II. From The Parton Model to QCD

1. Color and QCD
2. Field Theory Essentials
3. Infrared Safety
4. Summary

Enter the Gluon

If $\phi_{q/p}(x)$ = prob. for q with momentum xp

Then $$F_q = \sum_q \int_0^1 dx\, x\, \phi_{q/p}(x)$$

= total fractional momentum carried by quarks

est: $\boxed{0.5}$

What else? Quanta of force field that holds N together

'Gluons'

But what are they?

- Color

- Quark model problem

  $S_q = 1/2 \rightarrow$ fermion

  $\rightarrow$ antisymmetric wave function

  (but)

  (uud) state symmetric in spin/isospin

  expect lowest-lying $\psi(\vec{x}_u, \vec{x}_u', \vec{x}_d)$ to be symmetric

  where's the antisymmetry?

- Solution (Han Nambu 1968)

  Color

  b g r

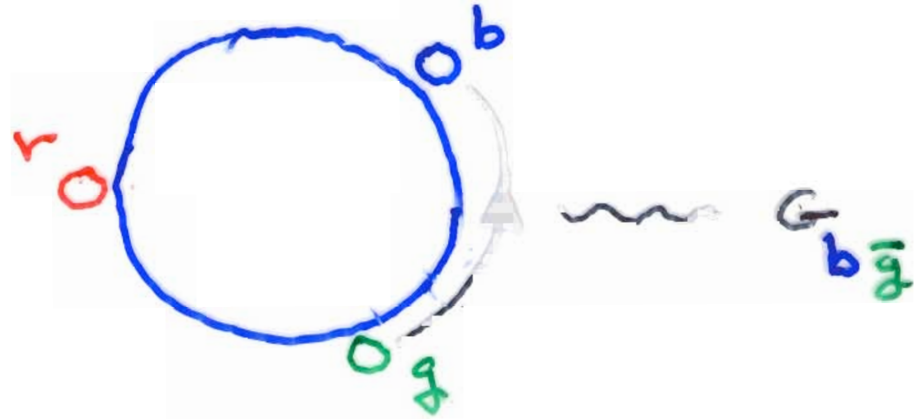
  a new quantum number

  $\psi^{sym}(u, u, d)$

  here's the antisymmetry

- Quantum Chromodynamics: dynamics of Color

representation: with no North Pole



on hyperglobe (↔ phase of wave function)

local rotation ↔ emission of gluon

$S=1$

- Yang Mills 1954

QCD (gluons coupled to color)

- Fritzsch, Gell Mann, Leutwyler
- Weinberg
- Gross, Wilczek

1973

- Fields and Lagrange Density for QCD

$\psi_f(x)$: Quark fields. Dirac fermion (like electron). Color triplet. $f = u, d, s, c, b, t$. Varying masses.

$A(x)$: Gluon field. Vector (like photon). Color octet massless

$$\mathcal{L}(\psi, A) = \sum_f \bar{\psi}_f \left([i\partial_\mu - g A_\mu^a t_a]\gamma^\mu - m_f\right)\psi_f$$

$$-\frac{1}{4}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g C_{abc} A_\mu^b A_\nu^c\right)^2$$

$$[t_a, t_b] = i C_{abc} t_c$$

- Schematic Pert. Theory Rules

$$\mathcal{L} = \bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi$$ 

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$ 

$$-g\bar{\psi} A_\mu^a t_a \gamma^\mu \psi$$ 

$$-\frac{1}{2}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) g C_{abc} A^{\mu b} A^{\nu c}$$ 

$$-\frac{1}{4} g^2 C_{abc} A_\mu^b A_\nu^c C_{ade} A^{\mu d} A^{\nu e}$$ 

From Lagrangian Cross Sections

$$
\begin{array}{ccc}
\text{Fields} & \longrightarrow \ \text{Lagrangian} \ \longleftarrow & \text{Symmetries} \\
 & \downarrow & \\
 & \text{Pert. Theo. Rules} & \\
 & \downarrow & \\
\text{Renorma-lization} & \longrightarrow \ \text{Green Functions} & \\
 & \downarrow & \\
 & \boxed{\begin{array}{c}\text{S-Matrix} \\ \downarrow \\ \text{Cross Sections}\end{array}} & \longleftarrow \ \text{Observables}
\end{array}
$$

