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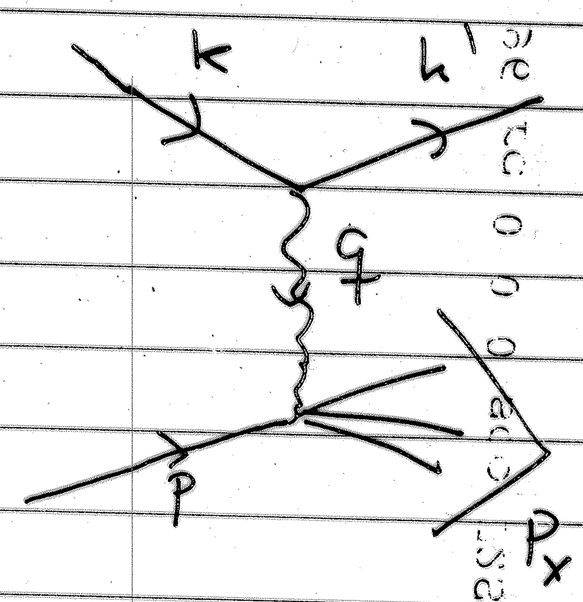
The Structure of the proton

F.M. Steffens

We will be interested in the following process:



The differential cross section for the "electron" to be scattered into a solid angle $d\Omega$ with an energy range dE' is:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4}\frac{E'}{E} L_{\mu\nu} W^{\mu\nu} \qquad (1.1)$$

See Bjorken and Drell, formula B1

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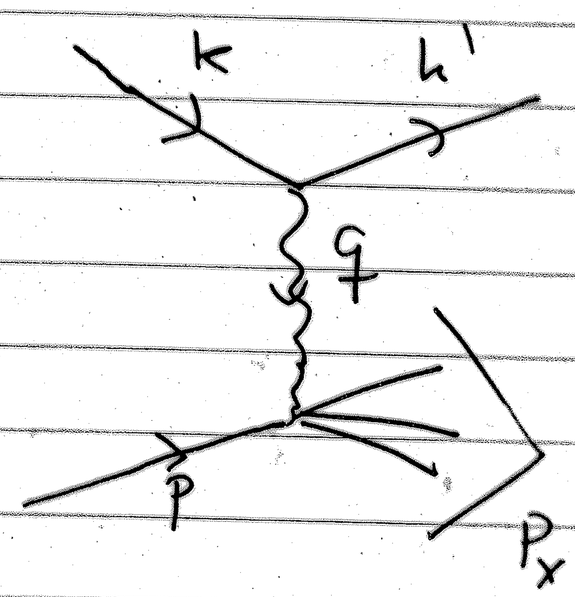

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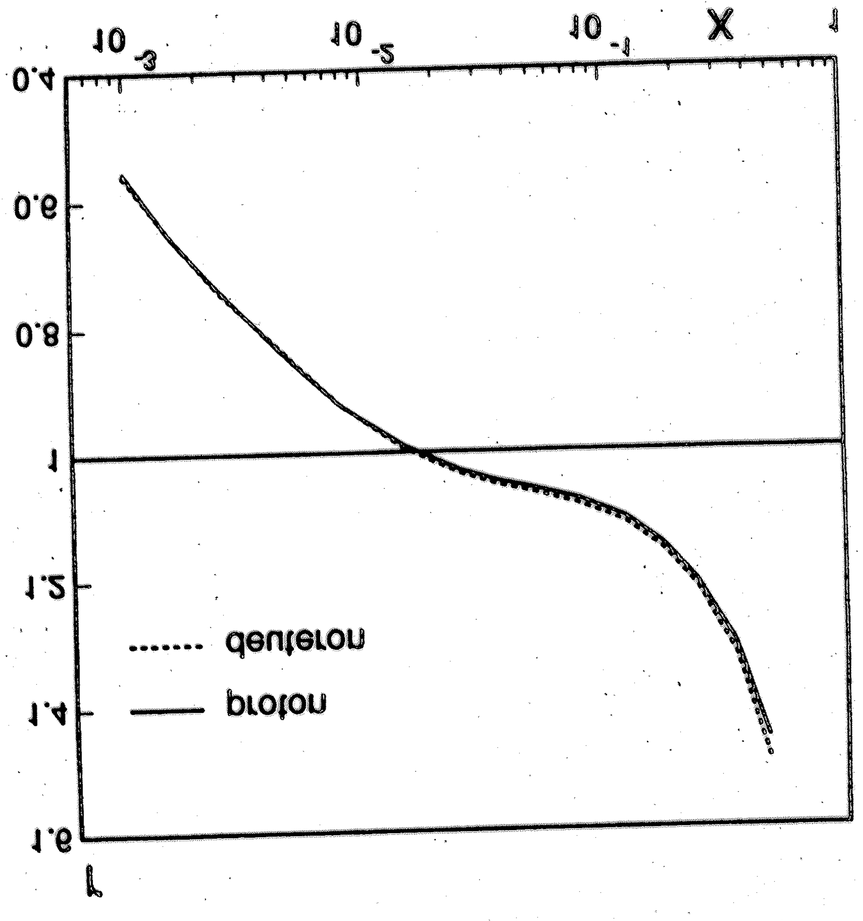


FIG. 6. The ratio r of statistical errors for A_1 from hadron-tagged events and from inclusive events as a function of x, for proton and for deuteron.

effect of changes in the acceptance for both the scattered muon and for the hadrons.

