# Overview of nucleon form factor measurements

Focus on theoretical calculations of form factors

Mark Jones

Jefferson Lab

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# Many approaches to calculating form factors

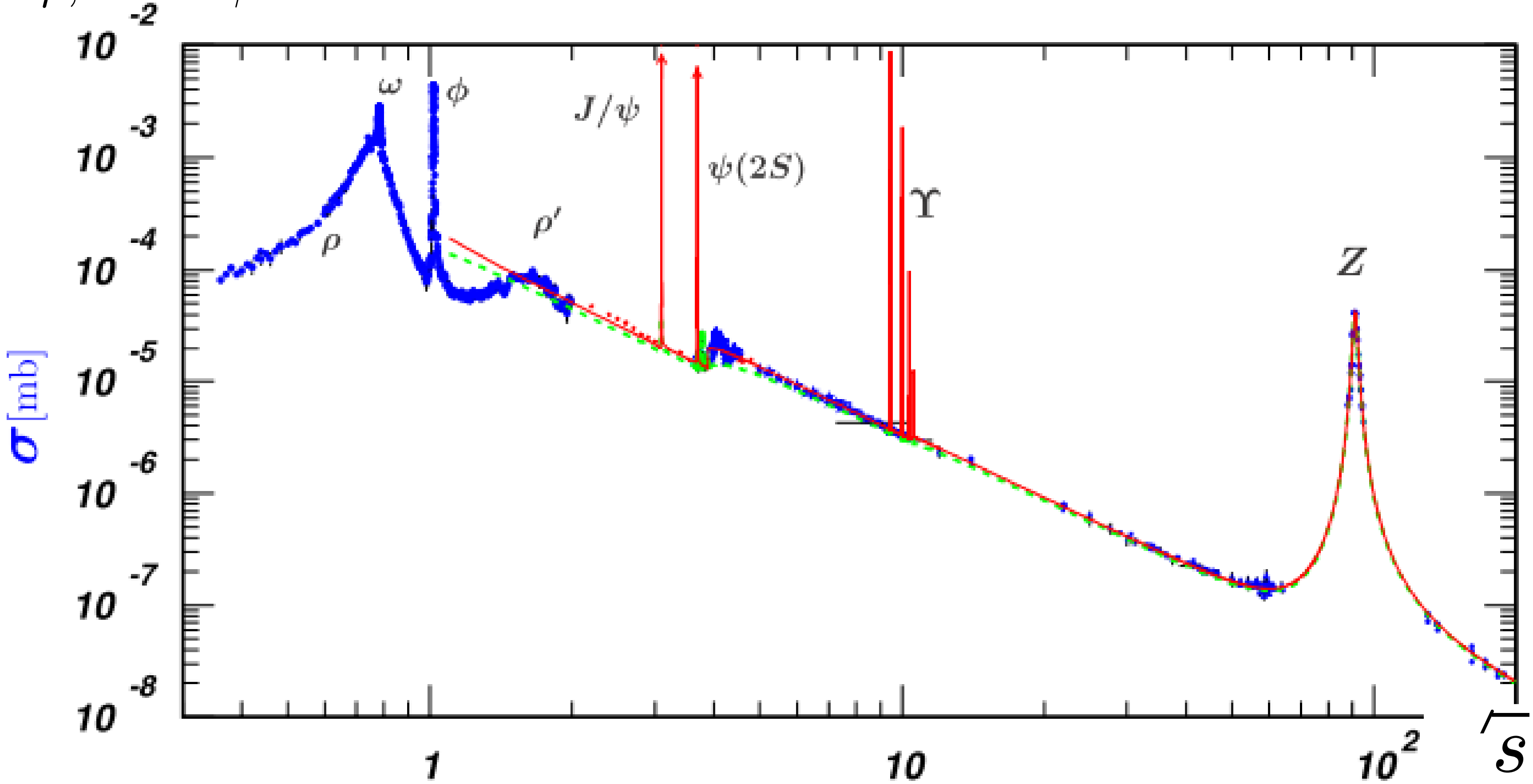
Major difficulty is how to describes colorless nucleons being built out of gluons and quarks.

A few of the many approaches

- Vector meson dominance model
- Dispersion relations
- Constituent quark model
- Lattice Gauge Theory
- Field theoretical approaches
- Perturbative QCD

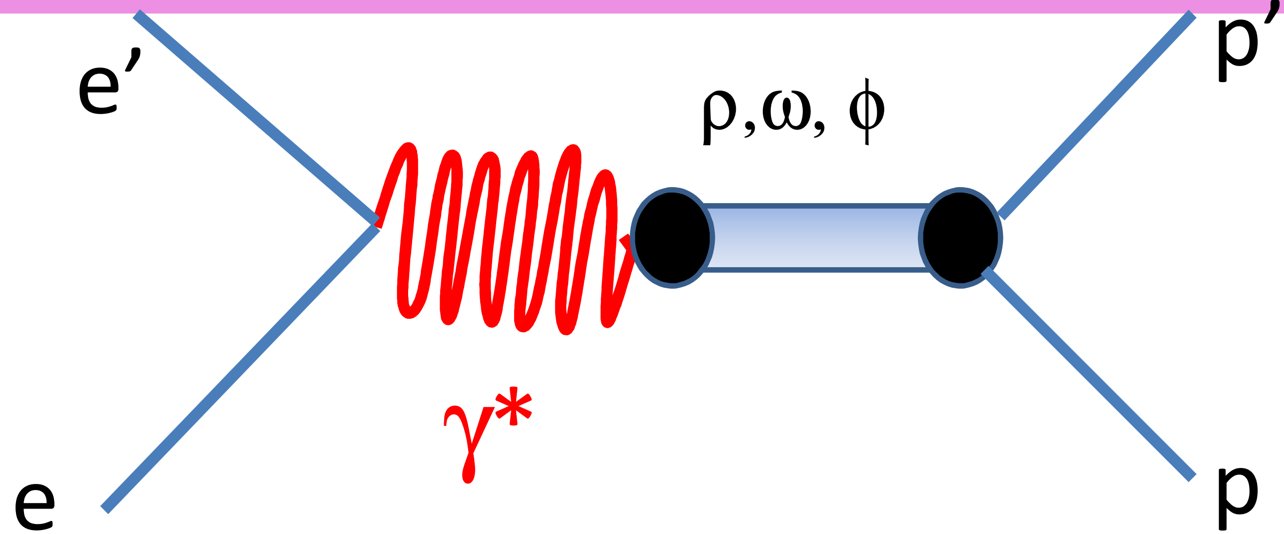
# Vector meson resonances in $e^+e^-$

The total cross section $e^+e^- \rightarrow hadrons$ shows resonances for vector meson $\rho, \omega$ and $\phi$




