Overview of nucleon form factor measurements

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HUGS 2009
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Review articles


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In the beginning ...

- 1918, Rutherford discovers the proton
- 1932, Chadwick discovers the neutron and measures the mass as 938 +/- 1.8 MeV
- 1933, Frisch and Stern measure the proton’s magnetic moment $= 2.6 +/- 0.3 \mu_B = 1 + \kappa_p$
- 1940, Alvarez and Bloch measure the neutron’s magnetic moment $= 1.93 +/- 0.02 \mu_B = \kappa_n$
In the beginning ...

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• 1933, Frisch and Stern measure the proton’s magnetic moment = 2.6 +/- 0.3 \( \mu_B = 1 + \kappa_p \)
• 1940, Alvarez and Bloch measure the neutron’s magnetic moment = 1.93 +/- 0.02 \( \mu_B = \kappa_n \)

Proton and neutron have anomalous magnetic moments a finite size.

Frisch, R.O., Stern, O.; Z. Phys. 85, 4 (1933)
Electron as probe of nucleon elastic form factors

\[ J_\mu = \langle p' | \Gamma_\mu | p \rangle \]

Known QED coupling
Electron as probe of nucleon elastic form factors

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Known QED coupling

Unknown $\gamma^* \Lambda$ coupling
Electron as probe of nucleon elastic form factors

Known QED coupling

Unknown $\gamma^* N$ coupling

Nucleon vertex:

$$\Gamma_\mu(p', p) = F_1(Q^2) \gamma_\mu + \frac{i\kappa_p}{2M_p} \sigma_{\mu\nu} q^\nu$$

Dirac

Pauli

Elastic form factors

$F_1$ is the helicity conserving (non spin-flip)

$F_2$ is helicity non-conserving (spin-flip)
**Electron-Nucleon Scattering kinematics**

- **Incident Electron beam**
  \[ \mathcal{P}_e = (E_e, \vec{k}) \]

- **Scattered electron**
  \[ \Theta_e \quad \mathcal{P}'_e = (E'_e, \vec{k}') \]

- **Fixed nucleon target with mass M**

**Virtual photon kinematics**

\[ Q^2 = - (\mathcal{P}_e - \mathcal{P}'_e)^2 = 4E_e E'_e \sin^2 (\theta_e/2) \quad (m_e = 0) \]

\[ \nu = E_e - E'_e \]
**Electron-Nucleon Scattering kinematics**

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**\( \gamma^* N \) center of mass energy**

\[ W = \sqrt{M^2 + 2M\nu - Q^2} \]
Electron-Nucleon Scattering kinematics

Incident Electron beam
\( \mathcal{P}_e = (E_e, \vec{k}) \)

Scattered electron
\[ \Theta_e \quad \mathcal{P}'_e = (E'_e, \vec{k}') \]

Final Elastic scattering
\( W = M \)

Inelastic scattering
\( W > M + m_\pi \)

Resonance scattering
\( W = M_R \)

Virtual photon kinematics

\[ Q^2 = -(\mathcal{P}_e - \mathcal{P}'_e)^2 = 4E_eE'_e \sin^2(\theta_e/2) \quad (m_e = 0) \]

\[ \nu = E_e - E'_e \]

\( \gamma^*N \) center of mass energy

\[ W = \sqrt{M^2 + 2M\nu - Q^2} \]
Electron-Nucleon cross section

Single photon exchange (Born) approximation

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{E'_e}{E_e} \left\{ F_1^2(Q^2) \right\} \\
+ \tau \left[ \kappa^2 F_2^2(Q^2) + 2(F_1(Q^2) + \kappa F_2(Q^2))^2 \tan^2 \frac{\theta_e}{2} \right] \}
\]

\[
\tau = \frac{Q^2}{4M^2}
\]

Low \(Q^2\)

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{E'_e}{E_e} F_1^2(Q^2)
\]
Early Form factor measurements

Proton is an extended charge potential \( \rho(\vec{r}) \)

\[
\sigma = \sigma_{\text{Mott}} \left| \int \rho(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3 \vec{r} \right|^2
\]

\[
\sigma = \sigma_{\text{Mott}} \left| F(\vec{q}) \right|^2
\]

Proton has a radius of \( 0.80 \times 10^{-13} \) cm

\[
\rho(\vec{r}) = \sqrt{3} a e^{a^2 \vec{r}}
\]

\[
F(\vec{q}) = (1 + \frac{q^2}{a^2})^{-2}
\]

“Dipole” shape

Q\(^2\) = 0.5 GeV\(^2\)

Hofstadter R., Rev. Mod. Phys. 28, 214 (1956).
Sach’s Electric and Magnetic Elastic Form Factors

