Through the looking glass: And what nucleons do there

HUGS, 5-23 June 2006
Lecture 2/2

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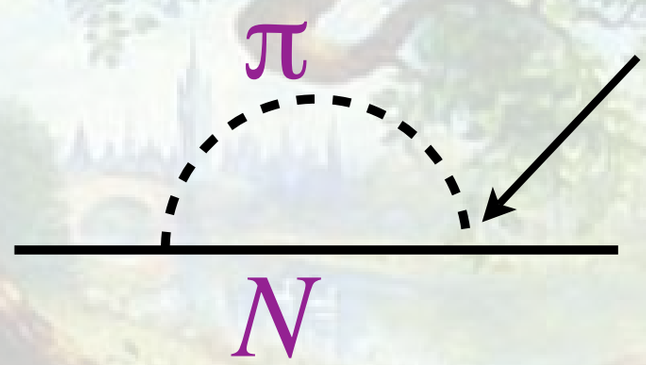
Jefferson Lab

Chiral correction

$m_\pi^2 \propto m_q$

$O(p^2): \quad M_0 + c_2 m_\pi^2$

$O(p^3):$



$\sim \dfrac{g_A}{f_\pi}$ **Nucleon axial charge** / **Pion decay constant**

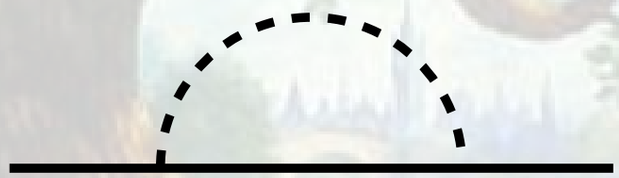
Quantum fluctuation, integrate over all intermediate pion energies

Do some spin algegra, isospin sum, heavy-nucleon limit, and temporal and angular integration:

$$= -\frac{3g_A^2}{16\pi^2 f_\pi^2}\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2}$$

Poor-man renormalisation

$$O(p^2): \quad M_0 + c_2 m_\pi^2$$



$$= -\frac{3g_A^2}{16\pi^2 f_\pi^2}\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2}$$

cubic divergence!

$$\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2} = \int_0^\infty dk\,\frac{k^4-m_\pi^4+m_\pi^4}{k^2+m_\pi^2}$$

$$= \int dk\,\left(k^2-m_\pi^2\right) + \int dk\,\frac{m_\pi^4}{k^2+m_\pi^2} = \frac{\pi}{2}m_\pi^3$$

$$M_N = M_0 + c_2 m_\pi^2 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\left[\int dk\,k^2 - m_\pi^2\int dk + \frac{\pi}{2}m_\pi^3\right]$$

Renormalised expansion

$$M_N = M_0 + c_2 m_\pi^2 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\left[\int dk\, k^2 - m_\pi^2 \int dk + \frac{\pi}{2} m_\pi^3\right]$$

$$M_0^{ren} = M_0 - \frac{3g_A^2}{16\pi^2 f_\pi^2}\int dk\, k^2$$

$$c_2^{ren} = c_2 + \frac{3g_A^2}{16\pi^2 f_\pi^2}\int dk$$

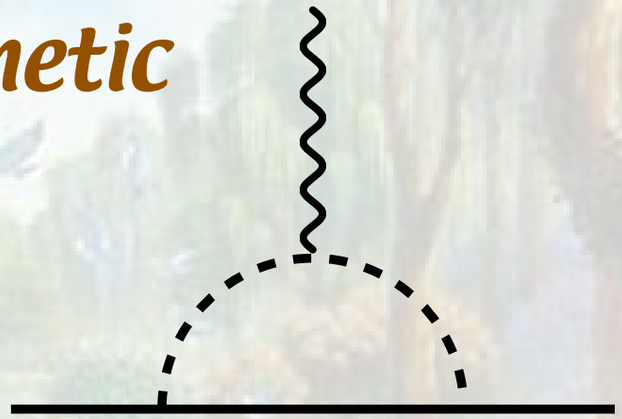
Absorb infinities into redefinition of expansion constants

$$M_N = M_0^{ren} + c_2^{ren} m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

Nonanalytic term: model-independent

Chiral structure of the nucleon

Chiral corrections to electromagnetic form factors



Leading nonanalytic corrections

$$\mu^{p(n)} \sim \mp \frac{g_A^2 M_N}{8\pi f_\pi^2} m_\pi$$

$$\langle r^2 \rangle_E^{p(n)} \sim \mp \frac{1+5g_A^2}{(4\pi f_\pi)^2} \log \frac{m_\pi}{\Lambda_\chi}$$

charge radii become infinite in the chiral limit

The Foldy Term

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$F_1^n(Q^2 = 0) = 0 \qquad \mu^n \equiv G_M^n(Q^2 = 0) = F_2^n(0)$$

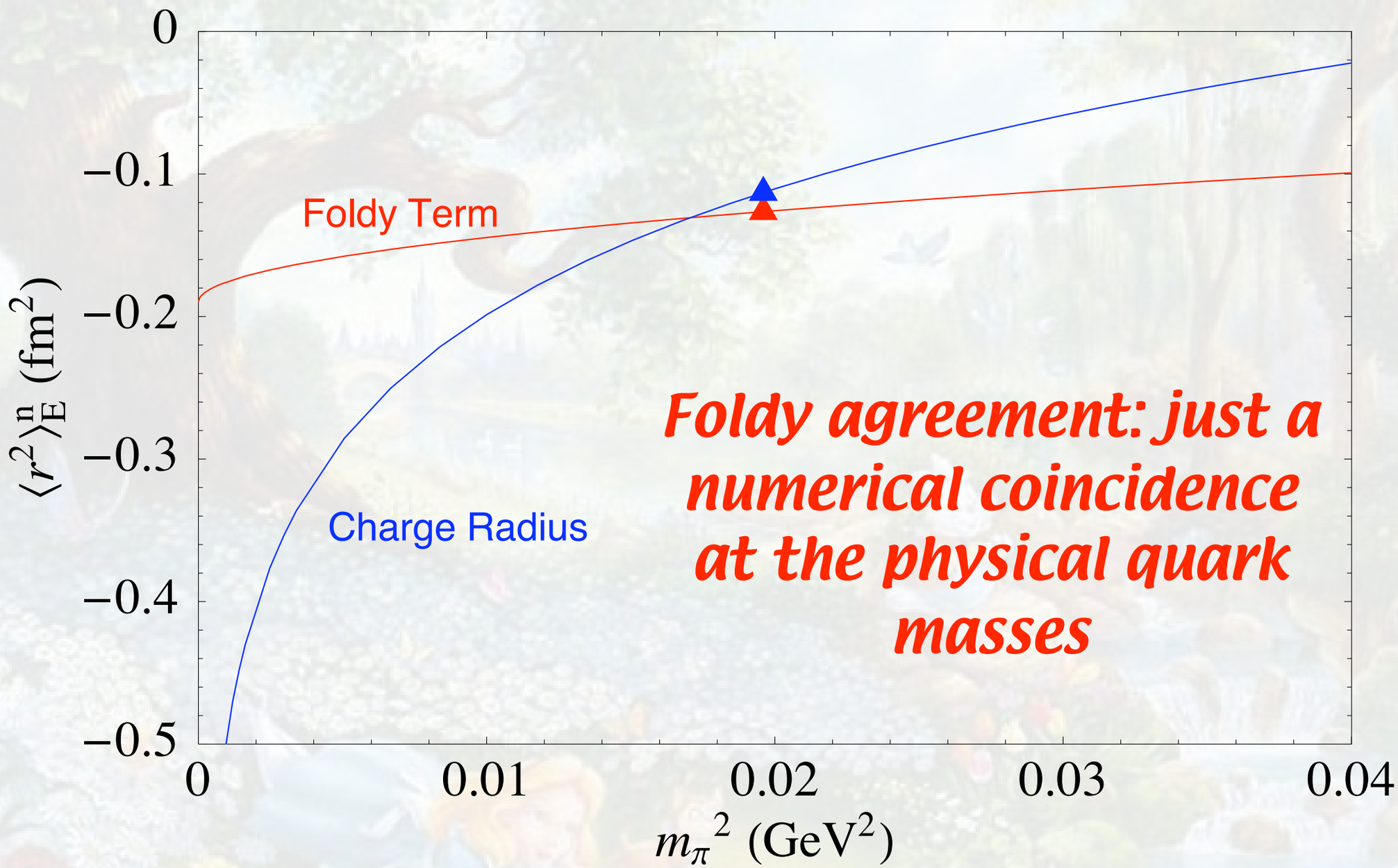
$$\langle r^2 \rangle_E^n = -6\frac{d}{dQ^2}G_E^n(Q^2 = 0) = -6\frac{d}{dQ^2}F_1^n(0) + \frac{3}{2M^2}\mu^n$$

$$\langle r^2 \rangle_E^n = -0.116 \pm 0.002\,\mathrm{fm}^2$$

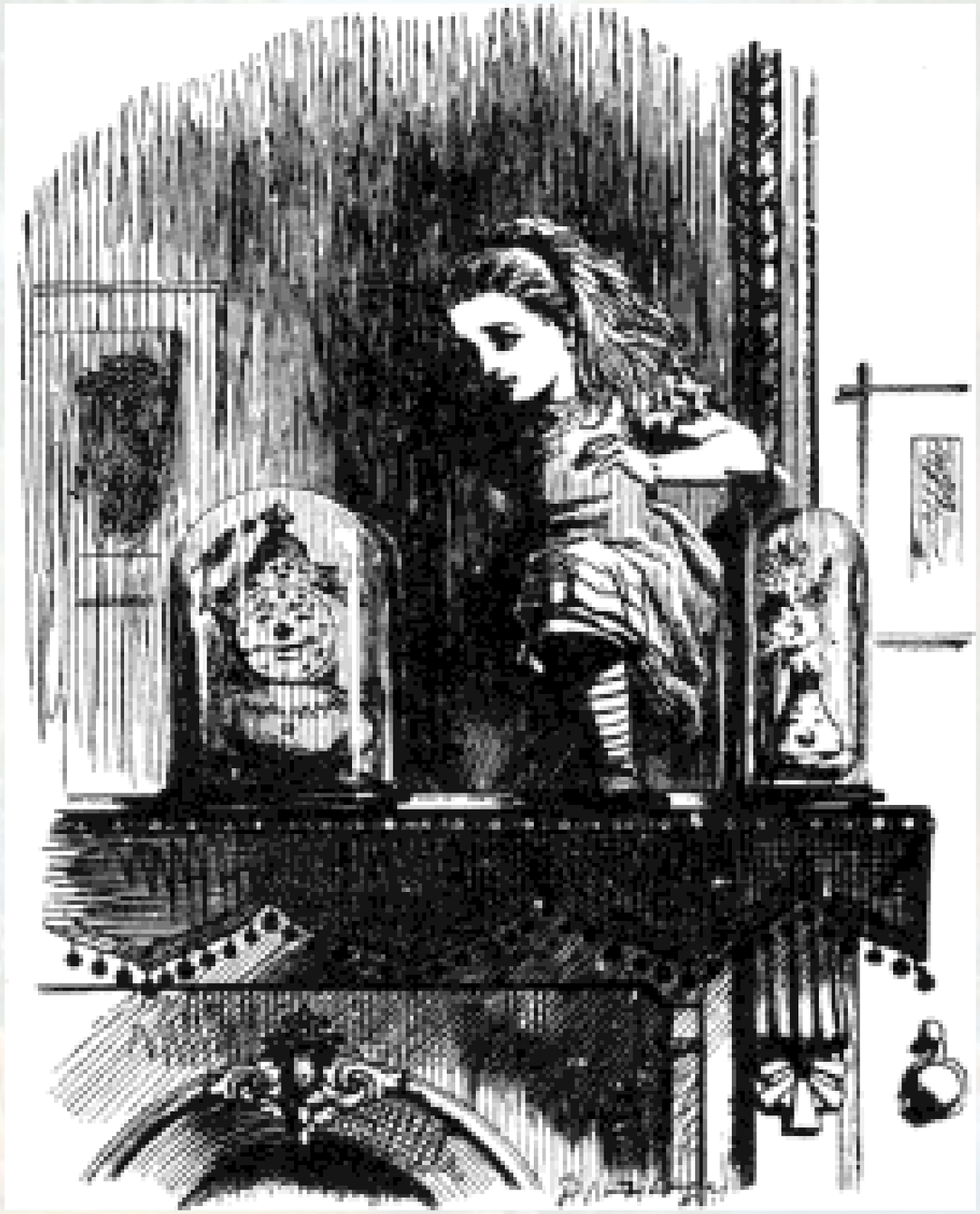
$$-0.126\,\mathrm{fm}^2$$

Intrinsic charge distribution interpreted to be zero, charge radius just a relativistic effect of anomalous magnetic moment

