Introduction to the Parton Model and Perturbative QCD

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Hampton U. Grad Summer School, June 8, 2008
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- The Parton Model and Deep-inelastic Scattering
- From the Parton Model to QCD
- Factorization, Evolution and Resummation
The Context of QCD: “Fundamental Interactions”

- Electromagnetic

- + Weak Interactions ⇒ Electroweak

- + Strong Interactions (QCD) ⇒ Standard Model

- + . . . = Gravity and the rest?

- QCD: A theory “off to a good start”. Think of . . .
  - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2 \Rightarrow$ elliptical orbits . . . 3-body problem . . .
  - $L_{QCD} = \bar{q} \not{D}q - (1/4)F^2 \Rightarrow$ asymptotic freedom
    . . . confinement . . .
The Parton Model and Deep-inelastic Scattering

• 1. Nucleons to Quarks (And the Standard Model)

• 2. DIS: Structure Functions and Scaling

• 3. Getting at the Quark Distributions

• 4. Extensions
1. From Nucleons to Quarks

- Nucleons, pions and isospin:

\[
\begin{pmatrix}
  p \\
  n
\end{pmatrix}
\]

- p: \( m=938.3 \text{ MeV}, S = 1/2, I_3 = 1/2 \)
- n: \( m=939.6 \text{ MeV}, S = 1/2, I_3 = -1/2 \)

\[
\begin{pmatrix}
  \pi^+ \\
  \pi^0 \\
  \pi^-
\end{pmatrix}
\]

- \( \pi^\pm: m=139.6 \text{ MeV}, S = 0, I_3 = \pm 1 \)
- \( \pi^0: m=135.0 \text{ MeV}, S = 0, I_3 = 0 \)
• Isospin space . . .

• With a “north star” set by electroweak interactions:

Analog: the rotation group (more specifically, $SU(2)$).
• ‘Modern’: $\pi$, $N$ common substructure: quarks
  – Gell Mann, Zweig 1964

• spin $S = 1/2,
  isospin doublet ($u, d$) & singlet ($s$)
  with approximately equal masses ($s$ heavier);

$$
\begin{pmatrix}
  u (Q = 2e/3, I_3 = 1/2) \\
  d (Q = -e/3, I_3 = -1/2) \\
  s (Q = -e/3, I_3 = 0)
\end{pmatrix}
$$

$$
\pi^+ = (u\bar{d}) , \quad \pi^- = (\bar{u}d) , \quad \pi^0 = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) , \\
p = (uud) , \quad n = (udd) , \quad K^+ = (u\bar{s}) \ldots
$$
• Requirement for $N$:
  symmetric spin/isospin wave function (!)

• $\mu_p/\mu_n = -3/2$ (good to %)

• and now, six: 3 ‘light’ ($u, d, s$), 3 ‘heavy’: ($c, b, t$)
Aside: quarks in the standard model: $SU(3) \times SU(2)_L \times U(1)$

- Quark and lepton fields: $L(eft)$ and $R(ight)$
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi$; $\psi = q, \ell$
  - Helicity: spin along $\vec{p}$ ($R=right$ handed) or opposite ($L=left$ handed) in solutions to Dirac equation

- $\psi^{(L)}$: contains only $L$ particle solutions to Dirac eqn. $R$ antiparticle solutions

- $\psi^{(R)}$: only $R$ particle solutions, $L$ antiparticle

- An essential feature: $L$ and $R$ have different interactions in general!
– L quarks come in “weak $SU(2)$” = “weak isospin” pairs: “generations”

\[
q_i^{(L)} = (u_i, d'_i = V_{ij}d_j ) \quad u_i^{(R)}, \ d_i^{(R)}
\]

\[
\begin{align*}
(u, d') & \quad (c, s') & \quad (t, b') \\
\ell_i^{(L)} = (\nu_i, e_i ) & \quad e_i^{(R)}, \ \nu_i^{(R)} \\
(\nu_e, e) & \quad (\nu_\mu, \mu ) & \quad (\nu_\tau, \tau )
\end{align*}
\]