UV Divergences (Towards Renormalization and The Renormalization Group)

As an example:

Use

$$\mathcal{L}_{\phi 4} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

UV divergences

$M(s,t) =$ 

$+ \ldots$

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)} \frac{1}{(p_1+p_2-k)^2-m^2}$$

$$\sim \int^{\infty} \frac{d^4k}{(k^2)^2} + \ldots$$

Interpretation: states of high 'mass'

Fact:

$E_i \quad E_I \quad E_i \qquad E_I'$

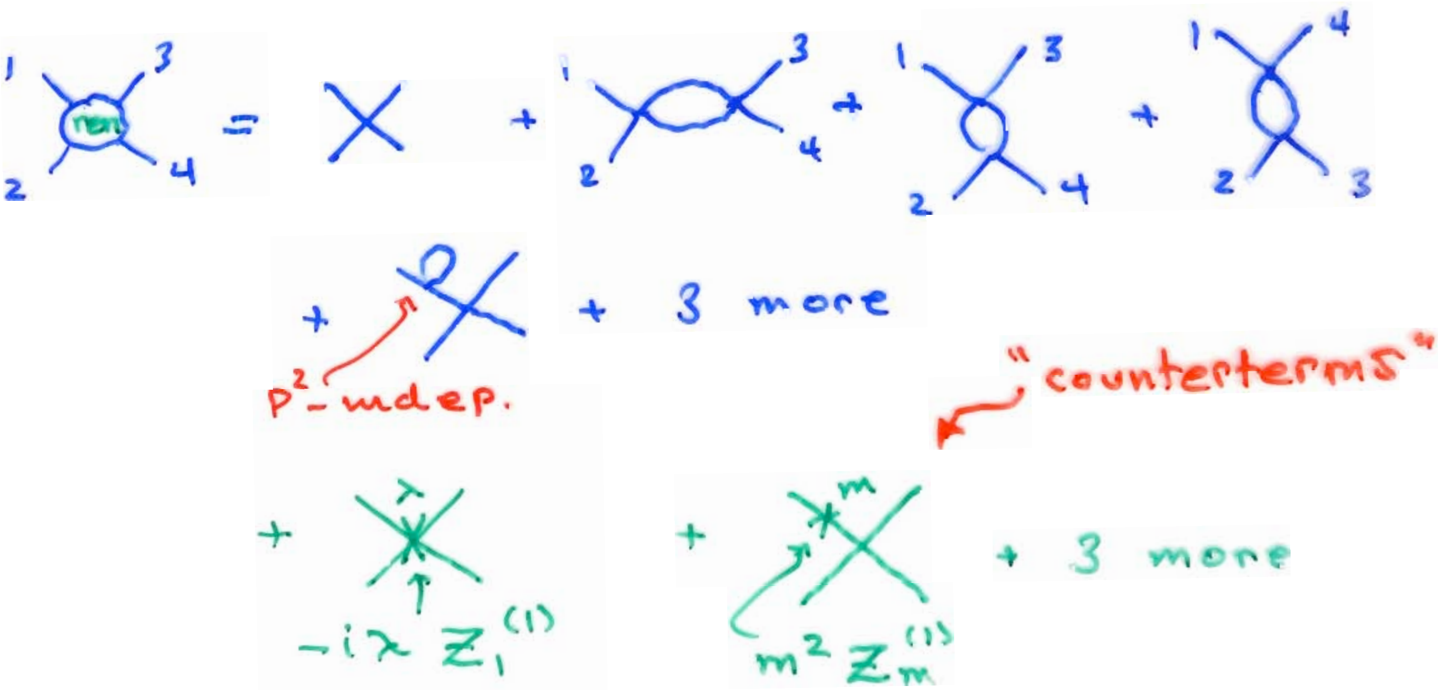
$E_s = \sum_{i \text{ in } S} \sqrt{\vec{p}_i^{\,2} + m^2}$

$$\sim \int (PS)_I \left(\frac{1}{E_i - E_I} + \frac{1}{E_i - E_I'} \right)$$

$\to \infty$ from $\vec{P}_I \to \infty$, $E_I \to \infty$

uncertainty $\to$ equivalent to $\Delta t = 0$ 'local' interaction

4-point: $G = \int \Pi e \; \langle 0 | T(\Pi \phi(x_i)) | 0 \rangle$



= UV finite

(N ...) ... dependence

...)