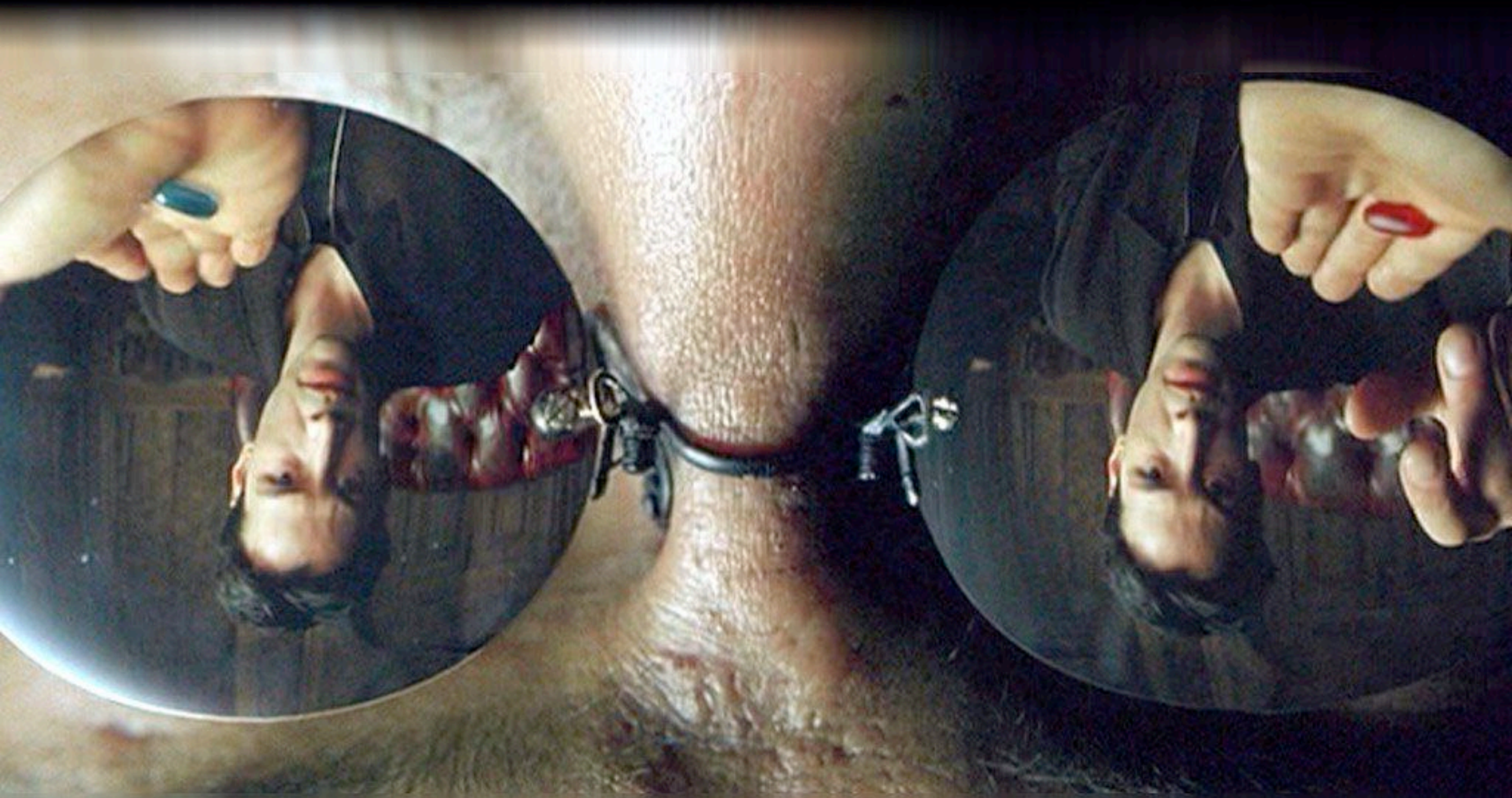
Foldy Term (1952)

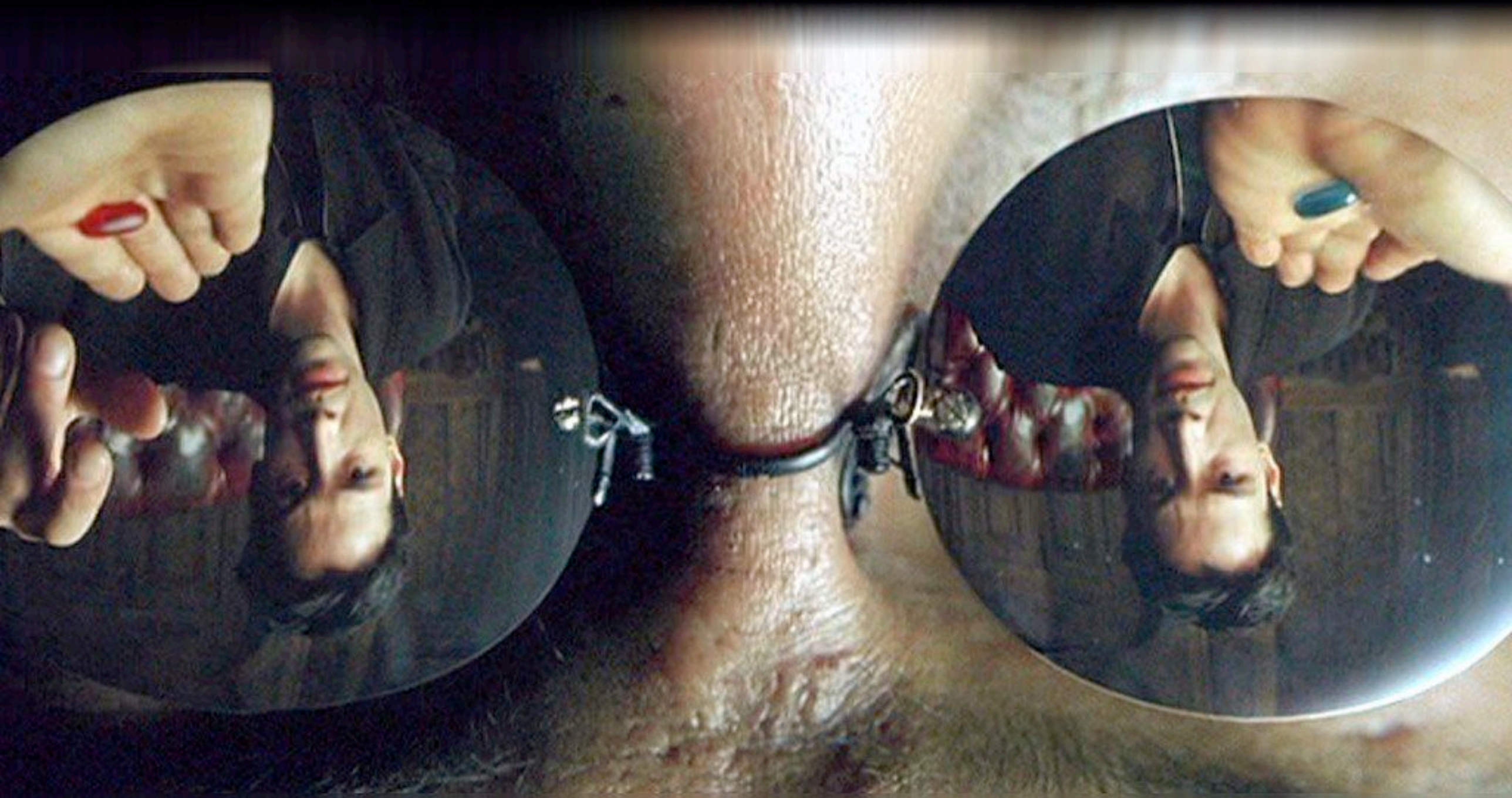


Foldy agreement: just a numerical coincidence at the physical quark masses



Entering the parity-violating world

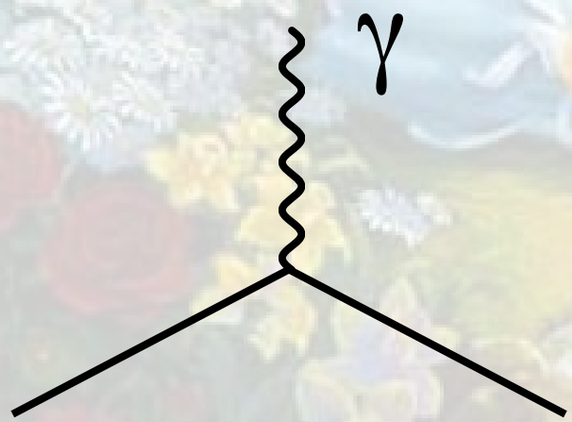


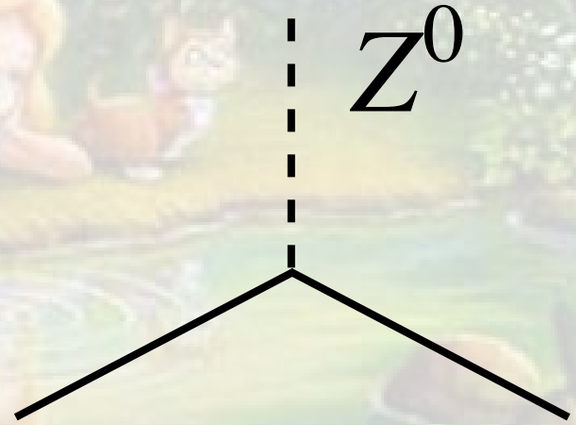


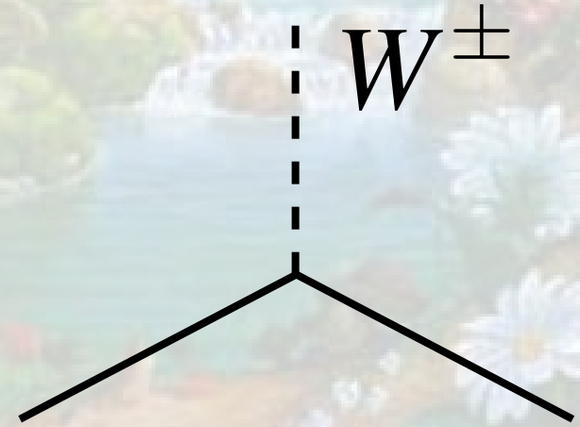
The weak-interaction

- Standard Model electroweak theory unified by Weinberg, Salam & Glashow
  - Local gauge symmetry group $SU(2) \times U(1)$
- Parity maximally broken by SU(2) sector, coupling to left-handed currents only

$$\mathcal{L}_{\text{int}} = -eJ^{\mu}_{EM}(x)\mathcal{A}_{\mu}(x) - \frac{g}{2\cos\theta_W}J^{\mu}_{NC}(x)\mathcal{Z}_{\mu}(x) - \frac{g}{2\sqrt{2}}J^{\mu}_{CC}(x)\mathcal{W}^{\dagger}_{\mu}(x) + \text{h.c.}$$







The weak-interaction... 2

- **Weak mixing angle**
  - **Physical photon is mixture of U(1) and SU(2) gauge fields**

$$\sin\theta_W = \frac{e}{g}, \quad \sin^2\theta_W = 0.2230 \pm 0.0004$$

- **The Fermi interaction**

$$G_F = \frac{\sqrt{2}g^2}{8M_W^2} = 1.16639 \pm 0.00001 \times 10^{-5}\,\mathrm{GeV}^{-2}$$

Neutral weak current

- **Electromagnetic current**

$$J^{\mu}_{EM} = \frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d - \frac{1}{3}\bar{s}\gamma^{\mu}s$$

