QCD and Rescattering in Nuclear Targets
Lecture 4

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Coherent Multiple Scattering

- Hard probe and its probing size
- Coherent multiple scattering and power corrections
  - —Resummation of power corrections to DIS SFs
- Universal nuclear dependence in nPDFs
  - —Resummation of power corrections to nPDFs
- Coherent multiple scattering in Drell-Yan production
- Summary

Hard probe and its probing size

- **Hard probe – process with a large momentum transfer:**

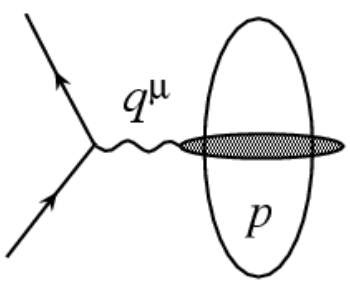
$$q^\mu \quad \text{with} \quad Q \equiv \sqrt{\left|q^2\right|} \gg \Lambda_{\text{QCD}}$$

- **Size of a hard probe is very localized and much smaller than a typical hadron at rest:**

$$\frac{1}{Q} \ll 2R \sim \text{fm}$$

- **But, it might be larger than a Lorentz contracted hadron:**

$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2R\left(\frac{m}{p}\right) \quad \text{or equivalently} \quad x \ll x_c \equiv \frac{1}{2mR} \sim 0.1$$

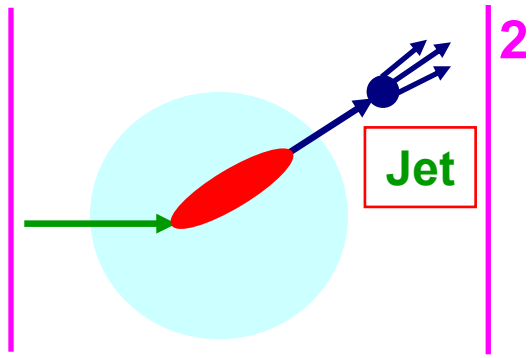


If an active parton *x* is small enough
the hard probe could cover several nucleons
In a Lorentz contracted large nucleus!

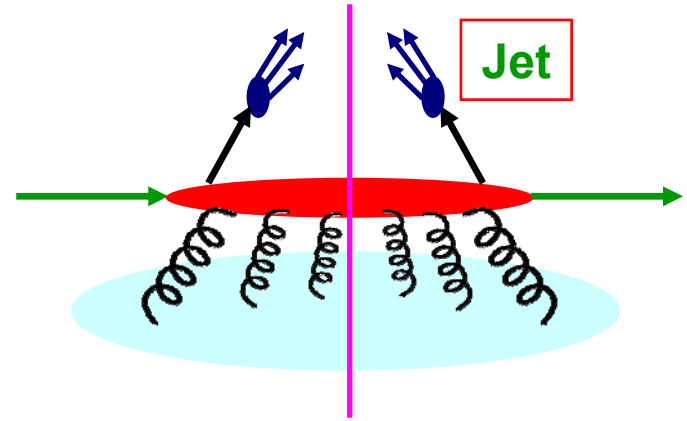
Coherence soft multiple scattering

- **Strong quantum interference between scattering centers**

$\sigma_{\text{Jet}} \propto$



$\propto$



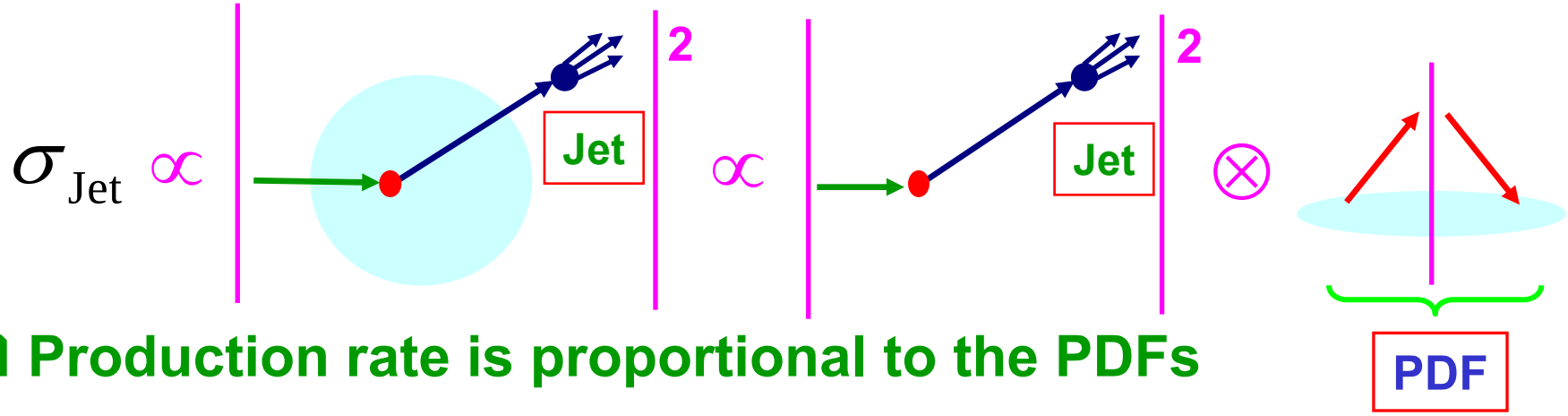
- **Modify production rate as well as jet spectrum**

- **Nuclear dependence from multi-parton correlations**
  - **Multi-parton correlation functions are process independent if pQCD factorization can be applied**
  - **Fourier transform from momentum to coordinate → universal matrix elements of multiple fields**
  - **no additional scale – power suppressed**

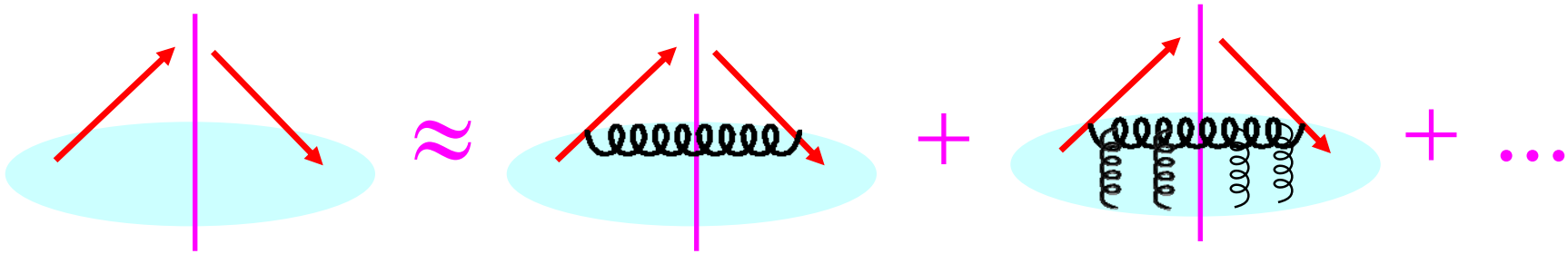
Qiu and Sterman, 2003

Single hard scattering

- **Non-perturbative dynamics is effectively frozen**



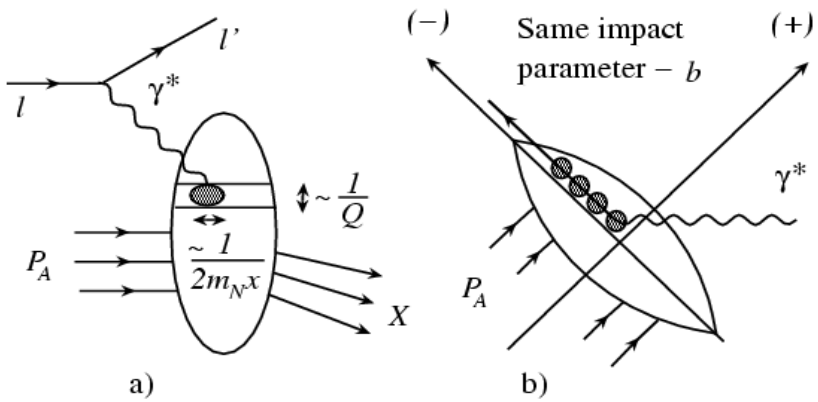
- **Production rate is proportional to the PDFs**
- **Nuclear dependence from nPDFs**
  - **modified DGLAP evolution**
  - **input nPDFs for the evolution**
  - **nPDFs are universal and process independent**



Mueller and Qiu, 1986

Size of the power correction

- Coherent multiple scattering leads to dynamical power corrections:



$$d\sigma \approx d\sigma^{(S)} + d\sigma^{(D)} + \ldots$$

Naïve power counting:

$$\frac{d\sigma^{(D)}}{d\sigma^{(S)}} \sim \alpha_s \frac{1/Q^2}{R^2} \left\langle F^{+\alpha} F_\alpha^{+} \right\rangle A^{1/3}$$

2D lightcone dynamics

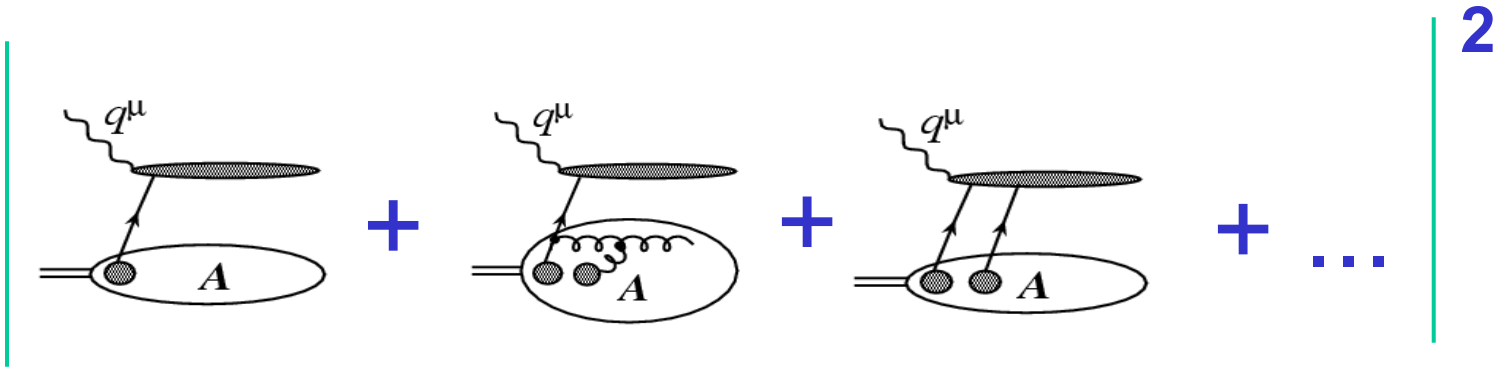
- Characteristic scale for the power corrections: $\left\langle F^{+\alpha} F_\alpha^{+} \right\rangle$
- For a hard probe: $\dfrac{\alpha_s}{Q^2 R^2} \ll 1$
- Enhanced by nuclear radius: $A^{1/3} \leq 6$
- Enhanced by the slope of small-x distribution: $-\dfrac{\partial}{\partial x}\varphi(x)$