C. Comparison of A_1 for inclusive and hadron-tagged events

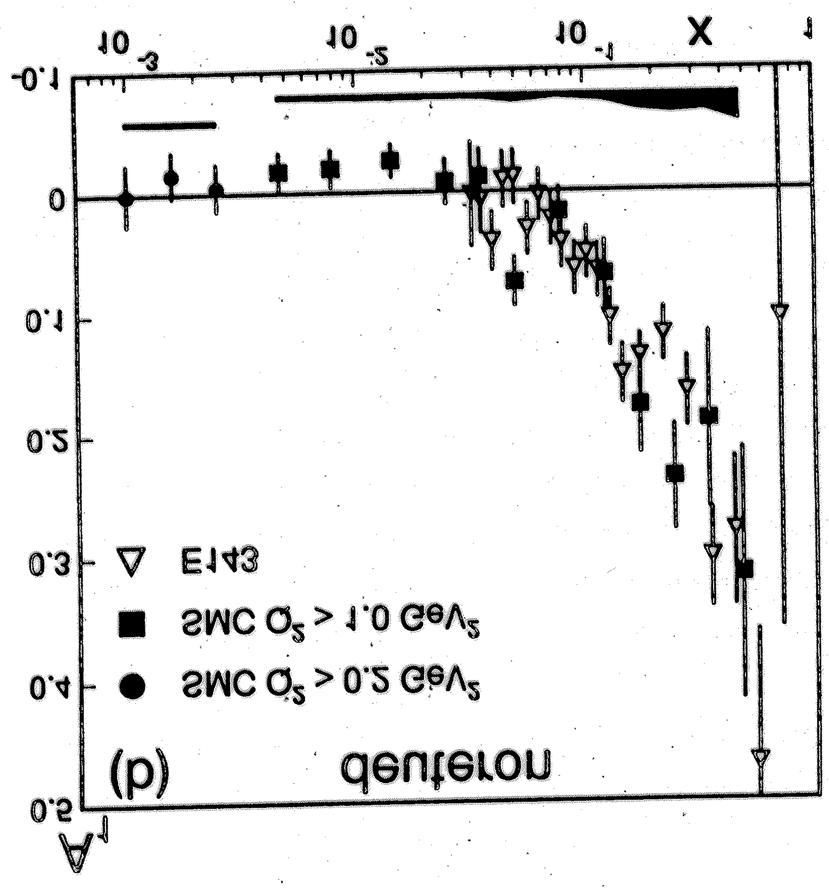
The A_1 asymmetries for the two types of event selections, inclusive and hadron tagged, are compared in Fig. 2. The differences are small except for the two lowest x points for the proton data.

As explained before, the results for the event selection with hadron tagging have smaller statistical errors at low x, while the inclusive event selection gives more precise results for high x. This can be seen in Fig. 6, which gives the ratio of the statistical errors for A_1 obtained with the two types of event selections as a function of x.

D. Optimal set of A_1 from SMC data

Figure 6 demonstrates that for $x<0.05$ the more accurate results for A_1 are obtained by using hadron-tagged events, while for $x>0.05$ the inclusive events give the more precise result. We therefore take as the optimal set of A_1 values the results from the hadron method for $x<0.05$ and the results from the inclusive method for $x>0.05$. This leads to the A_1 values in bins of x presented in Fig. 7 and Tables II and III. The hadron method is used for the lowest 6 x bins for the data shown in Fig. 7. Contributions to the systematic error are detailed in Tables VI and V for each x bin and their quadratic sum is shown as a band in Fig. 7.

The weak Q_5 dependence of A_p^1 and A_d^1 in each bin of x is presented in Figs. 8 and 9 and Tables IV and VII. From perturbative QCD a different Q_5 behavior is expected for the structure functions F_1 and g_1, hence $A_1=g_1/F_1$ should be Q_5 dependent. This dependence follows from the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [20]. It was determined in our QCD analysis, performed in next-to-leading order (NLO), which is presented in the following paper [14]. The results are shown as the solid



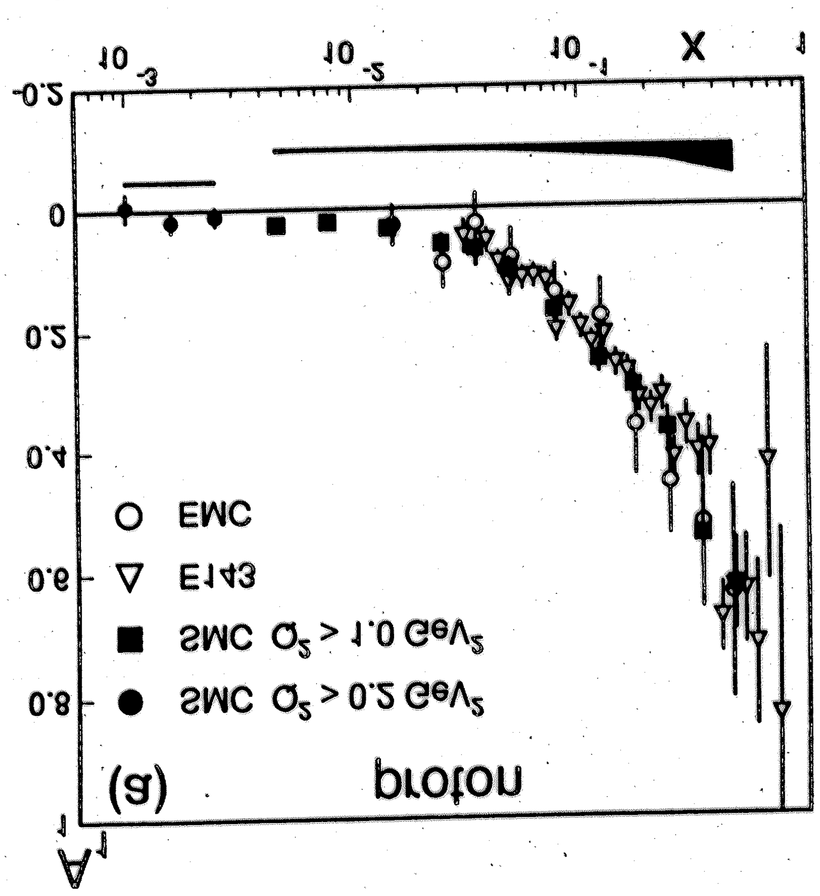


FIG. 7. The optimal set of SMC results for A_1 together with the results from other experiments. Statistical errors are shown as error bars. The shaded bands below indicate the systematic uncertainty for the SMC measurements.