The photon is a vector probe so natural to assume that these vector mesons may play a role in elastic electron proton scattering

# Vector meson dominance model



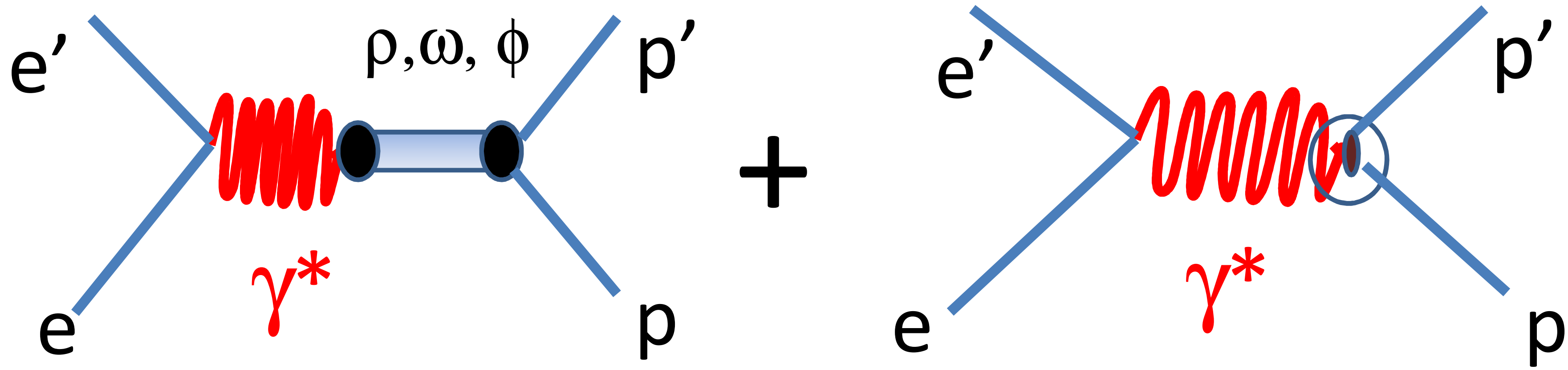

Masses of ρ,ω, ϕ are 770, 782 and 1020 MeV

Consider two vector poles with opposite contributions to form factor

$$F_{1,2} \sim \frac{a}{q^2 - m_\rho^2} + \frac{-a}{q^2 - m_\omega^2} = \frac{a(m_\rho^2 - m_\omega^2)}{(q^2 - m_\rho^2)(q^2 - m_\omega^2)}$$

Easily explain the dipole form $G_D = \frac{1}{(Q^2 - m^2)^2}$

# VMD + intrinsic structure




In 1973, Iachello, Jackson and Lande modeled the form factors assuming VMD and an intrinsic structure.

$F_1$ has VMD and intrinsic structure contribution

$F_2$ only has VMD part

F. Iachello, A.D. Jackson and A. Lande, Phys. Lett. B **43**, 191 (1973).

# VMD + intrinsic structure

Work in terms of isoscaler and isovector combinations of form factor

$$F_i^S = F_i^p + F_i^n \qquad F_1^V = F_1^p - F_1^n$$

$$F_1^S(Q^2) = \frac{1}{2}g(Q^2)\left[1 - \beta_\omega - \beta_\varphi + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2}\right],$$

$$F_1^V(Q^2) = \frac{1}{2}g(Q^2)\left[1 - \beta_\rho + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2}\right],$$

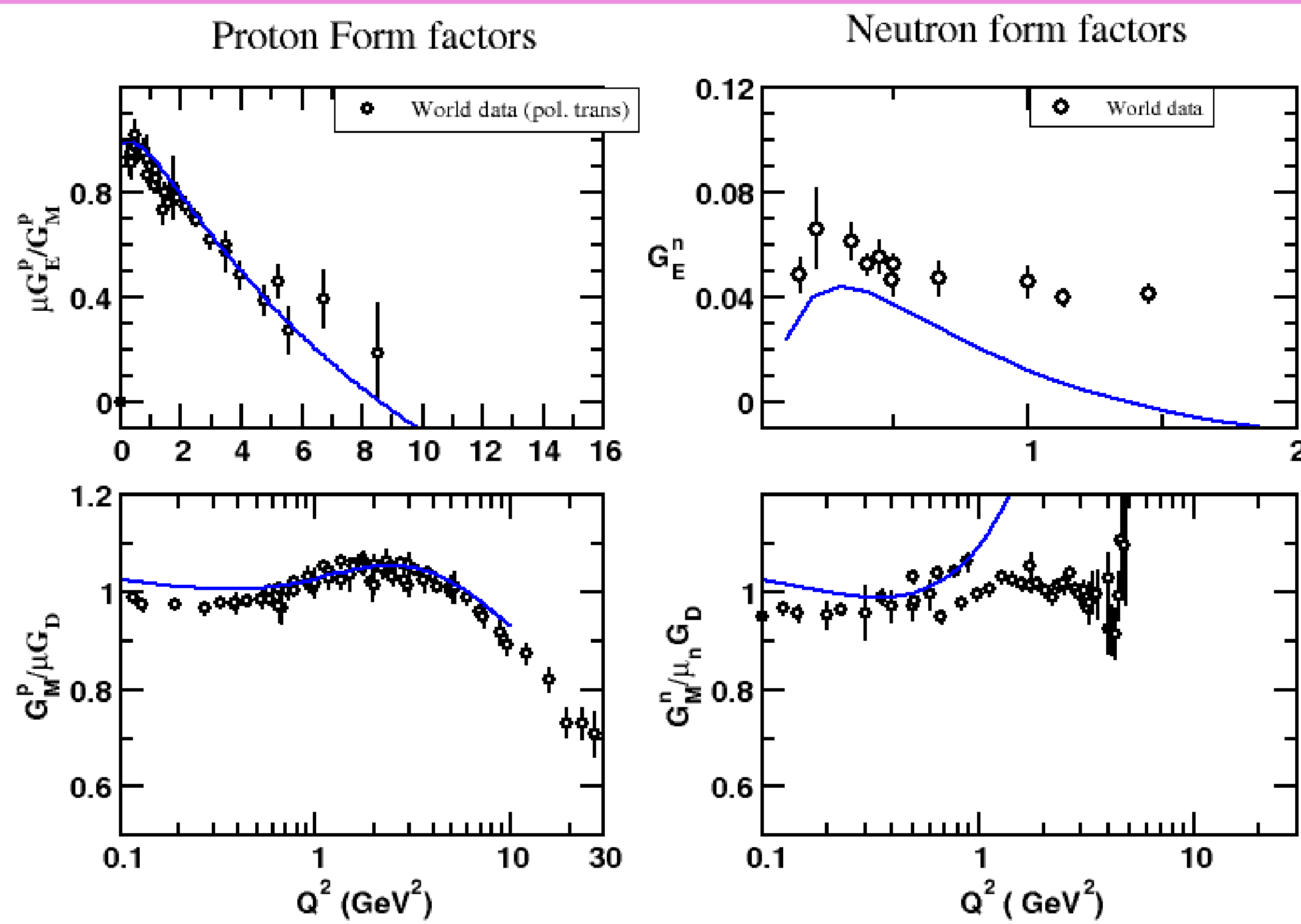
$$F_2^S(Q^2) = \frac{1}{2}g(Q^2)\left[(\mu_p + \mu_n - 1 - \alpha_\varphi)\frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2}\right],$$

