

Overview of nucleon form factor measurements

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HUGS 2009

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Review articles

C. F. Perdrisat, V. Punjabi, M. Vanderhaeghen **Prog.Part.Nucl.Phys.59:694,2007**

J. Arrington, C. D. Roberts, J. M. Zanotti, **J.Phys.G34:S23-S52,2007**

Hyde-Wright C. E. and K. de Jager, *Ann. Rev. Nucl. Part. Sci.* **54**, 217 (2004).

Gao H. Y., *Int. J. Mod. Phys. E* **12**, 1 (2003) [Erratum-ibid. *E* **12**, 567 (2003)].

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In the beginning ...

- 1918, Rutherford discovers the proton
- 1932, Chadwick discovers the neutron and measures the mass as $938 \pm 1.8 \text{ MeV}$
- 1933, Frisch and Stern measure the proton's magnetic moment = $2.6 \pm 0.3 \mu_B = 1 + \kappa_p$
- 1940, Alvarez and Bloch measure the neutron's magnetic moment = $1.93 \pm 0.02 \mu_B = \kappa_n$

Frisch, R.O., Stern, O.: *Z. Phys.* **85**, 4 (1933)

L. Alvarez and F. Bloch, *Phys. Rev.*, **57**, 111 (1940).

In the beginning ...

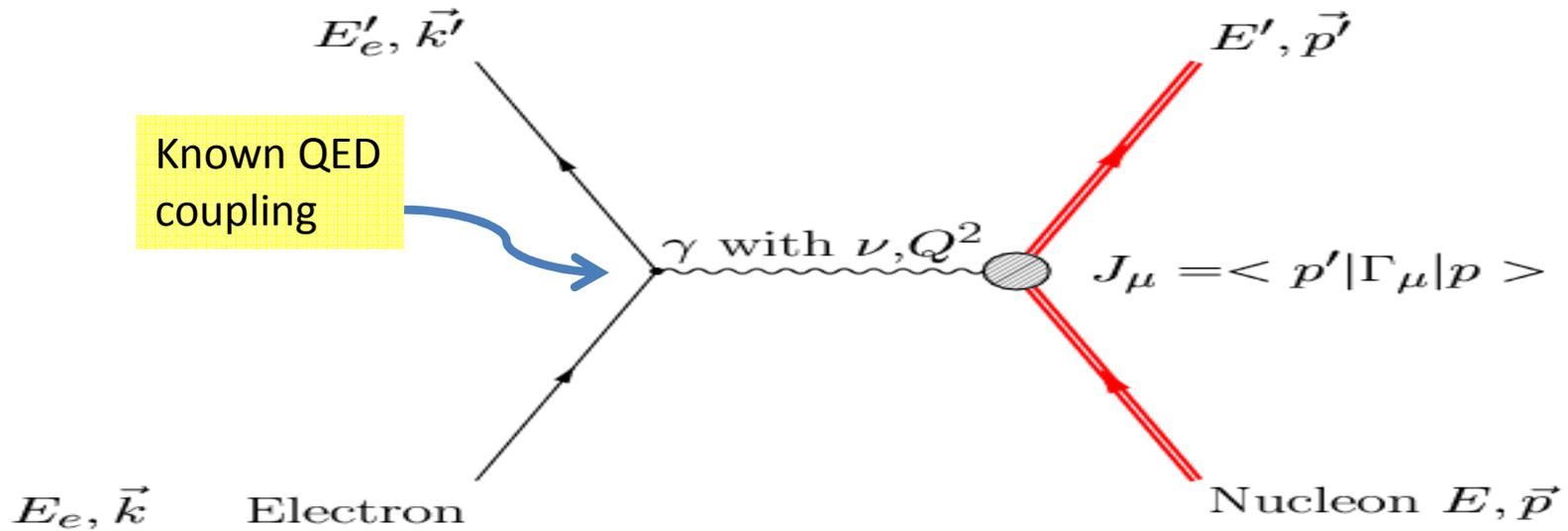
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Proton and neutron have anomalous magnetic moments  a finite size.

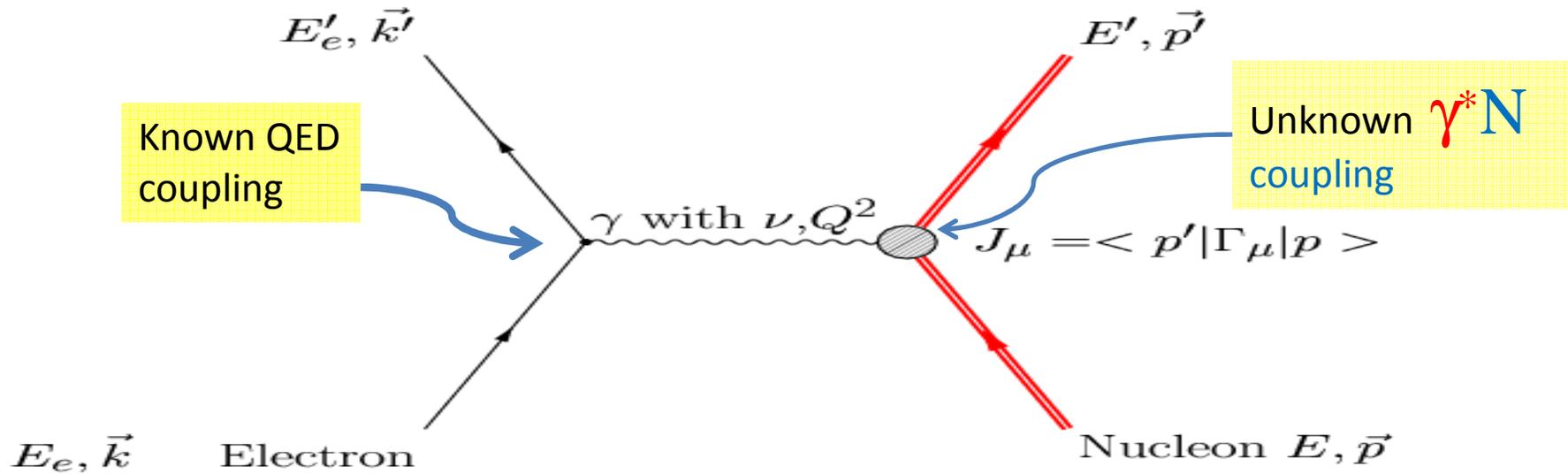
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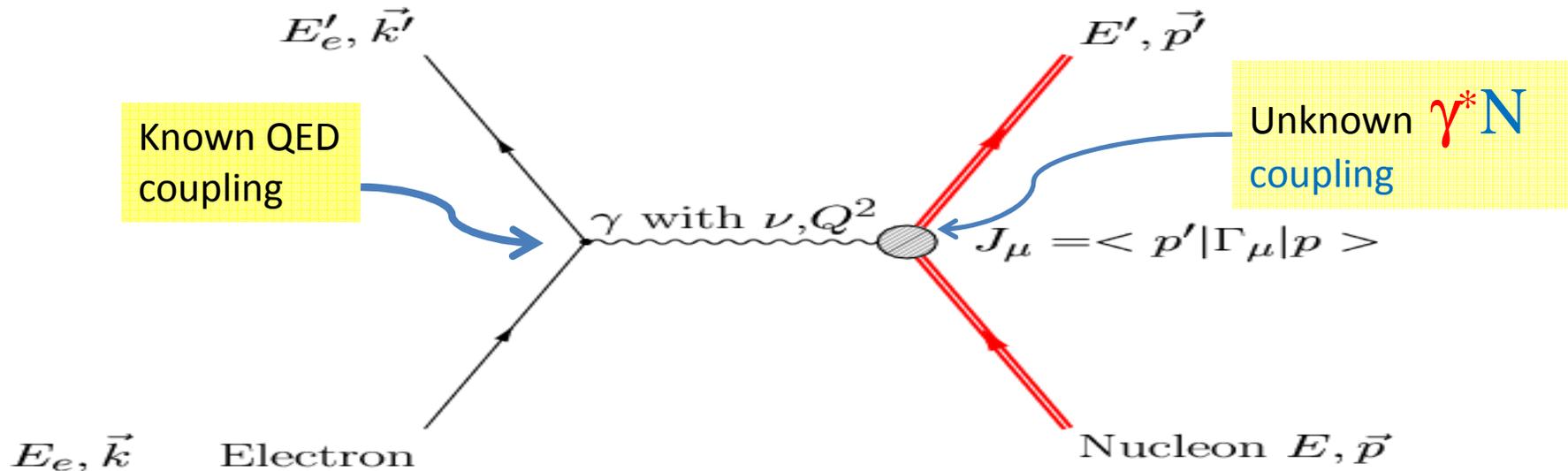
Electron as probe of nucleon elastic form factors