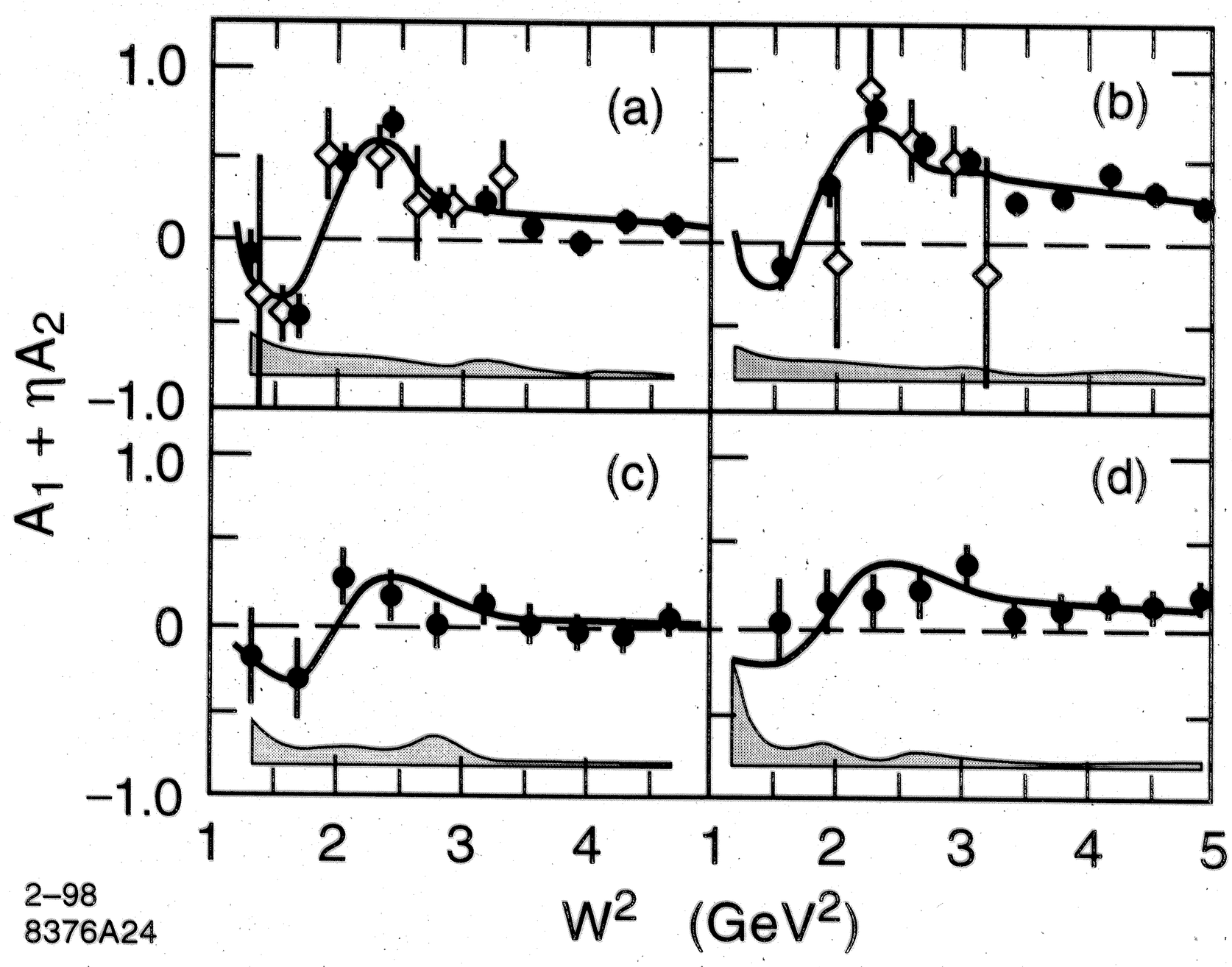


FIG. 23. Extracted values for $A_1 + \eta A_2$ (circles) in the resonance region for the proton at (a) 4.5° and (b) 7.0°; and for the deuteron at (c) 4.5° and (d) 7.0°. Also shown are the Monte Carlo predictions (solid line) and the data of Baum *et al.* [132] (diamonds). Error bars correspond to statistical errors only, whereas the bands below the data correspond to the systematic errors.

We can also get W_1 and W_2 by:

- Using unpolarized scattering
- Making an appropriate choice for θ

Lattest information on these "Structure functions" can be found in:

Usipanteo et al. PRD 71: 054007, 2005

<u>Example</u>: Suppose we polarize both target and beam

$\frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'}$ = target spin is paralell to the beam spin

$\frac{d^2\sigma^{\uparrow\Downarrow}}{d\Omega dE'}$ = now they are antiparalell

For $\uparrow\Uparrow$, $\alpha = 0°$; For $\uparrow\Downarrow$, $\alpha = 180°$

One can then measure asymmetries

$$\boxed{A_{=} = \frac{\uparrow\Downarrow - \uparrow\Uparrow}{\uparrow\Downarrow + \uparrow\Uparrow} = \frac{Q^2}{2}\frac{1}{EE'}\frac{(E+E'\cos\theta)MG_1 - Q^2G_2}{2\sin^2\frac{\theta}{2}W_1 + \cos^2\frac{\theta}{2}W_2}} \quad (1.22)$$

↓

The longitudinal asymmetry

$$\boxed{A_{\perp} = \frac{\uparrow\Leftarrow - \uparrow\Rightarrow}{\uparrow\Leftarrow + \uparrow\Rightarrow} = \frac{Q^2}{2E}\sin\theta\cos\phi\,\frac{MG_1 + 2EG_2}{2\sin^2\frac{\theta}{2}W_1 + \cos^2\frac{\theta}{2}W_2}} \quad (1.23)$$

↓

The transverse asymmetry

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The G_2 piece is a bit longer:

$$\ldots G_2 = -Q^2 \cos\alpha \, G_2 + 2EE' \sin\theta \sin\alpha \cos\phi \, G_2$$