$$F_2^V(Q^2) = \frac{1}{2}g(Q^2)\left[(\mu_p - \mu_n - 1)\frac{m_\rho^2}{m_\rho^2 + Q^2}\right],$$

$$g(Q^2) = (1 + \gamma Q^2)^{-2} \qquad \frac{m_\rho^2}{m_\rho^2 + Q^2} \to \frac{m_\rho^2 + 8\Gamma_\rho m_\pi/\pi}{m_\rho^2 + Q^2 + (4m_\pi^2 + Q^2)\,\Gamma_\rho \alpha(Q^2)/m_\pi}$$

Use known masses and fit α,β,γ to the form factor data (at that time 1973!)

# VMD + intrinsic structure




Follows the proton $G_E/G_M$ fall-off.

But neutron form factors are not well described

Of course most of data shown did not exist in 1973!

# Revisiting VMD + intrinsic structure

In 2004, Iachello and Bijker decide to redo the fit to new world data but modify form of $F_2^V$ to include the intrinsic structure with additional 1/Q$^2$ term

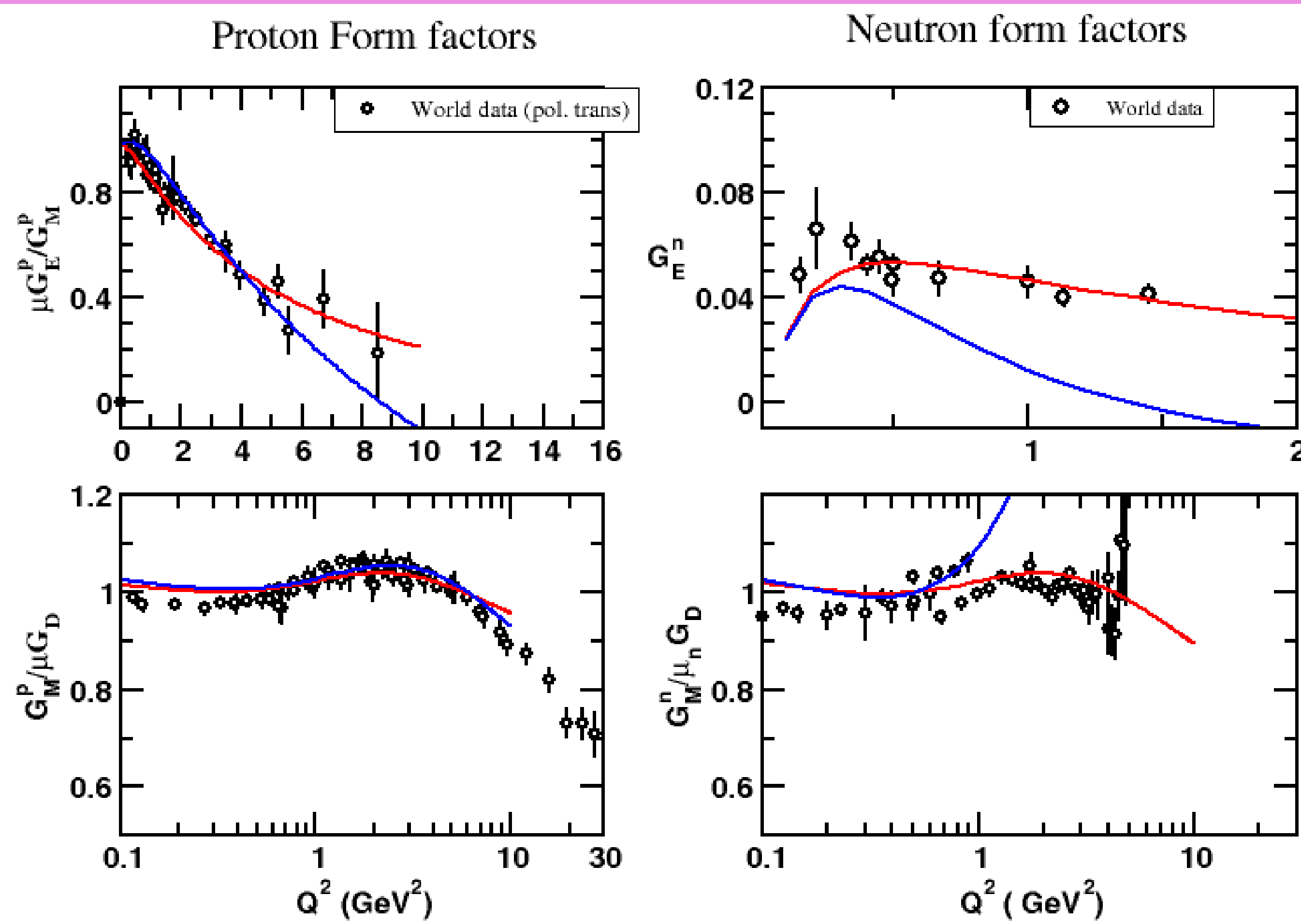
$$
\begin{aligned}
F_1^S(Q^2) &= \frac{1}{2}g(Q^2)\left[1-\beta_\omega-\beta_\varphi+\beta_\omega\frac{m_\omega^2}{m_\omega^2+Q^2}+\beta_\varphi\frac{m_\varphi^2}{m_\varphi^2+Q^2}\right], \\
F_1^V(Q^2) &= \frac{1}{2}g(Q^2)\left[1-\beta_\rho+\beta_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right], \\
F_2^S(Q^2) &= \frac{1}{2}g(Q^2)\left[(\mu_p+\mu_n-1-\alpha_\varphi)\frac{m_\omega^2}{m_\omega^2+Q^2}+\alpha_\varphi\frac{m_\varphi^2}{m_\varphi^2+Q^2}\right], \\
F_2^V(Q^2) &= \frac{1}{2}g(Q^2)\left[\frac{(\mu_p-\mu_n-1-\alpha_\rho)}{1+\gamma Q^2}+\alpha_\rho\frac{m_\rho^2}{m_\rho^2+Q^2}\right],
\end{aligned}
$$

