Overview of nucleon form factor measurements

Focus on theoretical calculations of form factors

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Many approaches to calculating form factors

Major difficulty is how to describe colorless nucleons being built out of gluons and quarks.

A few of the many approaches
- Vector meson dominance model
- Dispersion relations
- Constituent quark model
- Lattice Gauge Theory
- Field theoretical approaches
- Perturbative QCD
Vector meson resonances in $e^+e^-$

The total cross section $e^+e^- \rightarrow \text{hadrons}$ shows resonances for vector meson $\rho, \omega$ and $\phi$.

The photon is a vector probe so natural to assume that these vector mesons may play a role in elastic electron proton scattering.
Masses of $\rho, \omega, \phi$ are 770, 782 and 1020 MeV

Consider two vector poles with opposite contributions to form factor

$$F_{1,2} \sim \frac{a}{q^2-m_{\rho}^2} + \frac{-a}{q^2-m_{\omega}^2} = \frac{a(m_{\rho}^2-m_{\omega}^2)}{(q^2-m_{\rho}^2)(q^2-m_{\omega}^2)}$$

Easily explain the dipole form $G_D = \frac{1}{(Q^2-m^2)^2}$
In 1973, Iachello, Jackson and Lande modeled the form factors assuming VMD and an intrinsic structure.

$F_1$ has VMD and intrinsic structure contribution

$F_2$ only has VMD part

VMD + intrinsic structure

Work in terms of isoscaler and isovector combinations of form factor

\[ F^S_i = F^p_i + F^n_i \quad F^V_1 = F^p_1 - F^n_1 \]

\[
F^S_1(Q^2) = \frac{1}{2} g(Q^2) \left[ 1 - \beta_\omega - \beta_\varphi + \beta_\omega \frac{m^2_\omega}{m^2_\omega + Q^2} + \beta_\varphi \frac{m^2_\varphi}{m^2_\varphi + Q^2} \right],
\]

\[
F^V_1(Q^2) = \frac{1}{2} g(Q^2) \left[ 1 - \beta_\rho + \beta_\rho \frac{m^2_\rho}{m^2_\rho + Q^2} \right],
\]

\[
F^S_2(Q^2) = \frac{1}{2} g(Q^2) \left[ (\mu_p + \mu_n - 1 - \alpha_\varphi) \frac{m^2_\omega}{m^2_\omega + Q^2} + \alpha_\varphi \frac{m^2_\varphi}{m^2_\varphi + Q^2} \right],
\]

\[
F^V_2(Q^2) = \frac{1}{2} g(Q^2) \left[ (\mu_p - \mu_n - 1) \frac{m^2_\rho}{m^2_\rho + Q^2} \right],
\]

\[
g(Q^2) = (1 + \gamma Q^2)^{-2}
\]

Use known masses and fit \(\alpha,\beta,\gamma\) to the form factor data (at that time 1973!)
VMD + intrinsic structure

Follows the proton $G_E/G_M$ fall-off.
But neutron form factors are not well described
Of course most of data shown did not exist in 1973!
Revisiting VMD + intrinsic structure

In 2004, Iachello and Bijker decide to redo the fit to new world data but modify form of $F_2^V$ to include the intrinsic structure with additional $1/Q^2$ term

\[
F_1^S(Q^2) = \frac{1}{2}g(Q^2) \left[ 1 - \beta_\omega - \beta_\varphi + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2} \right],
\]

\[
F_1^V(Q^2) = \frac{1}{2}g(Q^2) \left[ 1 - \beta_\rho + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right],
\]

\[
F_2^S(Q^2) = \frac{1}{2}g(Q^2) \left[ (\mu_p + \mu_n - 1 - \alpha_\varphi) \frac{m_\omega^2}{m_\omega^2 + Q^2} + \alpha_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2} \right],
\]

\[
F_2^V(Q^2) = \frac{1}{2}g(Q^2) \left[ \frac{(\mu_p - \mu_n - 1 - \alpha_\rho)}{1 + \gamma Q^2} + \alpha_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2} \right],
\]

\[
g(Q^2) = (1 + \gamma Q^2)^{-2} \quad \frac{m_\rho^2}{m_\rho^2 + Q^2} \to \frac{m_\rho^2 + 8\Gamma_\rho m_\pi / \pi}{m_\rho^2 + Q^2 + (4m_\pi^2 + Q^2) \Gamma_\rho \alpha(Q^2)/m_\pi}
\]

