Non-Perturbative QCD at Finite Temperature

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Jlab Hugs student talk, 6-20-2008

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outline

1. Non-perturbative QCD at zero temperature
   - Introduction to non-perturbative physics
   - Tools for studying non-perturbative QCD
     - Diagrammatics
     - Schwinger Dyson Equation
   - Illustration: Gap Equation Revisited

2. Finite Temperature Field Theory
   - Survey of basic formalism of Finite Temperature of Field Theory
   - Finite Temperature Gap Equation

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Introduction

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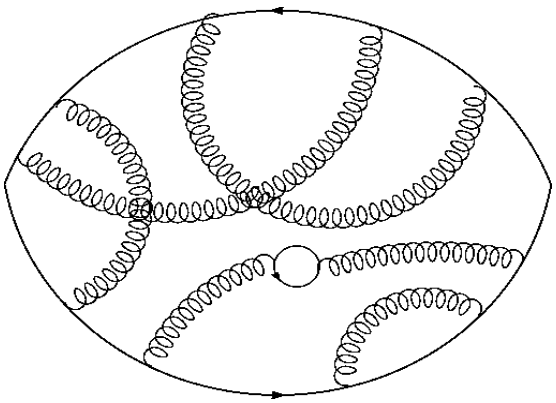
No body is too many!!

Introduction

Difficutly in calculating arbitarily complicated diagram



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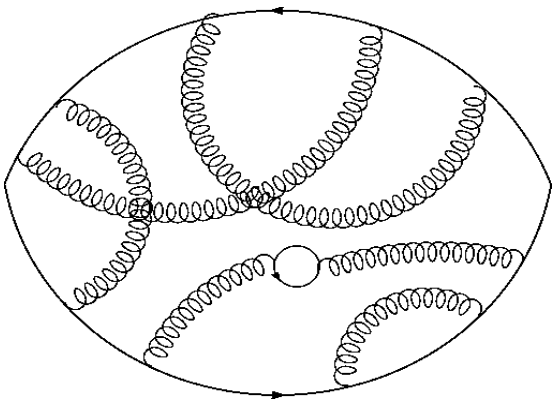

Impossible to **sum** all the diagrams

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  classical mechanics example: mass attached to a spring: $A\sin(\sqrt{\frac{k}{m}}t)$

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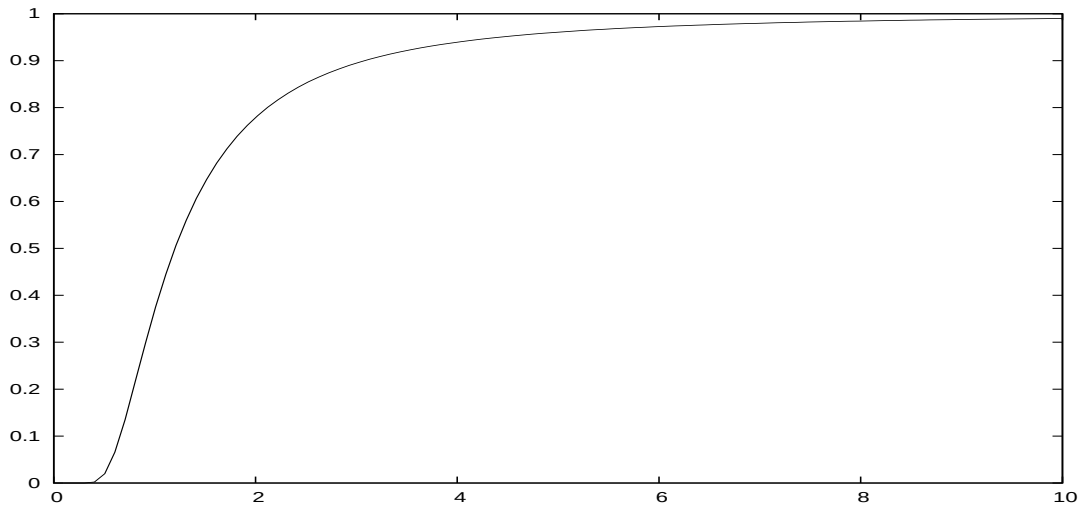
illustration of the failure of perturbation