$$g(Q^2) = (1+\gamma Q^2)^{-2} \qquad \frac{m_\rho^2}{m_\rho^2+Q^2} \to \frac{m_\rho^2+8\Gamma_\rho m_\pi/\pi}{m_\rho^2+Q^2+(4m_\pi^2+Q^2)\,\Gamma_\rho\alpha(Q^2)/m_\pi}$$

Use known masses and fit α,β,γ to the form factor data

Bijker R. and F. Iachello, Phys, Rev. C **69**, 068201 (2004).

# VMD + intrinsic structure




Now able to fit the neutron factors

Interesting to note that the proton GE/GM was fitted to data circa 2004 so three Hall C points did not exist. But new fit agrees well with these data points.

# Constituent quark models

- Nucleon is the ground state of a three quark system in a confining potential
- An example is the Isgur-Karl model which combines a linear confining potential with an interquark force mediated by one gluon exchange
- Non-relativistic CQM gives a good description of the baryon mass spectrum and static properties of baryons

# Constituent quark models

- Nucleon is the ground state of a three quark system in a confining potential
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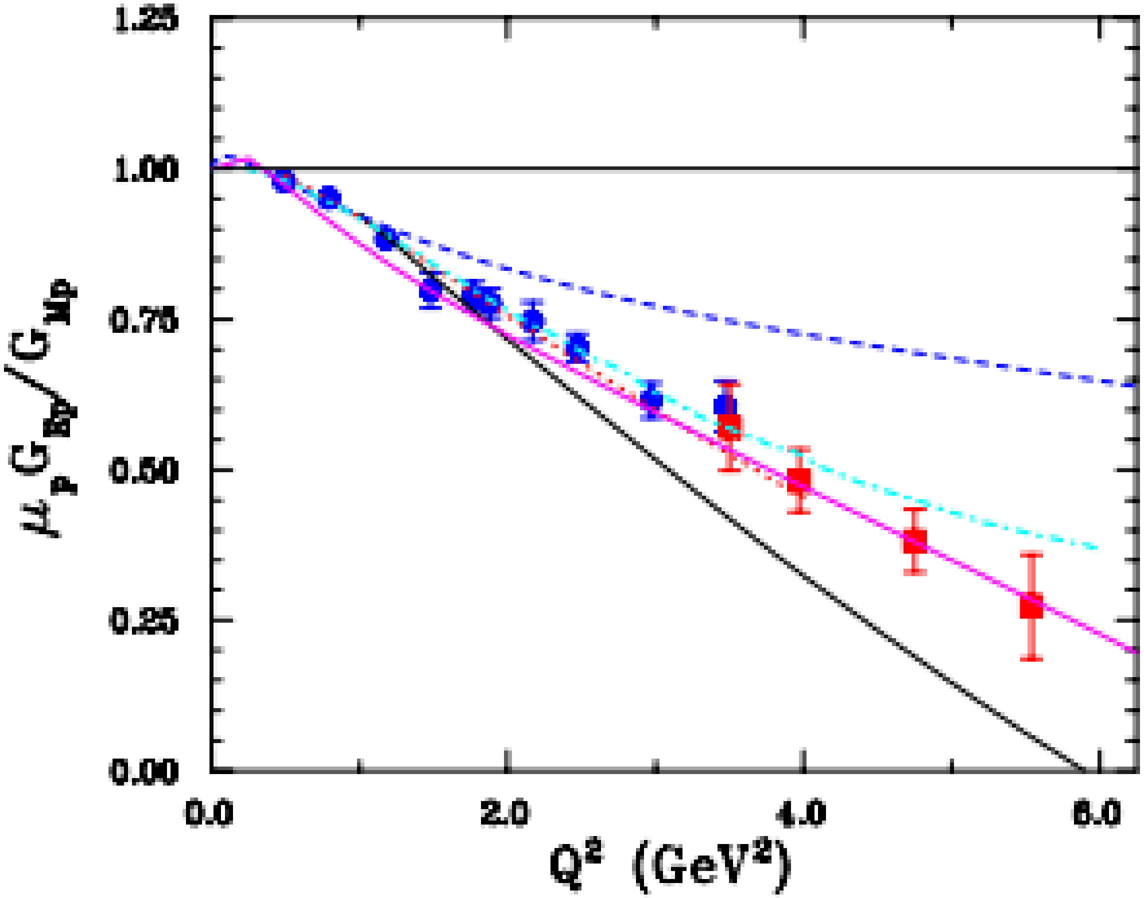
- Extending CQM to calculate form factors requires a relativistic treatment.
- Need a relation between the spin and momenta in the rest frame wave function and that in the moving frame
- No natural way to include pion cloud unless constituent quark has a form factor

# Various CQM calculations



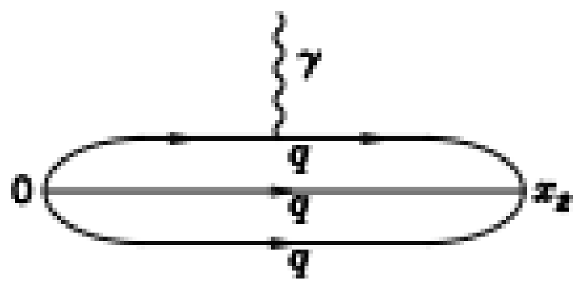

All models attribute the fall-off in proton $G_E/G_M$ is to relativistic effects on the constituent quark spin due to rotations