← $O(\lambda^3)$

can choose

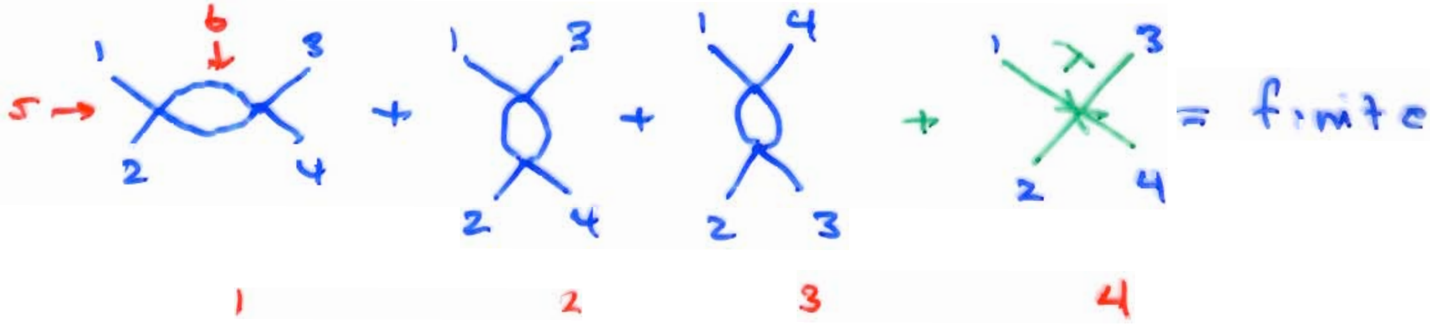
$= 0 \quad (m = m_{phys})$

So concentrate on

What is it going to be?

• Renormalization Schemes

Choose counterterms so that combination



how? for example:

define 1+2+3 by cutting off $\int d^4k$ at $k^2 = \Lambda^2$ (regularization) then

$$1+2+3 = a \ln \frac{\Lambda^2}{s} + b$$

(a, b finite fns. of s, t, u, m^2)

now <u>choose</u>

$$4 = -a \ln \frac{\Lambda^2}{\mu^2}$$

so that

$$1+2+3+4 = -a \ln \frac{s}{\mu^2} + b$$

independent of Λ

Critereon for choosing μ is a "renormalization scheme"

MOM scheme: $\mu = s_0$, some point in mtm. space

$\overline{MS}$ scheme: same μ for all graphs

But the value of μ is still arbitrary

μ = renormalization scale

Modern view (Wilson)

we hide our ignorance

high-E ($E \gg \mu$) physics

particles

lanized

effective theory

with the same low energy

behavior as the true theory

(= SUSY, String...?)

μ-dependence is the price for

working with an effective theory

But it has its advantages too...

- The Renormalization Group

As μ changes, mass m and coupling g change in value.

$m = m(\mu)$ $g = g(\mu)$ "renormalized"

but...

Physical quantities can't depend on μ:

$$\mu \frac{d}{d\mu} \sigma \left(\frac{t_{ij}}{\mu^2}, \frac{m^2}{\mu^2}, g(\mu), \mu \right) = 0$$

invariants

The "group" is the set of all changes in μ.