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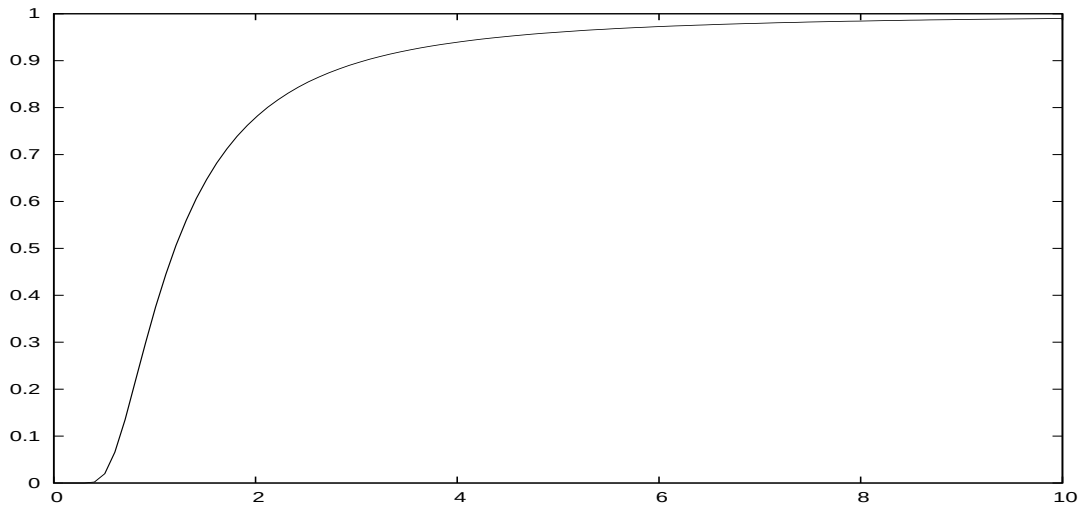

...an innocent looking function
using Taylor expansion, we write

$$f(x) = A_0 + A_1 x + A_2 x^2 + ...$$

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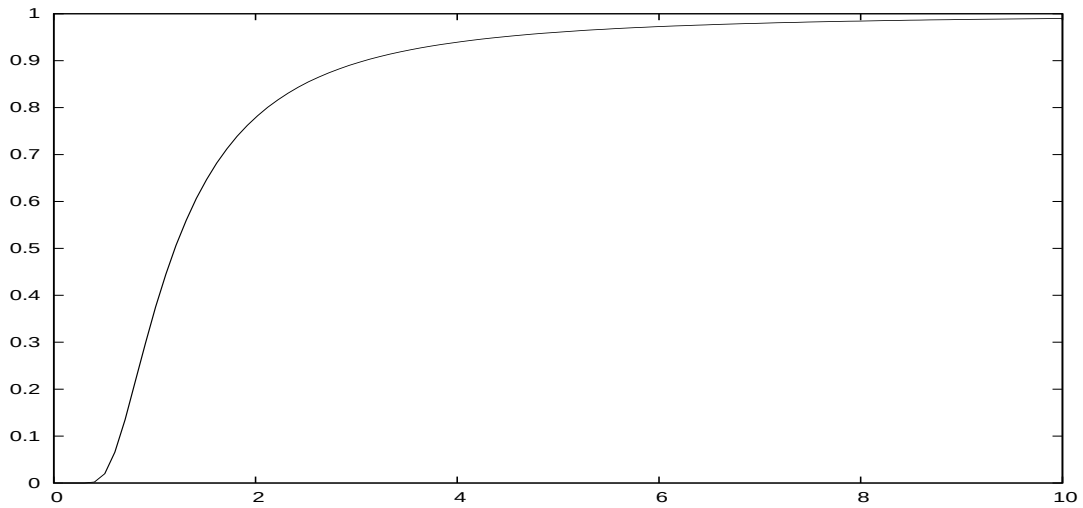

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using Taylor expansion, we write

$$f(x) = A_0 + A_1 x + A_2 x^2 + ...$$

$A_0, A_1, A_2, ...$ are all strightly 0!

illustration of the failure of perturbation

differential equation...

illustration of the failure of perturbation

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$$f(x) + x^3 f' = 0$$

illustration of the failure of perturbation

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any other method will work, other than perturbation!

Different Philosophy

Perturbative Vs Non-perturbative:

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approximation $\neq$ perturbation!
QFT contains more than just the S-matrix and perturbation!

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motivation

perturbation does not help as we want to sum diagrams to all order in α

new tools other than perturbation!

in some sense, we need exact relations among diagrams!

two particularly useful ones:

- Diagrammatics
- Schwinger Dyson Equation

method of partial sum

summing a particular class of diagrams to all order

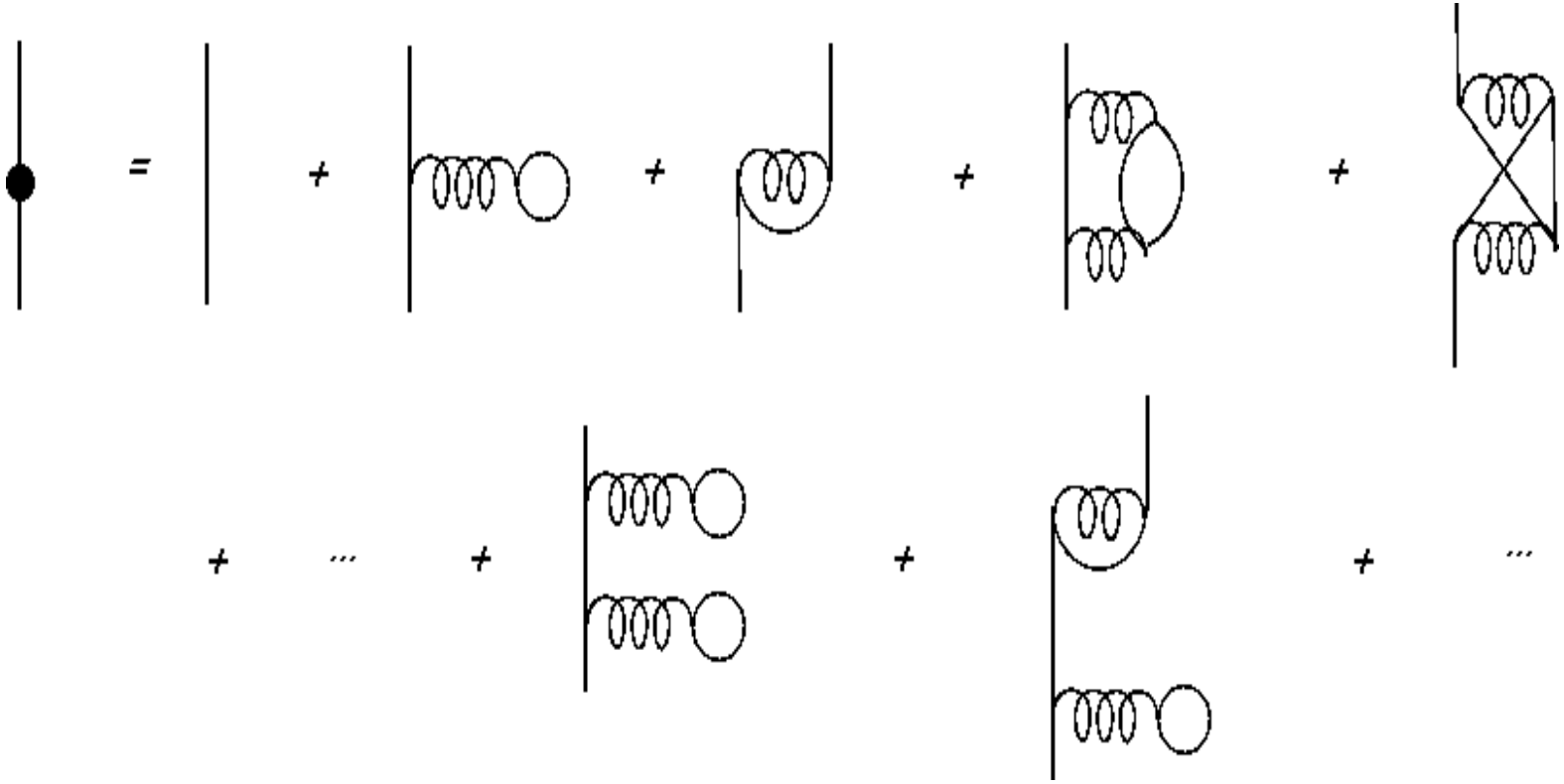
method of partial sum

summing a particular class of diagrams to all order

to be explicit, let's consider the diagrams for constructing a propagator

diagrammatics of propagator

in general, to construct the propagator, we need to sum...



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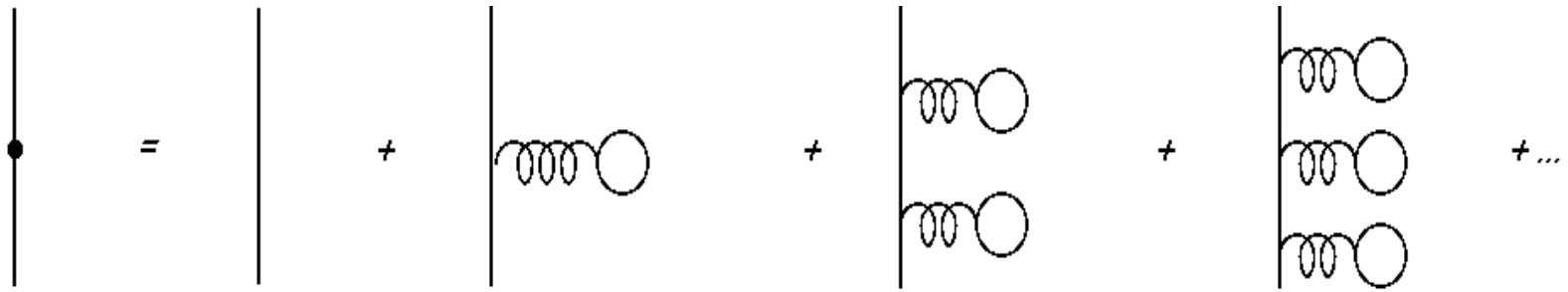