# Lattice gauge theory

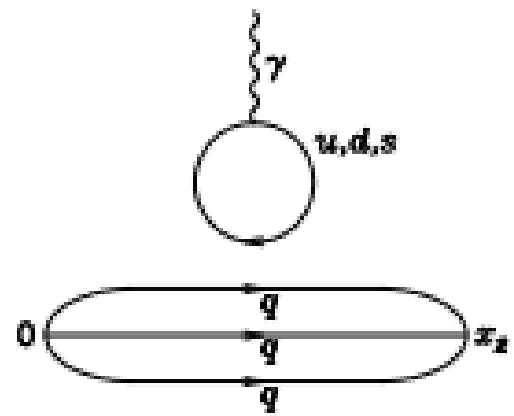
- Discretized version of QCD on a space-time lattice
- Calculations done on a lattice space $a$ . Then extrapolated to $a$ = 0
- Need a large enough box to contain the hadron size
  - Need to use quark mass larger than the real mass
  - Defined in terms of the pion mass. Typically pion mass larger than 360 MeV

- One major challenge is calculating the “disconnected” diagrams.
- With out these diagrams one can calculated only isovector form factor

$$F_1^V = F_1^p - F_1^n$$

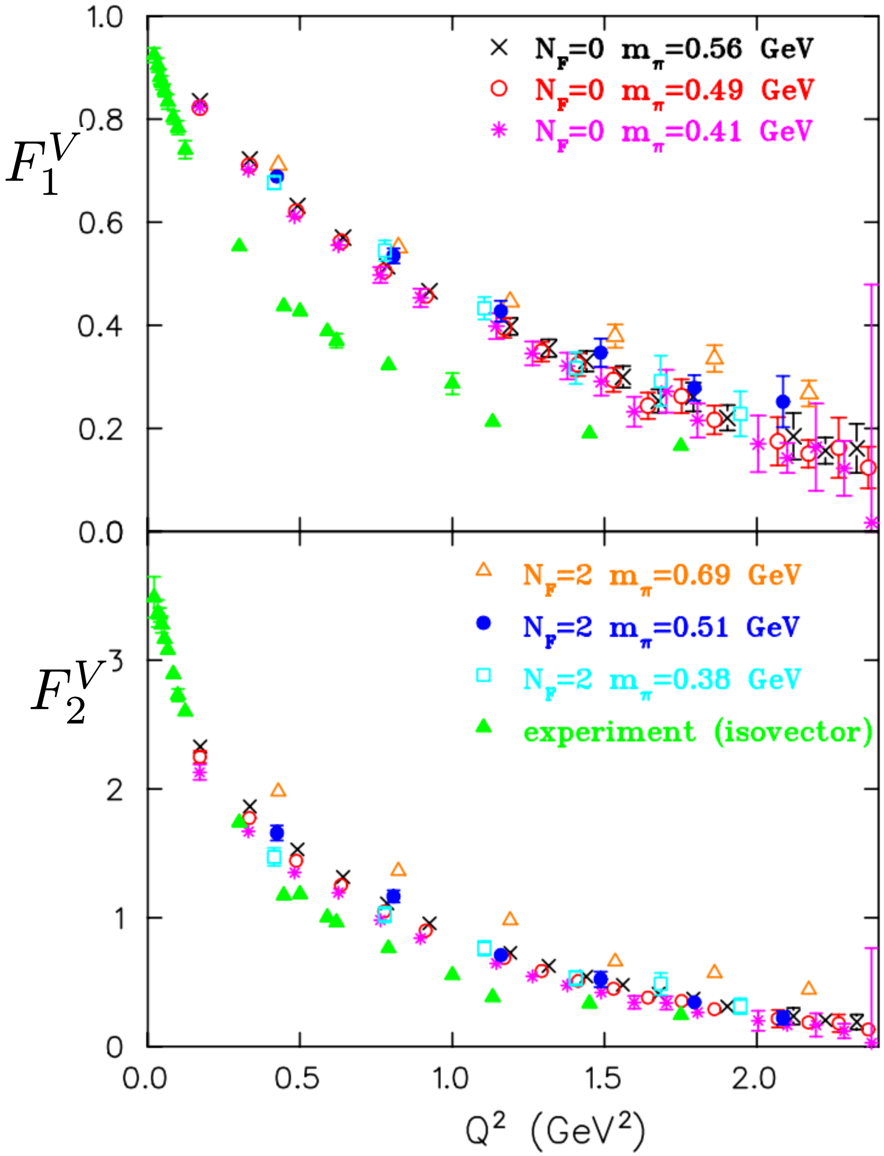



“Connected” diagrams
Photon couple to quark
which is directly connect
to nucleon




“Disconnected” diagrams
Photon couples to meson
loop which then couples to
nucleon by gluon exchange