"RG Equation" $[\sigma] = -\omega$

(let $m \equiv 0$)

$$\boxed{\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \omega \right) \sigma \left(\frac{t_{ij}}{\mu^2}, g(\mu) \right) = 0}$$

$$\boxed{\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu}}$$

• The "Running" Coupling

consider any $(m=0, w=0)$

$$\sigma\left(\frac{t_1}{\mu^2}, \frac{t_2}{t_1} \cdots g(\mu)\right)$$

$$\mu \frac{d\sigma}{d\mu} = 0 \rightarrow \quad \frac{\partial \sigma}{\partial \ln\mu} = -\beta(g)\frac{\partial\sigma}{\partial g} \qquad (1)$$

in PT:

$$\sigma = g^2(\mu)\,\sigma^{(1)} + g^4(\mu)\left[\sigma^{(2)}\left(\frac{t_2}{t_1}\right) + \tau^{(2)} \ln\frac{t_1}{\mu^2}\right] + \cdots \qquad (2)$$

(2) in (1) $\rightarrow$

$$g^4 \tau^{(2)} = 2g\sigma^{(1)}\beta(g) + \cdots$$

$$\beta(g) = \frac{g^3}{2}\frac{\tau^{(2)}}{\sigma^{(1)}} + \mathcal{O}(g^5)$$

$$\beta(g) \equiv -\frac{g^3}{16\pi^2}\beta_0 + \mathcal{O}(g^5)$$

In QCD:

$$-\beta_0 = -\left(11 - \frac{2}{3}n_f\right)$$

$\rightarrow \beta_0 < 0 \rightarrow \quad g$ decreases as μ increases

- Asymptotic Freedom

Solution for QCD running coupling

(= effective)

(= renormalized)

(= $g(\mu)$)

$$\mu \frac{\partial g}{\partial \mu} = - g^3 \frac{\beta_0}{16\pi^2} \qquad \frac{d\mu}{\mu} \equiv dt$$

$$\frac{dg}{g^3} = -\frac{\beta_0}{16\pi^2} dt \qquad \mu_2 = \mu_1 e^t$$

$$\frac{1}{g^2(\mu_1)} - \frac{1}{g^2(\mu_2)} = -\frac{\beta_0}{16\pi^2} 2 \cancel{d}t$$

$$g^2(\mu_2) = \frac{g^2(\mu_1)}{1 + \frac{\beta_0}{16\pi^2} g^2(\mu_1) 2t} \qquad (-\beta_0 < 0)$$

$$\xrightarrow[t \to \infty]{} 0 \quad \text{(Asymptotic freedom)}$$

$$g^2(\mu_2) = \frac{g^2(\mu_1)}{1 + \frac{\beta_0}{16\pi^2} g^2(\mu_1) \ln \frac{\mu_2^2}{\mu_1^2}}$$

$$\alpha_s(\mu_2^2) \equiv \frac{g^2(\mu_2)}{4\pi} = \frac{\alpha_s(\mu_1)}{1 + \frac{\beta_0}{4\pi} \alpha_s(\mu_1) \ln \frac{\mu_2^2}{\mu_1^2}}$$

- Reparameterization: Λ_{QCD}

Effective coupling $\equiv$ renormalized coupling

→ μ_1 and $g^2(\mu_1)$ not independent

→ define

$$\Lambda_{QCD} = \mu_1 \, e^{-\beta_0/\alpha_s(\mu_1)}$$

independent of μ_1

→ another useful form for $g(\mu)$

$$\boxed{\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2_{QCD})}}$$

'weak coupling at large momentum scales'

Suggests reln. to parton model in which partons act as if free, at short distances

But how to quantify this observation?

3 INFRARED SAFETY

- Would like to choose μ as 'large as possible' in calculations → small $g(\mu)$
- But how large *is* possible?

• Typical S-matrix elt

$$S\left(\frac{Q_i}{\mu^2}, \frac{p_i^2}{\mu^2}, \frac{m_f^2}{\mu^2}, \frac{m_g^2}{\mu^2}, \frac{Q_i^2}{Q_j^2}, g(\mu)\right)$$

$$= \sum_{n=1}^{\infty} a_n\left(\frac{Q_i^2}{\mu^2}, \frac{p_i^2}{\mu^2}, \frac{m_f^2}{\mu^2}, \frac{m_g^2}{\mu^2}, \frac{Q_i^2}{Q_j^2}\right) g^{2n}$$

Q_i^2 - large external invariants
p_i^2 - small external masses
m_f - 'light quark' mass
m_g = gluon mass (!!)

