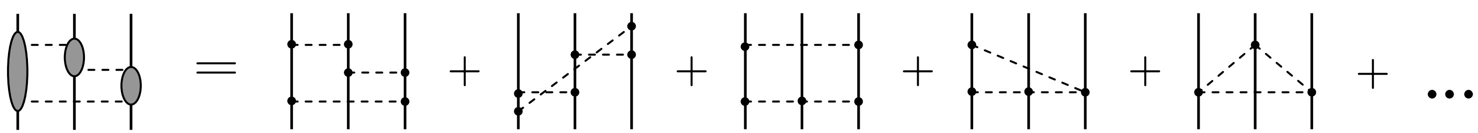
2π - exchange



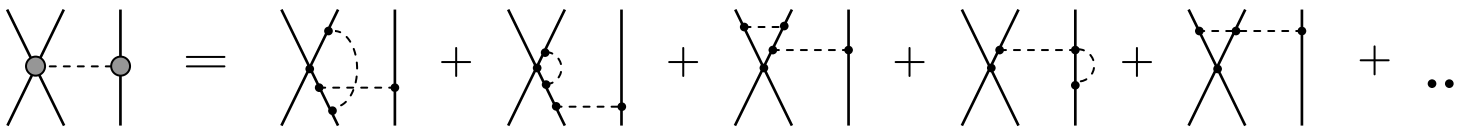
2π - 1π - exchange



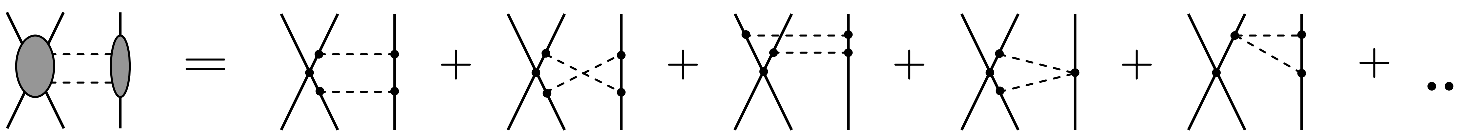
2π - exchange between all 3 nucleons



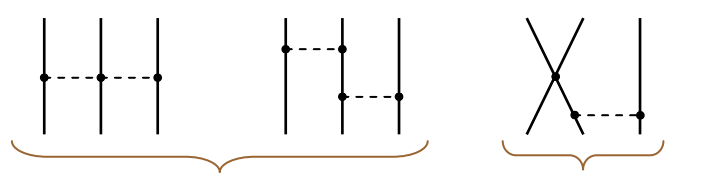
- contact - 1π - exchange



- contact - 2π - exchange



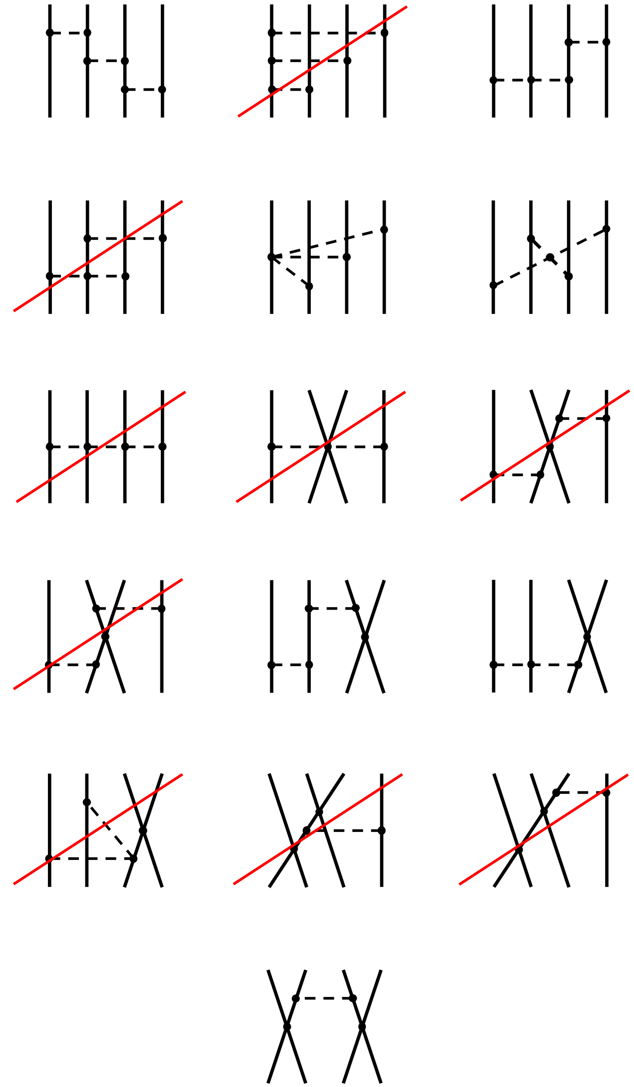
- one also has to take into account relativistic $1/m$ - corrections to:



<u>**Notice:**</u> 3NF at N^3LO is parameter-free !!

Four-nucleon force *(E.E. '05)*

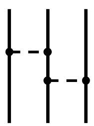
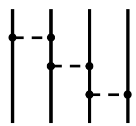
- first shows up at N^3LO
- chiral symmetry plays a crucial role
- parameter-free



Notice:

- disconnected and many connected graphs lead to vanishing contributions

- reducible-like diagrams contribute even in the static limit, for example:

 $= 0$ but:  $\neq 0$

$$\propto \frac{F}{[\vec{q}_1^{\,2} + M_\pi^2]\,[\vec{q}_{23}^{\,2} + M_\pi^2]^2\,[\vec{q}_4^{\,2} + M_\pi^2]}$$

- attractive contributions to the ^{4}He BE of the order of few 100 keV *(preliminary results by Rozpedzik, Golak et al., work in progress)*

Summary

- Various methods to derive nuclear potentials from the effective chiral Lagrangian have been introduced.
- The structure of 2N, 3N and 4N forces upto N^3LO in the chiral expansion has been outlined.
- Do these novel chiral forces really work ??

Applications of the chiral forces to few-nucleon systems will be considered in lecture 4 …