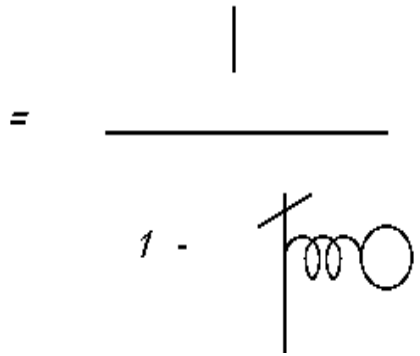

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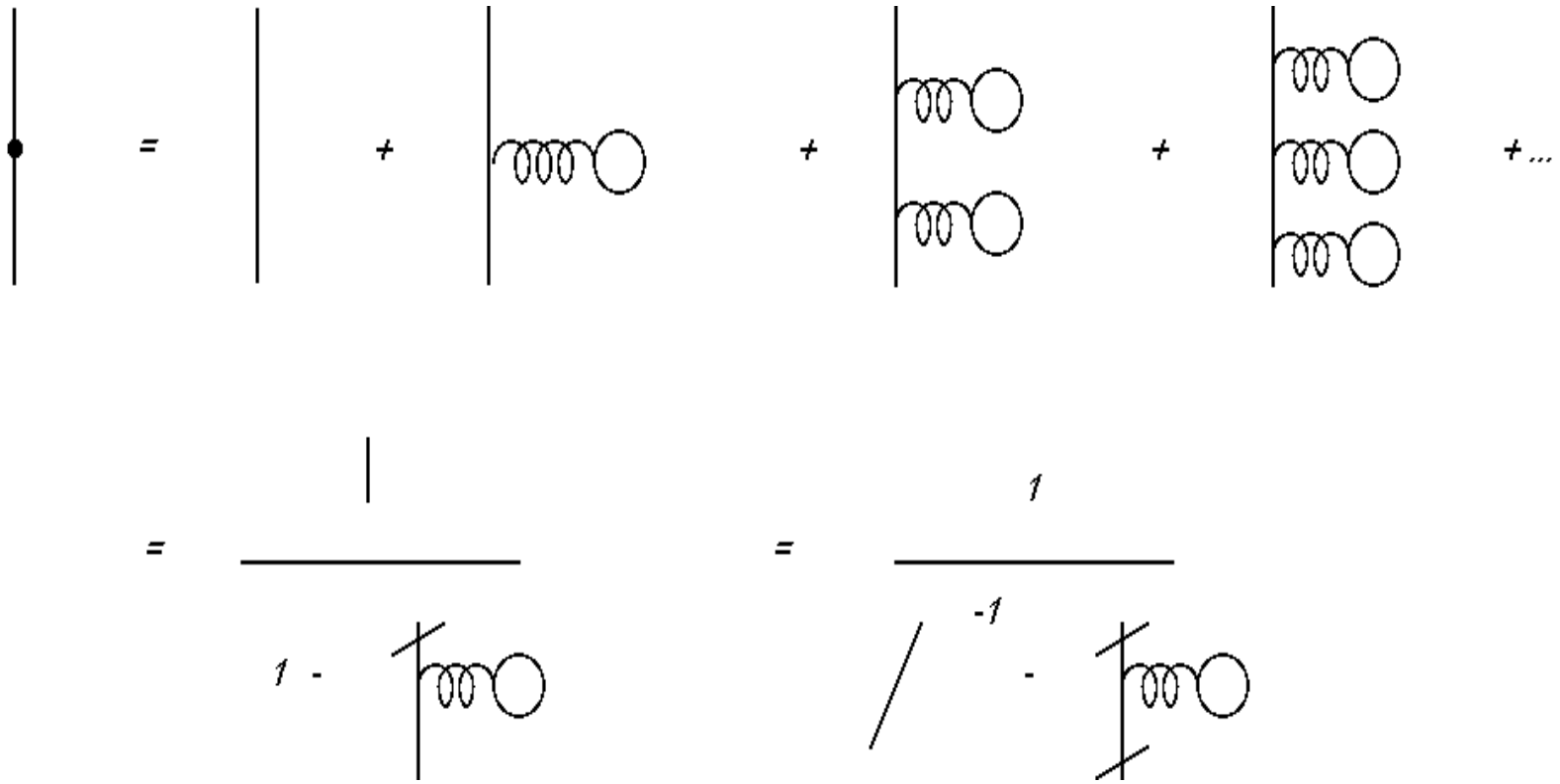

diagrammatics of propagator

the above procedure should be compared with the summing of geometric series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$$

diagrammatics of propagator

Two comments:

diagrammatics of propagator

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inspired by the series, we know we can do better:

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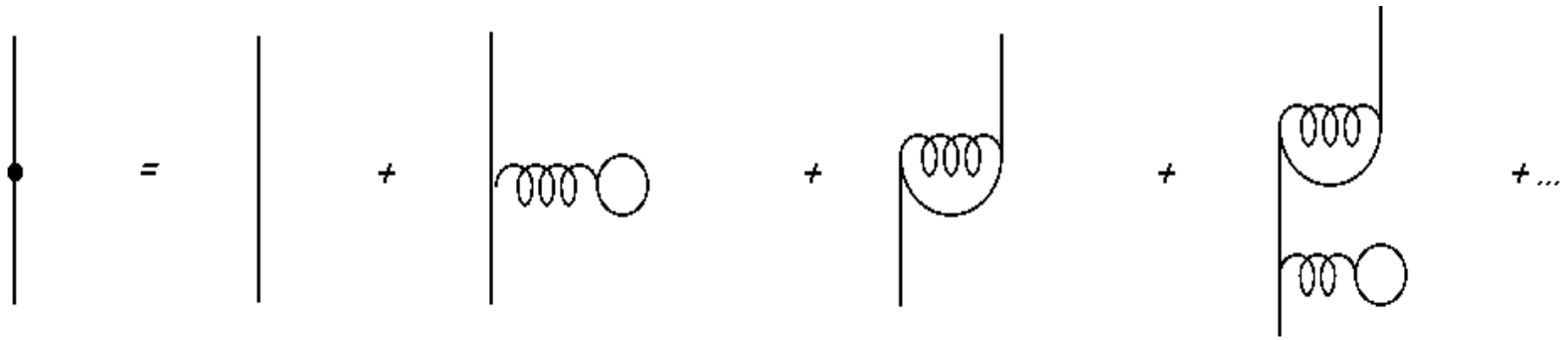

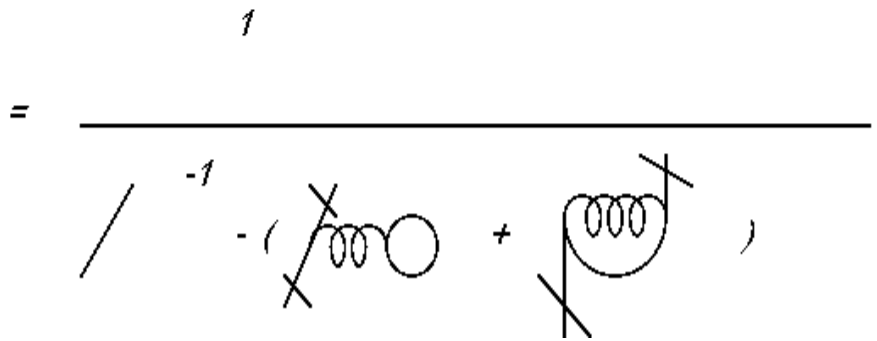

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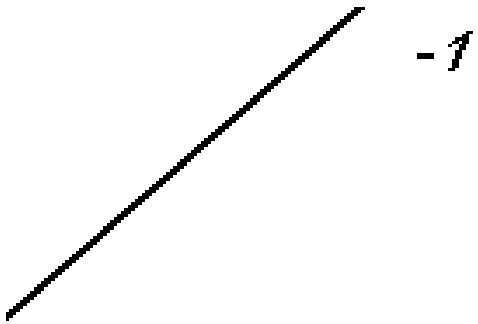

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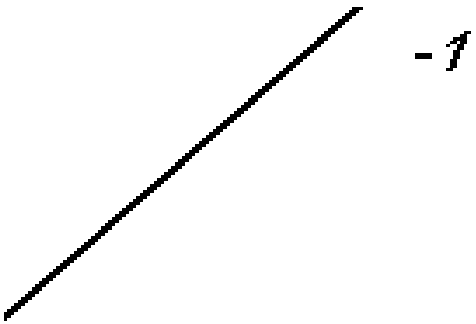
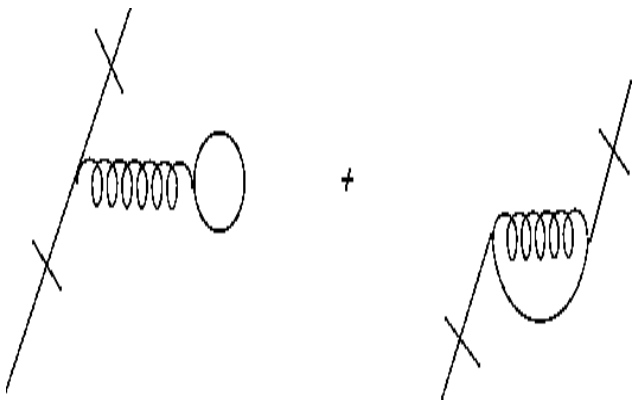