– $V_{ij}$ is the “CKM” matrix
• Weak vector bosons: electroweak gauge groups
  – SU(2): three vector bosons \( B_i \), coupling \( g \)
  – U(1): one vector boson \( C \), coupling \( g' \)

  – The physical bosons:

\[
W^\pm = B_1 \pm iB_2 \\
Z = -C \sin \theta_W + B_3 \cos \theta_W \\
\gamma \equiv A = C \cos \theta_W + B_3 \sin \theta_W
\]

\[
\sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad M_W = \frac{M_Z}{\cos \theta_W} \\
e = \frac{gg'}{\sqrt{g^2 + g'^2}} \quad M_W \sim \frac{g}{\sqrt{G_F}}
\]
• Weak isospin space: connecting $u$ with $d'$

• Only left handed fields move around this globe.
– The interactions of quarks and leptons with the photon, $W$, $Z$

$$L_{\text{EW}}^{(\text{fermion})} = \sum_{\text{all } \psi} \bar{\psi} \left( i\partial - e\lambda_{\psi} A - \left( \frac{gm_{\psi}}{2M_{W}} \right) h \right) \psi$$

$$-(g/\sqrt{2}) \sum_{q_{i}, e_{i}} \bar{\psi}^{(L)} \left( \sigma^{+} \mathcal{W}^{+} + \sigma^{-} \mathcal{W}^{-} \right) \psi^{(L)}$$

$$-(g/2 \cos \theta_{W}) \sum_{\text{all } \psi} \bar{\psi} \left( v_{f} - a_{f} \gamma_{5} \right) \mathcal{Z} \psi$$

– Interactions with the Higgs $h \propto \text{mass}$
– Interactions with $W$ are through $\psi_{L}$’s only
– Neutrino $Z$ exchange sensitive to $\sin^{2} \theta_{W}$, even at low energy. Observation made it clear by early 1970’s that $M_{W} \sim g/\sqrt{G_{F}}$ is large (need for colliders)
Symmetry violations in the standard model

- $W$'s interact through $\psi^{(L)}$ only, $\psi = q, \ell$.

- Left-handed quarks, leptons; right-handed antiquarks, leptons.

- Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.

- CP combination OK $L \rightarrow R \rightarrow L$ if all else equal, but it’s not (quite). Complex phases in CKM $V \rightarrow$ CP violation.
• Quarks as Partons: “Seeing” Quarks

No isolated fractional charges seen (“confinement.”)

Can such a particle be detected? (SLAC 1969)

Look closer: do high energy electrons bounce off anything hard? (Rutherford ‘prime’.)
• So look for:

\[ e(k) \quad ? \quad e(k') \]

“Point-like’ constituents.

The angular distribution: information about the constituents.
Kinematics \((e + N(P) → ℓ + X)\)

- \(V = γ, Z_0 \Rightarrow ℓ = e\) “neutral current” (NC).
- \(V = W^−(e^−, ν_e), V = W^+(e^+, \bar{ν}_e)\) “charged current” (CC).
$Q^2 = -q^2 = -(k - k')^2$ momentum transfer.

$x \equiv \frac{Q^2}{2p \cdot q}$ momentum fraction (from $p'^2 = (xp + q)^2 = 0$).

$y = \frac{p \cdot q}{p \cdot k}$ fractional energy transfer.

$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ squared final-state mass of hadrons.

$xy = \frac{Q^2}{S}$
Parton Interpretation (Feynman 1969, 72)
Look in the electron’s rest frame . . .