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Electron as probe of nucleon elastic form factors



Nucleon vertex:

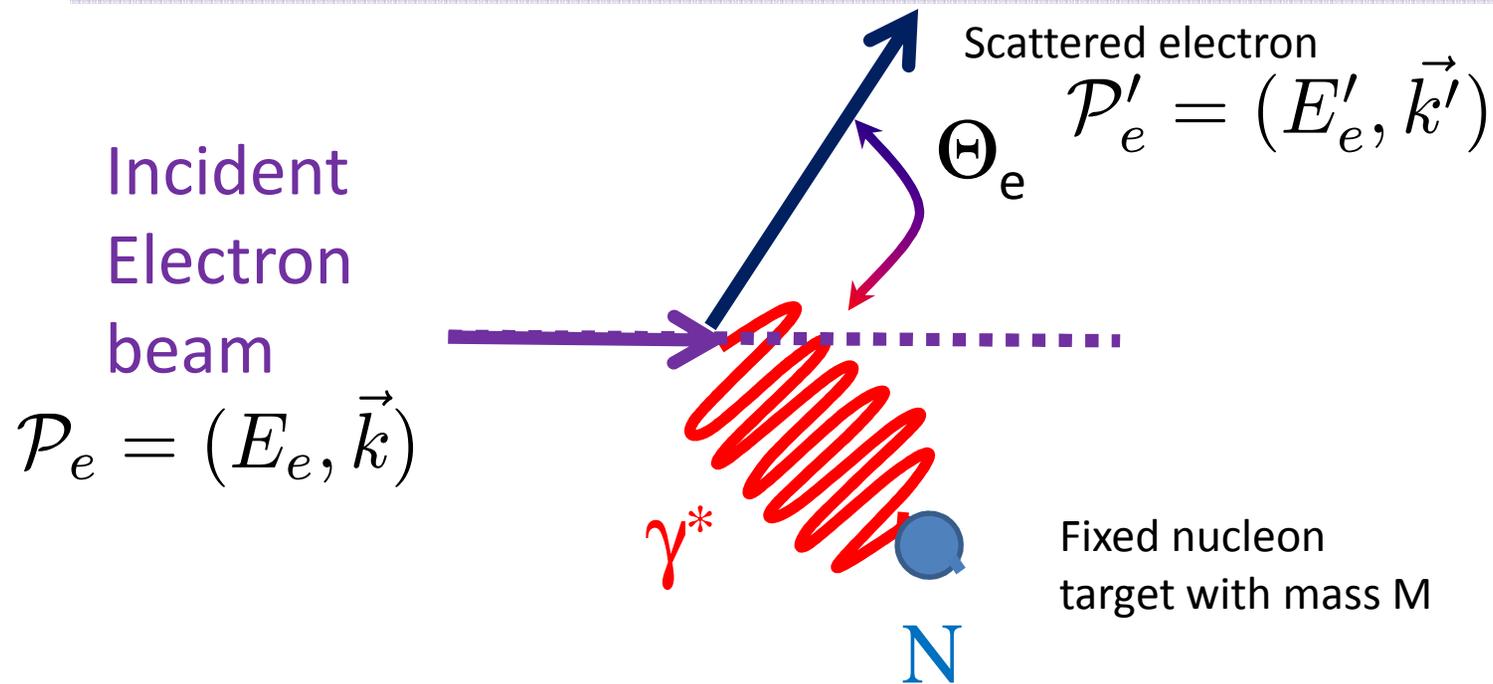
$$\Gamma_\mu(p', p) = \underbrace{F_1(Q^2)}_{Dirac} \gamma_\mu + \frac{i\kappa_p}{2M_p} \underbrace{F_2(Q^2)}_{Pauli} \sigma_{\mu\nu} q^\nu$$

Elastic form factors

F_1 is the helicity **conserving** (non spin-flip)

F_2 is helicity **non-conserving** (spin-flip)