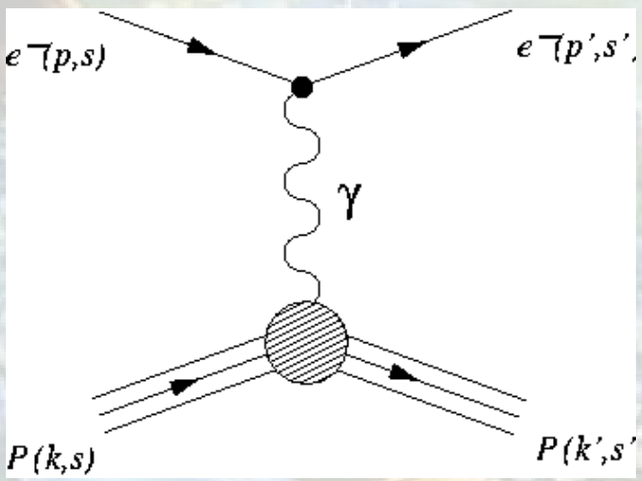
- **Neutral current**

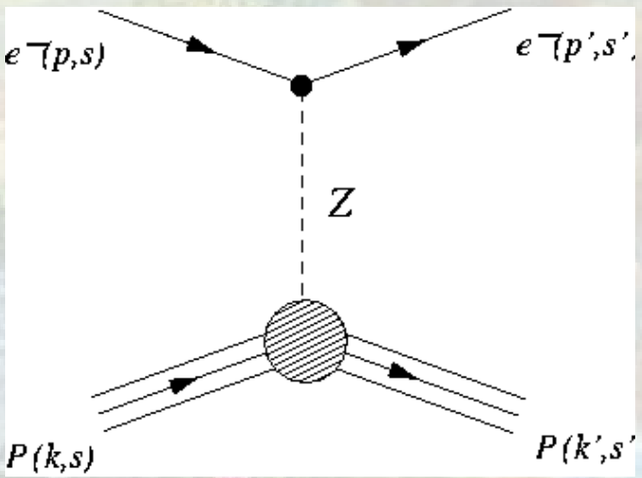
$$J^{\mu}_{NC} = \bar{u}\gamma^{\mu}\left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W - \frac{1}{2}\gamma_5\right)u + \bar{d}\gamma^{\mu}\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W + \frac{1}{2}\gamma_5\right)d$$
$$+\bar{s}\gamma^{\mu}\left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W + \frac{1}{2}\gamma_5\right)s$$

- **Charged current**
  - **More complicated: CKM mixing matrix**

Experimental Strategy

- *Measure the interference between photon and neutral currents in parity-violating electron scattering (PVES)*





$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \sim \frac{\left| \gamma \quad Z^0 \right|}{\left| \gamma \right|^2} \sim \frac{\left| M_{PV}^{NC} \right|}{\left| M^{EM} \right|} \sim \frac{Q^2}{(M_Z)^2}$$

PVES from a proton target

- *Total asymmetry*

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{p\gamma} G_E^{pZ} + \tau G_M^{p\gamma} G_M^{pZ} - \frac{1}{2}(1 - 4\sin^2\theta_W)\varepsilon' G_M^{p\gamma} \tilde{G}_A^p}{\varepsilon (G_E^{p\gamma})^2 + \tau (G_M^{p\gamma})^2}$$

Charge symmetry: neglect u-d mass difference, EM corrections
invariance under u-d exchange

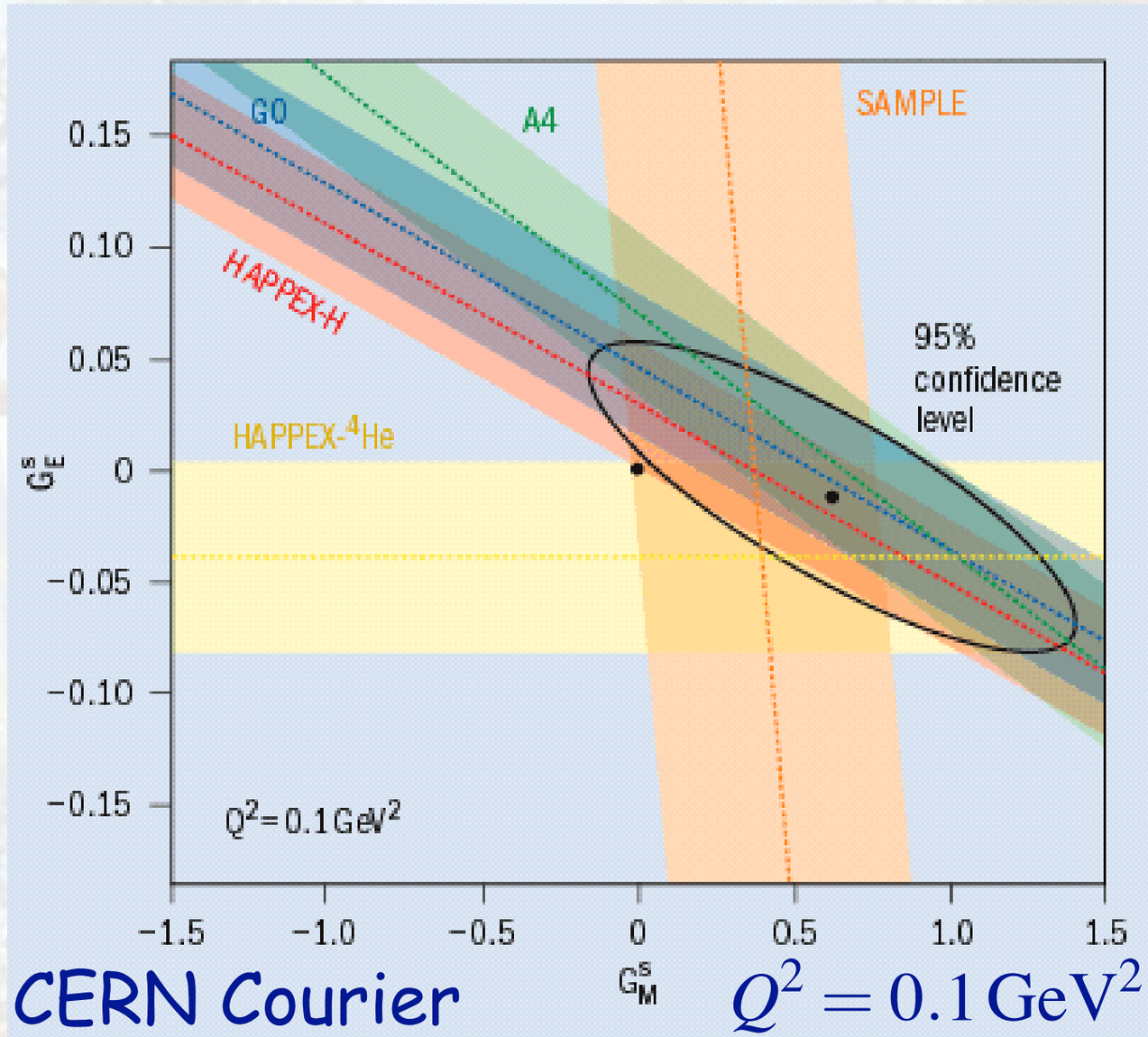
$$G_{E,M}^{p\,(u)} = G_{E,M}^{n\,(d)}\,, \qquad G_{E,M}^{p\,(s)} = G_{E,M}^{n\,(s)}\,, \ldots$$

$$4G_{E,M}^{pZ} = (1 - 4\sin^2\theta_W) G_{E,M}^{p\gamma} - G_{E,M}^{n\gamma} - G_{E,M}^{s}$$

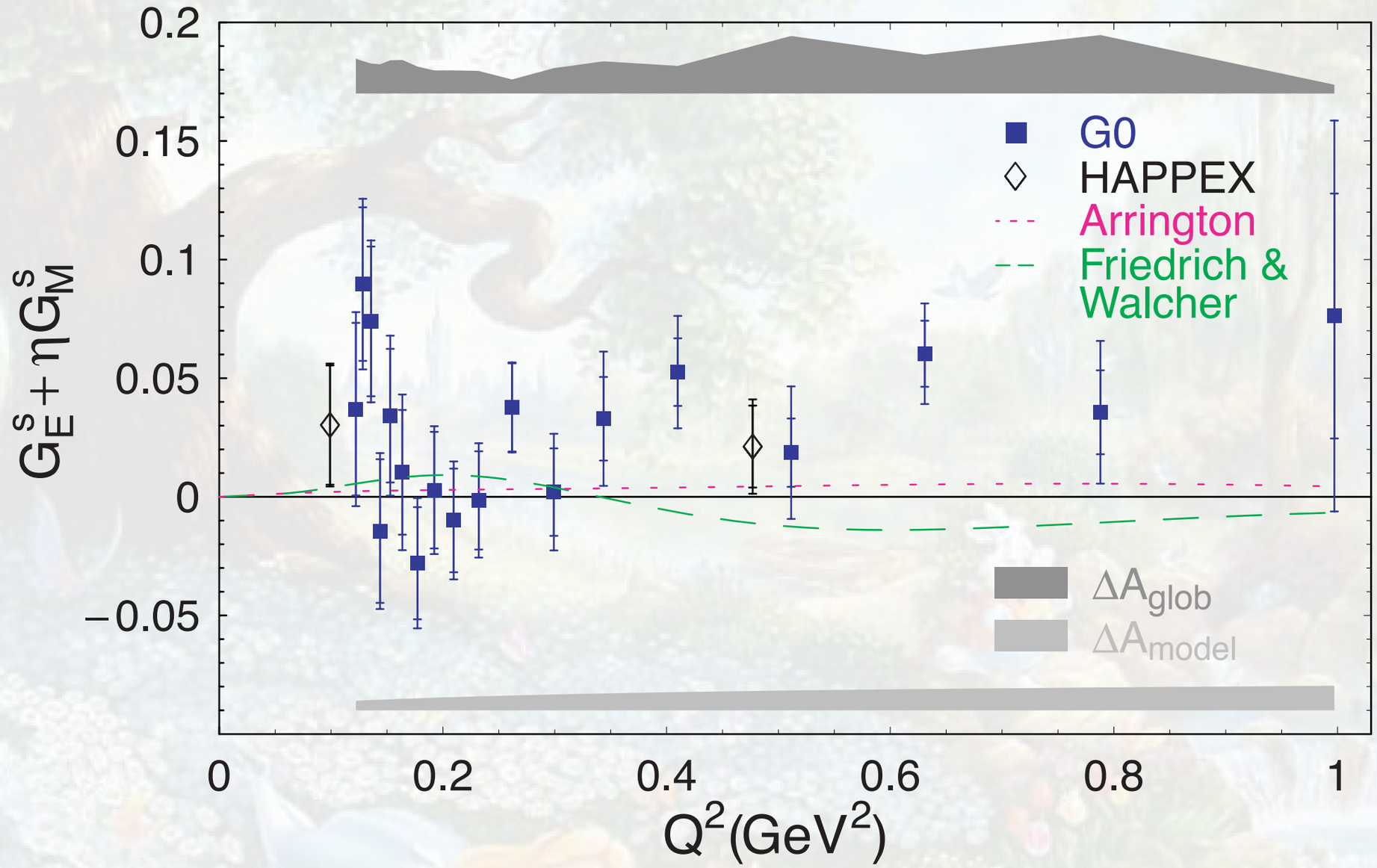
Strangeness

Experimental Programs

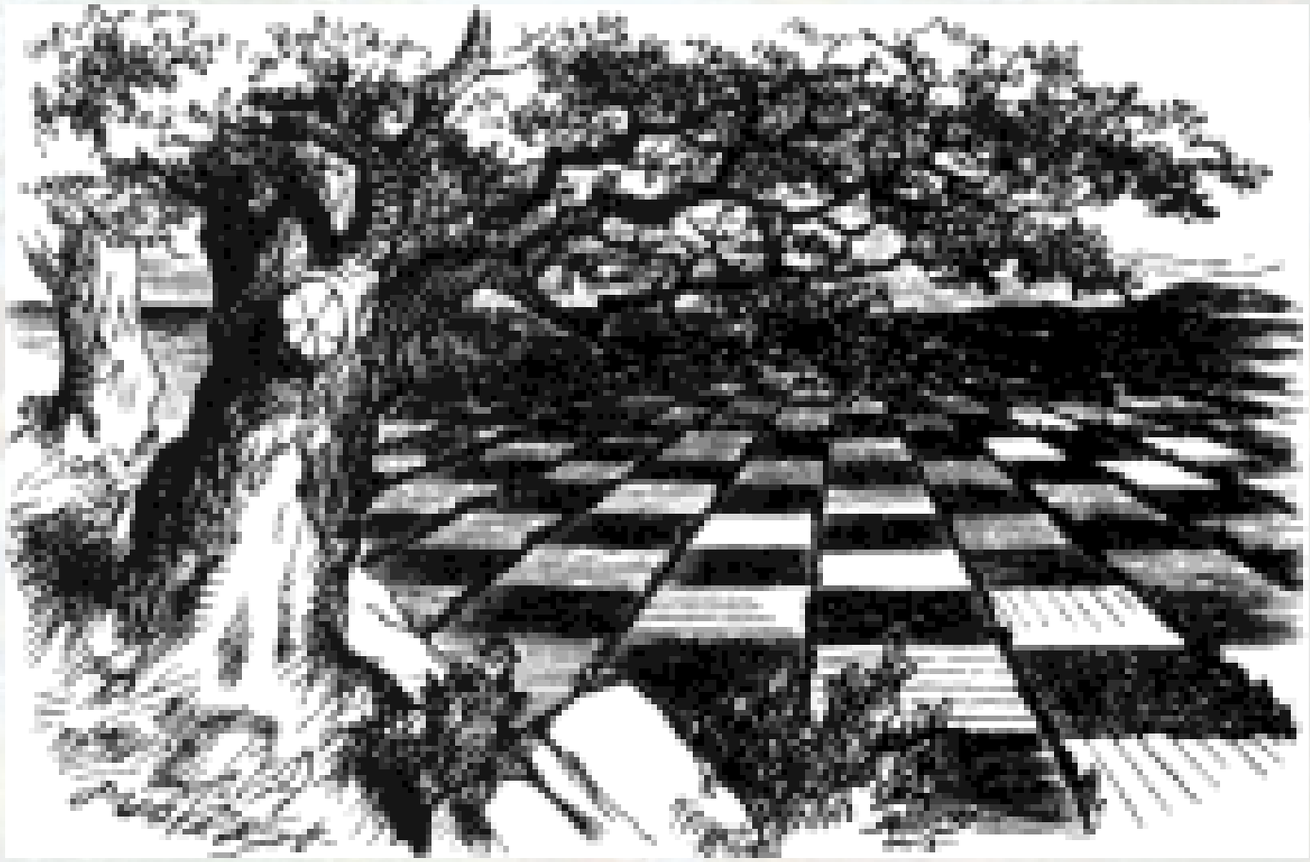
- SAMPLE Collaboration at MIT-Bates
  - Backward angle scattering: H & D @ Q^2~0.1 GeV^2
- HAPPEx Experiment at Jefferson Lab
  - Forward angle: H @ Q^2~0.5,0.1 GeV^2: He4 @ Q^2~0.1 GeV^2
- PVA4 at Mainz
  - Forward angle: H @ Q^2~0.23,0.1 GeV^2
  - Backward run: H @ Q^2~0.23 GeV^2 (in progress)
- G0 Experiment at Jefferson Lab
  - Forward angle: H @ Q^2~0.12–1.0 GeV^2
  - Backward run: H & D @ Q^2~0.23,0.6 GeV^2 (in progress)