I) Before:

\[ x_i p \quad \sum x_i = 1 \]
\[ 0 < x_i < 1 \]

Lorentz:
\{ flat frozen unexpected \}

II) After:

"Deep-inelastic Scattering"
Basic Parton Model Relation

\[ \sigma_{eh}(p, q) = \sum_{\text{partons } a} \int_0^1 d\xi \hat{\sigma}_{ea}^{\text{el}}(\xi p, q) \, \phi_{a/h}(\xi) \]

where: \( \sigma_{eh} \) is the cross section for
\[ e(k) + h(p) \rightarrow e(k' = k - q) + X(p + q) \]
and \( \hat{\sigma}_{ea}^{\text{el}}(xp, q) \) is the elastic cross section for
\[ e(k) + a(\xi p) \rightarrow e(k' - q) + a(\xi p + q) \] which sets
\[ (\xi p + q)^2 = 0 \rightarrow \xi = -q^2/2p \cdot q \equiv x. \]

and \( \phi_{a/h}(x) \) is the distribution of parton \( a \) in hadron \( h \), the “probability for a parton of type \( a \) to have momentum \( xp \)."
– **in words:** Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron’s parton distributions.

– **The nontrivial assumption:** quantum mechanical incoherence of large-\(q\) scattering and the partonic distributions. Multiply probabilities without adding amplitudes.

– **Heuristic justification:** the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. *Later we’ll see how this works in QCD.*
The familiar picture

\[ a, x P \]

\[ h(P) \]

\[ \phi_{a/h}^{(x)} \]
• Two modern parton distribution sets at moderate momentum transfer (note different weightings with $x$):
2. DIS: Structure Functions and Scaling

Photon exchange

\[ A_{e+N \rightarrow e+X}(\lambda, \lambda', \sigma; q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_{\lambda}(k) \]
\[ \times \frac{-ig^{\mu\mu'}}{q^2} \]
\[ \times \langle X | eJ_{\mu'}^{EM}(0) | p, \sigma \rangle \]
\[
\frac{d\sigma_{\text{DIS}}}{d^3k'} = \frac{1}{2^2 2s (2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda, \lambda', \sigma} |A|^2 (2\pi)^4 \delta^4(p_X + k' - p - k)
\]

**In \( |A|^2 \), let’s separate the known leptonic part from the “unknown” hadronic part:**

- **The leptonic tensor:**

\[
L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda, \lambda'} (\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^* (\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k))
\]

\[
= \frac{e^2}{2\pi^2} (k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu}k \cdot k')
\]
• The hadronic tensor:

\[ W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma, X} \langle X| J_\mu |p, \sigma \rangle^* \langle X| J_\nu |p, \sigma \rangle (2\pi)^4 \delta^4(p_X - p - q) \]

• And the cross section:

\[ 2\omega_{k'} \frac{d\sigma}{d^3 k'} = \frac{1}{s(q^2)^2} L^{\mu\nu} W_{\mu\nu} \]

• \( W_{\mu\nu} \) has sixteen components, but known properties of the strong interactions constrain \( W_{\mu\nu} \) . . .
• An example: current conservation

\[ \partial^\mu J^\text{EM}_\mu(x) = 0 \]

\[ \Rightarrow \langle X | \partial^\mu J^\text{EM}_\mu(x) | p \rangle = 0 \]

\[ \Rightarrow (p_x - p)^\mu \langle X | J^\text{EM}_\mu(x) | p \rangle = 0 \]

\[ \Rightarrow q^\mu W_{\mu\nu} = 0 \]

• With parity, time-reversal, etc . . .
\[ W_{\mu\nu} = - \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(x, Q^2) \]
\[
+ \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) W_2(x, Q^2)
\]

- Often given in terms of the dimensionless structure functions
\[
F_1 = W_1 \quad F_2 = p \cdot q W_2
\]

- Note that if there is no other mass scale the \( F \)’s cannot depend on \( Q \) except indirectly through \( x \).
Structure functions in the Parton Model: The Callan-Gross Relation

From the “basic parton model formula”:

\[
\frac{d\sigma_{eh}}{d^3k'} = \int d\xi \frac{d\sigma_{eq}^\text{el}(\xi)}{d^3k'} \phi_{q/h}(\xi) \tag{1}
\]