Putting everything together:

$$\boxed{\begin{aligned} \frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \Big(& 2\sin^2\frac{\theta}{2} W_1 + \cos^2\frac{\theta}{2} W_2 \Big) \\ & \pm (-1) \frac{2\alpha^2}{Q^2} \frac{E'}{E} \Big\{ \cos\alpha \left[(E + E'\cos\theta) M G_1 - Q^2 G_2 \right] \\ & + \sin\theta \sin\alpha \cos\phi \, E' \left[M G_1 + 2E G_2 \right] \Big\} \end{aligned}}$$

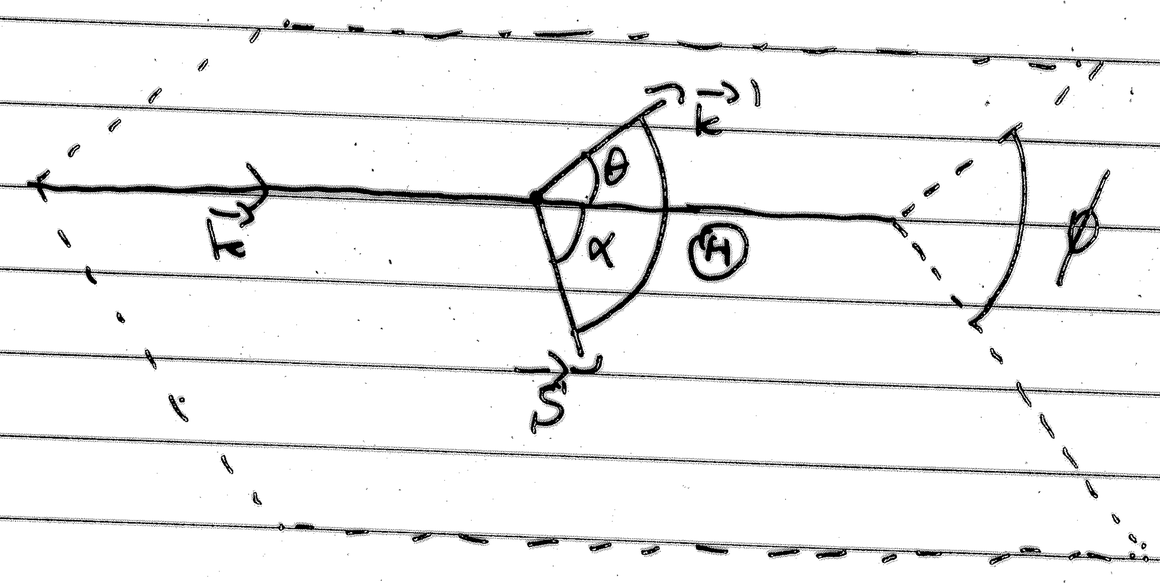
(1.21)

Measure the cross section and we will have information on

W_1, W_2, G_1, G_2

$\Rightarrow$ **Proton Structure**

The angles are such that:



with

$$\cos\Theta = \sin\theta \sin\alpha \cos\phi + \cos\theta \cos\alpha$$

Then:

$$L^{(a)}_{\mu\nu} W^{\mu\nu(a)} = \pm (-1) 2MQ^2 \left(E \cos\alpha + E' \cos\Theta \right) G_1 + \dots G_2$$

$$= \pm (-1) 2Q^2 \left\{ \left[\cos\alpha (E + E' \cos\theta) M G_1 + \sin\theta \sin\alpha \cos\phi \, E' M G_1 \right] + \dots G_2 \right\}$$

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$$L^{(0)}_{\mu\nu} W^{\mu\nu(0)} = \pm(-2)(-2)(\delta_\rho^\alpha \delta_\sigma^\beta - \delta_\rho^\beta \delta_\sigma^\alpha) \cdot$$

$$\cdot \left\{ M k^\rho q^\sigma q_\alpha S_\beta G_1 + \dots G_2 \right\}$$

$$= \pm 4M \left\{ \left((k.q)(q.S) - (k.S) q^2 \right) G_1 + \dots G_2 \right\}$$

but $k.q = k.k - k.k' = m^2 - k.k' = \frac{q^2}{2}$

$$\Rightarrow L^{(0)}_{\mu\nu} W^{\mu\nu(0)} = \pm 2Mq^2 \left\{ q.S - 2k.S \right\} G_1 + \dots G_2$$

$$= \pm 2M(-q^2) \left\{ (k.S) + (k'.S) \right\} G_1 + \dots G_2$$

- $q^2 \simeq -2EE'(1-\cos\theta) < 0 \Rightarrow Q^2 = -q^2 > 0$

Also:

- $k.S = -\frac{\vec{k}.\vec{S}}{|\vec{S}|} = -|\vec{k}|\cos\alpha \simeq -E\cos\alpha$ if $m \simeq 0$

- $k'.S = -E'\cos\Theta$ $\quad (S = (0, \hat{s}))$

Hence, the unpolarized cross section is given by:

$$\boxed{\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{q^4}\left[2\sin^2\frac{\theta}{2}\, W_1 + \cos^2\frac{\theta}{2}\, W_2\right]} \qquad (1.20)$$

Now to the antisymmetric part.

Remember:

$$L_{\mu\nu}W^{\mu\nu} = L^{(s)}_{\mu\nu}W^{\mu\nu(s)} + L^{(a)}_{\mu\nu}W^{\mu\nu(a)}$$

$$L^{(a)}_{\mu\nu}W^{\mu\nu(a)} = \pm 2i \cdot i \cdot \varepsilon_{\mu\nu\rho\sigma}\, k^\rho q^\sigma \varepsilon^{\mu\nu\alpha\beta} q_\alpha \cdot$$

$$\cdot \left\{ M S_\beta G_1 + \left[(P\cdot q) S'_\beta - (S'\cdot q) P_\beta\right] \frac{G_2}{M} \right\}$$

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Let's look at the coefficients of W_1

$$-2k'\cdot k - 4m^2 + 4k'\cdot k + \frac{2}{q^2}(k'\cdot q)(k\cdot q)$$

$$+\frac{1}{2}(2m^2 - 2k\cdot k')$$

$$= k'\cdot k - 3m^2 + \frac{2}{q^2}(k'\cdot k - k'\cdot k')(k\cdot k - k\cdot k')$$

$$= k'\cdot k - 3m^2 - \frac{q^2}{2}$$

$$= 2k\cdot k' - 4m^2 \simeq 2k\cdot k'$$ in the relativistic approximation

$$= 2(EE' - |\vec{k}||\vec{k}'|\cos\theta) \underset{m\to 0}{\simeq} 2EE'(1-\cos\theta)$$

$$= 4EE'\sin^2\frac{\theta}{2} \qquad (1.18)$$

A similar calculation for the coefficient of W_2:

$$(\qquad)W_2 \simeq 2EE'\cos^2\frac{\theta}{2}\, W_2 \qquad (1.19)$$

$\downarrow$

Exercise!