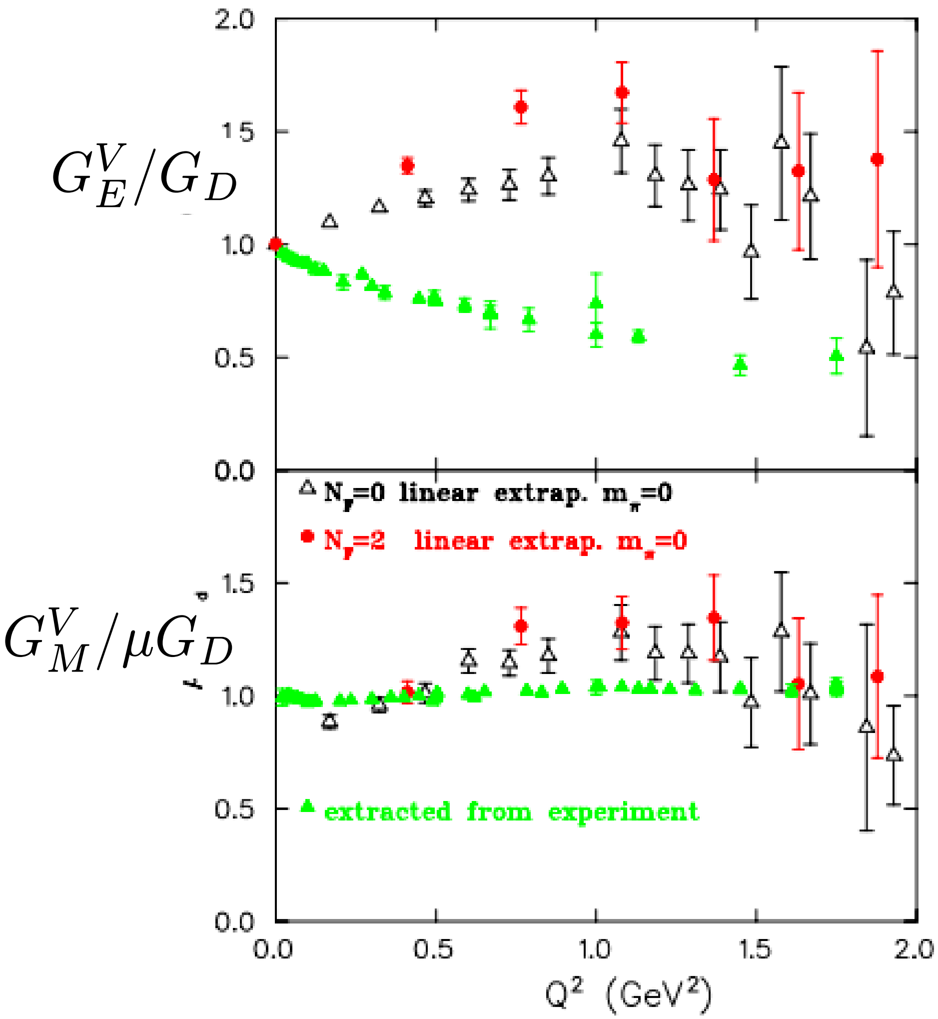
# Nicosia-MIT Lattice gauge calculation




Alexandrou C., G. Koutsou, J. W. Negele and A. Tsapalis, Phys. Rev. D **74**, 034508 (2006)

- Green points are data for isovector from factor
- $N_F$ = 0 is quenched approximation. ( No gluon fluctuations into mesons)
- $N_F$ = 2 is unquenched approximation.
- Small dependence on pion mass

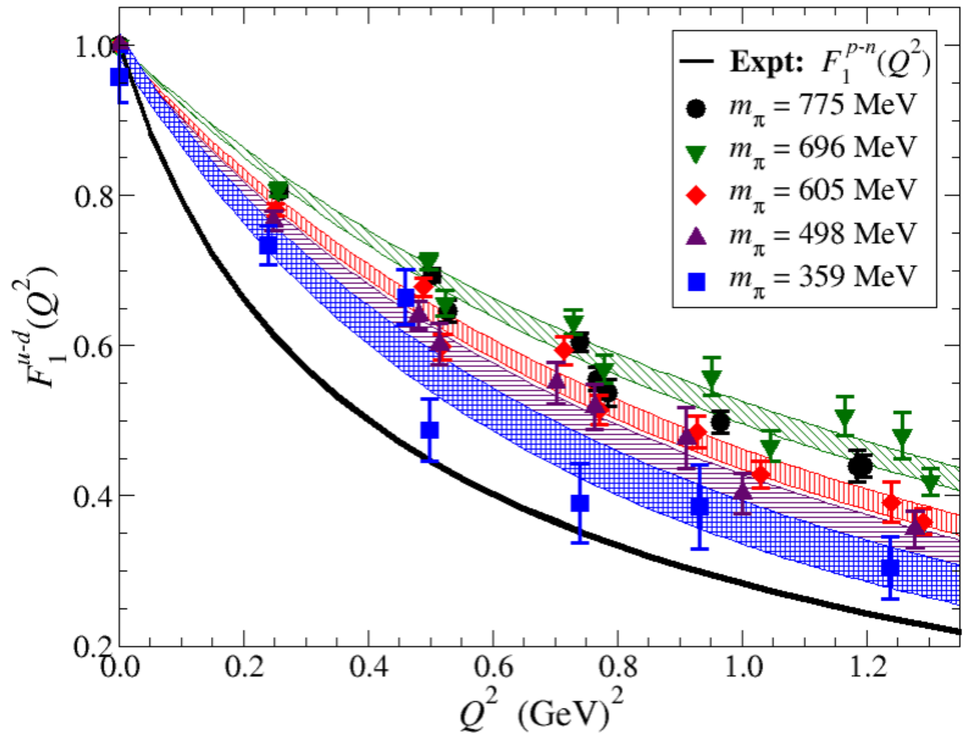
# Nicosia-MIT Lattice gauge calculation

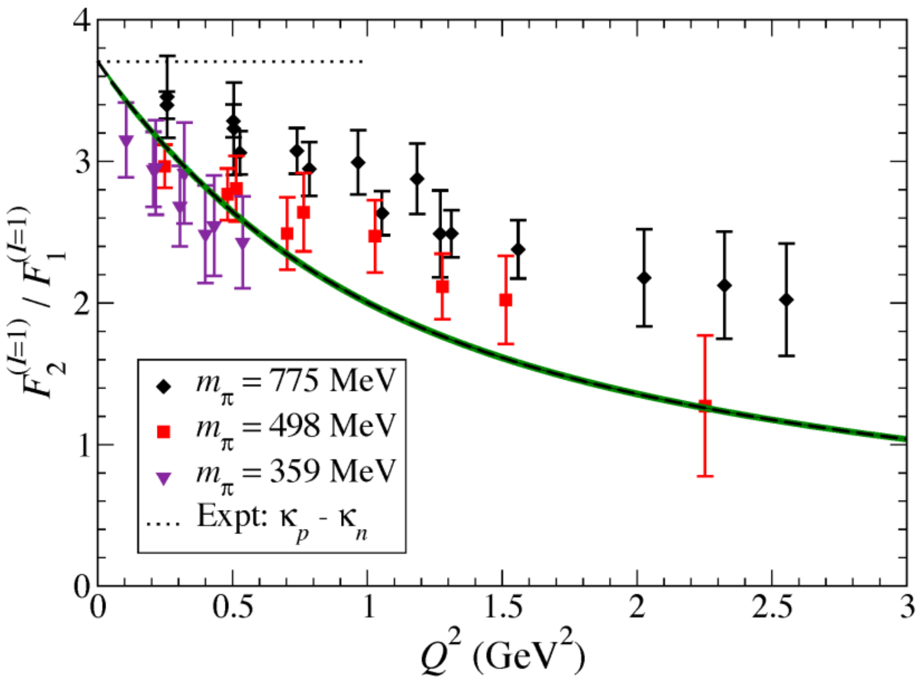



- Green points are data for isovector from factor
- Black points are LQCD is quenched approximation.
  $m_\pi$ = 410-560
- Red points are LQCD is unquenched approximation
  $m_\pi$ = 380-690
- Both LQCD calculations use a linear extrapolation of $m_\pi$ to 0