Electron-Nucleon Scattering kinematics

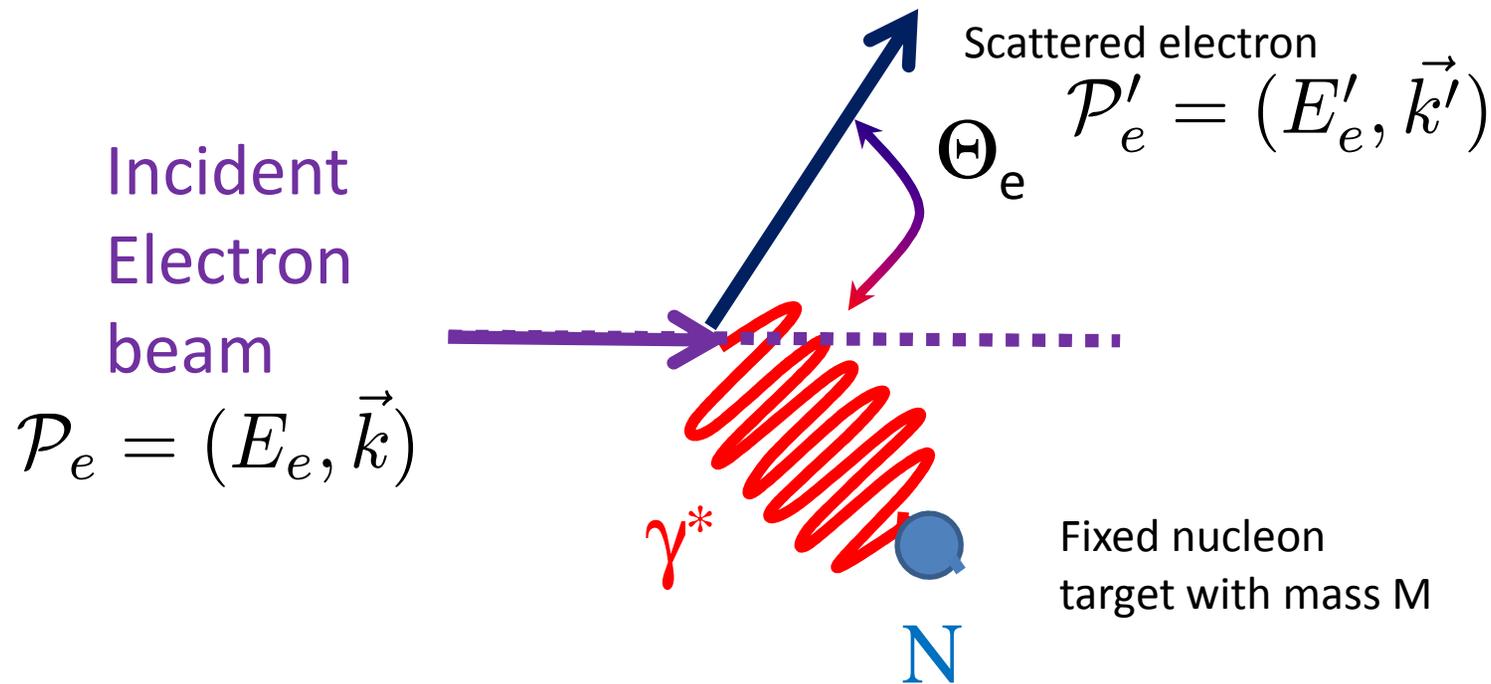


Virtual photon kinematics

$$Q^2 = -(\mathcal{P}_e - \mathcal{P}'_e)^2 = 4E_e E'_e \sin^2(\theta_e/2) \quad (m_e = 0)$$

$$\nu = E_e - E'_e$$

Electron-Nucleon Scattering kinematics



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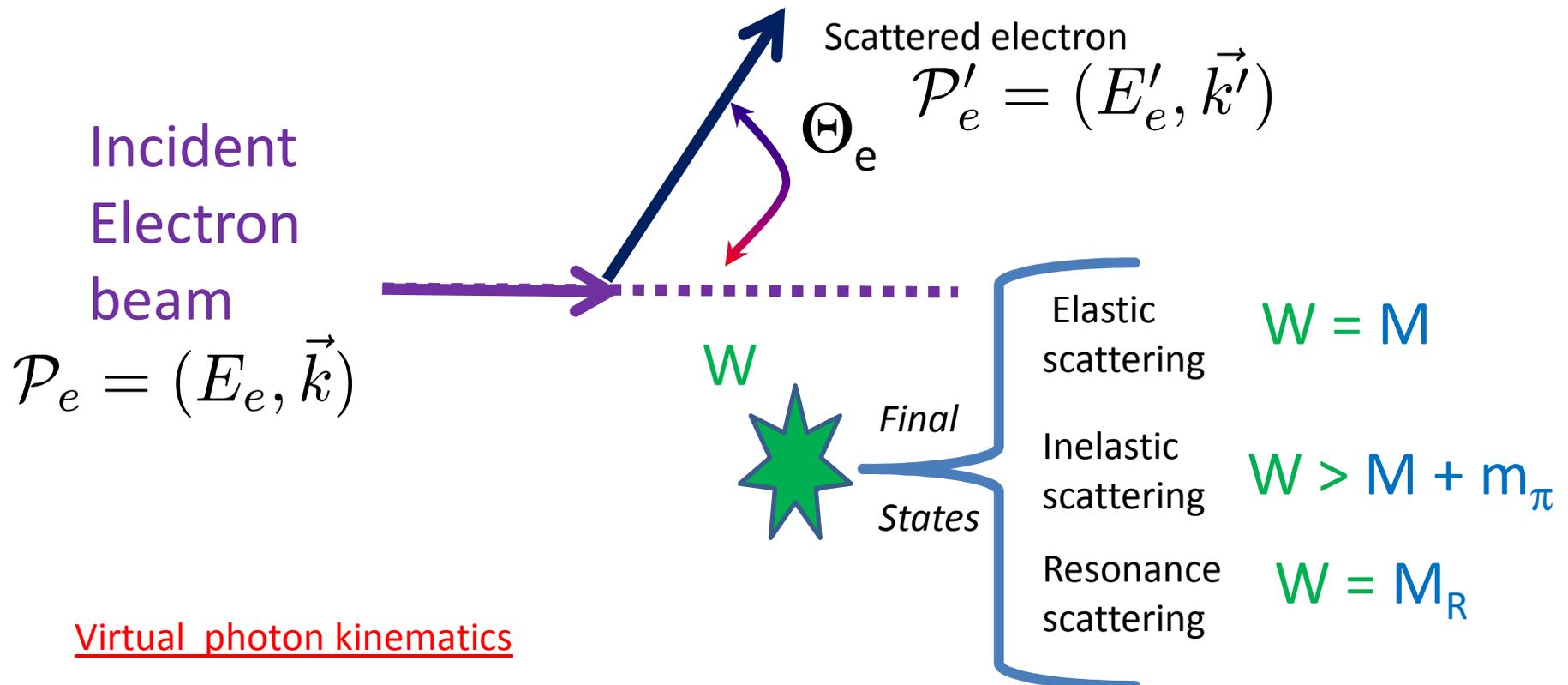
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$\gamma^* N$ center of mass energy

$$W = \sqrt{M^2 + 2M\nu - Q^2}$$

Electron-Nucleon Scattering kinematics



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Electron-Nucleon cross section

Single photon exchange (Born) approximation

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{E'_e}{E_e} \left\{ F_1^2(Q^2) + \tau \left[\kappa^2 F_2^2(Q^2) + 2(F_1(Q^2) + \kappa F_2(Q^2))^2 \tan^2 \frac{\theta_e}{2} \right] \right\}$$

$$\tau = Q^2 / 4M^2$$

Low Q^2



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{E'_e}{E_e} F_1^2(Q^2)$$

Early Form factor measurements

Proton is an extended charge potential $\rho(\vec{r})$



$$\sigma = \sigma_{\text{Mott}} \left| \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r} \right|^2$$