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Contractions

The cross section is proportional to:

$$W_{\mu\nu} L^{\mu\nu} = L^{(s)}_{\mu\nu} W^{\mu\nu(s)} + L^{(a)}_{\mu\nu} W^{\mu\nu(a)}$$

From (1.13) and (1.15):

$$L^{(s)}_{\mu\nu} W^{\mu\nu(s)} = 2\Big[\Big(-k'\cdot k - k\cdot k' - 2q^2\Big) W_1$$

$$+ \frac{1}{q^2}\Big(k'\cdot q\, k\cdot q + k'\cdot q\, k\cdot q + \frac{q^4}{2}\Big) W_1$$

$$+ \Big(2k'\cdot\Big(p - \frac{(p\cdot q)}{q^2} q\Big) k\cdot\Big(p - \frac{(p\cdot q)}{q^2} q\Big)$$

$$+ \frac{q^2}{2}\Big(p - \frac{p\cdot q}{q^2} q\Big)\cdot\Big(p - \frac{q\cdot p}{q^2} q\Big)\Big) W_2\Big] \qquad (1.17)$$

Now, we have:

$$q^2 = (k - k')^2 = k^2 + k'^2 - 2k\cdot k' = 2m^2 - 2k\cdot k'$$

$$k\cdot k' = EE' - \vec{k}\cdot\vec{k}' = EE' - |\vec{k}||\vec{k}'|\cos\theta$$

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For the antisymmetric part, remember that $W_{\mu\nu}$ <u>is not</u> an axial.

In general

$$W^{(a)}_{\mu\nu} = i\,\varepsilon_{\mu\nu\alpha\beta}\left[A(q^2,\nu)\,q^\alpha S^\beta + B(q^2,\nu)\,q^\alpha P^\beta + C(q^2,\nu)\,P^\alpha S^\beta\right]$$

<u>Exercise</u>. Derive that

$$W^{(a)}_{\mu\nu} = i\,\varepsilon_{\mu\nu\alpha\beta}\,q^\alpha\left[M S^\beta G_1(q^2,\nu) - (S\cdot q)\,P^\beta\,\frac{G_2(q^2,\nu)}{M}\right] \qquad (1.16)$$

with

$$A = M G_1 + \frac{(P\cdot q)}{M}\,G_2$$

$$B = -\frac{(S\cdot q)}{M}\,G_2$$

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$$\Rightarrow \left(A + \frac{B}{M^2} q^2 + \frac{C}{M^2} p\cdot q\right) q_\mu + \left(\frac{C}{M^2} q^2 + \frac{D}{M^2} p\cdot q\right) p_\mu = 0$$

because they are independent vectors, we have:

$$B = \frac{(p\cdot q)^2}{q^4} D - \frac{M^2}{q^2} A$$

$$C = -\frac{p\cdot q}{q^2} D$$

Going back to Eq. (1.14):

$$N_{\mu\nu} = \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] A + \left[\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\right]\frac{D}{M^2}$$

Let: $A = -W_1$; $D = W_2$

$$\boxed{W_{\mu\nu} = \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] W_1 + \left[\left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\right]\frac{W_2}{M^2}}$$

(1.15)

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- Current Conservation

$$\partial_\mu j^\mu = 0 \Rightarrow \langle B | \partial_\mu j^\mu | A \rangle = 0$$

$$i \langle B | [\hat{P}_\mu, j^\mu] | A \rangle = 0$$

$$i \underbrace{(p_\mu(B) - p_\mu(A))}_{q_\mu} \langle B | j_\mu | A \rangle = 0$$

$$\Rightarrow \boxed{q_\mu \langle B | j_\mu | A \rangle = 0}$$

Or $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$, because the hadronic tensor is a product of currents.

We now apply this condition on Eq. (1.14) for the hadronic tensor

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$$\Rightarrow \boxed{W_{\mu\nu}^{(s)} = A(q^2,\nu)\, g_{\mu\nu} + \frac{B(q^2,\nu)}{M^2} q_\mu q_\nu + \frac{C(q^2,\nu)}{M^2}\left(q_\mu P_\nu + q_\nu P_\mu\right) + \frac{D(q^2,\nu)}{M^2} P_\mu P_\nu} \tag{1.14}$$

Parity: It excludes terms like $\varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$ because the $\varepsilon_{\mu\nu\alpha\beta}$ tensor behaves as a axial under the parity transformation.

$$\gamma_0\, \varepsilon_{\mu\nu\alpha\beta}\, \gamma^0 \rightarrow -\varepsilon_{\mu\nu\alpha\beta}$$

Exercise: generalize $W_{\mu\nu}$ to the case where interactions which allow parity violations are included. (as neutrino proton scattering)

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The Hadronic Tensor

$$W_{\mu\nu} = W^{(S)}_{\mu\nu} + W^{(A)}_{\mu\nu}$$

symmetric in $\mu\nu$ — antisymmetric in $\mu\nu$

We impose:

- Lorentz invariance

Available four vectors

$$\underline{P}, \; q, \; S$$

$\underline{P}$: vector; q: vector; S: proton spin → pseudo vector

Symmetric part: must involve $\underline{P}, q$

Antisymmetric part: must involve $\underline{P}, q, S$

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We go back to the leptonic tensor and rewrite it:

$$\boxed{L_{\mu\nu} = 2\left(k_\mu k'_\nu + k_\nu k'_\mu + \frac{q^2}{2} g_{\mu\nu}\right) \pm 2i\, \varepsilon_{\mu\nu\rho\sigma}\, k^\rho q^\sigma} \qquad (1.13)$$

For a relativistic lepton $\left(\frac{m}{E} \to 0\right)$

What about the hadronic tensor, $W^{\mu\nu}$?