Suggestion of a large & positive strangeness magnetic moment



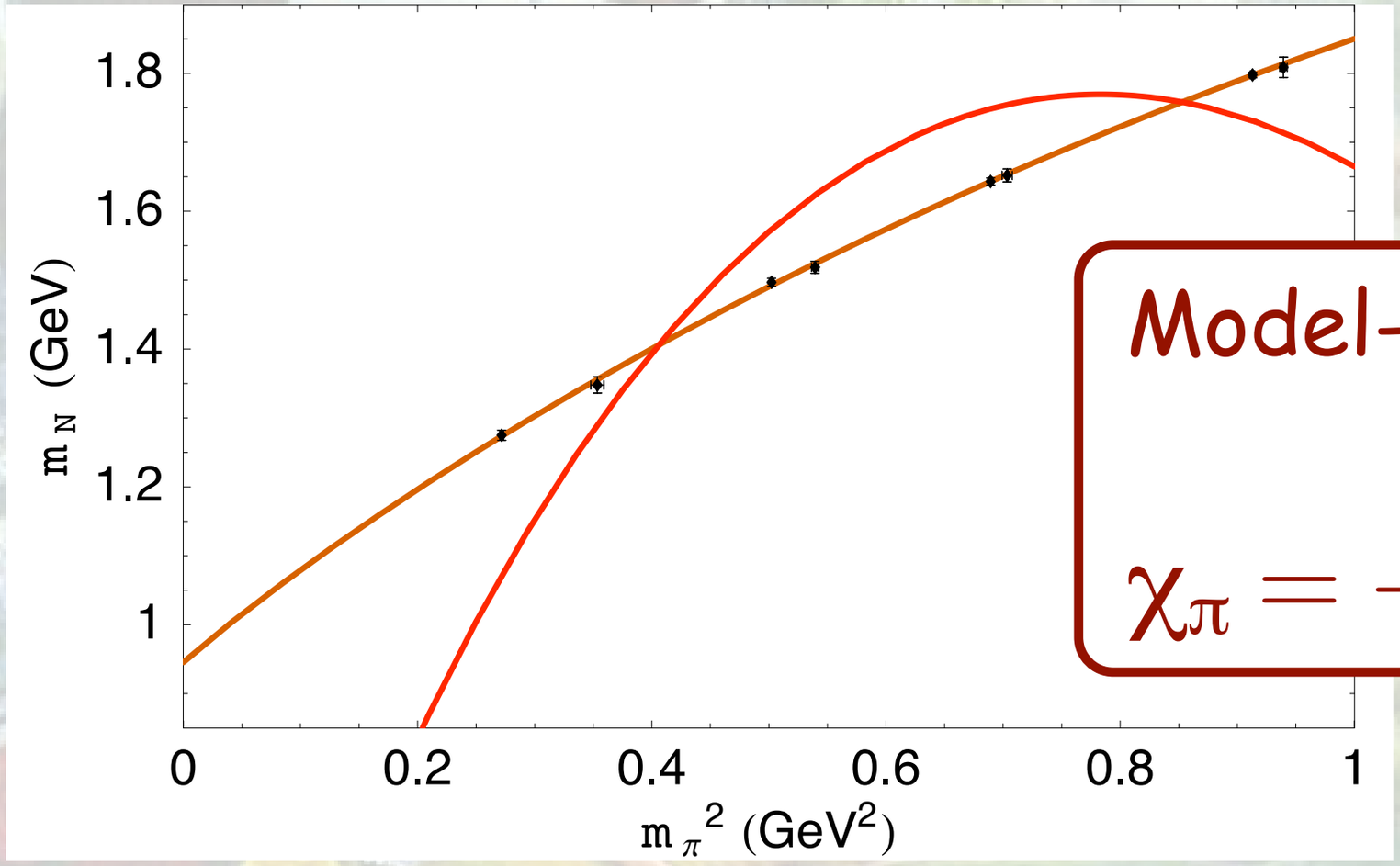
G0 Collaboration



THEORY:
Strangeness content in a QCD-based approach.

Chiral Extrapolation

ChPT: $M_N = c_0 + c_2 m_\pi^2 + \chi_\pi m_\pi^3 + \ldots$



$m_\pi^2 \propto m_q$

Model-independent value

$\chi_\pi = -5.5\mathrm{GeV}^{-2}$

$\chi_\pi \simeq -0.63\mathrm{GeV}^{-2}$

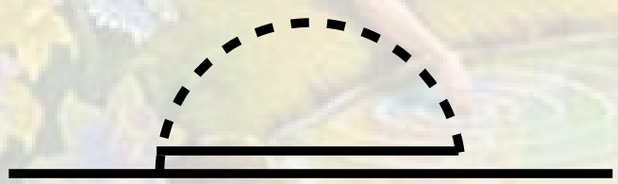
Finite-range regularisation

$$= -\frac{3g_A^2}{16\pi^2 f_\pi^2}\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2}$$

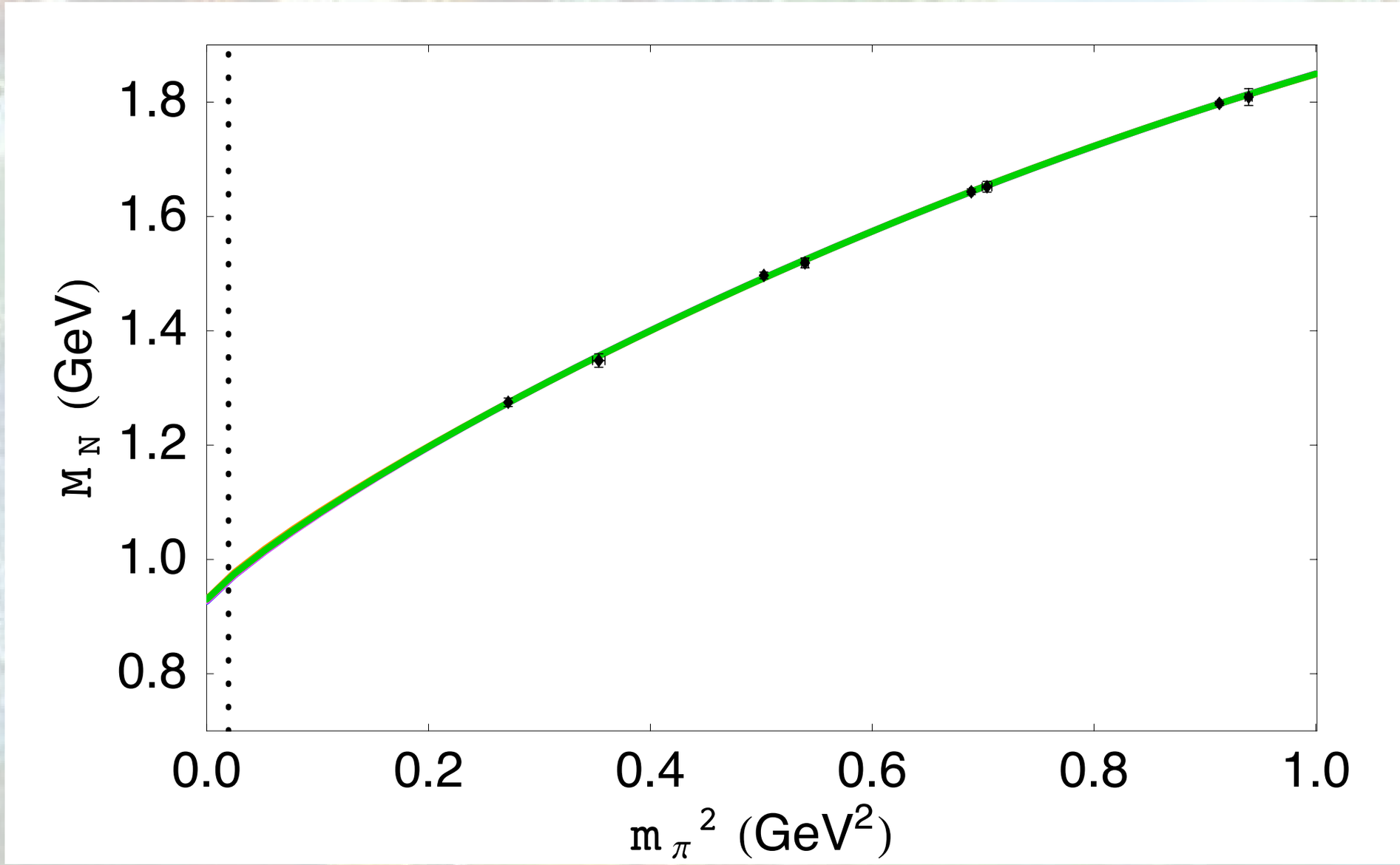
Use functional form to regularise ultra-violet behaviour of loop integral, eg. dipole

$$-\frac{3g_A^2}{16\pi^2 f_\pi^2}\int_0^\infty dk\,\frac{k^4}{k^2+m_\pi^2}\left(\frac{\Lambda^2}{\Lambda^2+k^2}\right)^4$$

Exactly the same model-independent nonanalytic contribution!