Nuclear dependence of observables

- At small x, measured nuclear dependence include both

nuclear dependence from coherent multiple scattering

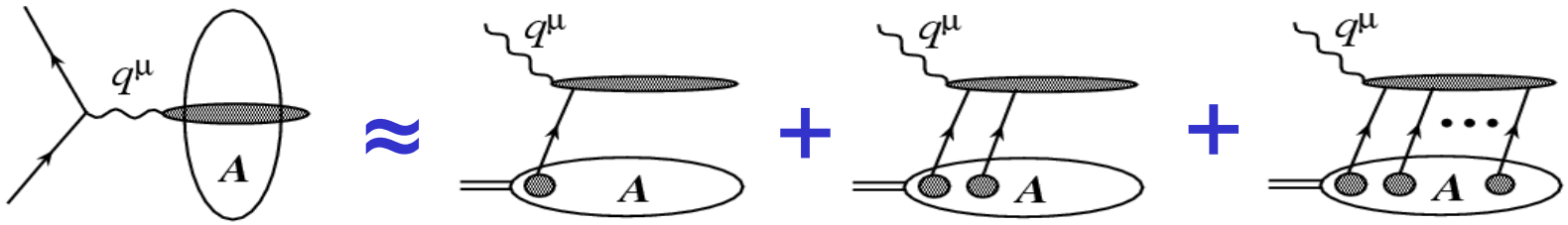
nuclear dependence from nPDFs



- Factorization to separate these two contributions

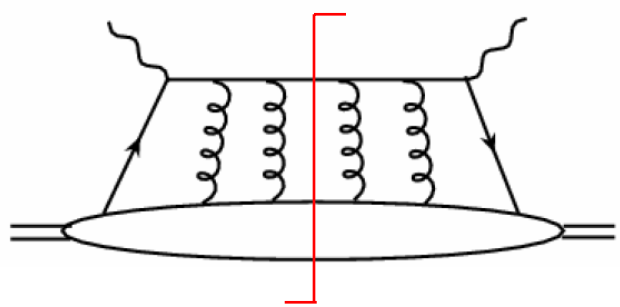
Calculate multi-parton interactions

At small x, the hard probe covers several nucleons, coherent multiple scattering could be equally important at relatively low Q



To take care of the coherence, we need to sum over all cuts for a given forward scattering amplitude

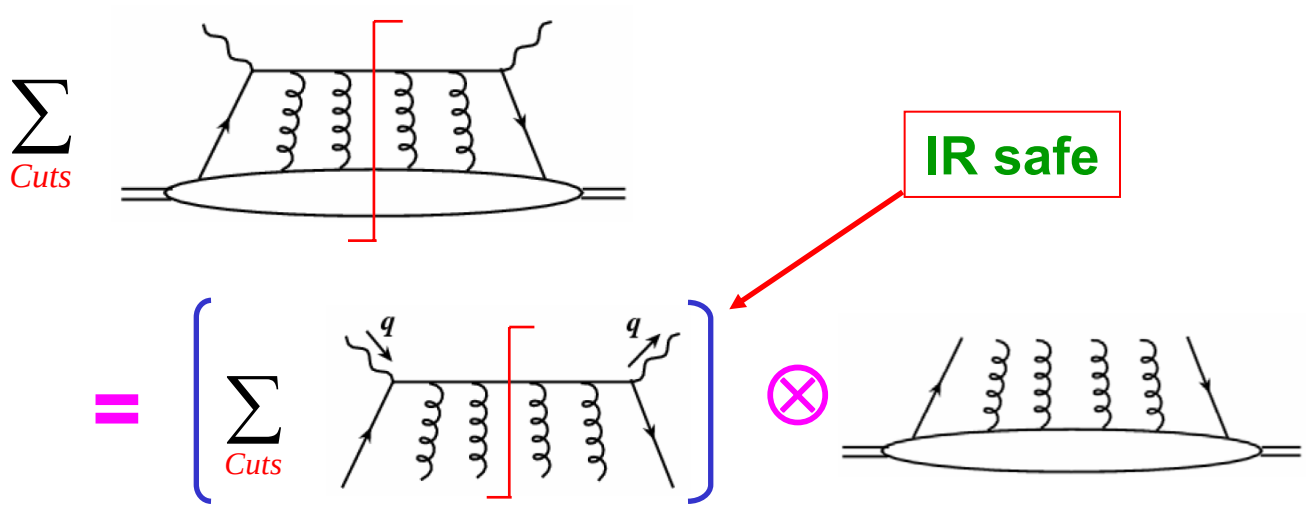
$$\sum_{Scatterings} \sum_{Cuts}$$



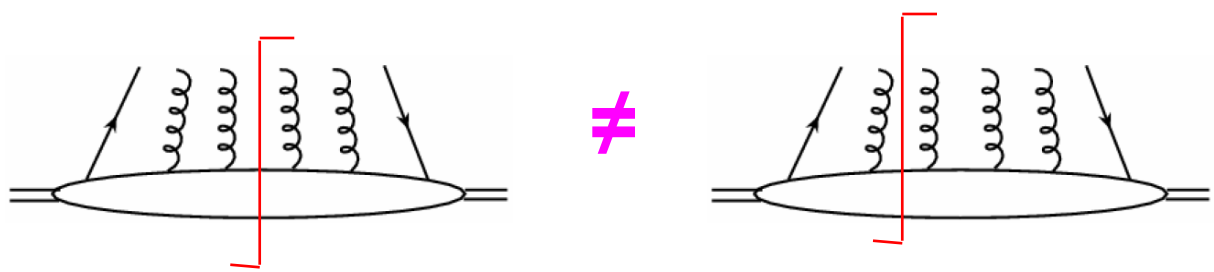
Summing over all cuts is also necessary for IR cancellation

Collinear approximation is important

With collinear approximation:

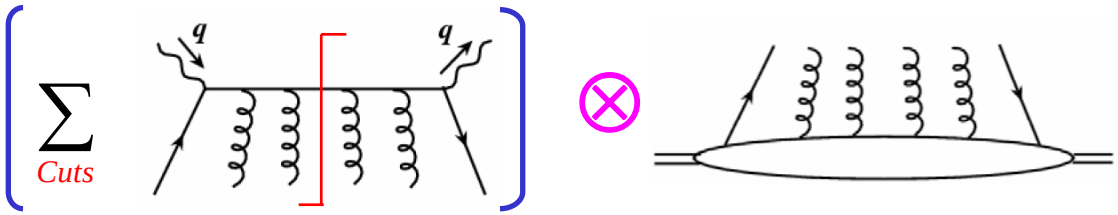


Different cuts for matrix elements of partons with k_T are not equal:



Multiple momentum integrations

- **Parton momentum convolution:**



$$\propto \int \prod_i dy_i^- \; e^{i x_i p^+ y_i^-} \left\langle P_A \middle| \prod_i F^{+\perp}\left(y_i^-\right) \middle| P_A \right\rangle$$

All coordinate space integrals are localized if x is large

- **Leading-pole approximation for dx_i integrals :**
  - dx_i integrals are fixed by the poles (no pinched poles)
  - $x_i=0$ removes the exponentials
  - dy integrals can be extended to the size of nuclear matter

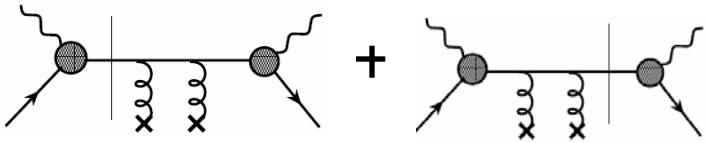
Leading-pole leads to highest powers in medium length, a much smaller number of diagrams to worry about

Multiple soft scattering to inclusive DIS

- **LO contribution to DIS cross section:**

$\Rightarrow \delta(x - x_B)$

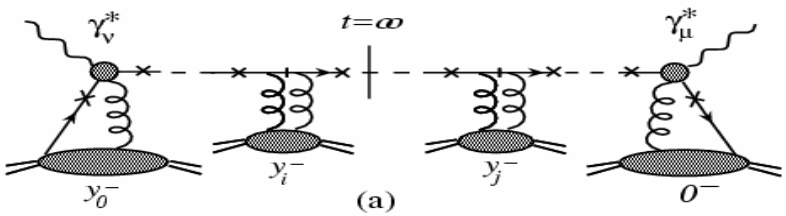
- **NLO contribution:**



$$\Rightarrow \frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right] x_B \lim_{x_1\to x}\left[\frac{1}{x-x_1}\delta(x_1-x_B)+\frac{1}{x_1-x}\delta(x-x_B)\right]$$

$$\int \frac{dy_2^- dy_1^-}{(2\pi)^2}\left[F^{+\alpha}(y_2^-)F_\alpha{}^+(y_1^-)\right]\theta(y_2^-) \qquad x_B\left[-\frac{d}{dx}\delta(x-x_B)\right]$$

- **Nth order contribution:**



$\Rightarrow$

$$\left[\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]\right]^N x_B^N \lim_{x_i\to x}\sum_{m=0}^{N}\delta(x_m-x_B)\left[\prod_{i=1}^{m}\left(\frac{1}{x_{i-1}-x_m}\right)\right]\left[\prod_{j=1}^{N-m}\left(\frac{1}{x_{m+j}-x_m}\right)\right]$$

Infrared safe!