- Not a point particle
- No Dirac Equation

$\Rightarrow$ We parametrize it!

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As Eq. (1.8) can be rewritten as:

$$|\vec{s}| = \frac{k^0}{\sqrt{E^2 - \vec{k}\cdot\vec{k}}} = \frac{E}{m}$$

$$\Rightarrow \boxed{\vec{s} = \pm \frac{E}{m\beta}\vec{\beta} = \pm \frac{\vec{k}}{m\beta}} \quad (1.11)$$

Hence:

$$\boxed{s^\mu = \pm \frac{k^\mu}{m\beta} \mp \frac{\sqrt{1-\beta^2}}{\beta} g^{\mu 0}} \quad (1.12)$$

↳ By construction

In the relativistic limit:

$$\frac{m}{E} \to 0 \Rightarrow \beta = 1$$

and

$$\boxed{s^\mu = \pm \frac{k^\mu}{m}} \quad (1.12)$$

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So, for the electron polarization $\underline{\underline{S}}$, and momentum $\underline{k}$

$$S \cdot k = 0 \Rightarrow S^0 k^0 - \vec{S} \cdot \vec{k} = 0$$

$$\boxed{S^0 = \frac{\vec{S} \cdot \vec{k}}{k^0} \equiv \vec{S} \cdot \vec{\beta}} \quad (1.7)$$

From $S \cdot S = -1 \Rightarrow \boxed{|\vec{S}| = \frac{1}{\sqrt{1 - (\hat{S} \cdot \vec{\beta})^2}}}$ (1.8)

And: $\vec{S} \cdot \vec{S} = (\vec{S} \cdot \vec{\beta})^2 + 1$

$$(\hat{S} \cdot \vec{\beta})^2 = \frac{(S^0)^2}{|\vec{S}|^2} = \frac{(\vec{S} \cdot \vec{\beta})(\vec{S} \cdot \vec{\beta})}{|\vec{S}|^2} =$$

$$= \hat{S}_a \beta_a \hat{S}_b \beta_b = \beta^2$$

$$\Rightarrow \boxed{\hat{S} \cdot \vec{\beta} = \pm \beta}$$ (1.9)

$+$ = Polarization paralell to mom.

$-$ = Pol. antiparalell to mom.

from (1.7) and (1.9):

$$\boxed{S_0 = \pm \beta |\vec{S}|} \xrightarrow[(1.7)]{} \boxed{\vec{S} = \pm \frac{|\vec{S}|}{\beta} \vec{\beta}} \quad (1.10)$$

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Bjorken and Drell: Relativistic quantum Mechanics

Take the polarization vector in the rest frame:

$$S^\mu = (0, \vec{s}) \quad , \text{such that } \vec{s}\cdot\vec{s} = 1$$

At any frame:

$$S^{\mu'} = a^\mu{}_\nu \underbrace{S^\nu}_{\text{rest}}$$

Lorentz transformation: $a^\mu{}_\nu a^\rho_\mu = g^\rho_\nu$

Same for the momentum:

$$p^\mu = (m, \vec{0}) \quad \text{at rest} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \quad p^{\mu'} = a^\mu{}_\nu \underbrace{p^\nu}_{\text{rest}}$$

$$\Rightarrow \quad S_\mu' p^{\mu'} = 0 \quad ; \quad S\cdot p = 0 \qquad (1.6)$$

$$S\cdot S = S_\mu S^\mu = -1$$

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Hence:

$$L_{\mu\nu} = 2\left[k_\mu k'_\nu + k_\nu k'_\mu + (m^2 - k.k')g_{\mu\nu}\right]$$

$$+ 2im\,\varepsilon_{\mu\nu\rho\sigma}\, s^\rho q^\sigma$$

as $q^2 = (k-k')^2 = k^2 + k'^2 - 2k.k' =$

$$= 2m^2 - 2k.k'$$

$$\boxed{L_{\mu\nu} = 2\left[k_\mu k'_\nu + k_\nu k'_\mu + q^2 \frac{g_{\mu\nu}}{2}\right] + 2im\, s^\rho q^\sigma\, \varepsilon_{\mu\nu\rho\sigma}} \quad (1.5)$$

What about the electron polarization vectors s??

So, $L_{\mu\nu} = \bar{u}_\alpha(k,s)(\gamma_\mu)_{\alpha\beta}(\not{k}'+m)_{\beta a}(\gamma_\nu)_{ab}\, u_b(k,s)$

Dirac Projector II

$$u_b(k,s)\,\bar{u}_\alpha(k,s) = \left[(\not{k}+m)\,\frac{(1+\gamma_5\not{s})}{2}\right]_{b\alpha}$$

$$\Rightarrow L_{\mu\nu} = \left[(\not{k}+m)\,\frac{1+\gamma_5\not{s}}{2}\right]_{b\alpha}(\gamma_\mu)_{\alpha\beta}(\not{k}'+m)_{\beta a}(\gamma_\nu)_{ab}$$

$$= \frac{1}{2}\,\mathrm{Tr}\left\{(\not{k}+m)(1+\gamma_5\not{s})\gamma_\mu(\not{k}'+m)\gamma_\nu\right\} \qquad (1.4)$$

$$= \frac{1}{2}\,\mathrm{Tr}\left\{(\not{k}+m)\gamma_\mu(\not{k}'+m)\gamma_\nu\right\} +$$

$$+ \frac{1}{2}\,\mathrm{Tr}\left\{(\not{k}+m)\gamma_5\not{s}\gamma_\mu(\not{k}'+m)\gamma_\nu\right\}$$

$\mathrm{Tr}\{\text{odd number of } \gamma\} = 0$

$\mathrm{Tr}\{\not{a}\not{b}\} = 4(a.b)$

$\mathrm{Tr}\{\not{a}\not{b}\not{c}\not{d}\} = 4\left[(a.b)(c.d)+(a.d)(b.c)-(a.c)(b.d)\right]$