Finite-range regularisation... 2



Dipole

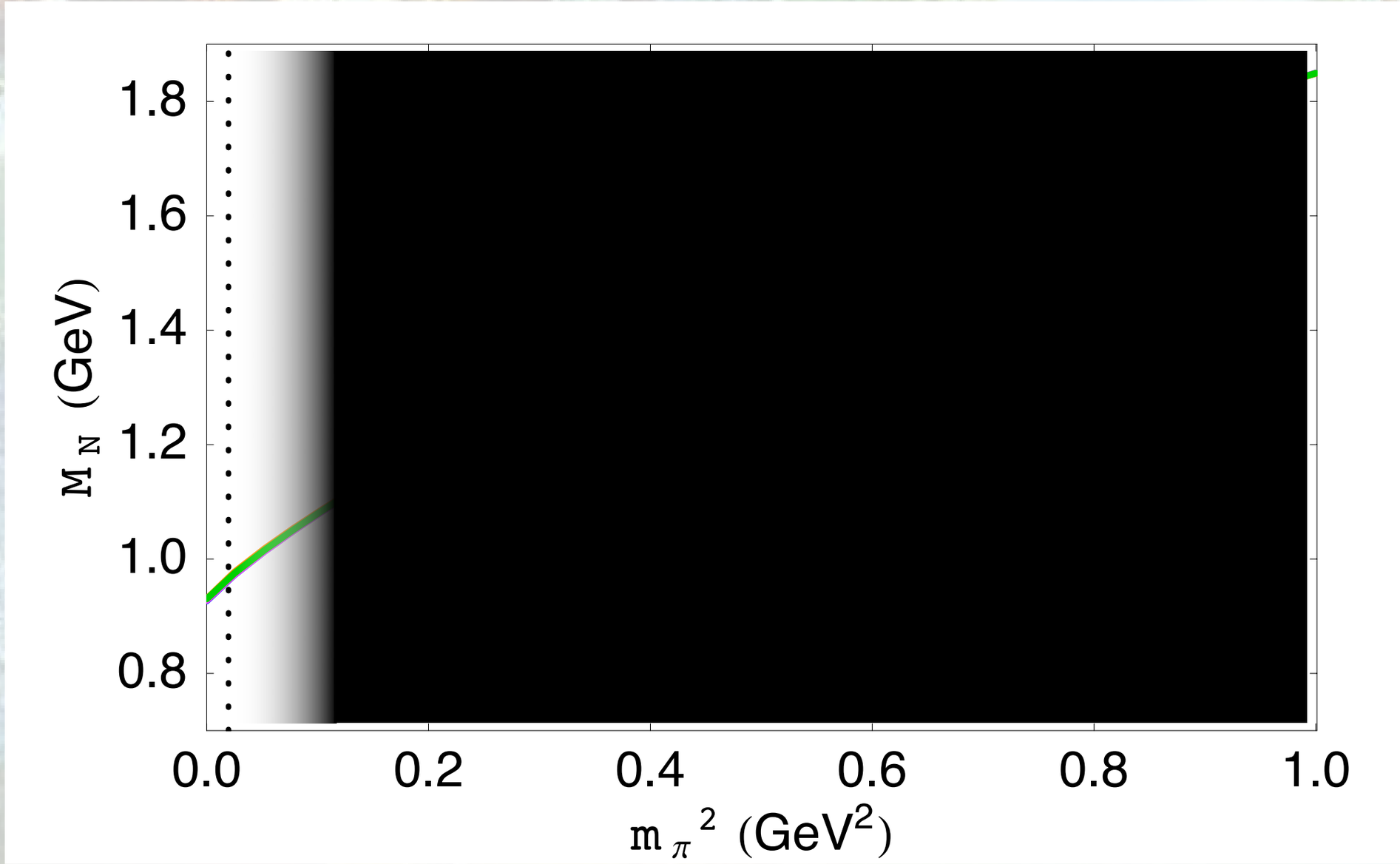
Theta

Monopole

Gaussian

Independent-of-model chiral extrapolation

Finite-range regularisation... 2



Dipole

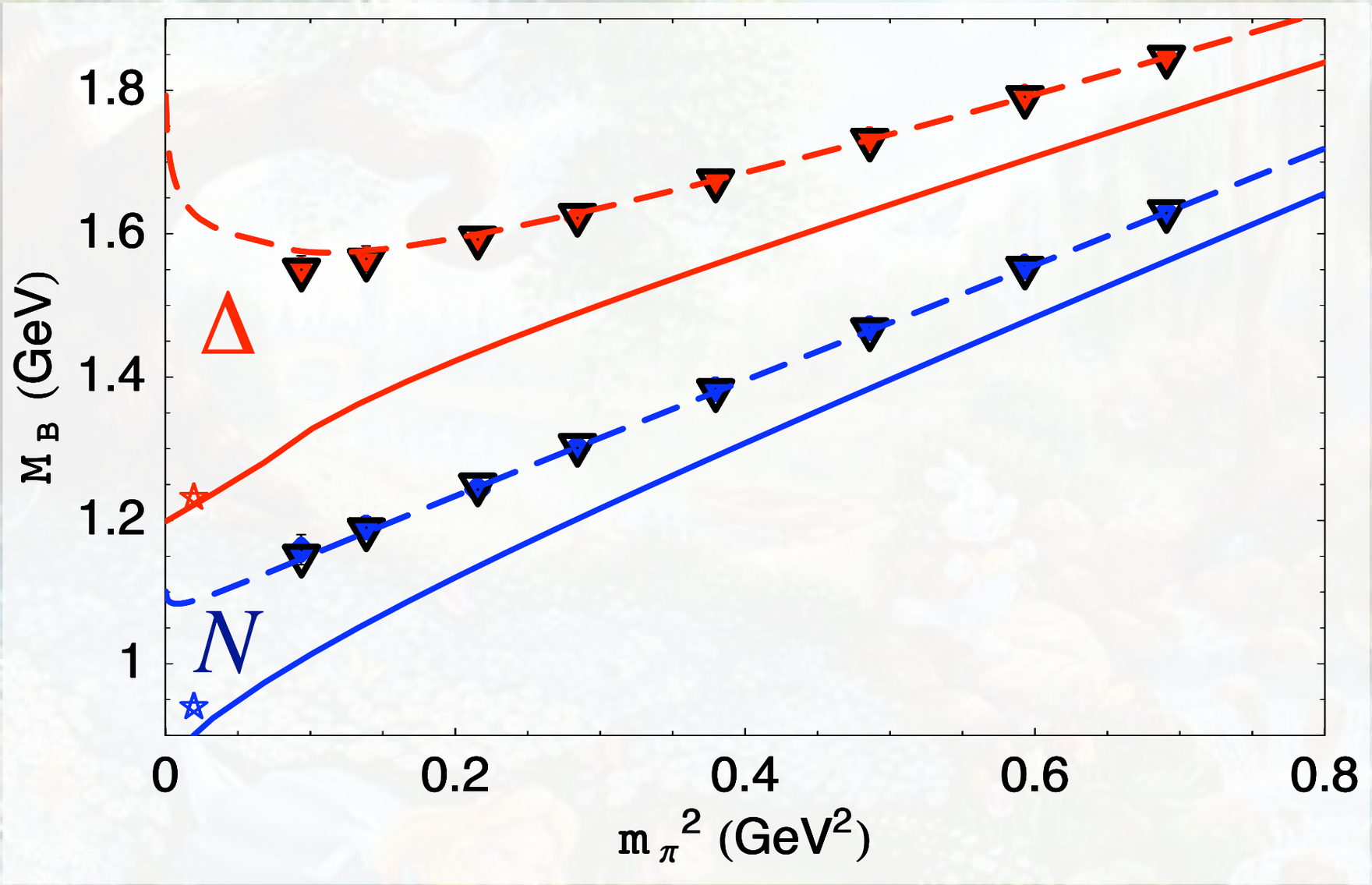
Theta

Monopole

Gaussian

Independent-of-model chiral extrapolation

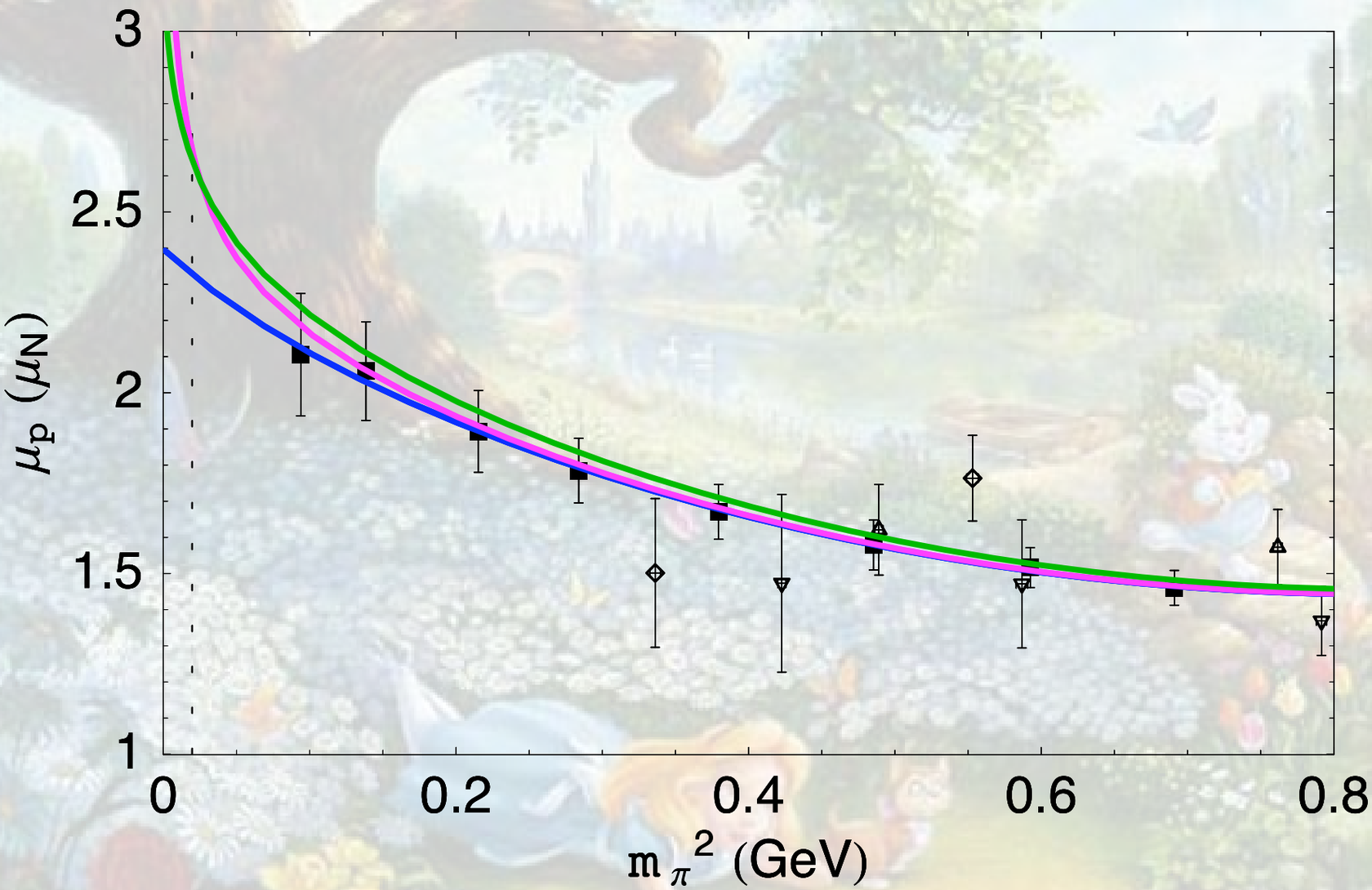
Baryon Masses



Establishing a connection between quenched simulations and QCD

Differences described by pion-loop corrections

Proton Magnetic Moment



Finite-volume

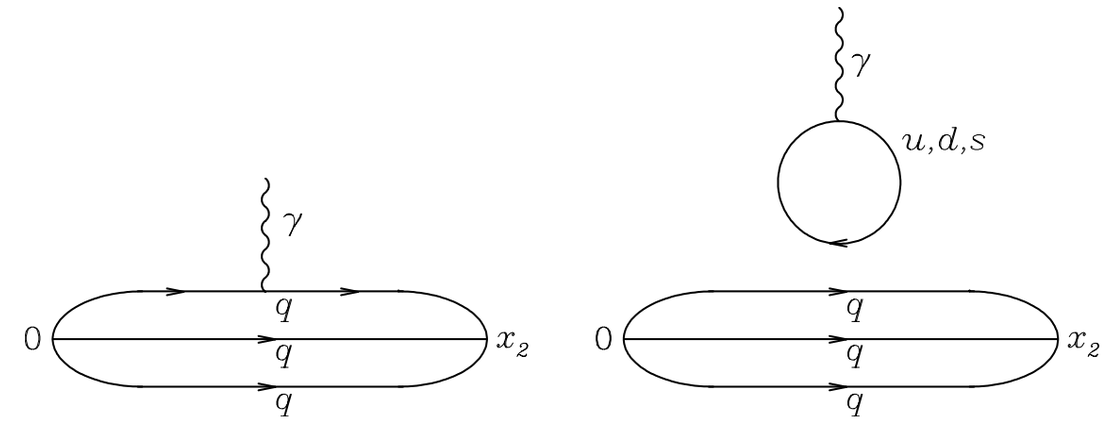
Quenched

QCD

Apply unquenching technique to EM structure

Strangeness

Assume charge symmetry



$$p = \frac{2}{3}u^p - \frac{1}{3}u^n + O_N$$

$$n = -\frac{1}{3}u^p + \frac{2}{3}u^n + O_N$$

$$\Sigma^+ = \frac{2}{3}u^\Sigma - \frac{1}{3}s^\Sigma + O_\Sigma$$

$$\Sigma^- = -\frac{1}{3}u^\Sigma - \frac{1}{3}s^\Sigma + O_\Sigma$$

$$3O_N = 2p + n - u^p$$

$$\Sigma^+ - \Sigma^- = u^\Sigma$$

$$3O_N = 2p + n - \frac{u^p}{u^\Sigma}(\Sigma^+ - \Sigma^-)$$