In center of mass of the eN system (Breit frame), no energy transfer $\nu_{\text{CM}} = 0$ so $|q|^2 = |\vec{q}|^2$

\[ \rho(\vec{r}) = \text{charge distribution} \quad \mu(\vec{r}) = \text{magnetization distribution} \]

\[ G_E = \int \rho(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3\vec{r} \quad G_M = \int \mu(\vec{r}) e^{i\vec{q} \cdot \vec{r}} d^3\vec{r} \]

\[ \frac{<r^2>}{6} = - \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \quad \frac{<r^2>}{6} = - \left. \frac{1}{\mu_N} \frac{dG_M(Q^2)}{dQ^2} \right|_{Q^2=0} \]

At $Q^2 = 0$

\[ G_{Mp} = 2.79 \quad G_{Mn} = -1.91 \]

\[ G_{Ep} = 1 \quad G_{En} = 0 \]
Electron-Nucleon cross section

Single photon exchange (Born) approximation

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \frac{E'_e}{E_e} \left\{ F_1^2(Q^2) \right. \\
+ \tau \left[ \kappa^2 F_2^2(Q^2) + 2(F_1(Q^2) + \kappa F_2(Q^2))^2 \tan^2 \frac{\theta_e}{2} \right]\left\} \right.
\]

\[G_E(Q^2) = F_1(Q^2) - \kappa_N \tau F_2(Q^2)\]

\[G_M(Q^2) = F_1(Q^2) + \kappa_N F_2(Q^2)\]

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \times \left( G_E^2 + \tau \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right] G_M^2 \right) / (1 + \tau)
\]
Elastic cross section in $G_E$ and $G_M$

\[
\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \times \left(G_E^2 + \tau \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2}\right] G_M^2\right) / (1 + \tau)
\]

Experiments from the 1960s to 1990s gave a cumulative data set

\[ \frac{G_E}{G_D} \approx \frac{G_M}{(\mu_p G_D)} \approx 1 \]

\[ G_D = (1 + \frac{Q^2}{0.71})^{-2} \]
Proton Form Factors: $G_{Mp}$ and $G_{Ep}$

$G_E$ contribution to $\sigma$ is small then large error bars

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At large $Q^2$, $G_E$ contribution is smaller so difficult to extract
GE > 1 then large error bars and spread in data.

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\(G_M\) measured to \(Q^2 = 30\)

\(G_E\) measured well only to \(Q^2 = 1\)
$Q^2$ dependence of elastic and inelastic cross sections

As $Q^2$ increases, \( \sigma_{\text{elastic}} / \sigma_{\text{Mott}} \) drops dramatically.
Q² dependence of elastic and inelastic cross sections

As Q² increases

\( \sigma_{\text{elastic}} / \sigma_{\text{Mott}} \) drops dramatically

At W = 2 GeV

\( \sigma_{\text{inel}} / \sigma_{\text{Mott}} \) drops less steeply
As $Q^2$ increases

$\frac{\sigma_{\text{elastic}}}{\sigma_{\text{Mott}}}$ drops dramatically

At $W = 2$ GeV

$\frac{\sigma_{\text{inel}}}{\sigma_{\text{Mott}}}$ drops less steeply

At $W=3$ and 3.5

$\frac{\sigma_{\text{inel}}}{\sigma_{\text{Mott}}}$ almost constant
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Point object inside the proton
Asymptotic freedom to confinement

• “point-like” objects in the nucleon are eventually identified as quarks
• Theory of Quantum Chromodynamics (QCD) with gluons mediating the strong force.
• At high energies, the quarks are asymptotically free and perturbative QCD approaches can be used.
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- At high energies, the quarks are asymptotically free and perturbative QCD approaches can be used.

No free quarks  

Confinement

- The QCD strong coupling increases as the quarks separate from each other
- Quantitative QCD description of nucleon’s properties remains a puzzle
- Study of nucleon elastic form factors is a window see how the QCD strong coupling changes
Elastic FF in perturbative QCD

Infinite momentum frame
- Nucleon looks like three massless quarks
- Energy shared by two hard gluon exchanges
- Gluon coupling is $1/Q^2$

\[ F_1(Q^2) \propto 1/Q^4 \]
Elastic FF in perturbative QCD

Infinite momentum frame

- Nucleon looks like three massless quarks
- Energy shared by two hard gluon exchanges
- Gluon coupling is $1/Q^2$

$$F_1(Q^2) \propto 1/Q^4$$

$F_2$ requires an helicity flip the spin of the quark.
Assuming the $L = 0$

$$F_2(Q^2) \propto 1/Q^6$$

In $e^- p \rightarrow e^- p$ scattering the nature of the interaction of the virtual photon with the proton depends strongly on wavelength.

- At very low electron energies $\lambda \gg r_p$:
  the scattering is equivalent to that from a “point-like” spin-less object.

- At low electron energies $\lambda \sim r_p$:
  the scattering is equivalent to that from a extended charged object.

- At high electron energies $\lambda < r_p$:
  the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks.

- At very high electron energies $\lambda \ll r_p$:
  the proton appears to be a sea of quarks and gluons.