$\mathrm{Tr}\{\gamma_5\not{a}\not{b}\not{c}\not{d}\} = 4i\,\varepsilon_{\alpha\beta\gamma\delta}\,a^\alpha b^\beta c^\gamma d^\delta$

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$L_{\mu\nu}$ is the leptonic tensor:

$$L_{\mu\nu} = \frac{1}{2} \sum_{s'} \langle k,s | j_\mu(0) | k',s' \rangle \langle k',s' | j_\nu(0) | k,s \rangle \quad (1.2)$$

↳ Eletric current

Suppose we are in the LAB frame:

$$P = (M, \vec{0}) \; ; \; k = (E, \vec{k}) \; ; \; k' = (E', \vec{k}') \quad (1.3)$$

$$q = k - k'$$

As $\langle k,s | j_\mu(0) | k',s' \rangle \propto \bar{u}(k,s) \gamma_\mu u(k',s')$

Dirac Matrixes Dirac Spinors positive energy

$$\Rightarrow L_{\mu\nu} = \sum_{s'} \bar{u}_\alpha(k,s) (\gamma_\mu)_{\alpha\beta} u_\beta(k',s') \; \bar{u}_a(k',s') (\gamma_\nu)_{ab} u_b(k,s)$$

Dirac projector I

$$\sum_{s'} u_\beta(k',s') \bar{u}_a(k',s') = (\not{k}' + m)_{\beta a} \; ,$$

$$\not{k}' = k'^\mu \gamma_\mu$$

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The Parton model

- First measurements of DIS cross sections were performed at SLAC, at the end of 60's

- Measurement for various values of $W^2 = (P+q)^2 = M^2 + 2M\nu + Q^2$ and Q^2

- Let $R = \frac{W_2}{W_1} \frac{Q^2+\nu^2}{Q^2}$

$$\Rightarrow \frac{d^2\sigma}{d\Omega dE'} = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_M}_{\frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}} \underbrace{W_2(Q^2,\nu)\left(1 + 2\frac{Q^2+\nu^2}{(1+R)Q^2} tg^2\frac{\theta}{2}\right)}_{}$$

Mott cross section

when W^2 gets away from the elastic scattering region, $W^2 = M^2$, the term above is almost Q^2 independent!!!

$\Rightarrow$ Imply that one variable only is relevant!!!

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Hence:

$$\boxed{\begin{aligned} F_2(x) &= \sum_i e_i^2 x f_i(x) \\ F_1(x) &= \sum_i e_i^2 f_i(x) \end{aligned}} \qquad (1.25)$$

and:

$$\boxed{F_2(x) = 2x F_1(x)} \qquad (1.26)$$

Callan - Gross relation

and:

$$R = \frac{4M^2x^2}{Q^2} \rightarrow 0 \text{ in the Bjorken limit}$$

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Comparing with (1.20): (with $m_i = xM$)

$$W_1^i = e_i^2 \frac{Q^2}{4x^2M^2} \delta\left(\nu - \frac{Q^2}{2xM}\right)$$

$$W_2^i = e_i^2 \delta\left(\nu - \frac{Q^2}{2xM}\right)$$

assumption: Total cross section is an incoherent sum of the partonic cross sections

$$\Rightarrow W_1(\nu, Q^2) = \sum_i \int_0^1 dx' f_i(x') e_i^2 \frac{Q^2}{4x'^2M^2} \delta\left(\nu - \frac{Q^2}{2x'M}\right) =$$

$$= \sum_i e_i^2 \frac{f_i(x)}{2M} = \frac{1}{M} F_1(x)$$

$$W_2(\nu, Q^2) = \sum_i \int_0^1 dx' f_i(x') e_i^2 \delta\left(\nu - \frac{Q^2}{2x'M}\right) =$$

$$= \sum_i e_i^2 \frac{x}{\nu} f_i(x) = \frac{1}{\nu} F_2(x)$$

Probability for the i parton to be found with a certain fraction x of the nucleon mass

We <u>can</u> calculate the elastic electron-parton cross section

The partonic tensor is (see Eq. (1.5)):

$$W^{i}_{\mu\nu} = \frac{1}{m_i}\left(p_\mu p'_\nu + p_\nu p'_\mu + \frac{q^2}{2} g_{\mu\nu}\right)\delta\left(q^2 + 2p\cdot q\right)$$

mass of the i parton

Elastic scattering

$W^2 = m^2 = (p+q)^2$

$$\boxed{\frac{d^2\sigma^i}{d\Omega dE'} = \frac{\alpha^2}{q^4} L_{\mu\nu} W^{\mu\nu(i)} = -\frac{4\alpha^2 E'^2}{Q^4}\left\{e_i^2 \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2} + e_i^2\cos^2\frac{\theta}{2}\right\}\delta\left(\nu - \frac{Q^2}{2m_i}\right)} \quad (1.24)$$

where e_i is the fraction of the electron charge $\underline{e}$ carried by the parton i

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Bjorken (PR 179 (1969) 1547) <u>before</u> the experiment that:

$$M W_1(Q^2, \nu) \longrightarrow F_1(x)$$

$$\nu W_2(Q^2, \nu) \longrightarrow F_2(x)$$

for $\nu, Q^2 \to \infty$, but $\frac{\nu}{Q^2}$ fixed $\left(x = \frac{Q^2}{2M\nu}\right)$

To have two variables is inelastic scattering

To have <u>one</u> variabe is elastic scattering

<u>Feynman</u>: We are observing electron–<u>parton</u> elastic scattering inside the proton

$\downarrow$

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So partons came to life ...

OBSERVED BEHAVIOR OF HIGHLY INELASTIC ELECTRON-PROTON SCATTERING

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Results of electron-proton inelastic scattering at 6° and 10° are discussed, and values of the structure function W_2 are estimated. If the interaction is dominated by transverse virtual photons, νW_2 can be expressed as a function of $\omega = 2M\nu/q^2$ within experimental errors for $q^2 > 1$ (GeV/c)^2 and $\omega > 4$, where ν is the invariant energy transfer and q^2 is the invariant momentum transfer of the electron. Various theoretical models and sum rules are briefly discussed.