$$\sigma = \sigma_{\text{Mott}} |F(\vec{q})|^2$$

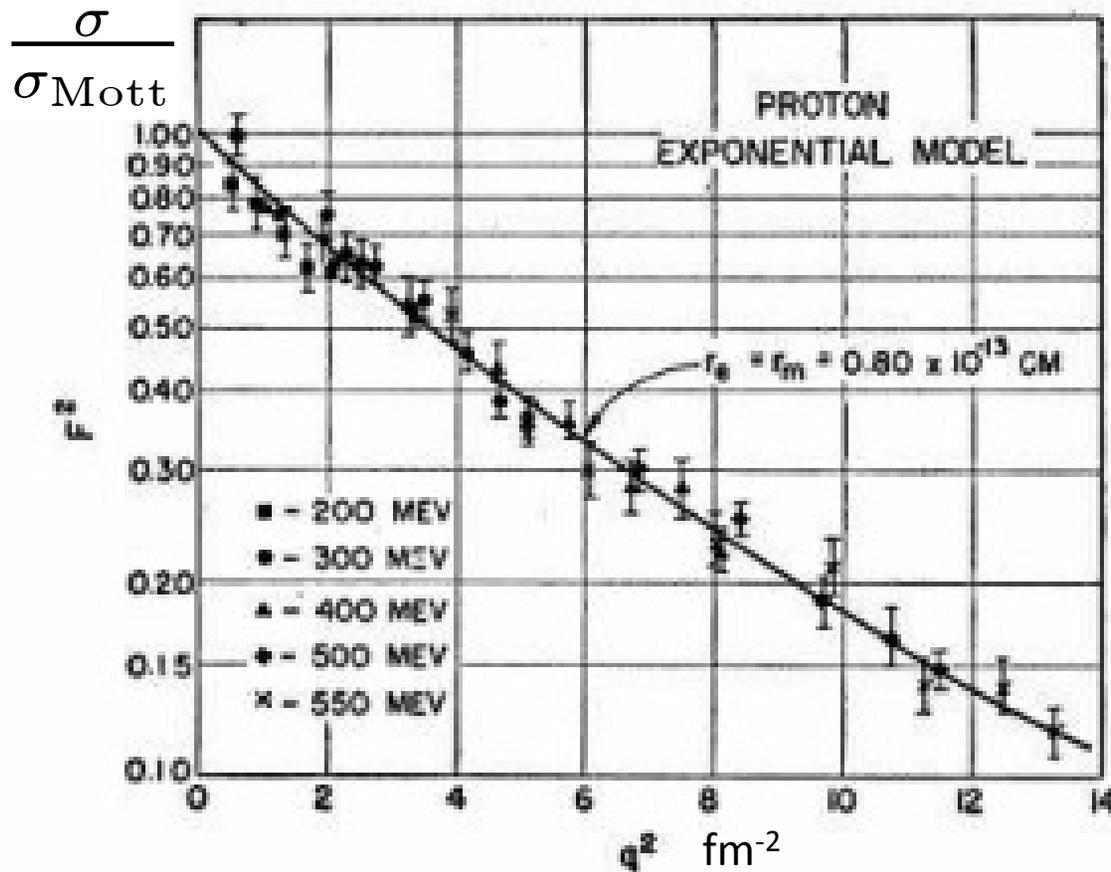
Proton has a radius of 0.80×10^{-13} cm

$$\rho(\vec{r}) = \sqrt{3} a e^{-a^2 r}$$

$$F(\vec{q}) = \left(1 + \frac{q^2}{a^2}\right)^{-2}$$

"Dipole" shape

$$Q^2 = 0.5 \text{ GeV}^2$$



Hofstadter R., Rev. Mod. Phys. **28**, 214 (1956).

Sach's Electric and Magnetic Elastic Form Factors

In center of mass of the eN system (Breit frame),

no energy transfer $v_{\text{CM}} = 0$ so $|\mathbf{q}|^2 = |\vec{q}|^2$

$\rho(\vec{r})$ = charge distribution

$\mu(\vec{r})$ = magnetization distribution

$$G_E = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$$

$$G_M = \int \mu(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}$$

$$\frac{\langle r^2 \rangle}{6} = -\frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

$$\frac{\langle r^2 \rangle}{6} = -\frac{1}{\mu_N} \frac{dG_M(Q^2)}{dQ^2} \Big|_{Q^2=0},$$

At $Q^2 = 0$

$$G_{Mp} = 2.79 \quad G_{Mn} = -1.91$$

$$G_{Ep} = 1 \quad G_{En} = 0$$

Electron-Nucleon cross section

Single photon exchange (Born) approximation

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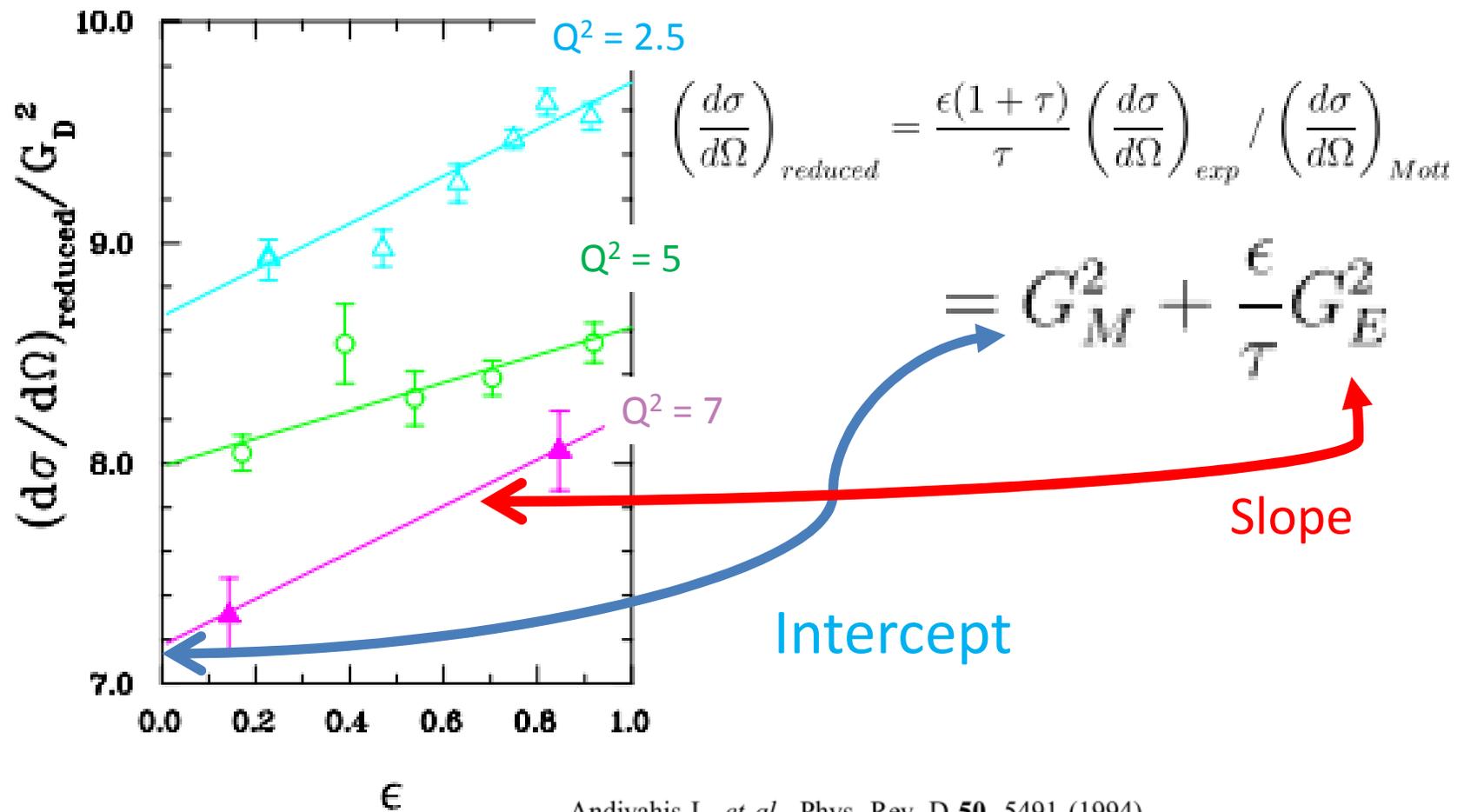
$$G_E(Q^2) = F_1(Q^2) - \kappa_N \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + \kappa_N F_2(Q^2)$$