Can treat a quark of “flavor” \( f \) just like any hadron and get

\[
\omega_{k'} \frac{d\sigma_{ef}^\text{el}(\xi)}{d^3k'} = \frac{1}{2(\xi s)Q^4} L^{\mu\nu} W_{\mu\nu}^\text{ef}(k + \xi p \rightarrow k' + p')
\]
Let the charge of $f$ be $e_f$. **Exercise 1**: Compute $W_{\mu\nu}^{ef}$ to find:

$$W_{\mu\nu}^{ef} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \delta \left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{2}$$

$$+ \left(\xi p_\mu - q_\mu \frac{\xi p \cdot q}{q^2}\right) \left(\xi p_\nu - q_\nu \frac{\xi p \cdot q}{q^2}\right) \delta \left(1 - \frac{x}{\xi}\right) \frac{e_f^2}{\xi p \cdot q}$$

**Ex. 2**: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks} f} e_f^2 x \phi_{f/p}(x) = 2xF_1(x)$$

**Ex. 3**: that this relation is quite different for scalar quarks.
• The Callan-Gross relation shows the compatibility of the quark and parton models.

• In addition: parton model structure functions are independent of $Q^2$, a property called “scaling”. With massless partons, there is no other massive scale. Then the $F$’s must be $Q$-independent; see above.

• Approximate properties of the kinematic region explored by SLAC in late 1960’s – early 1970’s.

• Explore corrections to this picture in QCD “evolution”.
Structure Functions and Photon Polarizations

In the P rest frame can take

\[ q^\mu = \left( \nu; 0, 0, \sqrt{Q^2 + \nu^2} \right), \quad \nu \equiv \frac{p \cdot q}{m_p} \]

In this frame, the possible photon polarizations \((\epsilon \cdot q = 0)\):

\[ \epsilon_R(q) = \frac{1}{\sqrt{2}} (0; 1, -i, 0) \]
\[ \epsilon_L(q) = \frac{1}{\sqrt{2}} (0; 1, i, 0) \]
\[ \epsilon_{\text{long}}(q) = \frac{1}{Q} \left( \sqrt{Q^2 + \nu^2}, 0, 0, \nu \right) \]
• Alternative Expansion

$$ W^{\mu\nu} = \sum_{\lambda=L,R,\text{long}} \epsilon^{\mu*}_{\lambda}(q)\epsilon^\nu_{\lambda}(q) F_{\lambda}(x, Q^2) $$

• For photon exchange (Exercise 4):

$$ F^{\gamma e}_{L,R} = F_1 $$

$$ F_{\text{long}} = \frac{F_2}{2x} - F_1 $$

• So $F_{\text{long}}$ vanishes in the parton model by the C-G relation.
- Generalizations: neutrinos and polarization

- Neutrinos: flavor of the “struck” quark is changed when a $W^\pm$ is exchanged. For $W^+$, a $d$ is transformed into a linear combination of $u, c, t$, determined by CKM matrix (and momentum conservation).

- $Z$ exchange leaves flavor unchanged but still violates parity.
The $Vh$ structure functions for $= W^+, W^-, Z$:

$$W_{\mu\nu}^{(Vh)} - \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1^{(Vh)}(x, Q^2)$$

$$+ \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \frac{1}{m_h^2} W_2^{(Vh)}(x, Q^2)$$

$$- i \epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)$$

with dimensionless structure functions:

$$F_1 = W_1, \quad F_2 = \frac{p \cdot q}{m_h^2} W_2, \quad F_3 = \frac{p \cdot q}{m_h^2} W_3$$
And with spin (back to the photon). Note equivalent expression for $W^{\mu\nu}$.

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z \; e^{i q \cdot z} \langle h(P, S) \mid J^{\mu}(z) J^{\nu}(0) \mid h(P, S) \rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^{\mu} q^{\nu}}{q^2}\right) F_1(x, Q^2) + \left(P^{\mu} - q^{\mu} \frac{P \cdot q}{q^2}\right) \left(P^{\nu} - q^{\nu} \frac{P \cdot q}{q^2}\right) F_2(x, Q^2)$$

$$+ i m_h \epsilon^{\mu\nu\rho\sigma} q_\rho \left[ \frac{S_\sigma}{P \cdot q} g_1(x, Q^2) + \frac{S_\sigma (P \cdot q) - P_\sigma (S \cdot q)}{(P \cdot q)^2} g_2(x, Q^2) \right]$$
• Parton model structure functions

\[ F_{2}^{(eh)}(x) = \sum_{f} e_{f}^{2} x \phi_{f/h}(x) \]

\[ g_{1}^{(eh)}(x) = \frac{1}{2} \sum_{f} e_{f}^{2} (\Delta \phi_{f/n}(x) + \Delta \bar{\phi}_{f/h}(x)) \]