In a previous Letter,[1] we have reported experimental results from a Stanford Linear Accelerator Center–Massachusetts Institute of Technology study of high-energy inelastic electron-proton scattering. Measurements of inelastic spectra, in which only the scattered electrons were detected, were made at scattering angles of 6° and 10° and with incident energies between 7 and 17 GeV. In this communication, we discuss some of the salient features of inelastic spectra in the deep continuum region.

One of the interesting features of the measurements is the weak momentum-transfer dependence of the inelastic cross sections for excitations well beyond the resonance region. This weak dependence is illustrated in Fig. 1. Here we have plotted the differential cross section divided by the Mott cross section, $(d^2\sigma/d\Omega dE')/(d\sigma/d\Omega)_{\text{Mott}}$, as a function of the square of the four-momentum transfer, $q^2 = 2EE'(1-\cos\theta)$, for constant values of the invariant mass of the recoiling target system, W, where $W^2 = 2M(E-E') + M^2 - q^2$. E is the energy of the incident electron, E' is the energy of the final electron, and θ is the scattering angle, all defined in the laboratory system; M is the mass of the proton. The cross section is divided by the Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{e^4}{4E^2}\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$

in order to remove the major part of the well-known four-momentum transfer dependence arising from the photon propagator. Results from both 6° and 10° are included in the figure for each value of W. As W increases, the q^2 dependence appears to decrease. The striking difference between the behavior of the inelastic and elastic cross sections is also illustrated in Fig. 1, where the elastic cross section, divided by the Mott cross section for $\theta = 10°$, is included. The q^2 dependence of the deep continuum is also consider-

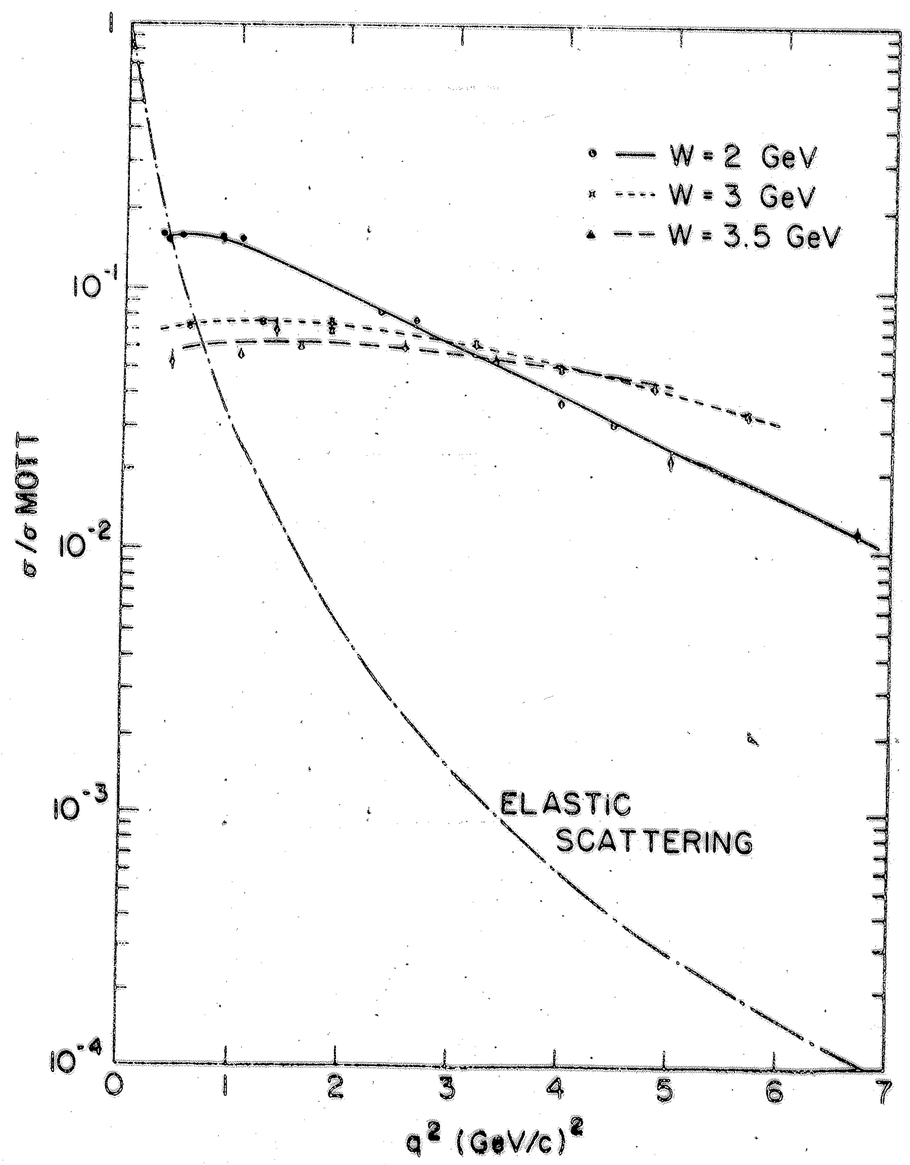

FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$, in GeV^{-1}, vs q^2 for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by σ_{Mott}, $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

We can also do the same thing for the polarized sector:

$$W_{\mu\nu}^{i\,(a)} = i\,\varepsilon_{\mu\nu\alpha\beta}\, S^{\alpha} q^{\beta}\, \delta(Q^2 - 2p\cdot q)$$

If we construct the electron-parton cross section, we will see that:

$$\boxed{\begin{aligned} G_1^i(\nu, Q^2) &= \frac{e_i^2}{2M^2\nu}\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \\ G_2^i(\nu, Q^2) &= 0 \end{aligned}} \qquad (1.27)$$

as before:

$$\boxed{\begin{aligned} M^2\nu\, G_1 &\longrightarrow g_1(x) = \frac{1}{2}\sum_i e_i^2\, \Delta f_i(x) \\ M\nu^2\, G_2 &\longrightarrow g_2(x) = 0 \end{aligned}} \qquad (1.28)$$