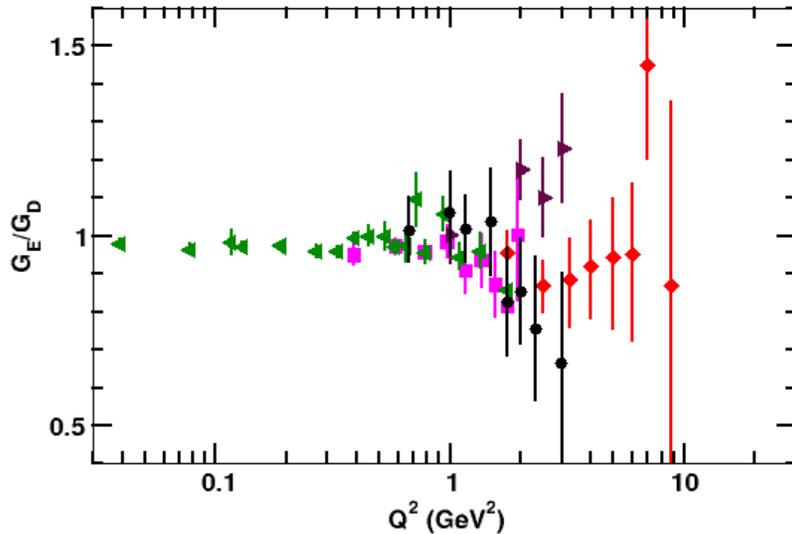
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \times \left(G_E^2 + \tau \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right] G_M^2 \right) / (1 + \tau)$$

Elastic cross section in G_E and G_M

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{Mott} \times \left(G_E^2 + \tau \left[1 + 2(1 + \tau) \tan^2 \frac{\theta_e}{2} \right] G_M^2 \right) / (1 + \tau)$$



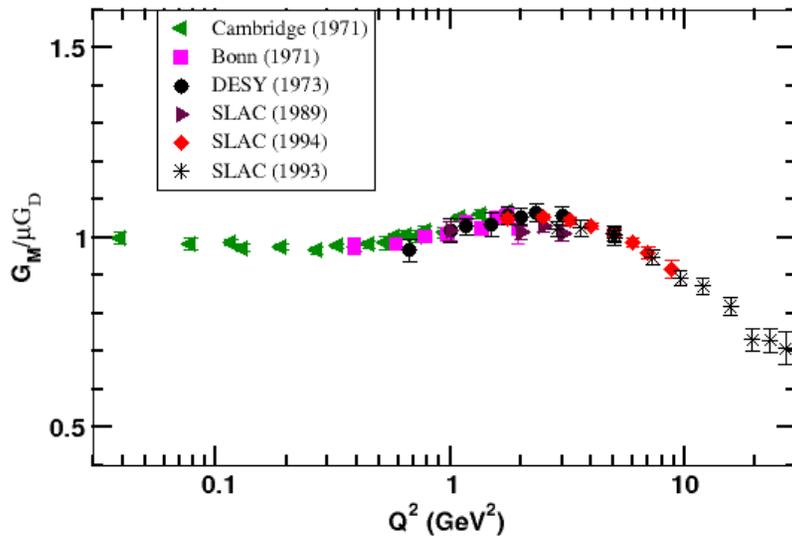
Proton Form Factors: G_{M_p} and G_{E_p}



Experiments from the 1960s to 1990s gave a cumulative data set

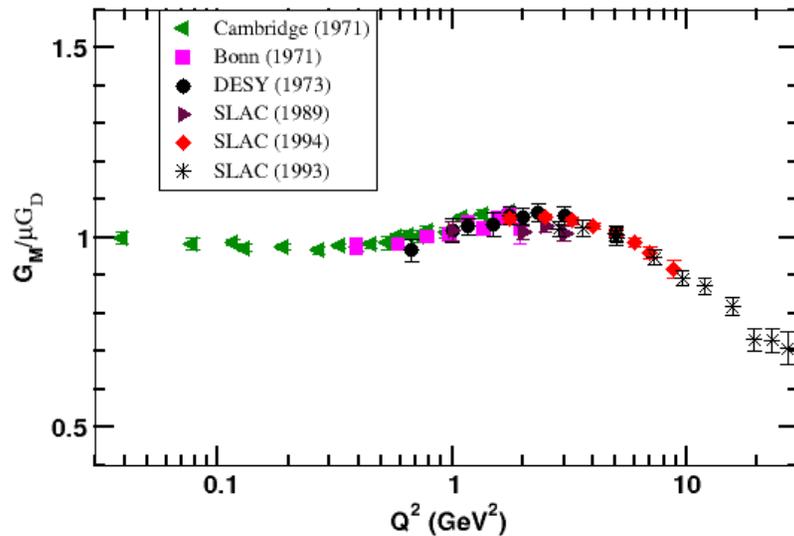
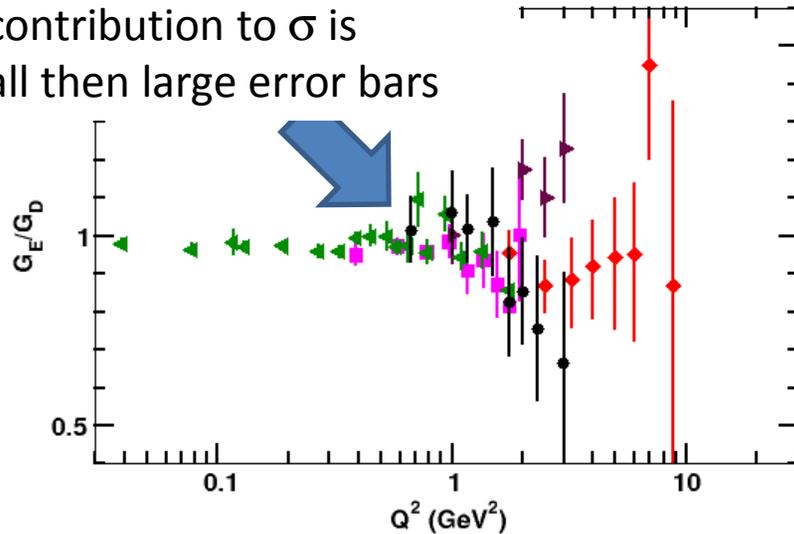
$$G_E/G_D \approx G_M/(\mu_p G_D) \approx 1$$