The a_n depend logarithmically on all ratios (standard lore: but see G.S., Phys Rev D18, 2773(78) for detailed discussion, also Collins, Soper, St in 'pQCD' ed. A. Mueller World S. (1978))

If choose $\mu^2 = Q_1^2$

get function of $x_{ij} = \frac{Q_i^2}{Q_j^2} = O(1)$

but also $\frac{m_f^2}{Q_i^2}, \frac{m_g^2}{Q_i^2} = 0, \frac{p_i^2}{Q_i^2}$

31 Ruins pert. expansion in general

Look for quantities <u>independent</u> of p_i^2, m_q^2, m_g^2

INFRARED SAFE QUANTITIES
(IRS)

RG Eqn for IRS σ

$$\sigma\left(\frac{Q_1^2}{\mu^2}, x_{ij}, g(\mu)\right) = \sigma\left(1, x_{ij}, g(Q_1^2)\right)$$

$$= \sum_{n=1}^{\infty} a_n(x_{ij})\, \alpha_s^n(Q_1^2)$$

$$\alpha_s \equiv g^2/4\pi$$

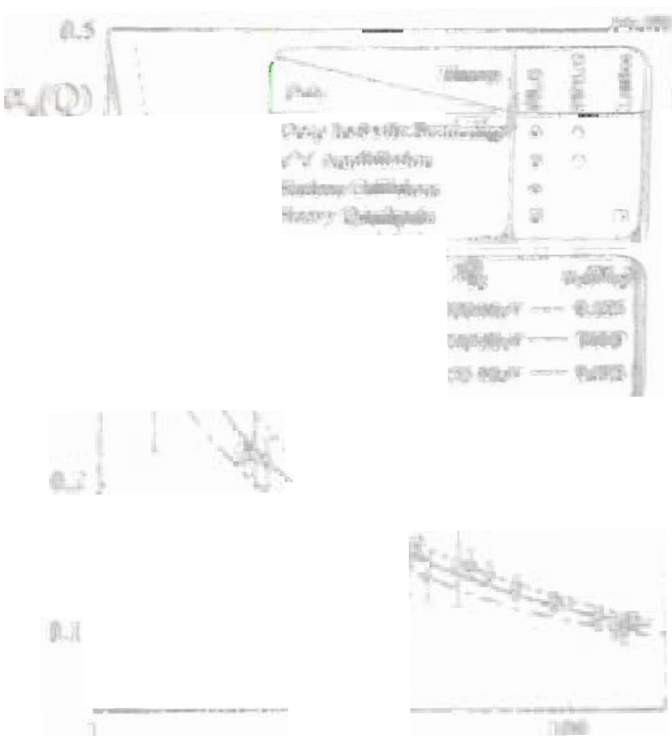
The majority of applications of pQCD are in the computation of IRS quantities.

IRS $\leftrightarrow$ momenta $\gg$ masses
"short distance"

MEASURE σ $\mapsto$ SOLVE FOR $\alpha_s(Q^2)$

Allows observation of 'running coupling'

Given f's (input where necessary), compare $\hat{\sigma}(\alpha_s)$ to experiment



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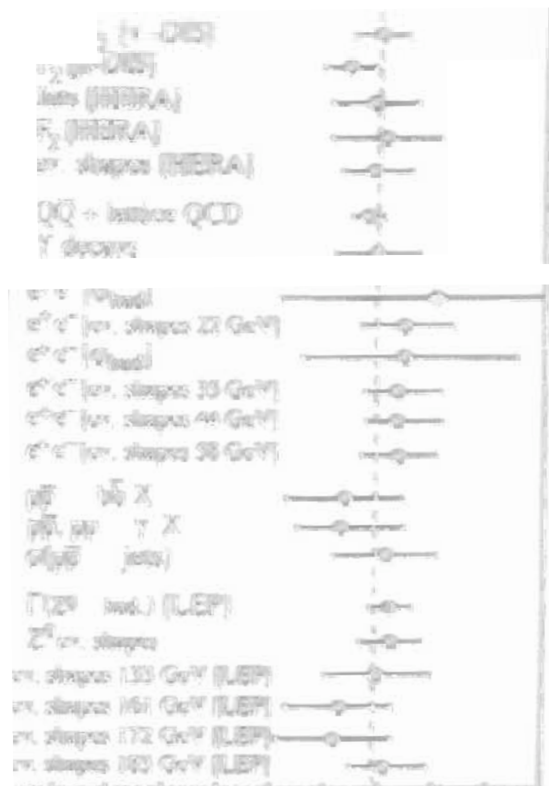