Use known masses and fit $\alpha, \beta, \gamma$ to the form factor data

VMD + intrinsic structure

Now able to fit the neutron factors

Interesting to note that the proton GE/GM was fitted to data circa 2004 so three Hall C points did not exist. But new fit agrees well with these data points.
Constituent quark models

- Nucleon is the ground state of a three quark system in a confining potential
- An example is the Isgur-Karl model which combines a linear confining potential with an interquark force mediated by one gluon exchange
- Non-relativistic CQM gives a good description of the baryon mass spectrum and static properties of baryons
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- Extending CQM to calculate form factors requires a relativistic treatment.
- Need a relation between the spin and momenta in the rest frame wave function and that in the moving frame
- No natural way to include pion cloud unless constituent quark has a form factor
Various CQM calculations

All models attribute the fall-off in proton $G_E/G_M$ is to relativistic effects on the constituent quark spin due to rotations.
Lattice gauge theory

- Discretized version of QCD on a space-time lattice
- Calculations done on a lattice space $a$. Then extrapolated to $a = 0$
- Need a large enough box to contain the hadron size
  - Need to use quark mass larger than the real mass
  - Defined in terms of the pion mass. Typically pion mass larger than 360 MeV

- One major challenge is calculating the “disconnected” diagrams.
- Without these diagrams one can calculate only isovector form factor
  \[ F_1^V = F_1^p - F_1^n \]

“Connected” diagrams
Photon couple to quark which is directly connected to nucleon

“Disconnected” diagrams
Photon couples to meson loop which then couples to nucleon by gluon exchange
Nicosia-MIT Lattice gauge calculation

Green points are data for isovector from factor

$N_F = 0$ is quenched approximation. (No gluon fluctuations into mesons)

$N_F = 2$ is unquenched approximation.

Small dependence on pion mass

Nicosia-MIT Lattice gauge calculation

- Green points are data for isovector from factor
- Black points are LQCD is quenched approximation.
  \[ m_\pi = 410-560 \]
- Red points are LQCD is unquenched approximation
  \[ m_\pi = 380-690 \]
- Both LQCD calculations use a linear extrapolation of \( m_\pi \) to 0

LHPC Lattice gauge calculation

- Different type of LQCD calculation
- See more dependence on the pion mass
- $F_2/F_1$ is better described than other LQCD calculations

Elastic FF in perturbative QCD

Infinite momentum frame

- Nucleon looks like three massless quarks
- Energy shared by two hard gluon exchanges
- Gluon coupling is $1/Q^2$

$$F_1(Q^2) \propto 1/Q^4$$
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$F_2(Q^2)$ requires an helicity flip the spin of the quark. Assuming the $L = 0$

$$F_2(Q^2) \propto 1/Q^6$$

Data do not support $F_2/F_1 \sim 1/Q^2$.

Calculations supported the idea that orbital angular momentum in nucleon wave function is needed to explain this dependence.


Q^2 dependence of F_2/F_1

Considering quarks in the nucleon with L=0 and L=1

Modifies pQCD counting rules

\[
\frac{F_2}{F_1} \propto \left[ \ln \frac{Q^2}{\Lambda^2} \right]^2
\]

\(\Lambda\) is not predicted by pQCD

Find \(\Lambda = 300\) MeV flattens data above \(Q^2 = 2\)