$$G_D = \left(1 + \frac{Q^2}{0.71}\right)^{-2}$$



Proton Form Factors: G_{M_p} and G_{E_p}

G_E contribution to σ is small then large error bars



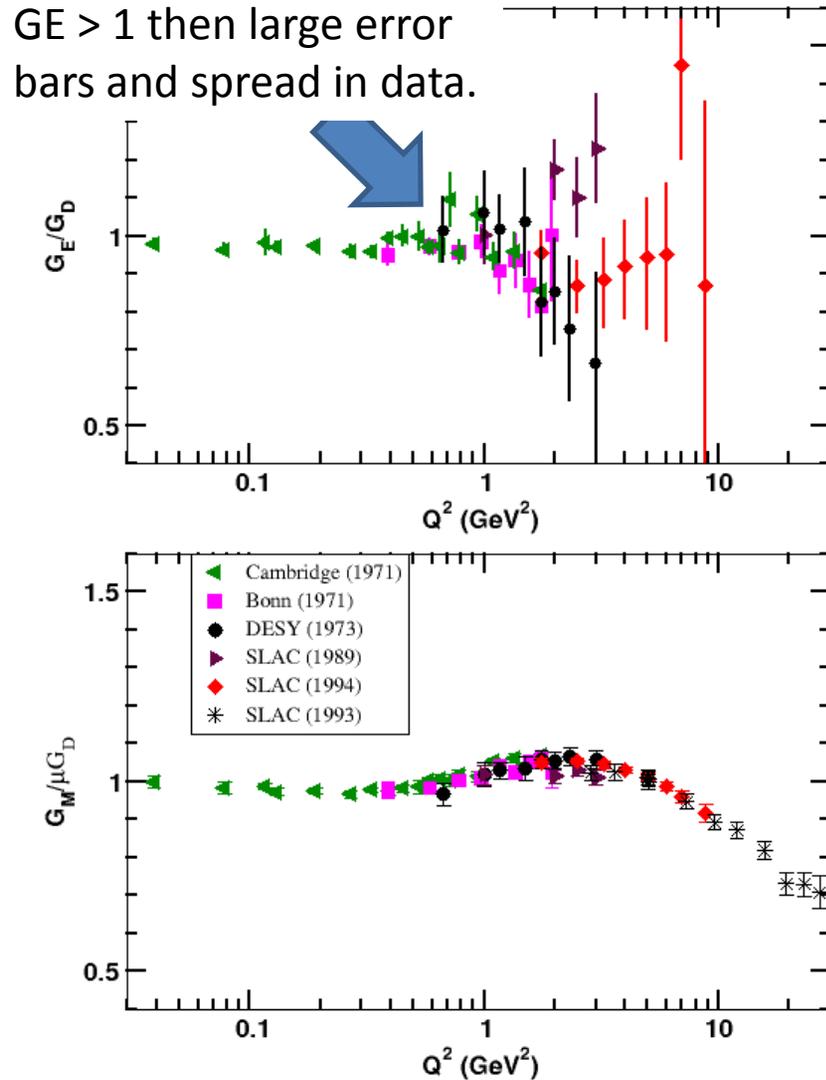
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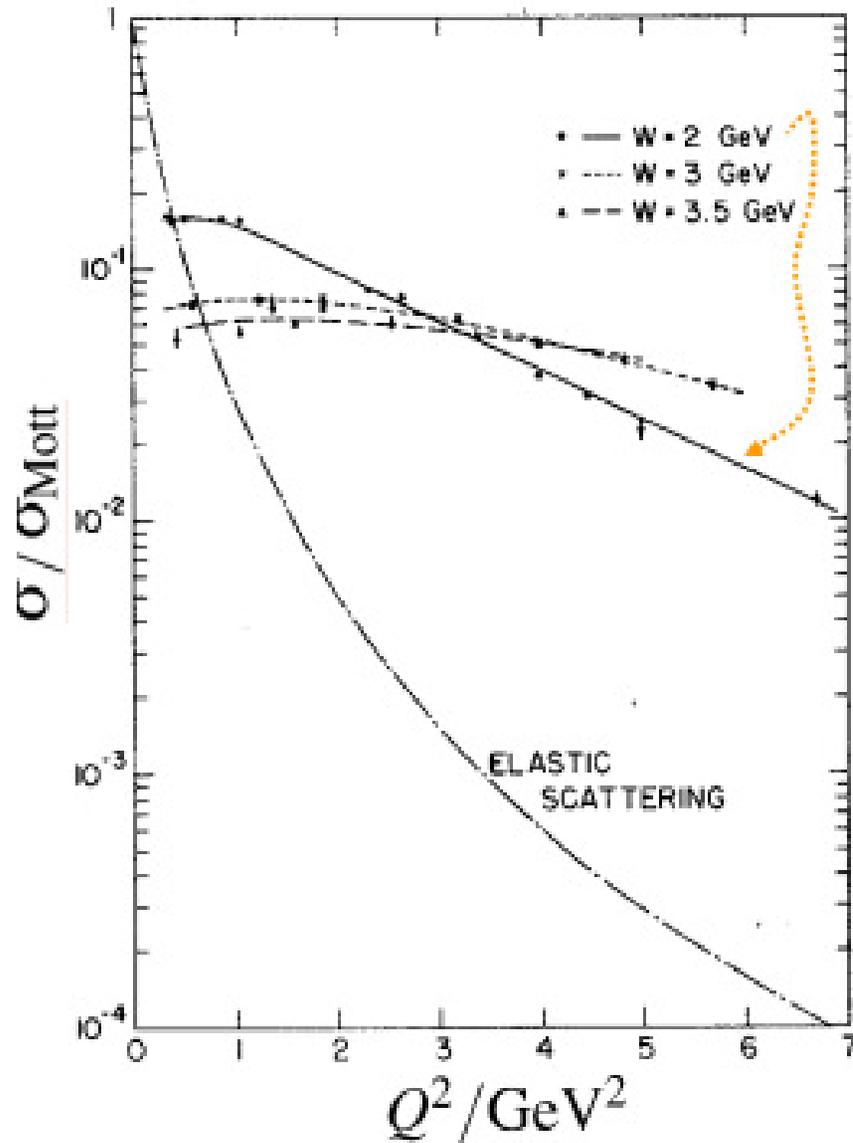
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G_M measured to $Q^2 = 30$

