QCD and Rescattering in Nuclear Targets
Lecture 4

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Coherent Multiple Scattering

- Hard probe and its probing size
- Coherent multiple scattering and power corrections
  - Resummation of power corrections to DIS SFs
- Universal nuclear dependence in nPDFs
  - Resummation of power corrections to nPDFs
- Coherent multiple scattering in Drell-Yan production
- Summary
Hard probe and its probing size

- **Hard probe** – process with a large momentum transfer:
  \[ q^\mu \quad \text{with} \quad Q = \sqrt{|q^2|} \gg \Lambda_{\text{QCD}} \]

- Size of a hard probe is very localized and much smaller than a typical hadron at rest:
  \[ \frac{1}{Q} \ll 2R \sim \text{fm} \]

- But, it might be larger than a Lorentz contracted hadron:
  \[ \frac{1}{Q} \sim \frac{1}{xp} \gg 2R \left( \frac{m}{p} \right) \quad \text{or equivalently} \quad x \ll x_c \equiv \frac{1}{2mR} \sim 0.1 \]

If an active parton \( x \) is small enough, the hard probe could cover several nucleons in a Lorentz contracted large nucleus!
Coherence soft multiple scattering

- **Strong** quantum interference between scattering centers

\[ \sigma_{\text{Jet}} \propto N \]

- Modify production rate as well as jet spectrum

- **Nuclear dependence from multi-parton correlations**
  - Multi-parton correlation functions are **process independent** if pQCD factorization can be applied
  - Fourier transform from momentum to coordinate
    - universal matrix elements of multiple fields
  - no additional scale – **power suppressed**

Qiu and Sterman, 2003
Single hard scattering

- **Non-perturbative dynamics is effectively frozen**

\[ \sigma_{\text{Jet}} \propto \sigma_{\text{Jet}}^2 \]

- **Production rate is proportional to the PDFs**

- **Nuclear dependence from nPDFs**
  - modified DGLAP evolution
  - input nPDFs for the evolution
  - nPDFs are universal and process independent

\[ \approx + \cdots \]

Mueller and Qiu, 1986
Size of the power correction

- Coherent multiple scattering leads to dynamical power corrections:

\[ d\sigma \approx d\sigma^{(S)} + d\sigma^{(D)} + \ldots \]

**Naïve power counting:**

\[ \frac{d\sigma^{(D)}}{d\sigma^{(S)}} \sim \alpha_s \frac{1/Q^2}{R^2} \left< F^{+\alpha} F_\alpha^+ \right> A^{1/3} \]

- Characteristic scale for the power corrections:

\[ \left< F^{+\alpha} F_\alpha^+ \right> \]

- For a hard probe:

\[ \frac{\alpha_s}{Q^2 R^2} \ll 1 \]

- Enhanced by nuclear radius:

\[ A^{1/3} \leq 6 \]

- Enhanced by the slope of small-x distribution:

\[ -\frac{\partial}{\partial x} \varphi(x) \]
At small $x$, measured nuclear dependence include both nuclear dependence from coherent multiple scattering and nuclear dependence from nPDFs.

Factorization to separate these two contributions.
Calculate multi-parton interactions

At small $x$, the hard probe covers several nucleons, coherent multiple scattering could be equally important at relatively low $Q$ ~

To take care of the coherence, we need to sum over all cuts for a given forward scattering amplitude

Summing over all cuts is also necessary for IR cancellation
Collinear approximation is important

With collinear approximation:

\[ \sum_{\text{Cuts}} \]

Different cuts for matrix elements of partons with $k_T$ are not equal:

IR safe
Multiple momentum integrations

- **Parton momentum convolution:**

  \[ \sum_{\text{Cuts}} \left( \sum_{\text{Cuts}} \right) \]

  \[ \propto \int \prod_{i} dy_i^- e^{ix_ip^+y_i^-} \left\langle P_A \left| \prod_{i} F^{+\perp} (y_i^-) \right| P_A \right\rangle \]

  All coordinate space integrals are localized if \( x \) is large

- **Leading-pole approximation for \( dx_i \) integrals:**
  - \( dx_i \) integrals are fixed by the poles (no pinched poles)
  - \( x_i=0 \) removes the exponentials
  - \( dy \) integrals can be extended to the size of nuclear matter

  **Leading-pole leads to highest powers in medium length, a much smaller number of diagrams to worry about**
Multiple soft scattering to inclusive DIS

- **LO contribution to DIS cross section:**
  \[ \delta(x - x_B) \]

- **NLO contribution:**
  \[ \frac{g^2}{Q^2} \left( \frac{1}{2N_c} \right) \left[ 2\pi^2 \tilde{F}^2(0) \right] x_B \lim_{x_1 \to x} \left[ \frac{1}{x - x_1} \delta(x_1 - x_B) + \frac{1}{x_1 - x} \delta(x - x_B) \right] \]
  \[ \int \frac{dy_2^- dy_1^-}{(2\pi)^2} \left[ F^{+\alpha}(y_2^-) F^{\alpha+}(y_1^-) \right] \theta(y_2^-) x_B \left[ -\frac{d}{dx} \delta(x - x_B) \right] \]