$\Longleftrightarrow \not{p} - m$

diagrammatics of propagator



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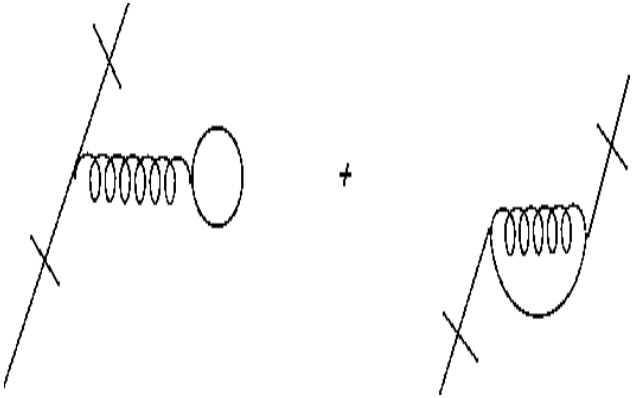

$\Longleftrightarrow$ dynamical mass

Schwinger Dyson Equation

an exact relations among n-point functions

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basic idea:

$$\int dx \frac{d}{dx} f(x) -> 0$$

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$$\mathcal{Z} = \int D\psi D\overline{\psi} DG e^{i(S + \int \overline{\eta}\psi + \overline{\psi}\eta + \ldots)} / \int D\psi D\overline{\psi} DG e^{i(S)}$$

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using the fact

$$\int D\psi D\overline{\psi} DG (\frac{\delta}{\delta \overline{\psi}} e^{i(S + \int \overline{\eta}\psi + \overline{\psi}\eta + \ldots)}) = 0$$

$$\mathcal{Z} = e^{i\mathcal{W}}$$

Schwinger Dyson Equation

as an illustration, for the Hamiltonian:

$$H = \int d^3x \psi^\dagger_{\vec{x}} \gamma^0 [-i\vec{\gamma}\cdot\nabla + m]\psi_{\vec{x}} - G\int d^3x d^3y V_{\vec{x}\vec{y}} \psi^\dagger_{\vec{x}} T^a \psi_{\vec{x}} \psi^\dagger_{\vec{y}} T^a \psi_{\vec{y}}$$

Schwinger Dyson Equation

$$(i\gamma\cdot\partial_x - m)\frac{\delta^2\mathcal{W}}{\delta\overline{\eta_x}\delta\eta_y} + 2G\int d^4z V\gamma^0 T^a i\frac{\delta^2\mathcal{W}}{\delta\overline{\eta_x}\delta\eta_z}\gamma^0 T^a \frac{\delta^2\mathcal{W}}{\delta\overline{\eta_z}\delta\eta_y} = \delta_{xy}$$

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Gap Equation Revisited

to illustrate the main idea above, we look at the gap equation

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consider the Hamiltonian:

$$H = \int d^3x \psi^\dagger_{\vec{x}} \gamma^0 [-i\vec{\gamma} \cdot \nabla + m] \psi_{\vec{x}} - G \int d^3x d^3y V_{\vec{x}\vec{y}} \psi^\dagger_{\vec{x}} T^a \psi_{\vec{x}} \psi^\dagger_{\vec{y}} T^a \psi_{\vec{y}}$$

for the contact case: $V \to \delta^{(3)}$

this corresonds to the case where the quark exchange an instantaneous gluon locally

Gap Equation Revisited

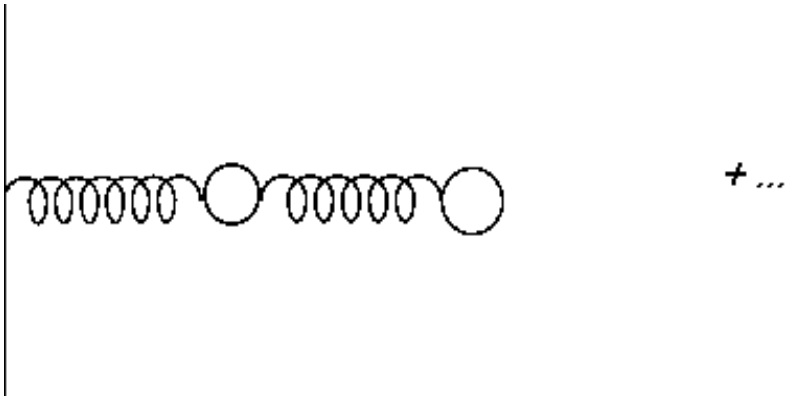
the gap equation tell you how the quark is dressed according to the Hamiltonian

namely, the dynamical mass generated:

before showing you the answer, we need to perform our final dressing...

the final dressing

in fact we miss some diagram...