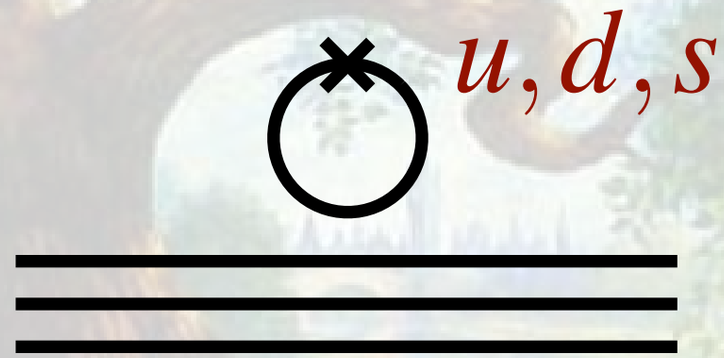
Lattice QCD

$$3O_N = p + 2n - u^n$$

$$\Xi^0 - \Xi^- = u^\Xi$$

$$3O_N = p + 2n - \frac{u^n}{u^\Xi}(\Xi^0 - \Xi^-)$$

The loop part

$O_N =$ 

"l" loop contribution

$$= \frac{2}{3}{}^{l}G_M^u - \frac{1}{3}{}^{l}G_M^d - \frac{1}{3}{}^{l}G_M^s$$

$${}^{l}G_M^u = {}^{l}G_M^d$$

$$O_N = -\frac{1}{3}\left({}^{l}G_M^d + {}^{l}G_M^s\right)$$

$$= \frac{{}^{l}G_M^s}{3}\left(\frac{1 - {}^{l}R_d^s}{{}^{l}R_d^s}\right)$$

QCD equality for $m_u = m_d$

$${}^{l}R_d^s = {}^{l}G_M^s / {}^{l}G_M^d = 0.139 \pm 0.042$$

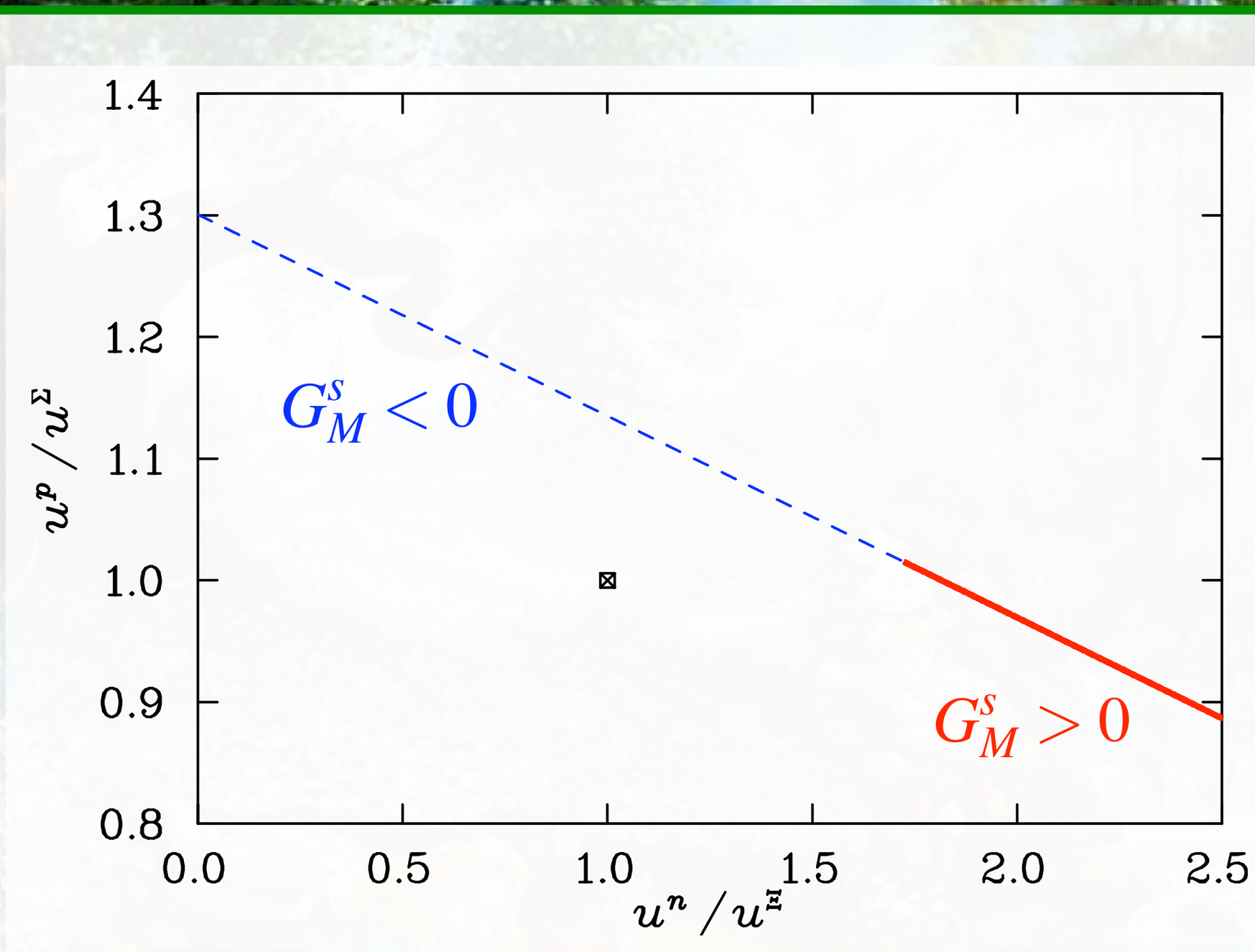
Kaon strength relative to pion

Strangeness Magnetic Moment

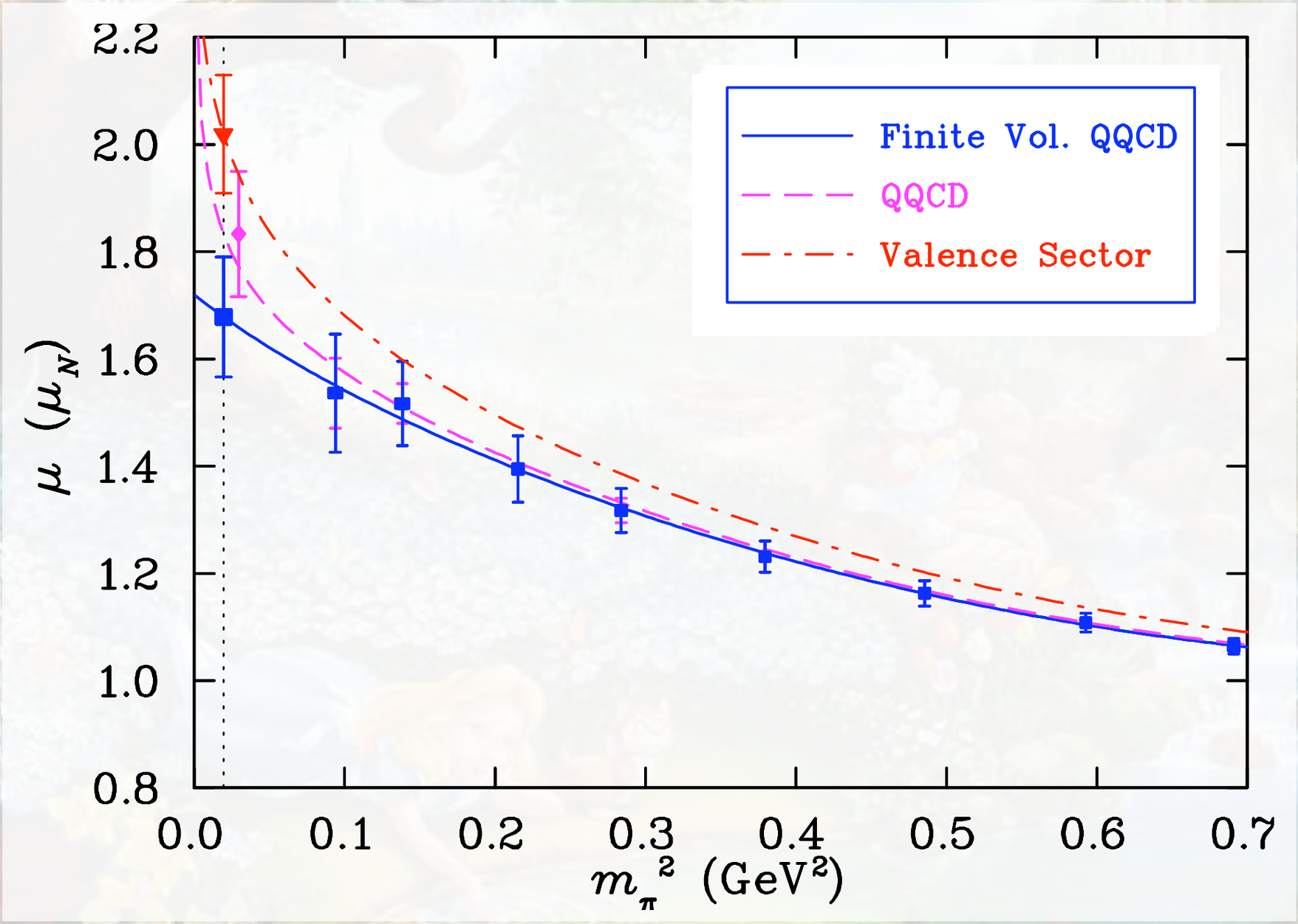
$$G_M^s = \left(\frac{{}^lR_d^s}{1-{}^lR_d^s}\right)\left[2p+n-\frac{u^p}{u^\Sigma}(\Sigma^+-\Sigma^-)\right]$$

$$G_M^s = \left(\frac{{}^lR_d^s}{1-{}^lR_d^s}\right)\left[p+2n-\frac{u^n}{u^\Xi}(\Xi^0-\Xi^-)\right]$$

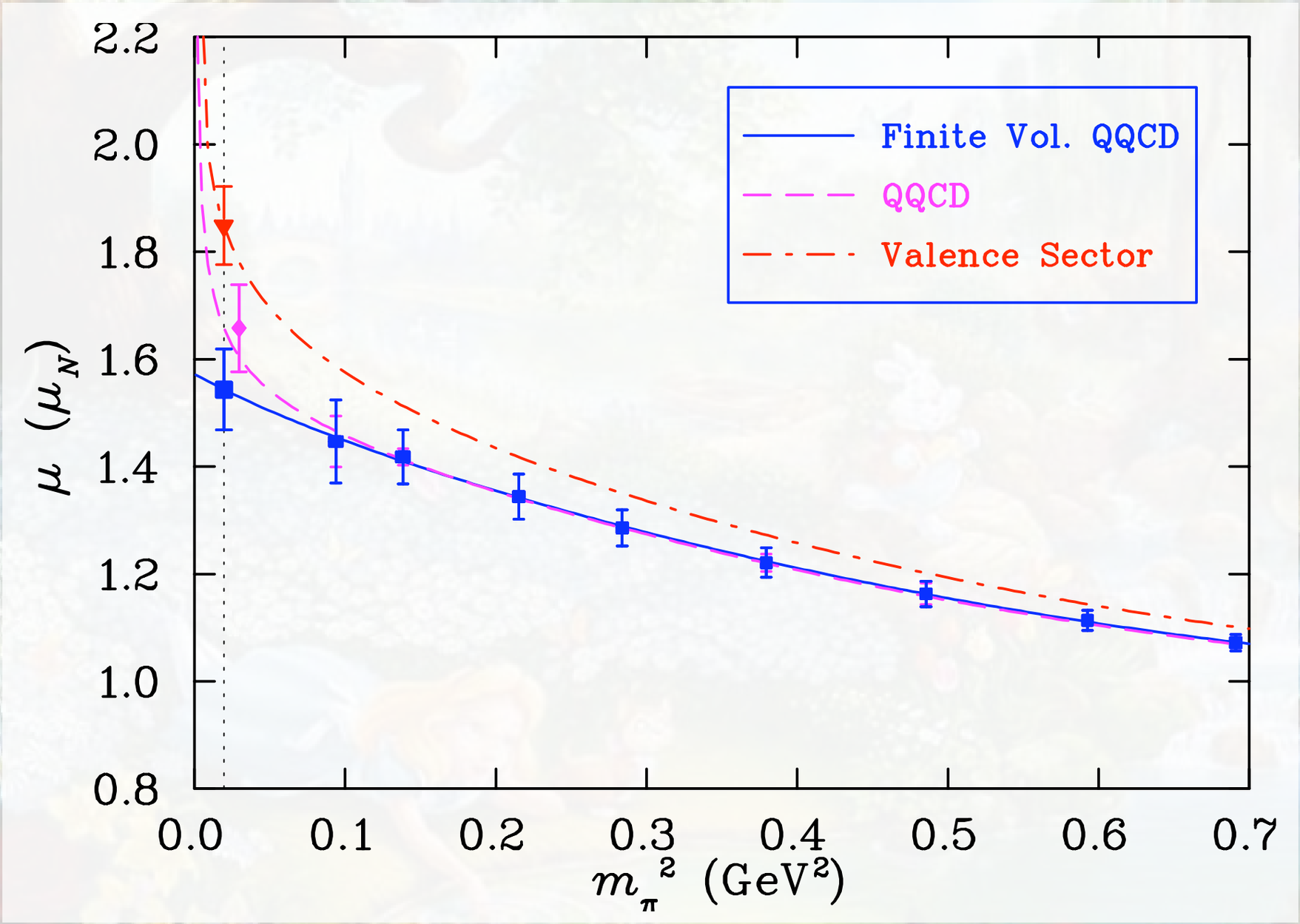
Two equations give linear constraint between p/Sigma and n/Xi ratios