G_E measured well only to $Q^2 = 1$

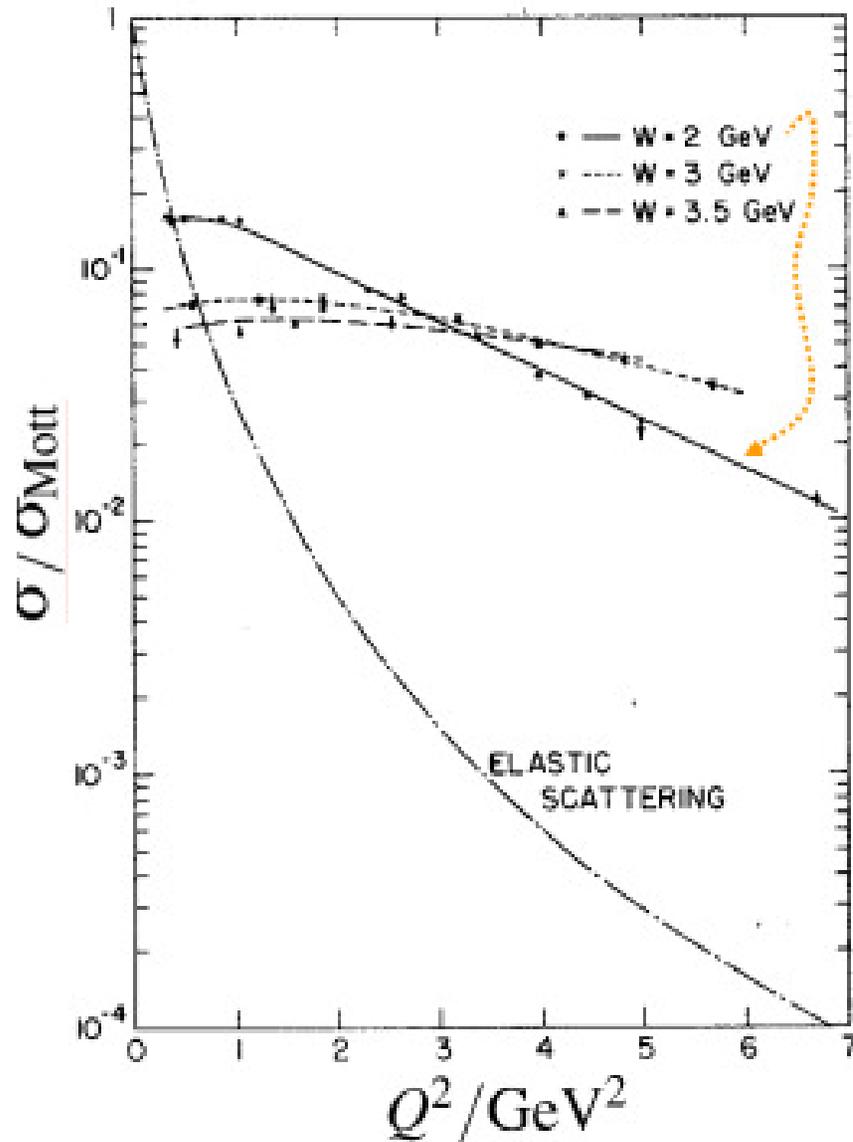
Q^2 dependence of elastic and inelastic cross sections



As Q^2 increases

$\sigma_{\text{elastic}} / \sigma_{\text{Mott}}$ drops dramatically

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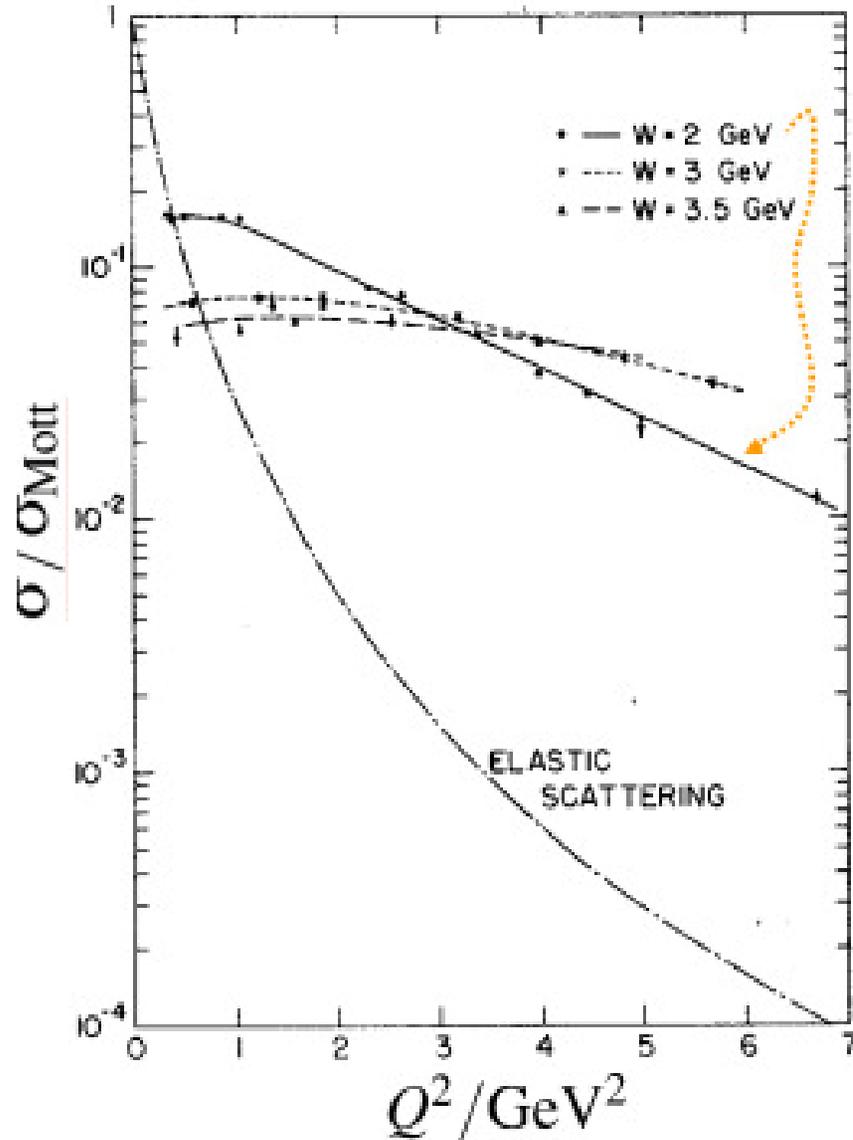
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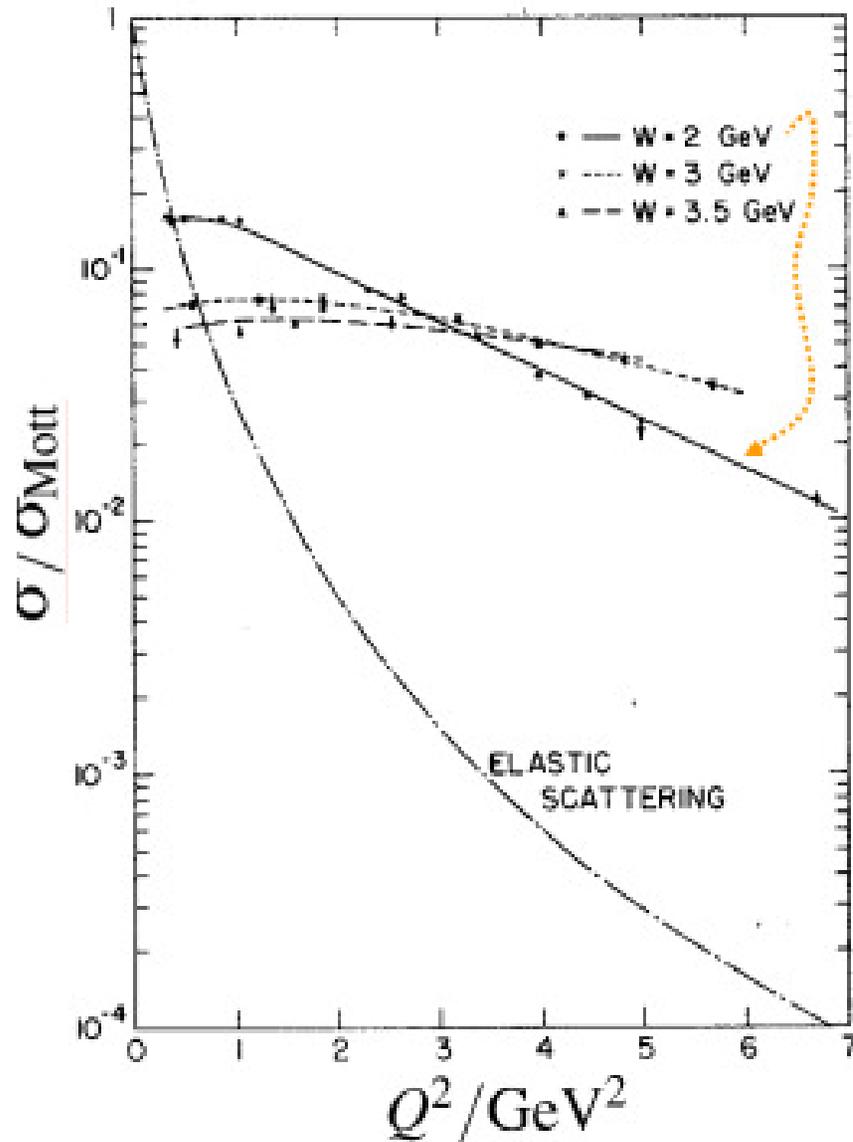
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$\sigma_{\text{inel}}/\sigma_{\text{Mott}}$ almost constant