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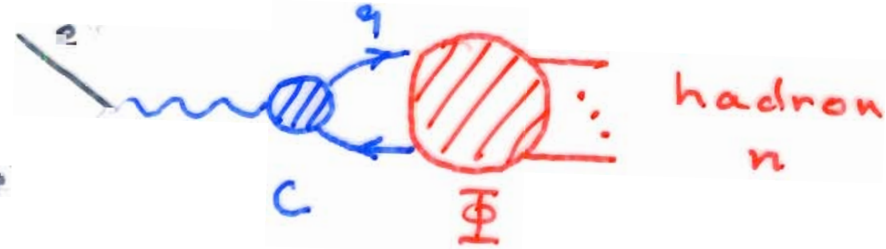
§ section

σ_{PT} = IR Safe ($\Phi = 1$ in notation ~~above~~ below)

cal results



'short-distance' C and 'long-distance' Φ have no quantum interference

$\Rightarrow$

$P_{q\bar{q}\to n}$

classical product of probabilities not amplitudes

$\Rightarrow \sigma_{tot} \quad \Sigma P$

$\boxed{\sum_n P_{q\bar{q}\to n}} \to = 1!$

KLN Theorem

this is C in this case

$= \sigma_{tot}^{(PT)}$

Note: $\sum_n P_{q\bar{q}\to n} = 1$ is 'unitarity'. Will hold in PT as well as in (hypothetical) exact calculation. But to calculate in PT will need IR REGULATION

33/34 (compare UV)

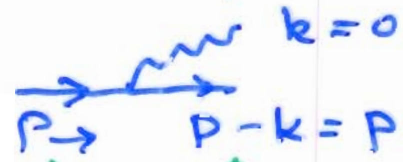
Test of IR sensitivity:

$-\ln\frac{m}{Q} \to \infty$ as $\frac{m}{Q} \to 0$

$\Rightarrow$ Look for problems in $m=0$ ~~theory~~ limit

Generic problems at $m^2 = 0 = p^2$

(i)



'infrared divergences'

both on-shell
→ long lived states

(ii)

$0 < \beta < 1$

'collinear divergences'

m = 0 particles are not stable in usual sense. Their interactions just won't quit!

In IR regulated version of theory we 'cut-off' IR (and collinear) divergences by modifying the theory.

Lets see how this works in e^+e^-

Note: IR regulated theory <u>not</u> the same <u>except</u> for IRS quantities

• IR Regularization Schemes for e^+e^-

– (i) $\frac{1}{k^2} \rightarrow \frac{1}{k^2 - m_g^2}$ for gluon

– (ii) dimensional (manifestly preserves gauge invariance)

(i) m_g is 'easy' – all integrals become finite at one loop

find

$$\sigma_3^{(m_g)} = \sigma_0 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} + \frac{5}{2} - \frac{\pi^2}{6}\right)$$

$$\sigma_2^{(m_g)} = \sigma_0\left(1 + \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(-2\ln^2\frac{Q}{m_g} + 3\ln\frac{Q}{m_g} - \frac{7}{4} + \frac{\pi^2}{6}\right)\right)$$

$$\sigma_{tot} = \sigma_0\left(1 + \frac{\alpha_s}{\pi}\right) + O(\alpha_s^2)$$

pretty simple! what about dim. regularization?

Results for Dimensional Regularization for IR and CO divergences!