$$x_B^N\left[(-1)^N\frac{1}{N!}\frac{d^N}{dx^N}\delta(x-x_B)\right]$$

Model for the correlation functions

- **Matrix elements:**

$$\left\langle P_A \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(y^-) \left[\prod_{i=1}^{N} \int \tilde{F}^2(0) \right] \right| P_A \right\rangle$$

- **Approximation:**

Nucleus is made of a group of loosely bound nucleons

$$\left| P_A \right\rangle \propto \prod_{i=1}^{A} \left| p \right\rangle \quad \text{with } p = \frac{P_A}{A}$$

$$\left\langle P_A \left| \hat{O}_0 \prod_{i=1}^{N} \hat{O}_i \right| P_A \right\rangle \propto A \left\langle p \left| \hat{O}_0 \right| p \right\rangle \prod_{i=1}^{N} \left\langle p \left| \hat{O}_i \right| p \right\rangle$$

- **Reduce the correlation functions to one unknown – a universal matrix element**

$$\left\langle p \left| F^{+\alpha} F_\alpha^{\ +} \right| p \right\rangle$$

Contributions to DIS structure functions

- **Transverse structure function:**

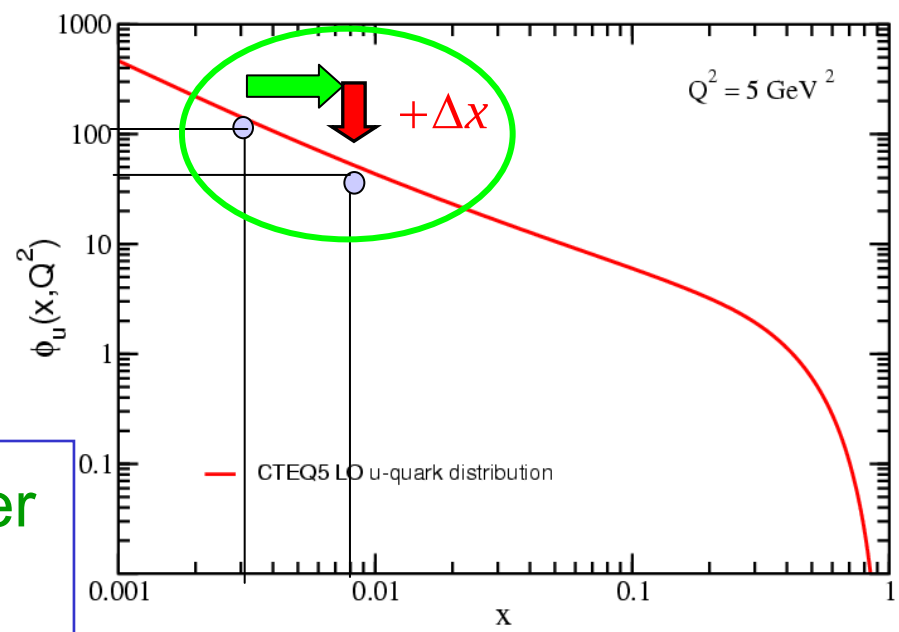
Qiu and Vitev, PRL (2004)

$$F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[\frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)$$

$$\approx F_T^{(0)}(x_B(1+\Delta), Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2} \left(A^{1/3} - 1 \right)$$

$$\xi^2 = \frac{3\pi\alpha_s}{8R^2} \langle F^{+\alpha} F_\alpha{}^{+} \rangle$$

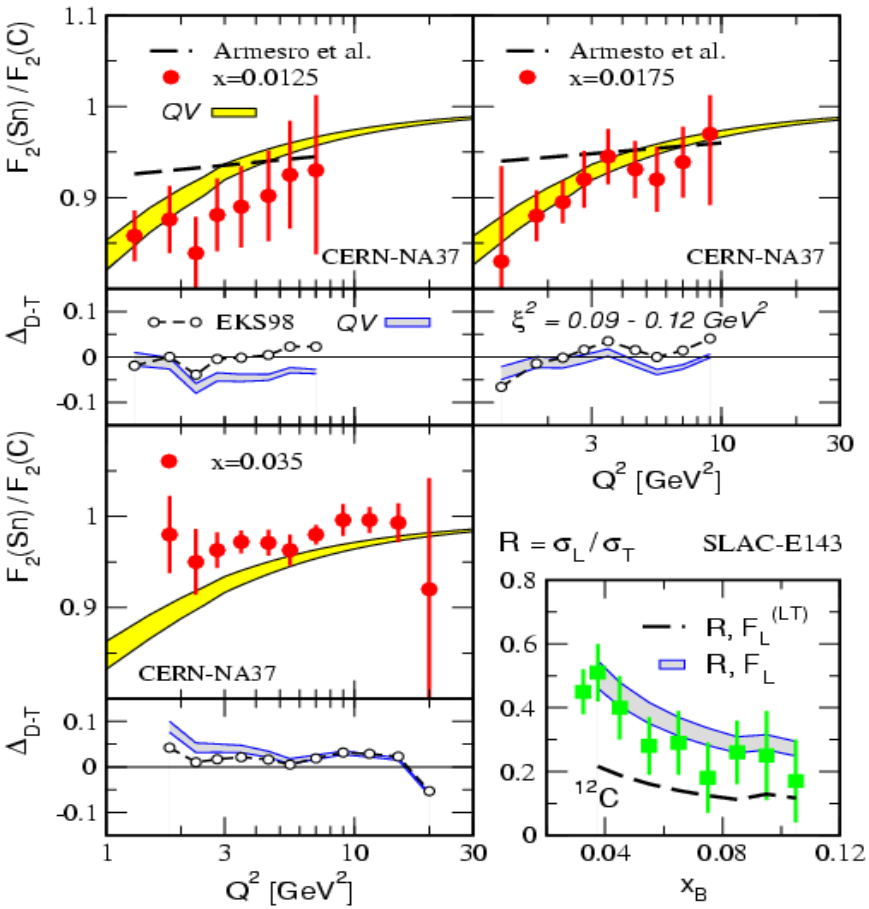


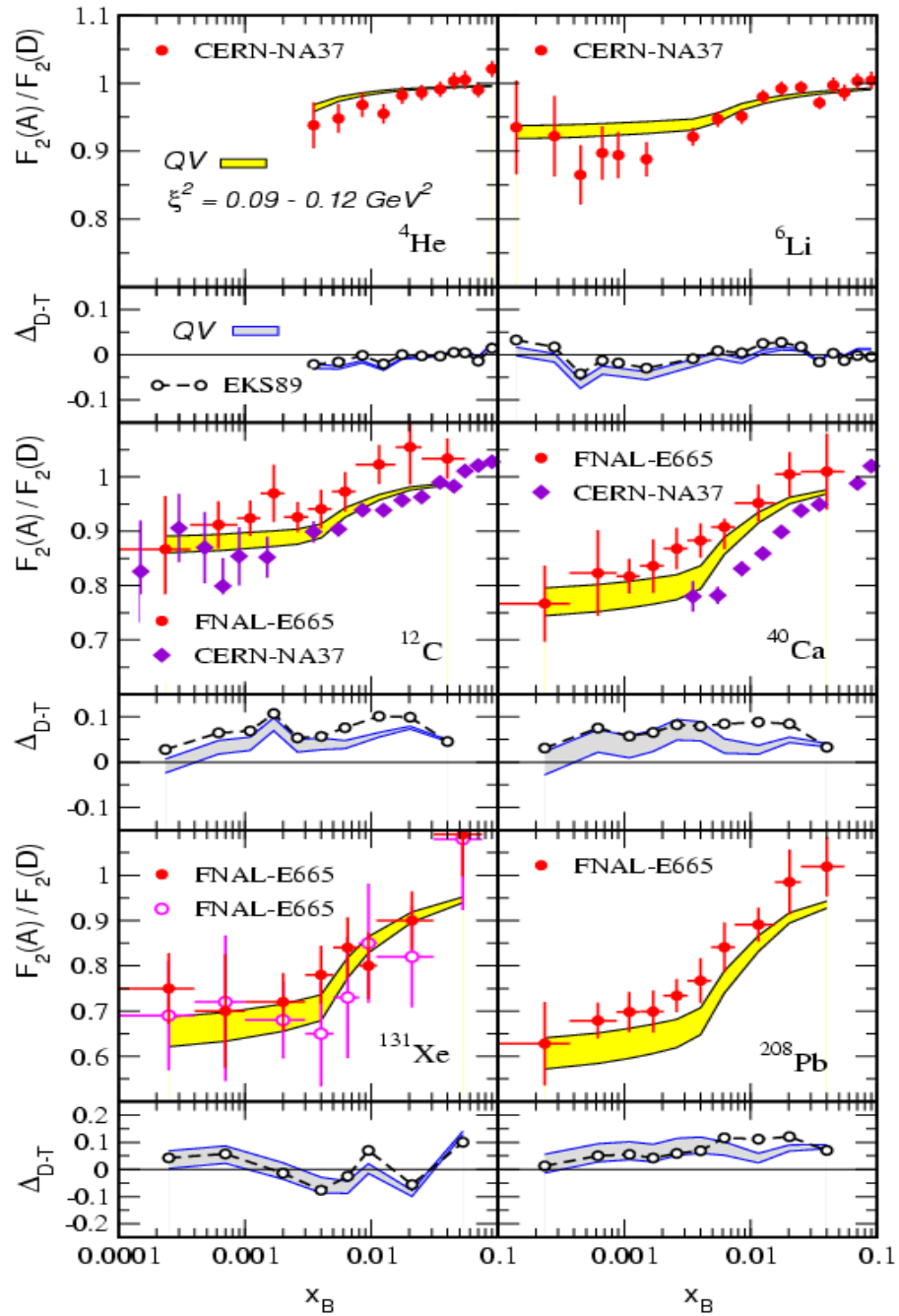
Single parameter for the power correction, and is proportional to the same characteristic scale