Alexandrou C., G. Koutsou, J. W. Negele and A. Tsapalis, Phys. Rev. D **74**, 034508 (2006)

# LHPC Lattice gauge calculation




- Different type of LQCD calculation
- See more dependence on the pion mass
- $F_2/F_1$ is better described than other LQCD calculations

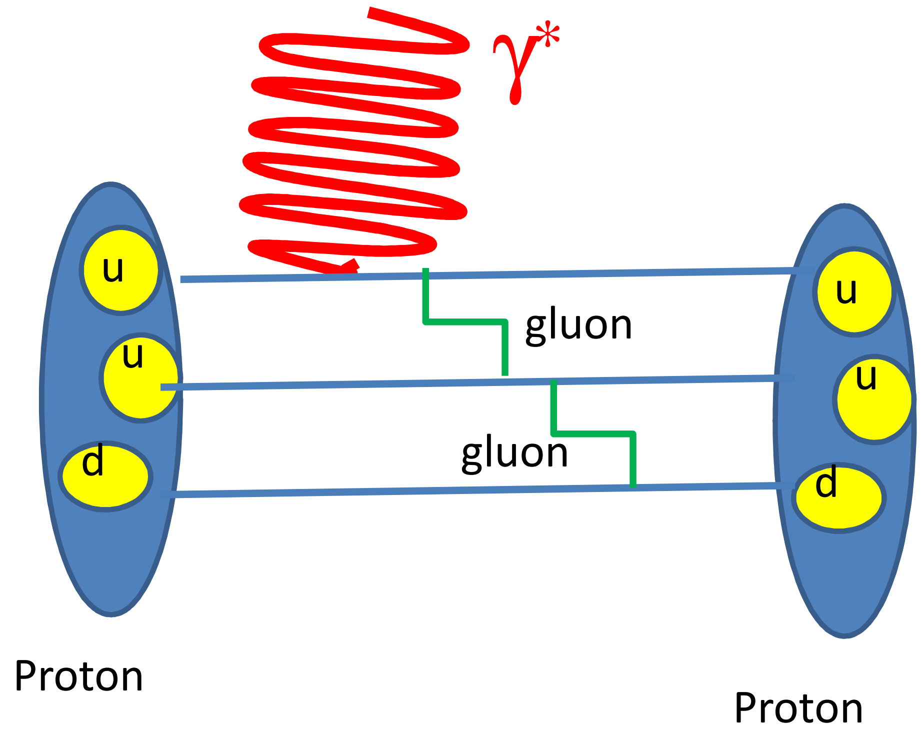
Edwards R. G. *et al.* [LHPC Collaboration], arXiv:hep-lat/0610007.

# Elastic FF in perturbative QCD

Infinite momentum frame

- Nucleon looks like three massless quarks
- Energy shared by two hard gluon exchanges
- Gluon coupling is $1/Q^2$

$$F_1(Q^2) \propto 1/Q^4$$



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Infinite momentum frame

- Nucleon looks like three massless quarks
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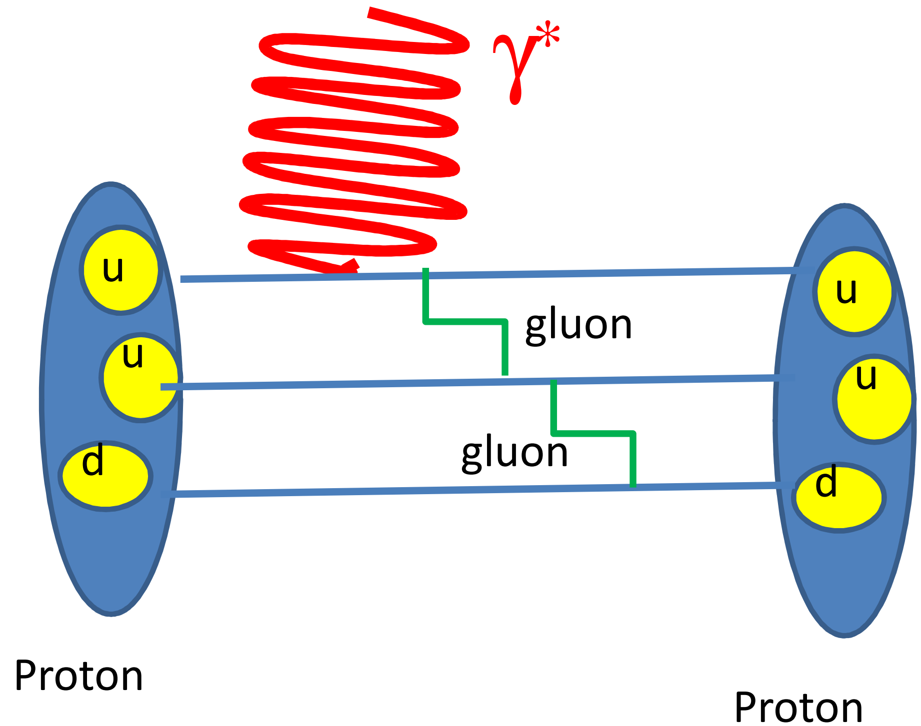
$$F_1(Q^2) \propto 1/Q^4$$

- $F_2$ requires an helicity flip the spin of the quark.