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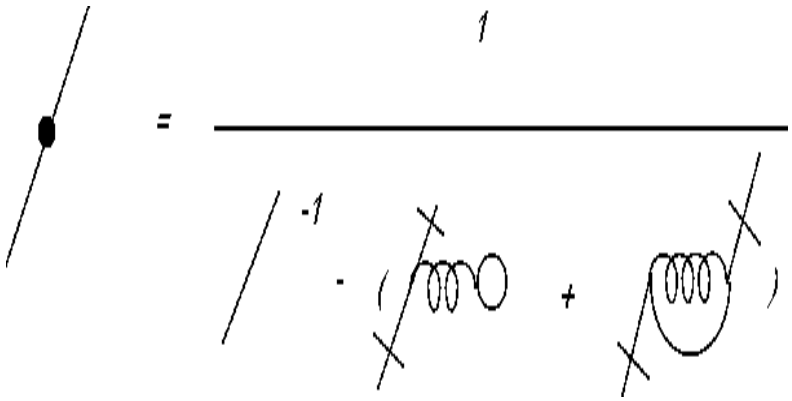

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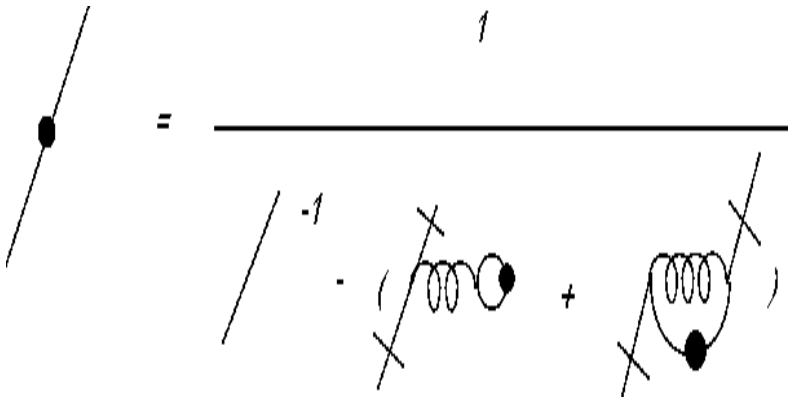


Gap Equation Revisited

the gap equation for the general case:

Generalized Gap Equation

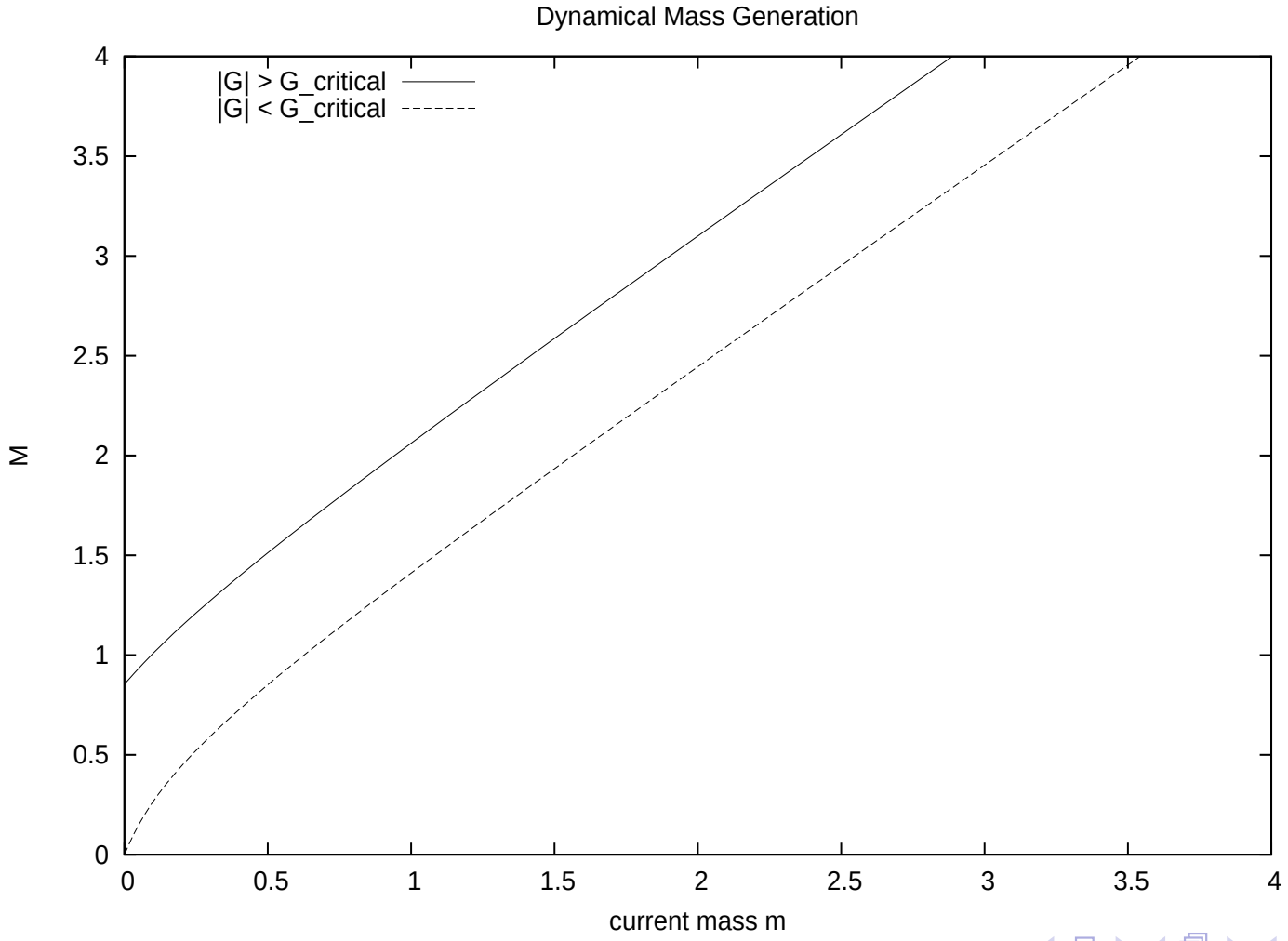
$$M(\vec{k}) = m - G\frac{Tr[TT]}{N_c}\int \frac{d^3k'}{(2\pi)^3} V_{\vec{k'}-\vec{k}}[\frac{M(\vec{k'})}{E_{\vec{k'}}} - \hat{k'}\cdot\hat{k}\frac{|\vec{k'}|M(\vec{k})}{E_{\vec{k'}}|\vec{k}|}]$$

$M(\vec{k})$ in general is a function of $\vec{k}$

it dictates how the mass is dynamically generated

Gap Equation Revisited

for the contact case, we have...



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motivation

FTFT is needed to study the physics of QGP, deconfinement and chiral restoration

when calculating observables in QFT, we only calculate the vacuum expectation value

at finite temperature, excited states start to contribute, the interesting quantity should be the **thermal average** of the observables

we expect $n_{E_{\vec{k}}}$ to enter QFT

partition function dictates the equilibrium Finite Temperature QFT

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basics of FTFT

the partition function:

$$\mathcal{Z} = Tr[e^{-\beta H}] = \int dq < q|e^{-\beta H}|q >$$

observables are given by

$$\mathcal{O} = Tr[e^{-\beta H}\mathcal{O}] =<< q|\mathcal{O}|q >$$

working in imaginary time $\tau = it$

a corresponding path integral representation of the partition function, with Periodic/Antiperiodic boundary condition

the boundary condition motivates the use of Matsubara Green's function:

$$\int dk^0 \rightarrow \frac{1}{\beta}\Sigma_{\omega_n}$$

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Gap equation in Finite Temperature

Generalized Gap Equation

$$M(\vec{k}) = m - G\frac{Tr[TT]}{N_c}\int \frac{d^3k'}{(2\pi)^3} V_{\vec{k'}-\vec{k}}[\frac{M(\vec{k'})}{E_{\vec{k'}}} - \hat{k'}\cdot\hat{k}\frac{|\vec{k'}|M(\vec{k})}{E_{\vec{k'}}|\vec{k}|}](1-2n_{E_{\vec{k'}}})$$

with $n_{E_{\vec{k}}} = \frac{1}{e^{\beta E_{\vec{k}}}+1}$

summary

- non-perturbative physics with $e^{\frac{-1}{x^2}}$
- summation of a class of diagram with $\frac{1}{1-x}$
- Schwinger Dyson Equation with $\int dx \frac{d}{dx} f(x)$
- Finite Temperature Field Theory with $Tr[e^{-\beta H}]$

thank you