- **Similar result for longitudinal structure function**

Neglect LT shadowing upper limit of ξ^2

$\xi^2 \sim 0.09 - 0.12\ \mathrm{GeV}^2$





The Gross-Llewellyn Smith Sum Rules

$$S_{\rm GLS} = \int_0^1 dx\ \frac{1}{2x}\left(xF_3^{\nu N}(x,Q^2) + xF_3^{\bar{\nu} N}(x,Q^2)\right)$$

$$\approx \#U + \#D = 3$$

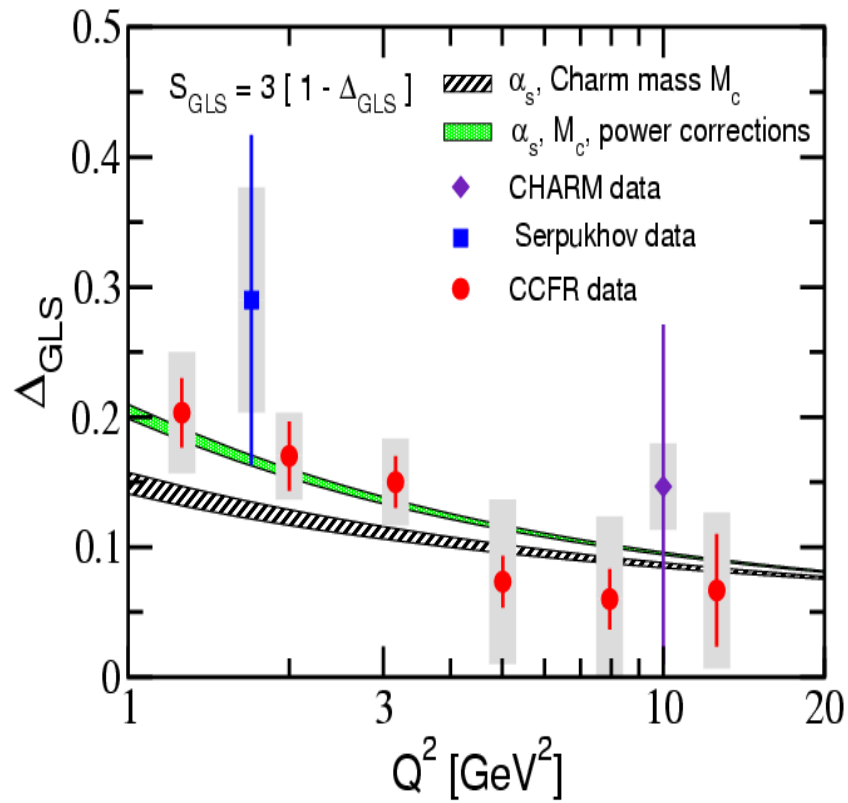
D.J.Gross and C.H Llewellyn Smith , Nucl.Phys. B 14 (1969)

$$\Delta_{\rm GLS} \equiv \frac{1}{3}\left(3 - S_{\rm GLS}\right) = \frac{\alpha_s(Q)}{\pi} + \frac{\kappa}{Q^2} + O\left(\frac{1}{Q^4}\right)$$

Fully coherent final-state power corrections to the sum rule almost cancel due to the unitarity:

$$\int_{-\infty}^{+\infty} dx\ \varphi(x+\Delta x) = \int_{-\infty}^{+\infty} dx\ \varphi(x)$$

But, nuclear enhanced power corrections only for a limited values of $x \in (0, 0.1)$



Qiu and Vitev, Phys.Lett.B 587 (2004)

Prediction is compatible with the trend in the current data

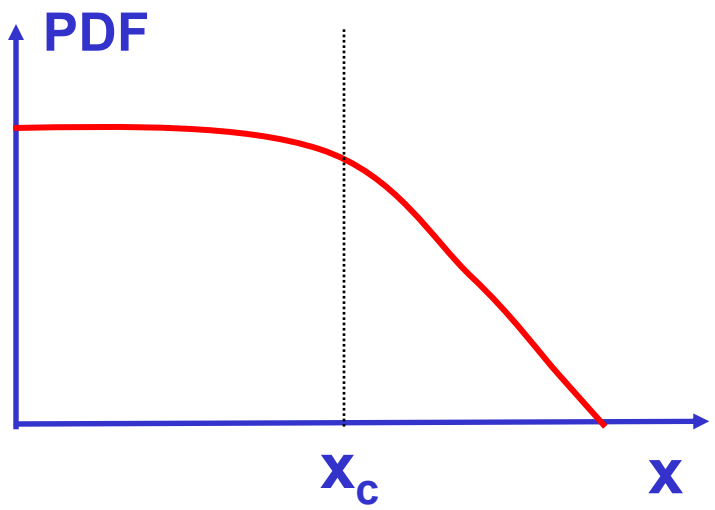
Process-dependent power corrections are important!

Leading twist shadowing

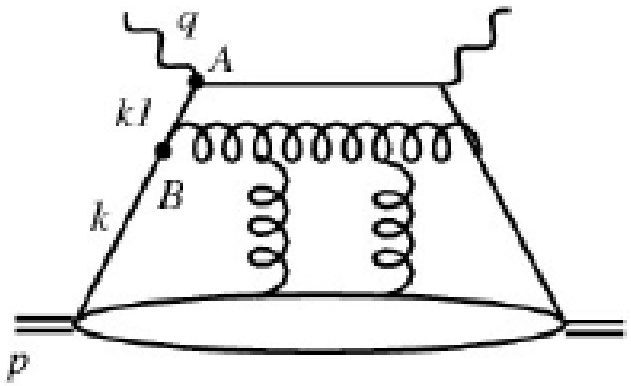
- Power corrections complement to the nuclear dependence in nPDFs:
  - Leading twist shadowing changes the x- and Q-dependence of the parton distributions
  - Power corrections to the DIS structure functions (or cross sections) are effectively equivalent to a shift in x
  - Power corrections vanish quickly as hard scale Q increases while the leading twist shadowing goes away much slower

- If leading twist shadowing is so strong that x-dependence of parton distributions saturates for $x < x_c$,

  additional power corrections, the shift in x, should have no effect to the cross section!



Beyond the tree-level

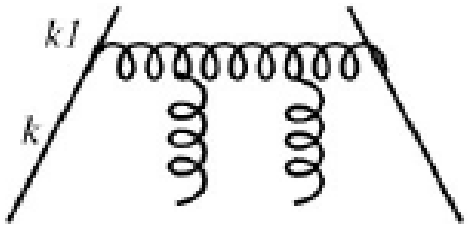


$$\int_0^{-Q^2} \frac{dk_1^2}{(k_1^2)^2}$$

$$k_1^2 \to 0$$

$$t_{AB} \to \infty$$

Collinear divergence



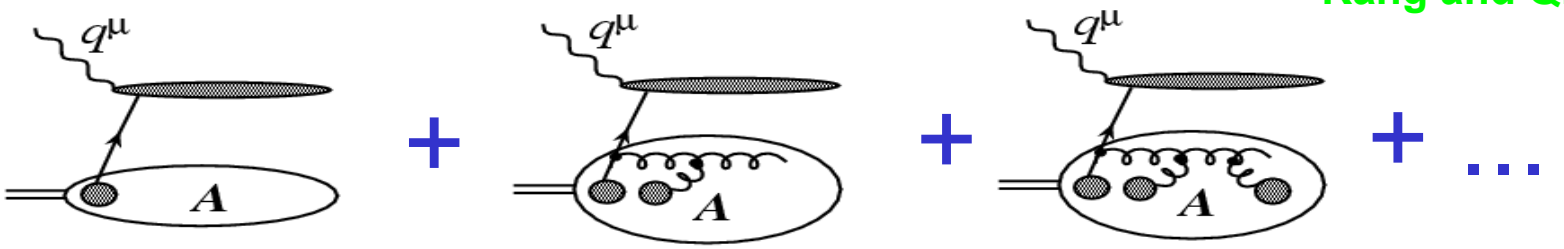
Should be a part of nuclear PDFs

- **But, DGLAP evolved nPDFs do not remove this singularity, nor any collinear divergences beyond single scattering**
- **Redefine nPDFs to include all collinear divergences of partonic subprocesses**

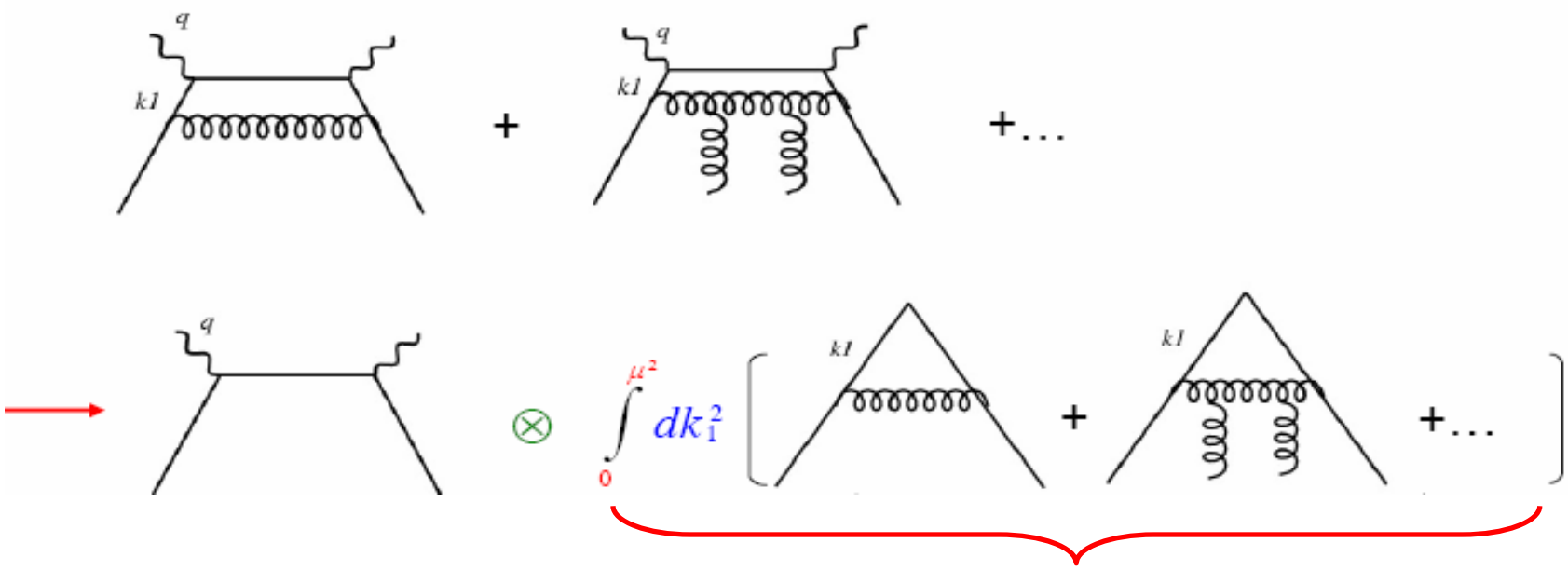
Power corrections to PDFs

- Hard probe sees only one effective parton:

Mueller and Qiu, 86
Kang and Qiu, 06

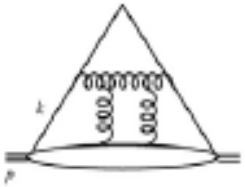


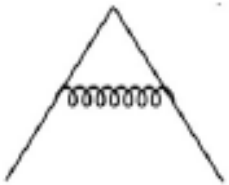
- Pinched poles in the ladder diagrams – corrections to evolution



PDF with modified evolution

Corrections to DGLAP evolution

$\varphi(x,\mu^2) =$  +  +  +…

$\varphi(x,\mu^2) -$  =  $\otimes$ [ +  +…]

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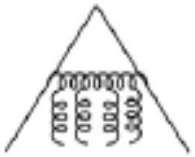
+  $\otimes$ [ +…] +…

Power Correction to evolution

$$\mu^2 \frac{\partial}{\partial \mu^2} \varphi(x,\mu^2) = \gamma\left(\frac{x}{x'},\alpha_s\right) \otimes \varphi(x',\mu^2) - \frac{1}{\mu^2} \overline{\gamma} \otimes \varphi^{(4)}(x',\mu^2) + \ldots$$

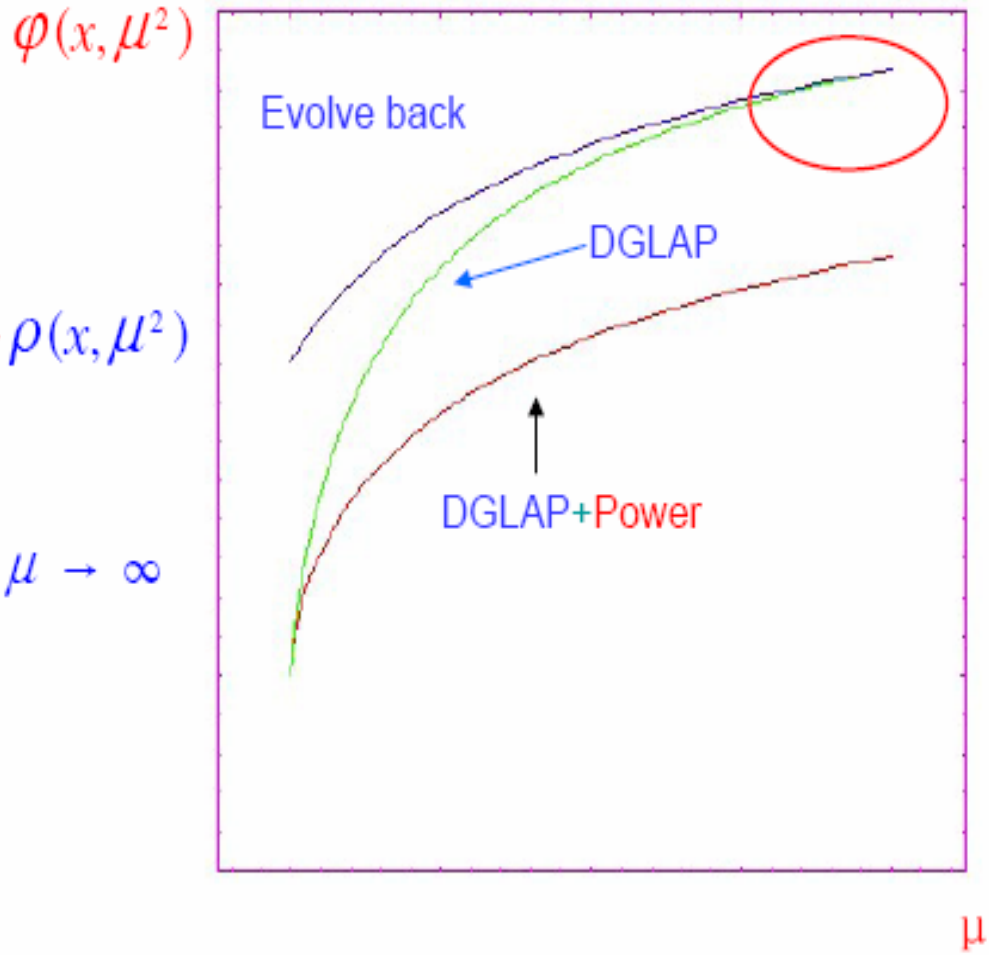
Corrections to PDFs not down by $1/Q^2$

Leading order power correction

$$\frac{\partial \varphi(x,\mu^2)}{\partial \ln \mu^2} = P(x) \otimes \varphi(x,\mu^2) - \frac{1}{\mu^2}\rho(x,\mu^2)$$

$$\int_{\mu_0^2}^{\mu^2} \frac{d\ln\bar{\mu}^2}{\bar{\mu}^2} = \frac{1}{\mu_0^2} - \frac{1}{\mu^2} \rightarrow \frac{1}{\mu_0^2} \quad as\ \mu \rightarrow \infty$$

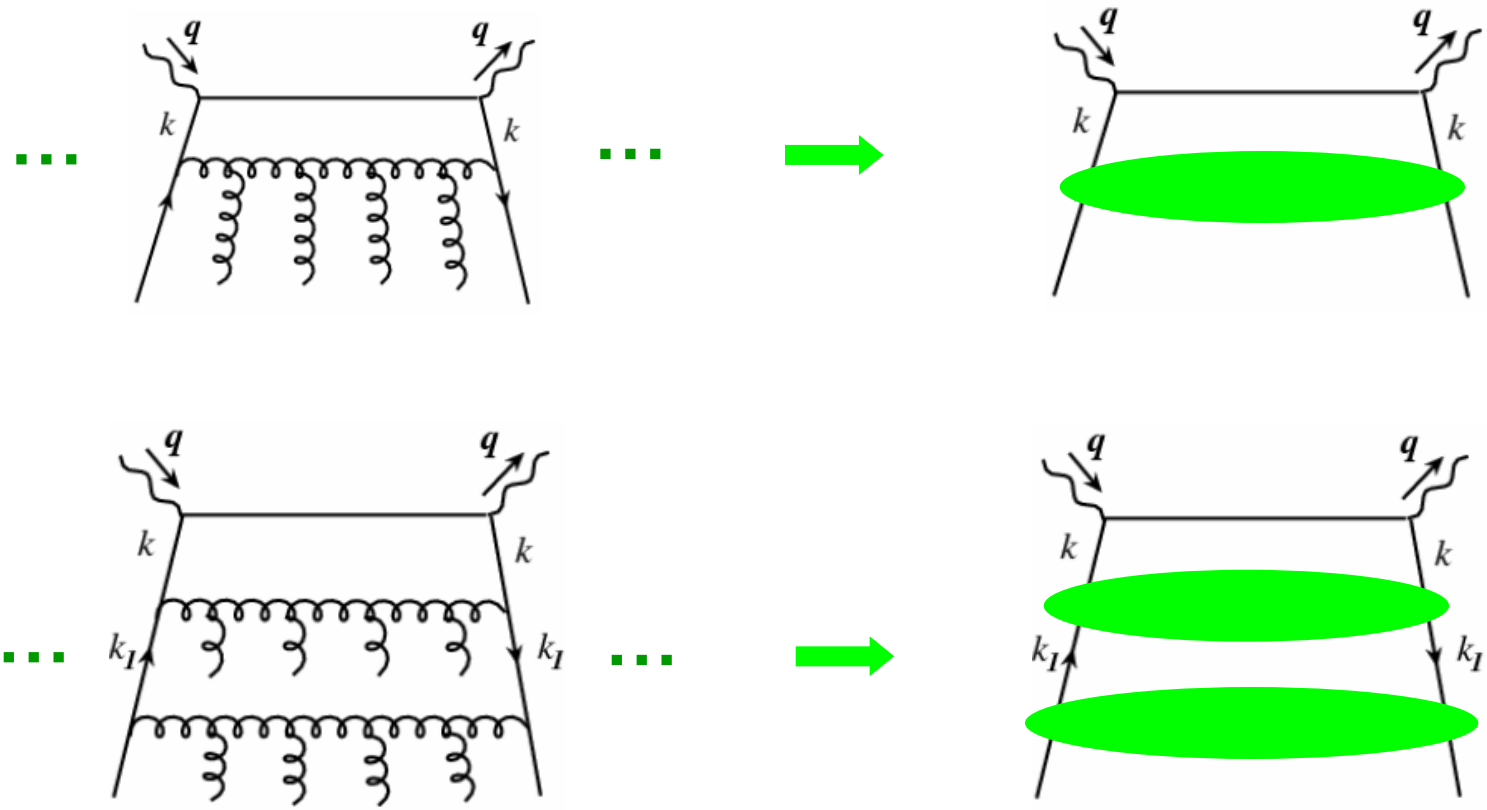
power correction can build up a big effect to low Q^2 distribution



What about high power corrections?