- **Nth order contribution:**
  \[ \left[ \frac{g^2}{Q^2} \left( \frac{1}{2N_c} \right) \left[ 2\pi^2 \tilde{F}^2(0) \right] \right]^N x_B^N \lim_{x_i \to x} \sum_{m=0}^N \delta(x_m - x_B) \left[ \prod_{i=1}^m \left( \frac{1}{x_{i+1} - x_m} \right) \right] \left[ \prod_{j=1}^{N-m} \frac{1}{x_{m+j} - x_m} \right] \]
  \[ (-1)^N \frac{1}{N!} \frac{d^N}{dx^N} \delta(x - x_B) \]

Infrared safe!
Model for the correlation functions

Matrix elements:

\[ \left\langle P_A \left| \bar{\Psi}(0) \frac{\gamma^+}{2p^+} \Psi(y^-) \left[ \prod_{i=1}^{N} \int \tilde{F}^2(0) \right] \right| P_A \right\rangle \]

Approximation:

Nucleus is made of a group of loosely bound nucleons

\[ \left| P_A \right\rangle \propto \prod_{i=1}^{A} \left| p \right\rangle \quad \text{with} \quad p = \frac{P_A}{A} \]

\[ \left\langle P_A \left| \hat{O}_0 \prod_{i=1}^{N} \hat{O}_i \right| P_A \right\rangle \propto A \left\langle p \left| \hat{O}_0 \right| p \right\rangle \prod_{i=1}^{N} \left\langle p \left| \hat{O}_i \right| p \right\rangle \]

Reduce the correlation functions to one unknown – a universal matrix element

\[ \left\langle p \left| F^{+\alpha} F^{+}_\alpha \right| p \right\rangle \]
Contributions to DIS structure functions

- **Transverse structure function:**

\[
F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[ \frac{\xi^2}{Q^2} (A^{1/3} - 1) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2)
\]

\[\approx F_T^{(0)}(x_B(1 + \Delta), Q^2)\]

\[\Delta \equiv \frac{\xi^2}{Q^2} (A^{1/3} - 1)\]

\[\xi^2 = \frac{3\pi\alpha_s}{8R^2} \langle F^{+\alpha} F^{\alpha} \rangle\]

Single parameter for the power correction, and is proportional to the same characteristic scale.

- **Similar result for longitudinal structure function**

Qiu and Vitev, PRL (2004)
Neglect LT shadowing upper limit of $\xi^2$

$\xi^2 \sim 0.09 - 0.12 \text{ GeV}^2$
The Gross-Llewellyn Smith Sum Rules

\[ S_{\text{GLS}} = \int_0^1 dx \frac{1}{2x} \left( xF_3^{\nu N}(x, Q^2) + xF_3^{\bar{\nu} N}(x, Q^2) \right) \]

\[ \approx \# U + \# D = 3 \]


\[ \Delta_{\text{GLS}} \equiv \frac{1}{3} \left( 3 - S_{\text{GLS}} \right) = \frac{\alpha_s(Q)}{\pi} + \frac{\kappa}{Q^2} + O \left( \frac{1}{Q^4} \right) \]

Fully coherent final-state power corrections to the sum rule almost cancel due to the unitarity:

\[ \int_{-\infty}^{+\infty} dx \varphi(x + \Delta x) = \int_{-\infty}^{+\infty} dx \varphi(x) \]

But, nuclear enhanced power corrections only for a limited values of \( x \in \left( 0, 0.1 \right) \)

Prediction is compatible with the trend in the current data

Process-dependent power corrections are important!

Leading twist shadowing

- Power corrections complement to the nuclear dependence in nPDFs:
  - Leading twist shadowing changes the x- and Q-dependence of the parton distributions
  - Power corrections to the DIS structure functions (or cross sections) are effectively equivalent to a shift in x
  - Power corrections vanish quickly as hard scale Q increases while the leading twist shadowing goes away much slower

- If leading twist shadowing is so strong that x-dependence of parton distributions saturates for $x < x_c$, additional power corrections, the shift in x, should have no effect to the cross section!
Beyond the tree-level

- But, DGLAP evolved nPDFs do not remove this singularity, nor any collinear divergences beyond single scattering

- Redefine nPDFs to include all collinear divergences of partonic subprocesses
Power corrections to PDFs

- Hard probe sees only one effective parton:

- Pinched poles in the ladder diagrams – corrections to evolution

PDF with modified evolution

Mueller and Qiu, 86
Kang and Qiu, 06
Corrections to DGLAP evolution

\[ \phi(x, \mu^2) = \] 

\[ \varphi(x, \mu^2) - \] 

\[ \mu^2 \frac{\partial}{\partial \mu^2} \phi(x, \mu^2) = \gamma \left( \frac{x}{x'}, \alpha_s \right) \otimes \varphi(x', \mu^2) - \frac{1}{\mu^2} \bar{\gamma} \otimes \varphi^{(4)}(x', \mu^2) + \ldots \]
Corrections to PDFs not down by $1/Q^2$