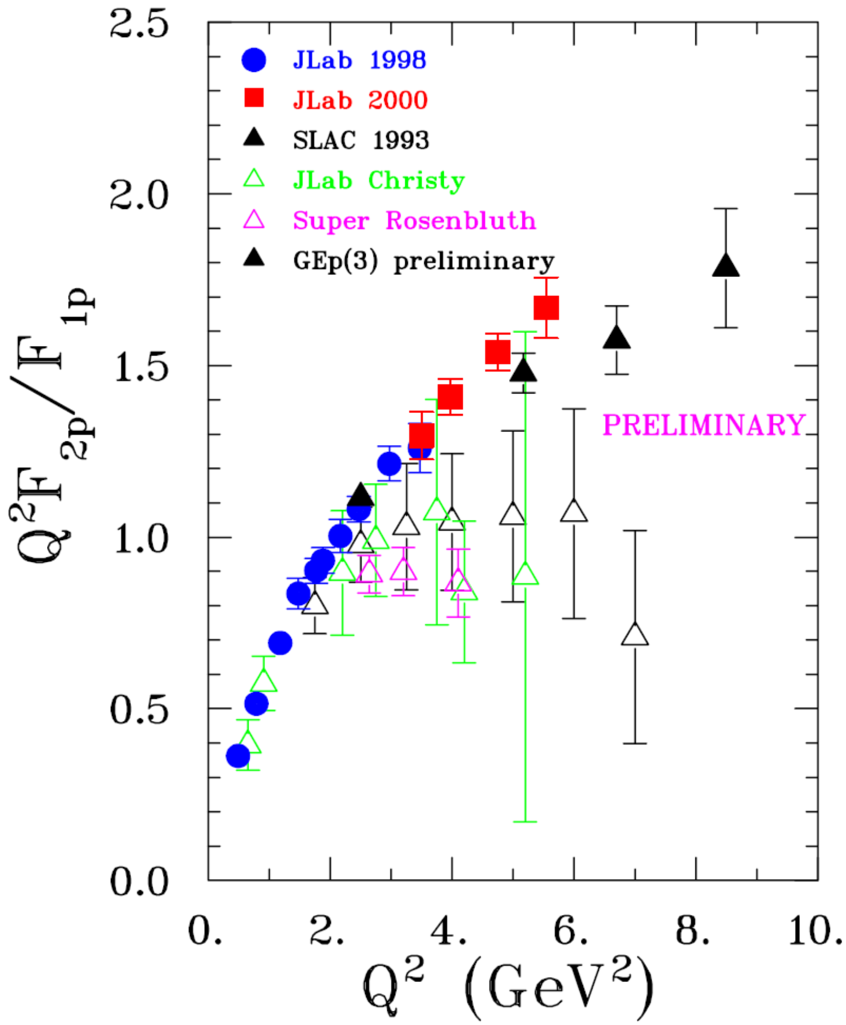
Assuming the L = 0

$$F_2(Q^2) \propto 1/Q^6$$




Brodsky S. J. and G.R. Farrar, Phys. Rev. D **11**, 1309 (1975).

# $Q^2$ dependence of $F_2/F_1$



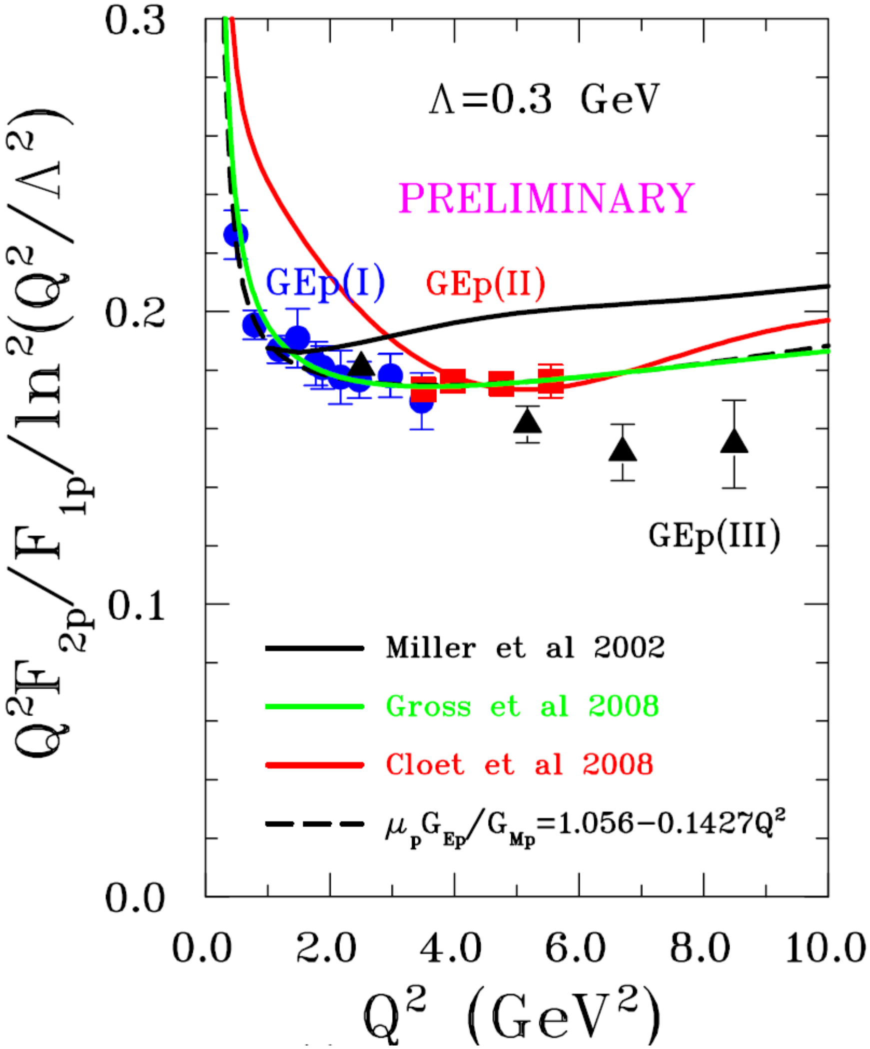

Data do not support $F_2/F_1 \sim 1/Q^2$

Calculations supported the idea that orbital angular momentum in nucleon wave function is needed to explain this dependence.

G. A. Miller and M. R. Frank. *Phys. Rev.*, C65:065205, 2002.

J. P. Ralston and P. Jain. *Phys. Rev.*, D69:053008, 2004.

# $Q^2$ dependence of $F_2/F_1$




Considering quarks in the nucleon with L=0 and L=1

Modifies pQCD counting rules

$$\frac{F^2}{F^1} \propto \frac{[\ln \frac{Q^2}{\Lambda^2}]^2}{Q^2}$$

$\Lambda$ is not predicted by pQCD

Find $\Lambda = 300$ MeV flattens data above $Q^2 = 2$

A. V. Belitsky, X-d. Ji, and F. Yuan. *Phys. Rev. Lett.*, 91:092003, 2003.