Modified ladder diagrams

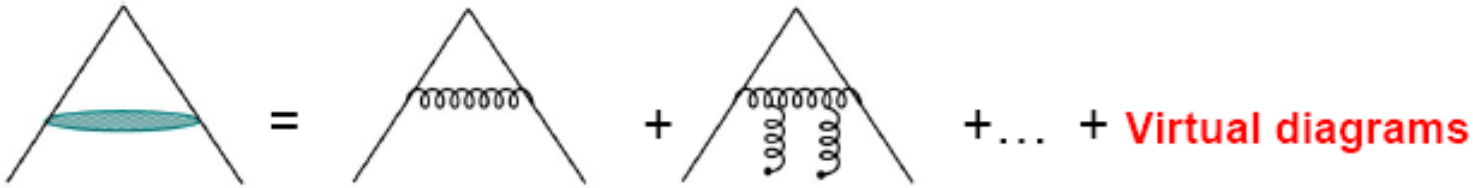
- Leading-pole – Leading $A^{1/3}$ term – less diagrams



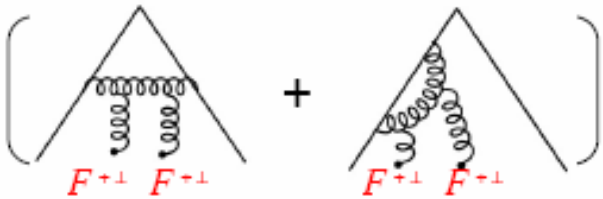
→ Modified DGLAP evolution equations at all powers

Evolution kennels for modified DGLAP

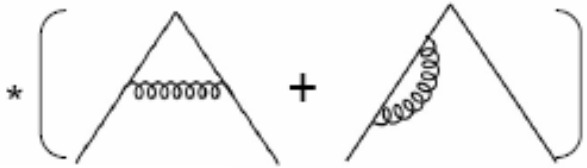
- Modified $q \to q$ evolution kennel:



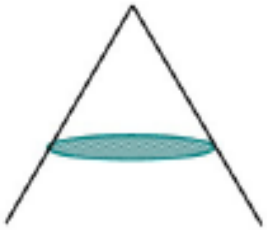
- First non-trivial term:



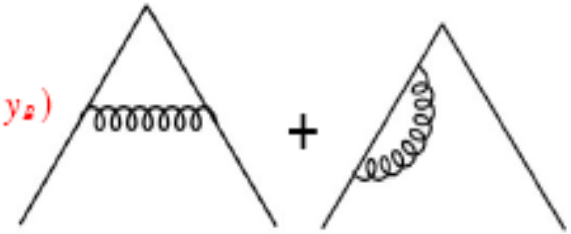
$$\sim \left[< F^{+\perp} F_{\perp}^{+} > \left(\frac{N_c}{N_c^2 - 1} \right) \cdot \left(\frac{1}{2} \right) (-1) \right] \frac{d}{dy} \delta(y - y_B)$$



- Sum of all power corrections:



$$\sim \sum_{N=0} \underbrace{\left[< F^{+\perp} F_{\perp}^{+} > \left(\frac{N_c}{N_c^2 - 1} \right) \cdot \left(\frac{1}{2} \right) (-1) \right]^N \frac{d^N}{dy^N}}_{\text{Translation Operator} \quad e^{-c \frac{d}{dy}}} \delta(y - y_B)$$

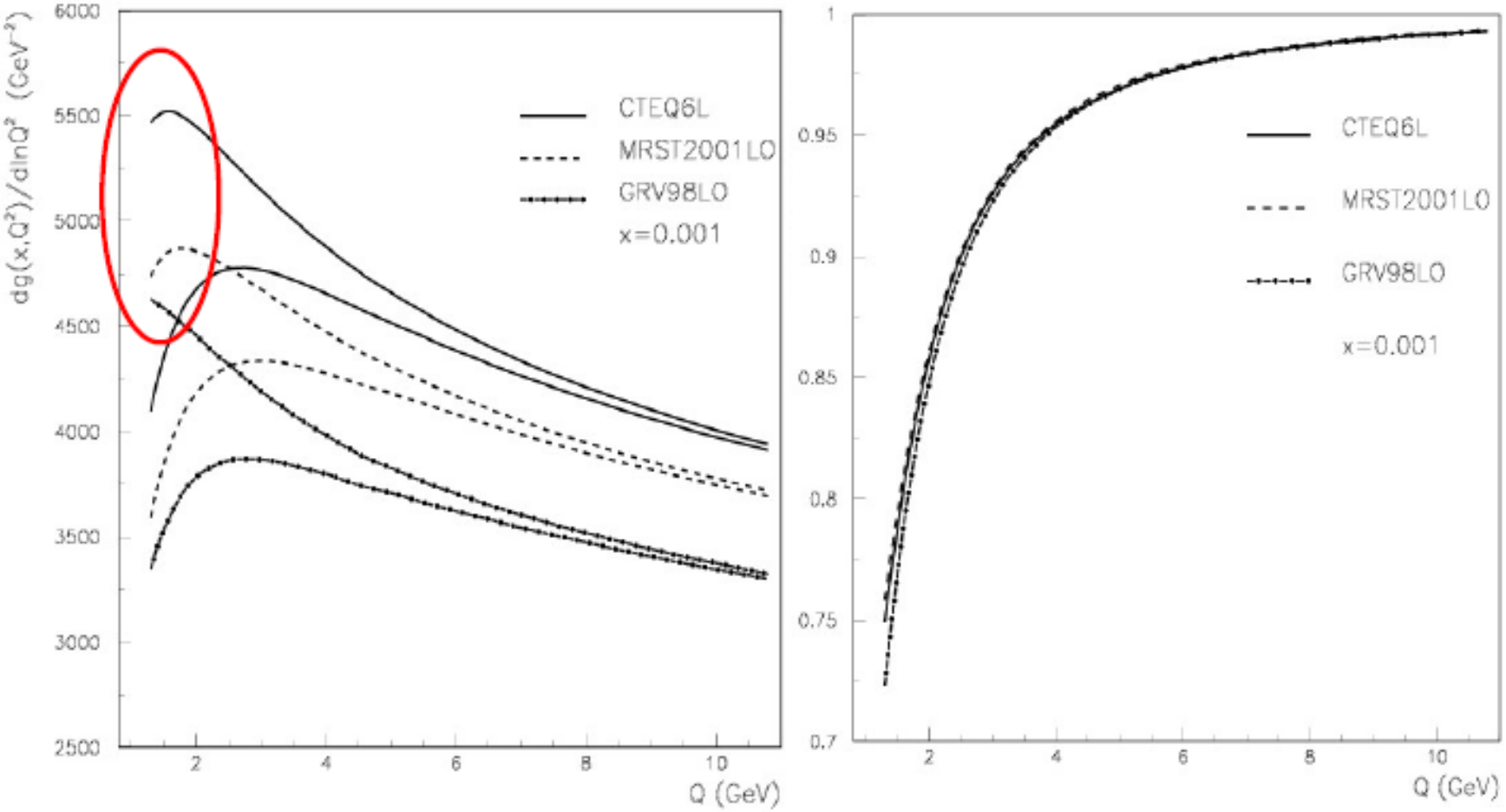


- Similar results for the other kennels

Numerical results

Gluon evolution slope

Kang and Qiu, 06



Important at small x and low Q^2

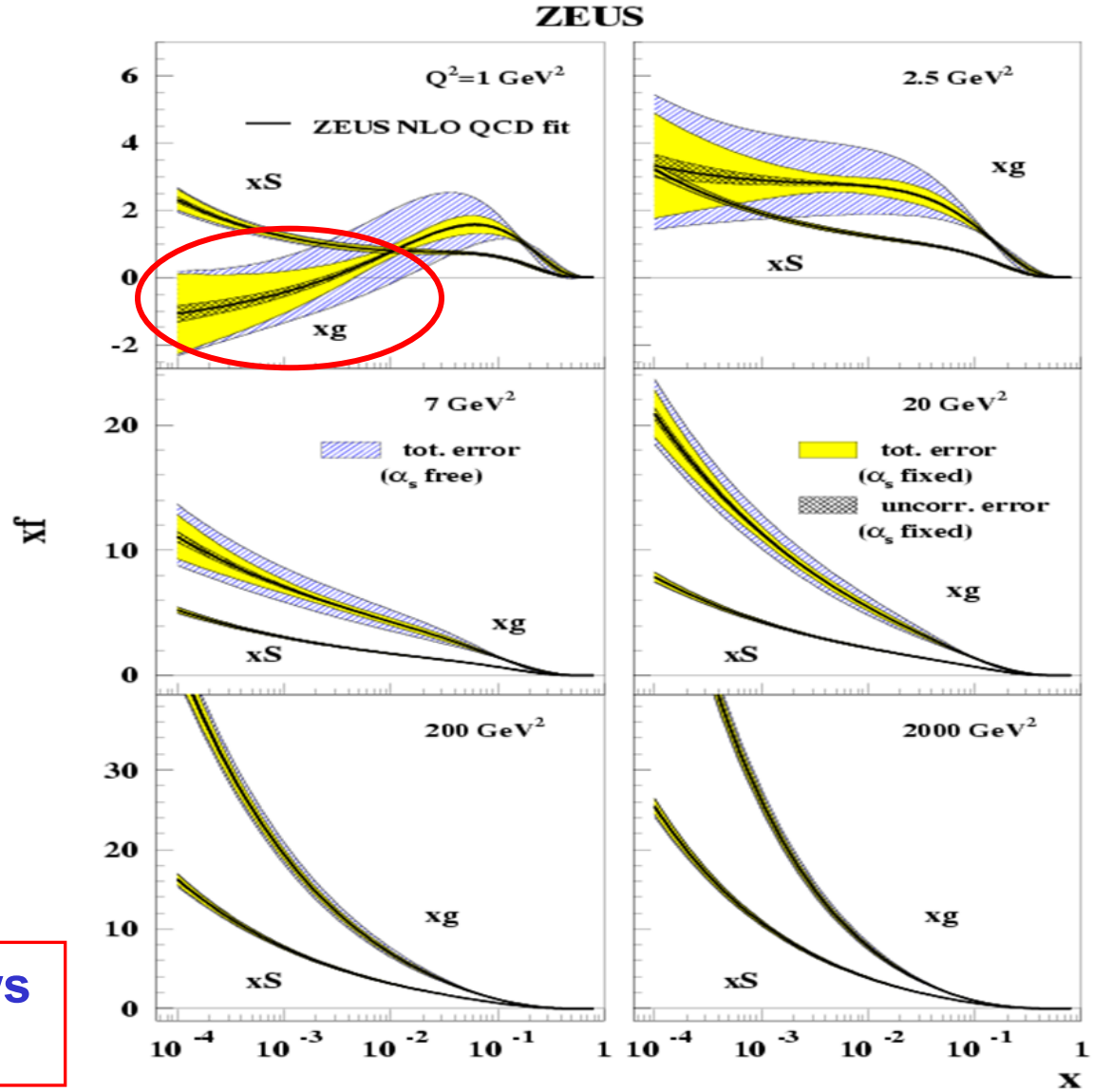
Negative gluon distribution at low Q?

- NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution
- MRST, CTEQ PDF's have the same features

Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

No!