1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.0
0.5
1.0
1.5
2.0
2.5
u^p / u^Σ
u^n / u^Ξ
$G_M^s < 0$
$G_M^s > 0$

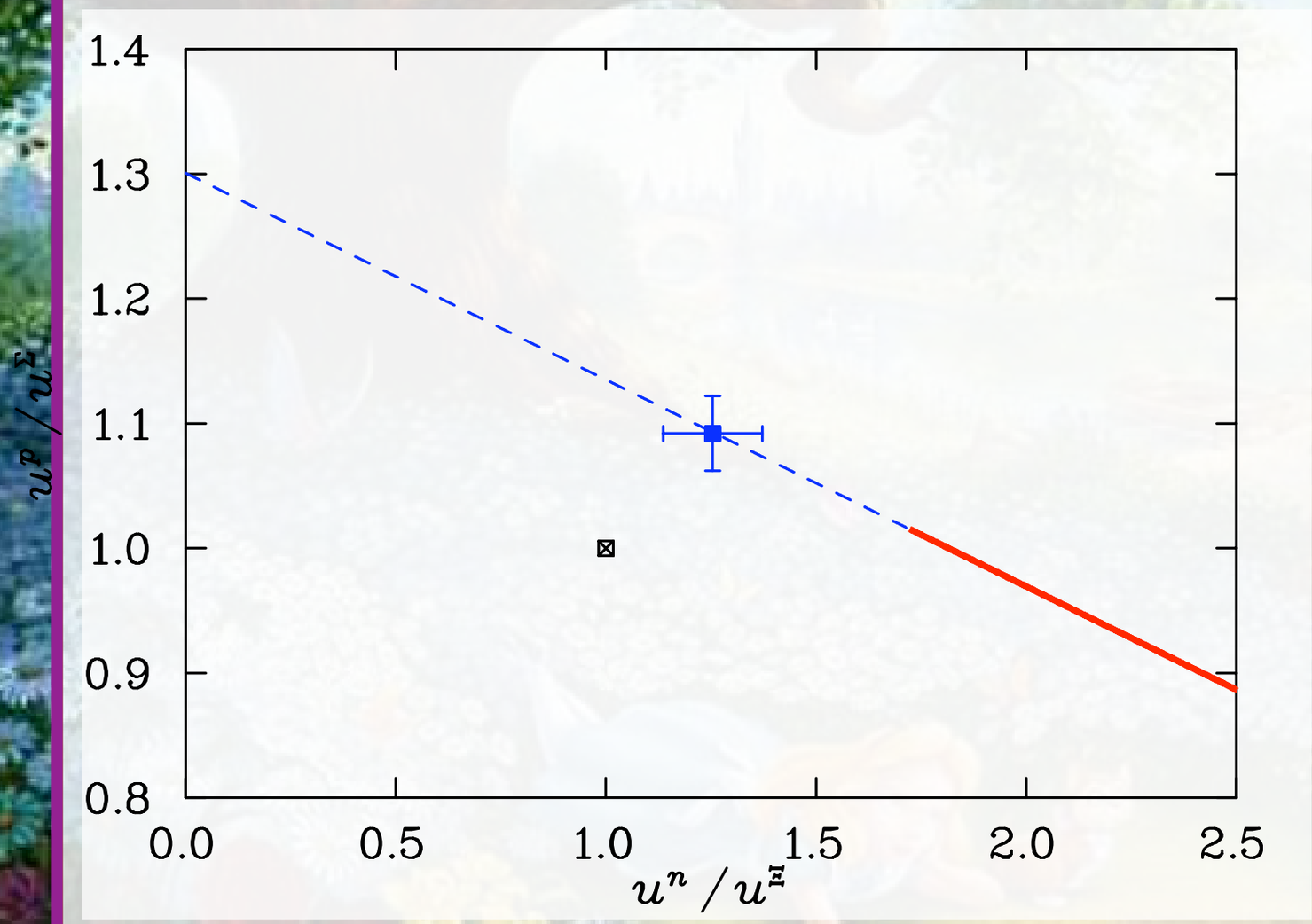
u-quark in the proton



u-quark in the Sigma



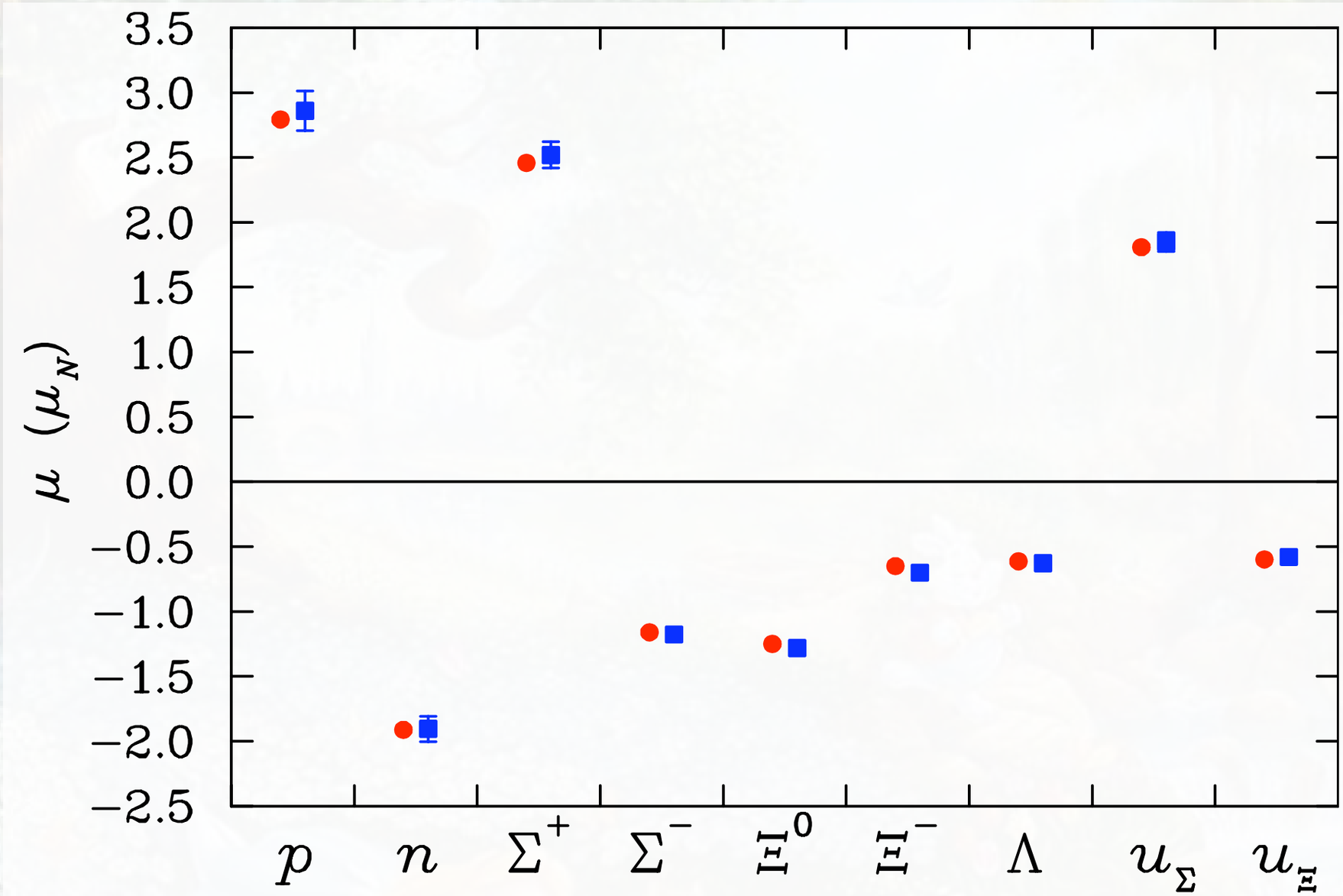
Final Result



$$\frac{u^p}{u^\Sigma} = 1.092 \pm 0.030$$

$$\frac{u^n}{u^\Xi} = 1.254 \pm 0.124$$

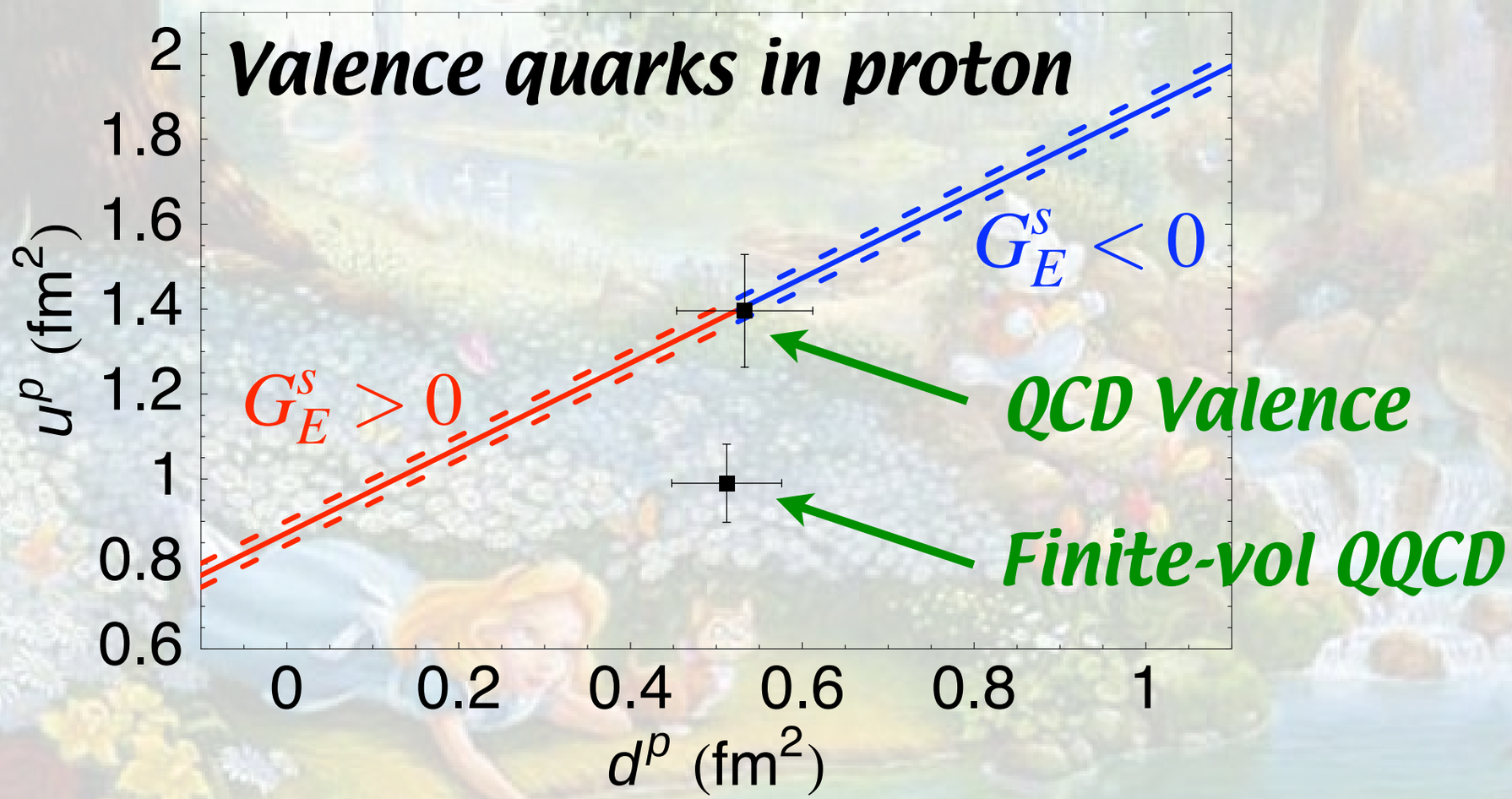
$$G_M^s = -0.046 \pm 0.022\,\mu_N$$



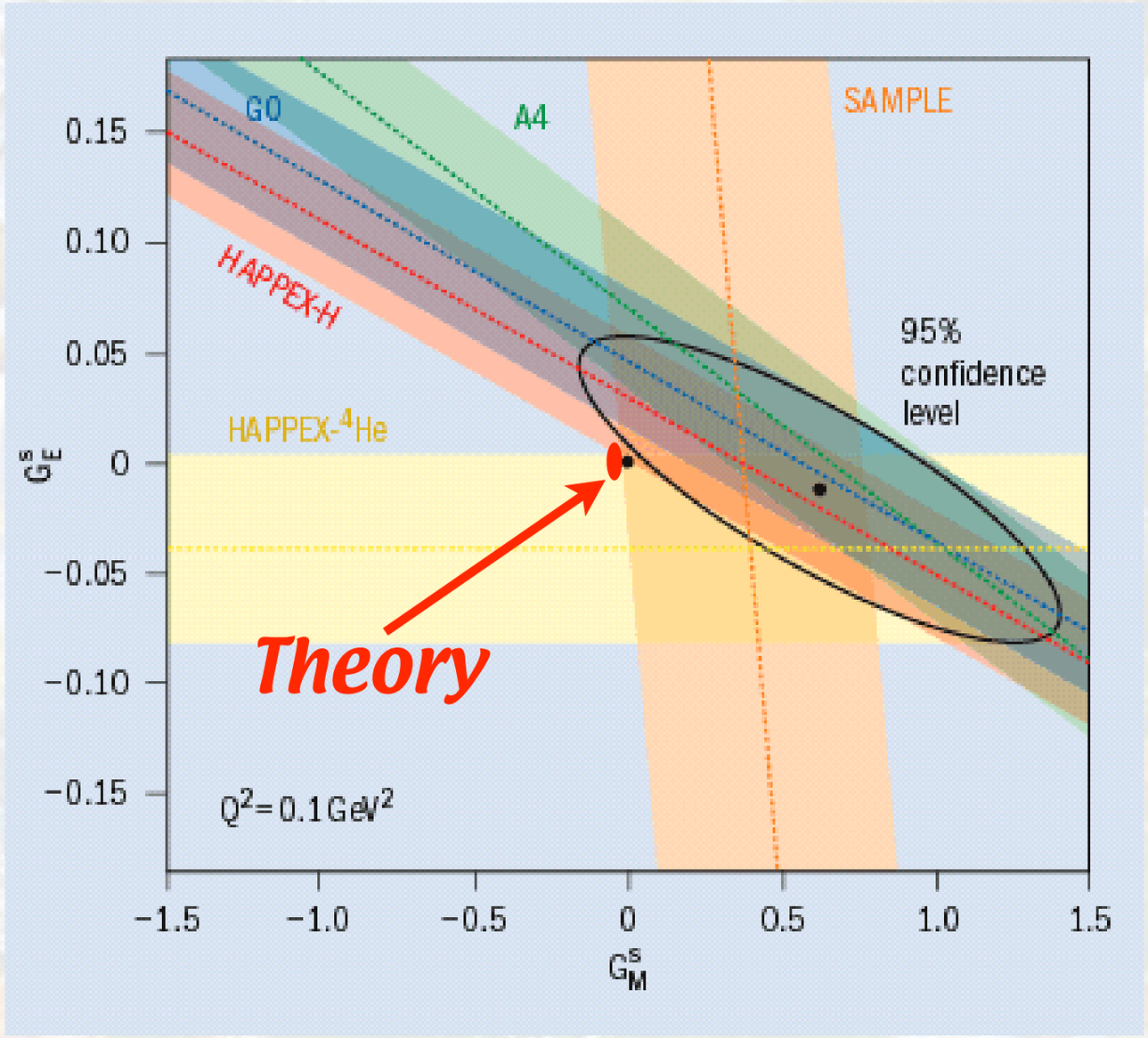
Hyperon magnetic moments

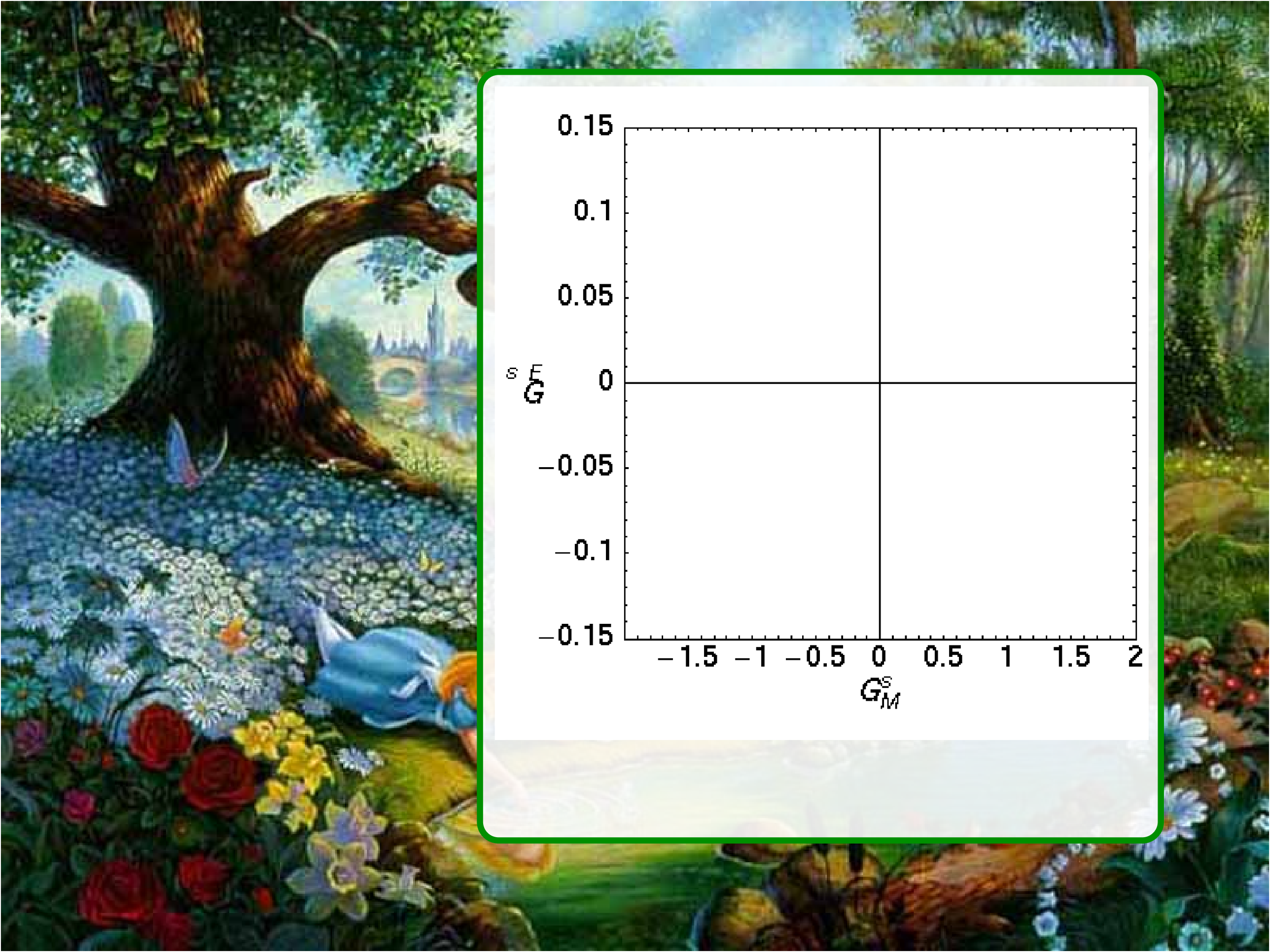
Electric Form Factor

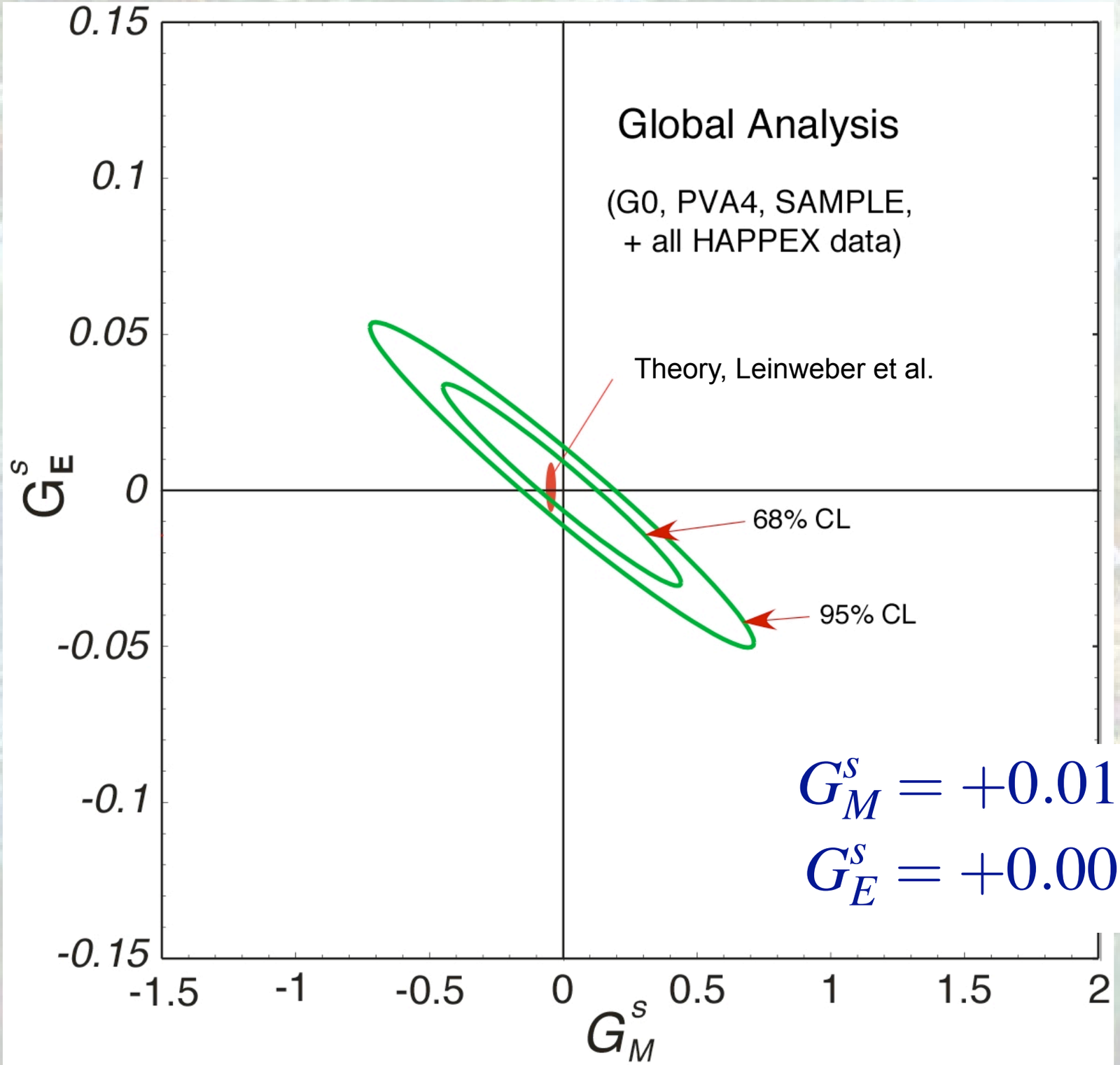
Limit knowledge of hyperon charge radii



$$G_E^s(Q^2 = 0.1) = +0.001 \pm 0.004 \pm 0.004$$


G0
A4
SAMPLE
HAPPEX-H
HAPPEX-4He
95% confidence level
Theory
Q2= 0.1 GeV2
0.15
0.10
0.05
0
−0.05
−0.10
−0.15
−1.5
−1.0
−0.5
0
0.5
1.0
1.5
GsE
GsM


0.15
0.1
0.05
0
−0.05
−0.1
−0.15
G_E^s
−1.5
−1
−0.5
0
0.5
1
1.5
2
G_M^s



New 2006 HAPPEx results + global analysis of all world data

$$G_M^s = +0.01 \pm 0.29\,\mu_N$$
$$G_E^s = +0.002 \pm 0.021$$

Electron scattering can probe charge and magnetisation distributions in the nucleon

Chiral symmetry important in low-energy QCD

Chiral symmetry provides vital information on nucleon structure in QCD

Weak-interaction violates parity symmetry, allows flavour separation of nucleon structure

Theoretical approach providing a significant advance in the understanding of the nucleon

THANKS!