Leading order power correction

$$\frac{\partial \varphi(x, \mu^2)}{\partial \ln \mu^2} = P(x) \otimes \varphi(x, \mu^2) - \frac{1}{\mu^2} \rho(x, \mu^2)$$

$$\int \frac{\mu^2}{\mu_0^2} \frac{d \ln \mu^2}{\mu^2} = \frac{1}{\mu_0^2} - \frac{1}{\mu^2} \to \frac{1}{\mu_0^2} \text{ as } \mu \to \infty$$

power correction can build up a big effect to low $Q^2$ distribution

What about high power corrections?
Modified ladder diagrams

- Leading-pole – Leading $A^{1/3}$ term – less diagrams

Modified DGLAP evolution equations at all powers
**Evolution kennels for modified DGLAP**

- **Modified** \( q \rightarrow q \) evolution kennel:

  \[
  \begin{align*}
  \begin{array}{c}
  \text{Translation Operator} \quad e^{-\epsilon \frac{d}{dy}} \\
  \sum_{N=0} \left[ \langle F^+ \langle F^+ \rangle \left( \frac{N}{N_c^2 - 1} \right) \cdot \left( \frac{1}{2} \right) (-1) \right]^N \frac{d^N}{dy^N} \delta(y - y_\perp) \right]
  \end{array}
  \end{align*}
  \]

- **First non-trivial term:**

  \[
  \begin{align*}
  \langle F^+ \langle F^+ \rangle \left( \frac{N_s^2}{N_c^2 - 1} \right) \cdot \left( \frac{1}{2} \right)(-1) \right] \frac{d}{dy} \delta(y - y_\perp)
  \end{align*}
  \]

- **Sum of all power corrections:**

  \[
  \begin{align*}
  \sum_{N=0} \left[ \langle F^+ \langle F^+ \rangle \left( \frac{N}{N_c^2 - 1} \right) \cdot \left( \frac{1}{2} \right) (-1) \right]^N \frac{d^N}{dy^N} \delta(y - y_\perp) \right]
  \end{align*}
  \]

- **Similar results for the other kennels**
Numerical results

Gluon evolution slope

Important at small x and low Q²

Kang and Qiu, 06
Negative gluon distribution at low $Q$?

- NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution
- MRST, CTEQ PDF’s have the same features

Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV? **No!**

Power corrections slows down small-$x$ evolution
Recombination prevents negative gluon

- In order to fit new HERA data, like MRST PDF’s, CTEQ6 gluon has to be much smaller than CTEQ5, even negative at $Q = 1$ GeV.

- The power correction to the evolution equation slows down the $Q^2$-dependence, prevents PDF’s to be negative.

$$\langle xG(x \to 10^{-5}) \rangle \sim 3$$

Eskola et al. NPB660 (2003)
Phase diagram of parton densities

- Experiments measure cross sections, not PDFs
- PDFs are extracted based on
  - factorization
  - truncation of perturbative expansion
- How to probe the boundary between different regions?
  - Look for where pQCD factorization fails
  - Power corrections – improve predictive power of factorization approach
Enhancement of low mass dileptons in heavy ion collisions:

Unlike DIS, coherent power corrections to Drell-Yan cross section are positive!

Figures from Shahoyan’s talk
Predicted power corrections to DY

Drell-Yan in h-nucleus collisions:

$\frac{d\sigma_{AB}}{dQ^2} \approx AB \frac{d\sigma_{NN}}{dQ^2} + \frac{d\sigma_{AB}}{dQ^2} + AB \frac{d\sigma_{NN}}{dQ^2} R_{AB}(Q)$

Depends on the same $\langle F^+ \alpha F_+ \rangle_{DY}$ and no additional free parameters

Qiu and Zhang, PLB525 (2002)

Greater than 60%?

About 30%  
Less than 4%
Intuition for the power corrections

- **DIS with a space-like hard scale:**
  
  \( \gamma^* \rightarrow q \rightarrow xp \)

  - LO
  - On-shell

  \[ \sum_i \approx x_p - \Delta \]

  - Resum all powers
  - Fixed

- **DY with a time-like hard scale:**
  
  \( xp \rightarrow q \rightarrow x'p' \)

  - LO
  - Fixed

  \[ \sum_i \approx x_p + \Delta x_p \]

  - Resum all powers
  - On-shell

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Jianwei Qiu, ISU
Summary and outlook

- Hard probe with an active small $x$ is not “local”
- Coherent QCD soft scattering – power corrections
- Leading-pole power corrections could be enhanced at small-$x$ (steep slope of PDFs)
- Leading-pole power corrections are expressed in terms of only ONE universal matrix element

$$\left\langle F^+ F^+ \right\rangle \sim \frac{1}{p^+} \int \frac{dy^-}{2\pi} \left\langle N | F^+ (0) F^+ (y^-) | N \right\rangle$$

- Power corrections to DGLAP evolution are important
- Leading-pole power corrections vanish for saturated nPDFs

Global fitting of nPDFs needs to include both power corrections to hard parts as well as the evolution kennels