Power corrections slows down small-x evolution

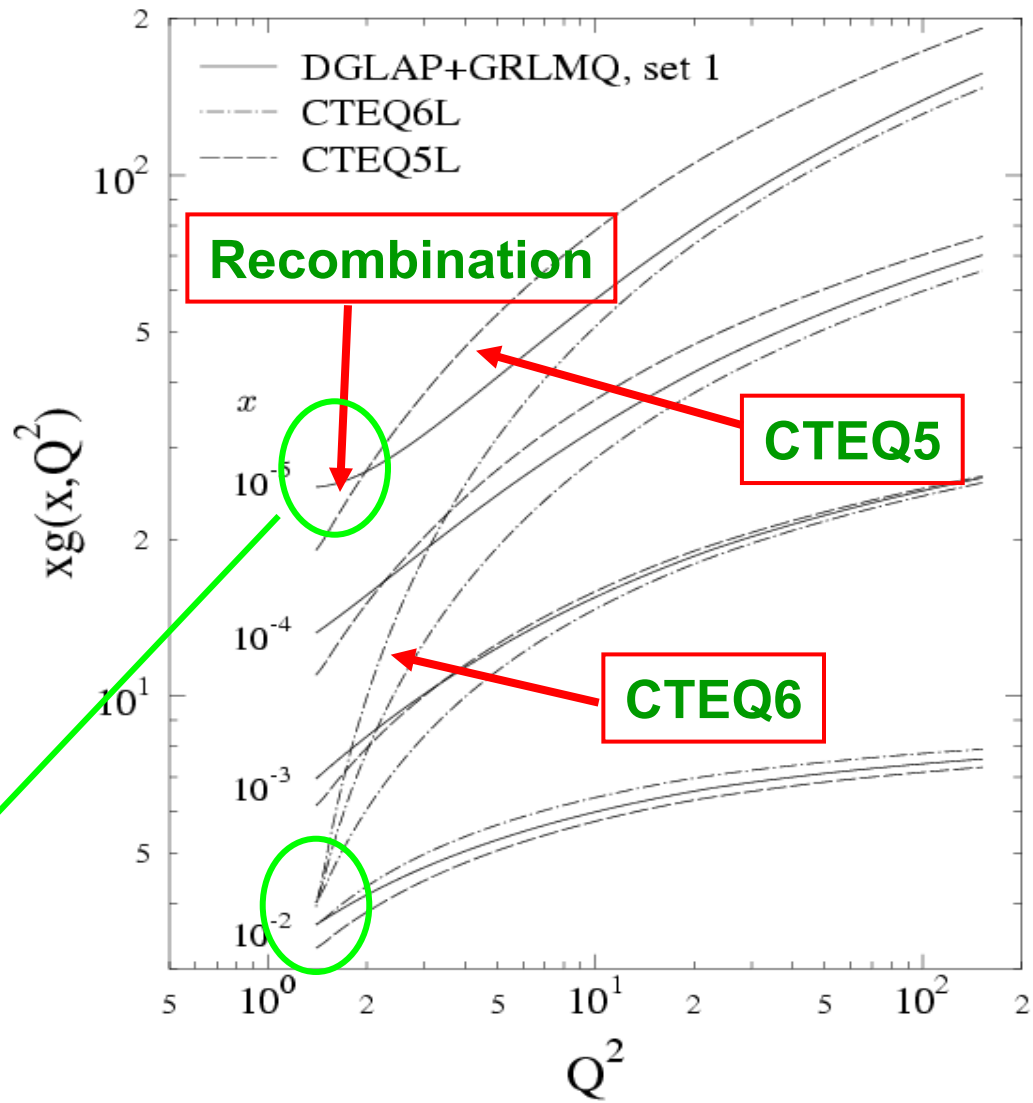


Recombination prevents negative gluon

- In order to fit new HERA data, like MRST PDF's, CTEQ6 gluon has to be much smaller than CTEQ5, even negative at Q = 1 GeV

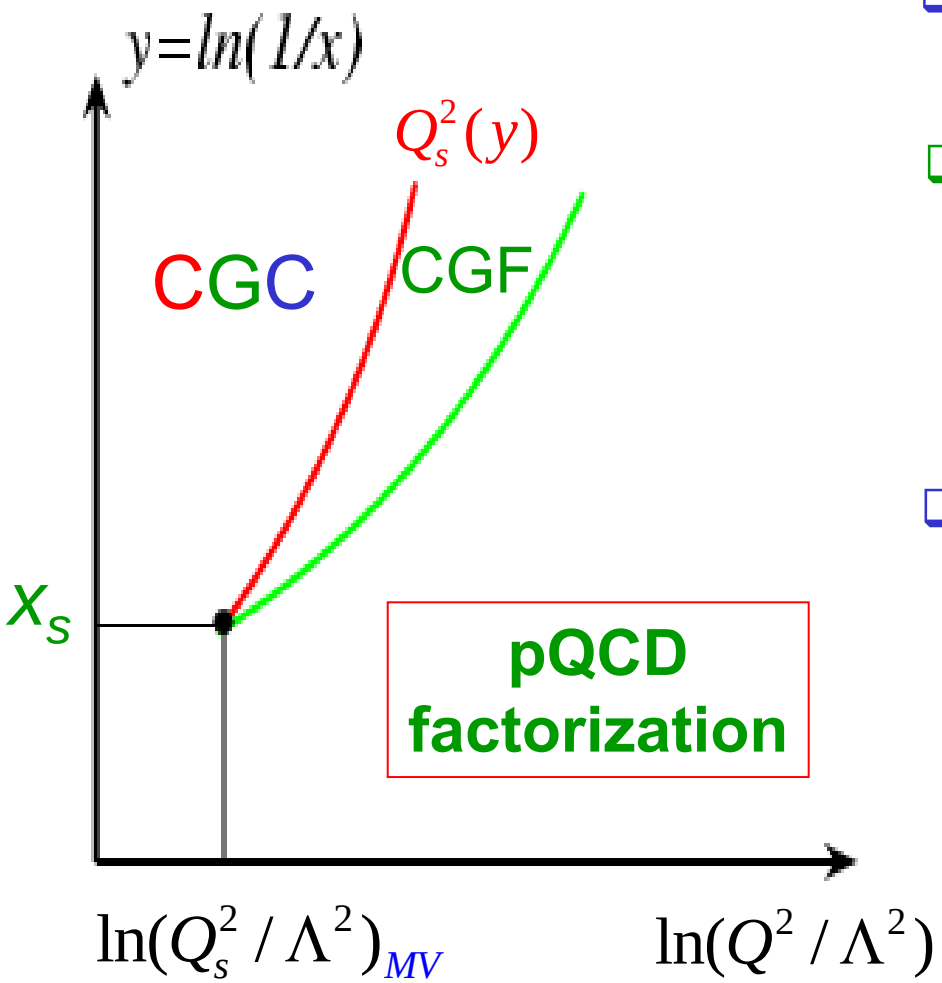
- The power correction to the evolution equation slows down the Q^2-dependence, prevents PDF's to be negative

$$\langle xG(x \to 10^{-5}) \rangle \sim 3$$



Eskola et al. NPB660 (2003)

Phase diagram of parton densities

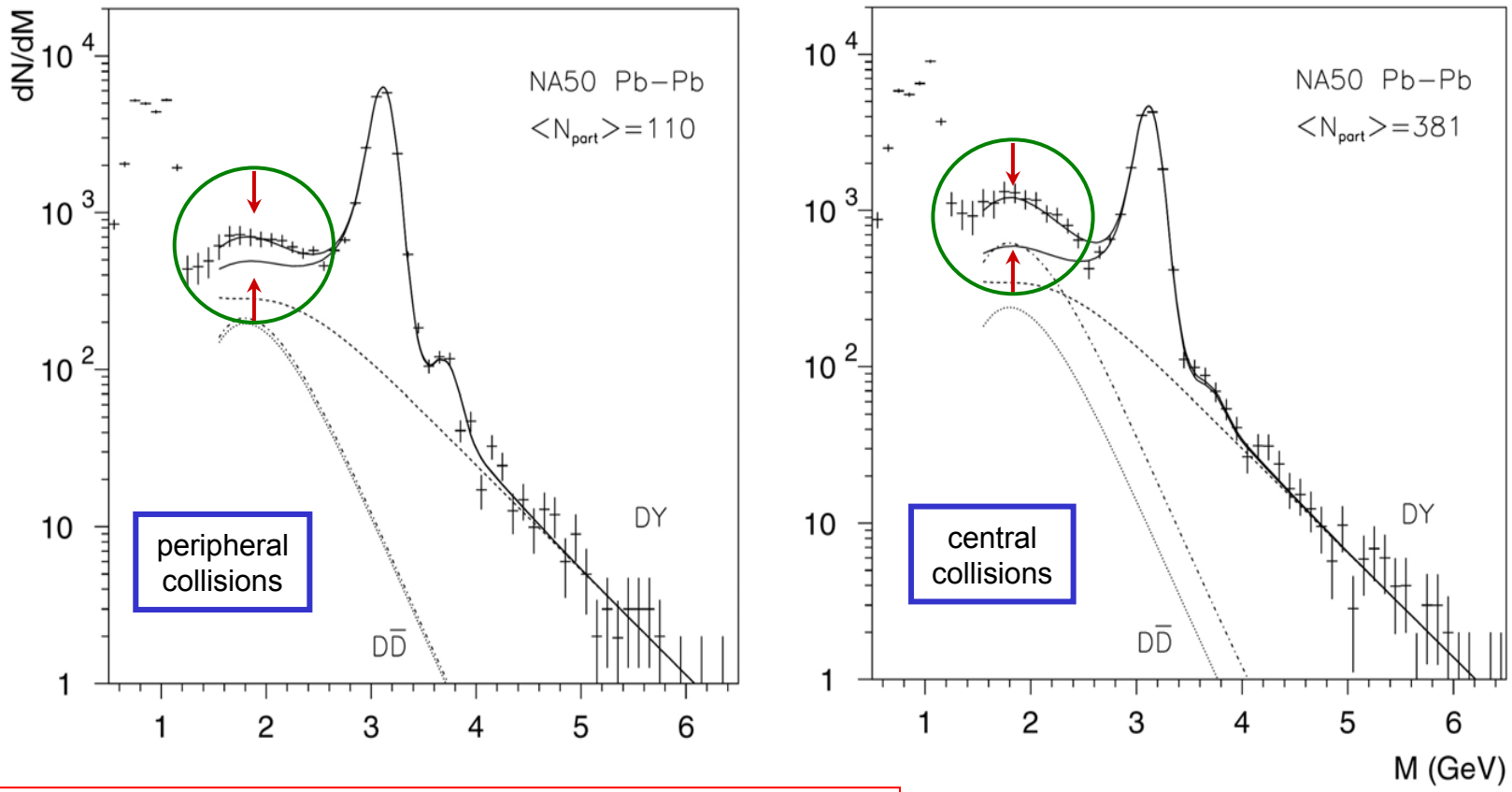


- Experiments measure **cross sections**, not **PDFs**
- PDFs are extracted based on
  - factorization
  - truncation of perturbative expansion
- How to probe the boundary between different regions?