• Notation: \( \Delta \phi_{f/h} = \phi_{f/h}^{+} - \phi_{f/h}^{-} \) with \( \phi_{f/h}^{\pm}(x) \) probability for struck quark \( f \) to have momentum fraction \( x \) and helicity with \( (+) \) or against \( (-) \) \( h \)'s helicity.
3. Getting at the Quark Distributions

• Relating the parton distributions to experiment

• Simplifying assumptions (adequate to early experiments; generally no longer adequate) that illustrate the general approach.

\[ \phi_{u/p} = \phi_{d/n} \quad \phi_{d/p} = \phi_{u/n} \quad \text{isospin} \]

\[ \phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \quad \text{symmetric sea} \]

\[ \phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \quad \text{no heavy quarks} \]
\[ F_2^{(\gamma N)}(x) = 2x F_1^{(\gamma N)}(x) = \sum_{f=u,d,s} e_F^2 x \phi_{f/N}(x) \]

\[ F_2^{(W^+ N)} = 2x \left( \sum_{D=d,s,b} \phi_{D/N}(x) + \sum_{U=u,c,t} \phi_{U/N}(x) \right) \]

\[ F_2^{(W^- N)} = 2x \left( \sum_{D} \phi_{\bar{D}/N}(x) + \sum_{U} \phi_{U/N}(x) \right) \]

\[ F_3^{(W^+ N)} = 2 \left( \sum_{D} \phi_{D/N}(x) - \sum_{U} \phi_{\bar{U}/N}(x) \right) \]

\[ F_3^{(W^- N)} = 2 \left( - \sum_{D} \phi_{\bar{D}/N}(x) + \sum_{U} \phi_{U/N}(x) \right) \]
• Overdetermined with the assumptions: checks consistency.

• Further consistency checks: Sum Rules, e.g.:

\[ N_{u/p} = \int_0^1 dx \left[ \phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2 \]

etc. for \( N_{d/p} = 1 \).

The most interesting ones make predictions on measurable structure functions . . .
The Adler Sum Rule:

\[ 1 = N_{u/p} - N_{d/p} = \int_0^1 dx \left[ \phi_{d/n}(x) - \phi_{d/p}(x) \right] \]

\[ = \int_0^1 dx \left[ \sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x) \right] \]

\[ - \int_0^1 dx \left[ \sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x) \right] \]

\[ = \int_0^1 dx \frac{1}{2x} \left[ F_2^{(\nu n)} - F_2^{(\nu p)} \right] \]

In the second equality, use isospin invar., in the third, all the extra terms cancel.
And similarly, the Gross-Llewellyn-Smith Sum Rule:

\[ 3 = N_{u/p} + N_{d/p} = \int_0^1 dx \, \frac{1}{2x} \left[ xF_3^{(\nu_n)} + xF_3^{(\nu_p)} \right] \]
4. Extensions

- Fragmentation functions

“Crossing” applied to DIS: “Single-particle inclusive” (1PI)
From scattering to pair annihilation.
Parton distributions become “fragmentation functions”.

\[ e^+ e^- \rightarrow X \]
\[ q^2 < 0 \]
\[ h(a, xP) \rightarrow D_{h/a}(z) \]
• Parton model relation for 1PI cross sections

\[
\frac{d\sigma_h(P, q)}{d^3P} = \sum_f \int_0^1 dz \frac{d\hat{\sigma}_f(P/z, q)}{d^3P} D_{h/f}(z)
\]

Heuristic justification: Formation of hadron $h$ from parton $f$ takes a time $\tau_0$ in the rest frame of $a$, but much longer in the CM frame – this “fragmentation” thus decouples from $\hat{\sigma}_f$, and is independent of $q$ (scaling).