(for now, just some formulas)

$$\sigma_3^{(\epsilon)} = \sigma_0 \, \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{(1-\epsilon)^2}{(3-2\epsilon)\,\Gamma(2-2\epsilon)}\right)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^{2\epsilon}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{2} + \frac{19}{4}\right)$$

$$\sigma_2^{(\epsilon)} = -\sigma_0 \, \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{(1-\epsilon)^2}{(3-2\epsilon)\,\Gamma(2-2\epsilon)}\right)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^{2\epsilon}\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{2} + 4\right)$$

again, one loop correction is

$$\sigma_0\left(\frac{\alpha_s}{\pi}\right)$$

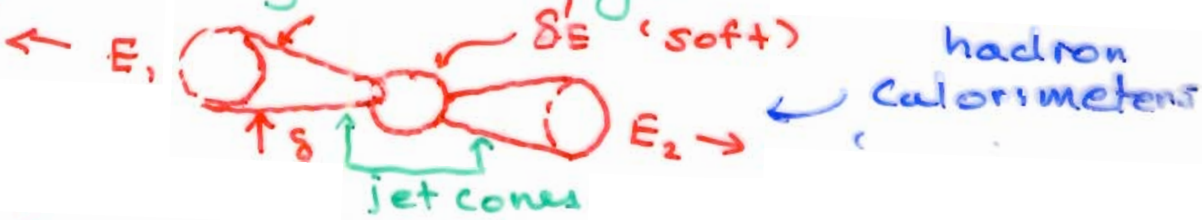
lesson σ_{tot} <u>is</u> IRS

(even if σ_2 and σ_3 are very sensitive to long-distance nature of IR regulated theory.)

- heuristic arguments very similar to e^+e^- total
- note: long-distance interactions possible only for <u>collinear</u> or (long-wavelength) <u>soft</u> particles
- suggests: summing over states with definite 'jets' of nearly collinear particles + soft particles → IRS cross section in e^+e^-
- can be made formal using KLN theorem-style arguments

Examples:

(i) energy into angular regions



(ii) jet mass; thrust

$T = \frac{1}{Q}\sum |p_i \cdot \hat{n}_T|$

'ancestor' of K_T, DURHAM, JADE & related algorithms

Typical Example:

Two-jet cross section in e^+e^-
(begins at α_s^0; dominates as $Q \to \infty$ since $\alpha_s(Q) \to 0$)

$$\sigma_{2J}^{(PM)} = \frac{3\sigma_0}{8}(1+\cos^2\theta)$$

$$\sigma_{2J}^{(pQCD)} = \frac{3\sigma_0}{8}(1+\cos^2\theta)\left(1+\sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n C_n\right)$$

$C_n = C_n(y)$ or $C_n(\delta)$

$y \sim m_J^2/s$

Example: Calorimeter 2-jet cross section

$$\sigma_{2J}^{(Q)} = \frac{3\sigma_0}{8}(1+\cos^2\theta) \cdot \left(1 - \frac{4\alpha_s(Q)}{3\pi}\left(4\ln\delta\ln\delta' + 3\ln\delta + \frac{\pi^2}{3} + \frac{5}{2}\right)\right)$$

only Q dependence

as $Q \to \infty$ $\quad \sigma_{tot} \to \sigma_{tot}^{2J}$

for p-p jets, often use
$\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ in place of δ, δ'

4. Classic applications of infrared safety (summary)

- Infrared Safe Cross Sections

- Generalizations of PM to Factorized Cross Sections

$$\text{PM}: \sigma_h^{DIS} = \int d\xi \, \sigma_{Born,a}(Q,\xi)\, \phi_a(\xi)$$

$$\text{pQCD} \quad \sigma_h^{DIS} = \int d\xi \; H_a(Q,\xi)\, \phi_a(\xi, Q)$$

$$\text{IRS}: \; H_a(Q,\xi) = \sigma_{Born} + \alpha_s(Q) H^{(1)} + \cdots$$

note

$\phi_a(\xi, Q)$ depends on Q

etc. for DY

- Evolution

$\phi_a(\xi, Q)$ obeys eq. of form

$$\frac{\partial}{\partial \ln Q} \phi_a(\xi, Q) = \int_\xi^1 d\xi' \, P_{ab}\left(\frac{\xi}{\xi'}, \alpha_s(Q)\right) \phi_b(\xi', Q)$$

$P_{ab}(\xi/\xi', \alpha_s(Q))$ IRS

Allows us to compute Q-dependence (scale breaking) of DIS structure functions, D-Y cross sections, etc...