- Look for where pQCD factorization fails
- Power corrections
  - improve predictive power of factorization approach

Drell-Yan at low mass

- **Enhancement of low mass dileptons in heavy ion collisions:**



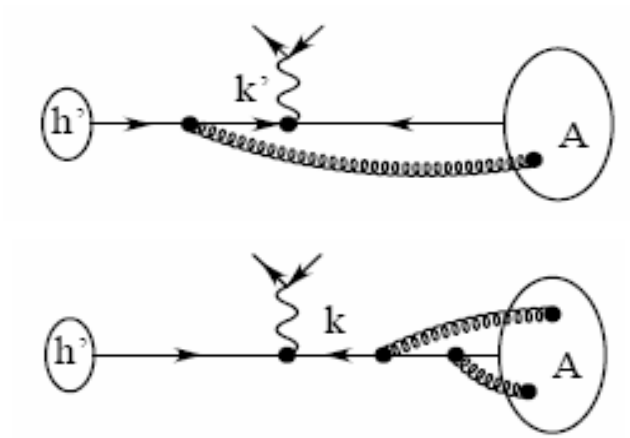
Unlike DIS, coherent power corrections to Drell-Yan cross section are positive!

Figures from Shahoyan's talk

Predicted power corrections to DY

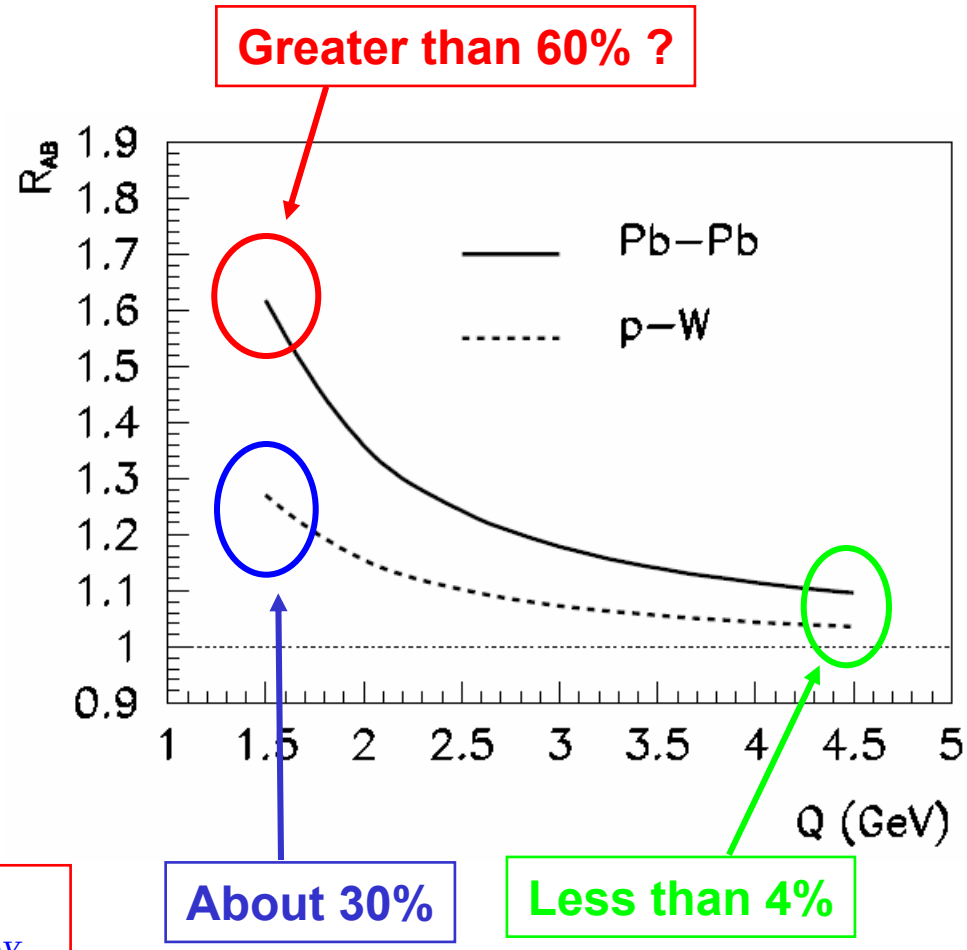
- **Drell-Yan in h-nucleus collisions:**

Qiu and Zhang, PLB525 (2002)



$$\frac{d\sigma_{AB}}{dQ^2} \approx AB\frac{d\sigma_{NN}^{(s)}}{dQ^2} + \frac{d\sigma_{AB}^{(d)}}{dQ^2} + \cdots$$

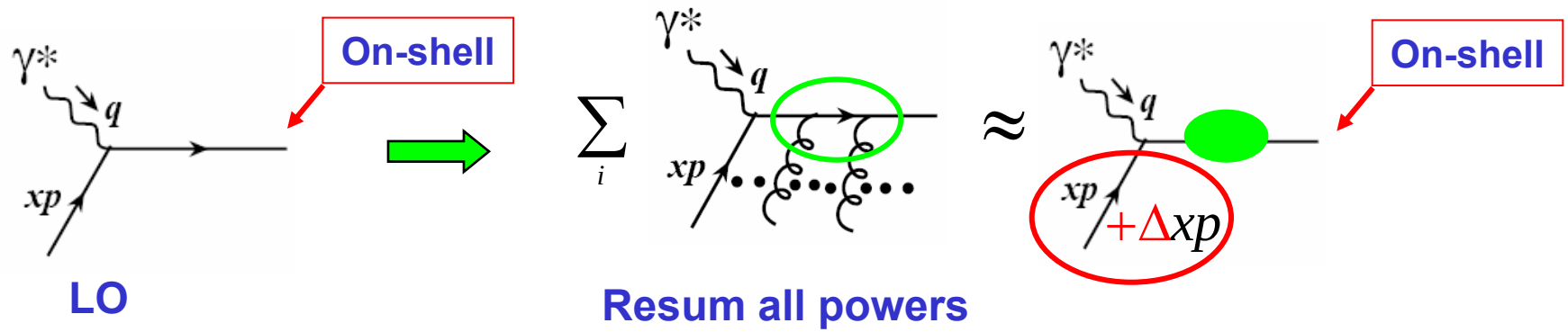
$$\equiv AB\frac{d\sigma_{NN}^{(s)}}{dQ^2} R_{AB}(Q)$$



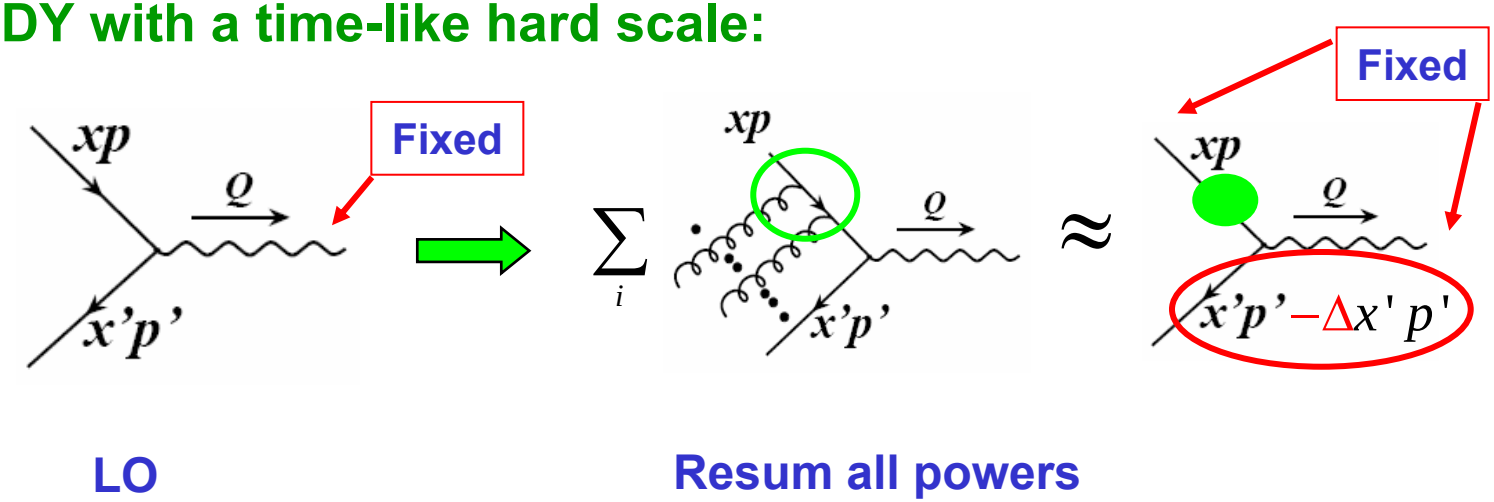
Depends on the same $\left\langle F^{+\alpha} F_{\alpha}^{+} \right\rangle_{DY}$ and no additional free parameters

Intuition for the power corrections

- DIS with a space-like hard scale:



- DY with a time-like hard scale:



Summary and outlook

- Hard probe with an active small *x* is not "local"
- Coherent QCD soft scattering – power corrections
- Leading-pole power corrections could be enhanced at small-x *(steep slope of PDFs)*
- Leading-pole power corrections are expressed in terms of only ONE universal matrix element

$$\left\langle F^{+\alpha} F_{\alpha}^{+} \right\rangle \sim \frac{1}{p^{+}} \int \frac{dy^{-}}{2\pi} \left\langle N \middle| F^{+\alpha}(0) F_{\alpha}^{+}\left(y^{-}\right) \middle| N \right\rangle$$

- Power corrections to DGLAP evolution are important
- Leading-pole power corrections vanish for saturated nPDFs

Global fitting of nPDFs needs to include both power corrections to hard parts as well as the evolution kennels