• Fragmentation picture suggests that hadrons are aligned along parton direction $\Rightarrow$ jets. And this is what happens.
For $e^+e^-$:
• And for DIS:

\[ Q^{**2} = 21475 \quad y = 0.55 \quad M = 198 \]

...just from the HOTLINE
• And in nucleon-nucleon collisions:

Run 178796 Event 67972991 Fri Feb 27 08:34:03 2004

Bins: 481
Mean: 2.32
Rms: 23.9
Min: 0.00933
Max: 384

mE_t: 72.1
phi_t: 223 deg
Finally: the Drell-Yan process


Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass $Q \ldots$ any electroweak boson in NN scattering.

$$d\sigma_{NN\rightarrow\mu\bar{\mu}+X}(Q,p_1,p_2)\sim\int d\xi_1 d\xi_2 \sum_{a=q\bar{q}} d\sigma_{aa\rightarrow\mu\bar{\mu}}^{\text{EW,Born}}(Q,\xi_1 p_1,\xi_2 p_2)$$

$\times$ (probability to find parton $a(\xi_1)$ in $N$)

$\times$ (probability to find parton $\bar{a}(\xi_2)$ in $N$)

The probabilities are $\phi_{q/N}(x_{i_i})$'s from DIS!
How it works (with colored quarks) …

- The Born cross section

\[ \sigma^{\text{EW,Born}} \]

is all from this diagram (\( \xi \)'s set to unity):

With this matrix element

\[ M = e_q \frac{e^2}{Q^2} \bar{u}(k_1) \gamma_\mu v(k_2) \bar{v}(p_2) \gamma^\mu u(p_1) \]

- First square and sum/average \( M \). Then evaluate phase space.
• Total cross section:

\[
\sigma_{q\bar{q} \rightarrow \mu \bar{\mu}}^{\text{EW, Born}}(x_1p_1, x_2p_2) = \frac{1}{2s} \int \frac{d\Omega}{32\pi^2} \frac{e_q^2 e_4^4}{3} (1 + \cos^2 \theta)
\]

\[
= \frac{4\pi \alpha^2}{9Q^2} \sum_q e_q^2 \equiv \sigma_0(M)
\]

With \( Q \) the pair mass and 3 for color average

Now we’re ready for the parton model **differential** cross section for NN scattering:

**Pair mass** \((Q)\) and **rapidity**

\[
\eta \equiv \frac{1}{2} \ln \left( \frac{Q^+}{Q^-} \right) = \frac{1}{2} \ln \left( \frac{(Q^0 + Q^3)}{(Q^0 - Q^3)} \right)
\]

overdetermined \( \rightarrow \) delta functions in the Born cross section
\[
\frac{d\sigma^{(PM)}_{NN \rightarrow \mu\bar{\mu} + X}(Q, p_1, p_2)}{dQ^2 d\eta} = \int \sum_{a=q\bar{q}} \sigma_{a\bar{a} \rightarrow \mu\bar{\mu}}^{\text{EW, Born}}(\xi_1 p_1, \xi_2 p_2) \\
\times \delta\left(Q^2 - \xi_1 \xi_2 S\right) \delta\left(\eta - \frac{1}{2} \ln \left(\frac{\xi_1}{\xi_2}\right)\right) \\
\times \phi_{a/N}(\xi_1) \phi_{\bar{a}/N}(\xi_2)
\]

and integrating over rapidity,

\[
\frac{d\sigma}{dQ^2} = \left(\frac{4\pi \alpha_{\text{EM}}^2}{9Q^4}\right) \int_0^1 d\xi_1 d\xi_2 \delta(\xi_1 \xi_2 - \tau) \sum_a \lambda_a^2 \phi_{a/N}(\xi_1) \phi_{\bar{a}/N}(\xi_s)
\]

Drell and Yan, 1970 (aside from 1/3 for color)

Analog of DIS: scaling in \(\tau = Q^2/S\)