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Point object inside
the proton

Asymptotic freedom to confinement

- “point-like” objects in the nucleon are eventually identified as quarks
- Theory of Quantum Chromodynamics (QCD) with gluons mediating the strong force.
- At high energies, the quarks are asymptotically free and perturbative QCD approaches can be used.

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**No free
quarks**



Confinement

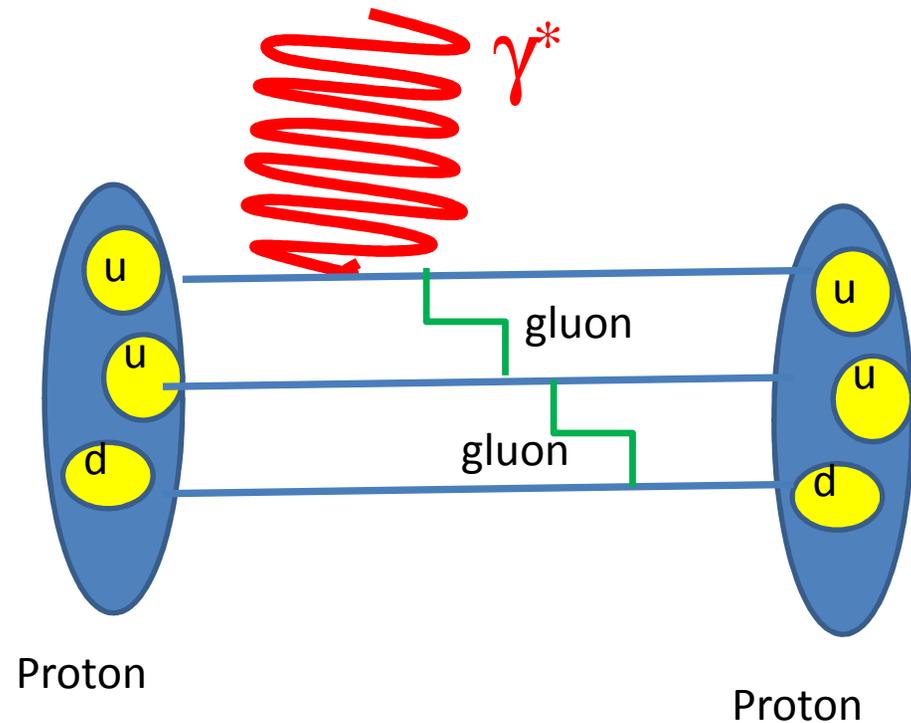
- The QCD strong coupling increases as the quarks separate from each other
- Quantitative QCD description of nucleon's properties remains a puzzle
- Study of nucleon elastic form factors is a window see how the QCD strong coupling changes

Elastic FF in perturbative QCD

Infinite momentum frame

- Nucleon looks like three massless quarks
- Energy shared by two hard gluon exchanges
- Gluon coupling is $1/Q^2$

$$F_1(Q^2) \propto 1/Q^4$$



Elastic FF in perturbative QCD

Infinite momentum frame

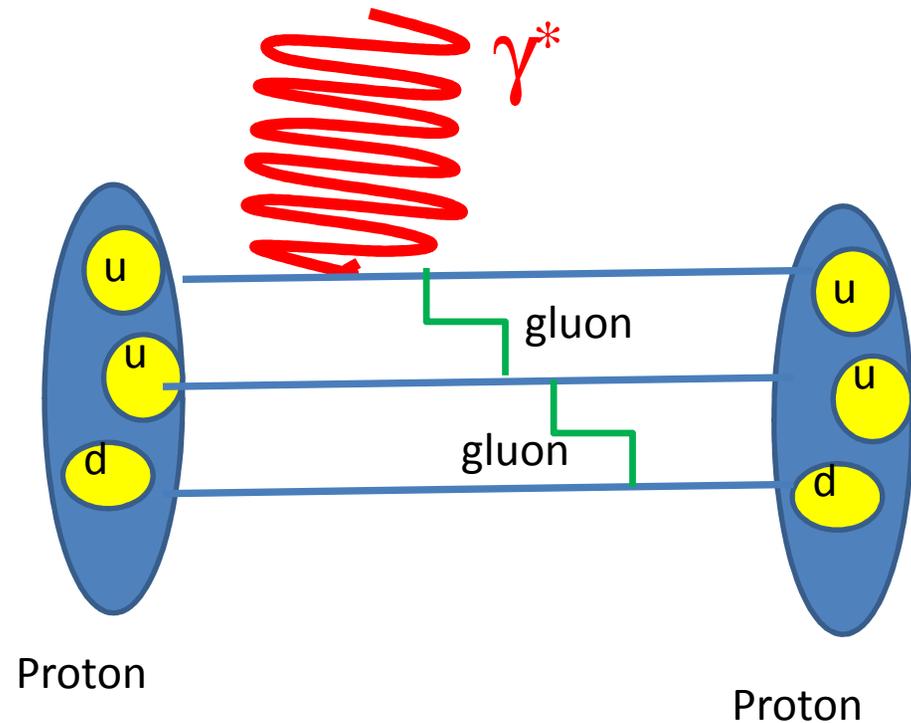
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- F_2 requires an helicity flip the spin of the quark.

Assuming the $L = 0$

$$F_2(Q^2) \propto 1/Q^6$$



Brodsky S. J. and G.R. Farrar, Phys. Rev. D **11**, 1309 (1975).

Electron as probe of nucleon structure

★ In $e^-p \rightarrow e^-p$ scattering the nature of the interaction of the virtual photon with the proton depends strongly on wavelength

- ♦ At **very low** electron energies $\lambda \gg r_p$:
the scattering is equivalent to that from a “point-like” spin-less object
- ♦ At **low** electron energies $\lambda \sim r_p$:
the scattering is equivalent to that from an extended charged object
- ♦ At **high** electron energies $\lambda < r_p$:
the wavelength is sufficiently short to resolve sub-structure. Scattering from constituent quarks
- ♦ At **very high** electron energies $\lambda \ll r_p$:
the proton appears to be a sea of quarks and gluons.

