Particles and Deep Inelastic Scattering

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HUGS - JLab - June 2010

Sum rules

You can integrate the structure functions and recover quantities like the net number of quarks.

Momentum Sum Rule

$$x_{tot} = \int F_2(x)dx \simeq \int x(q(x) + \overline{q}(x))dx$$

which yields the total fraction of the proton momentum carried by quarks. (After adjustment for the charges) We find that x_{tot} for quarks is about 50%. Half of whatever is in the proton is not quarks - it turns out to be gluons but you can't detect a gluon directly with an electron beam.

We can also measure the momentum fraction of anti quarks from $\int xF_3(x)dx$ and x_{tot} and get 10-15%.

Sum rules that don't say much

The numerical sum rule

$$N_{quark} = \int \frac{dx}{x} F_2(x) \simeq \sum_i \int (q_i(x) + \overline{q}_i(x)) dx$$

can reach thousands as Q^2 decreases.

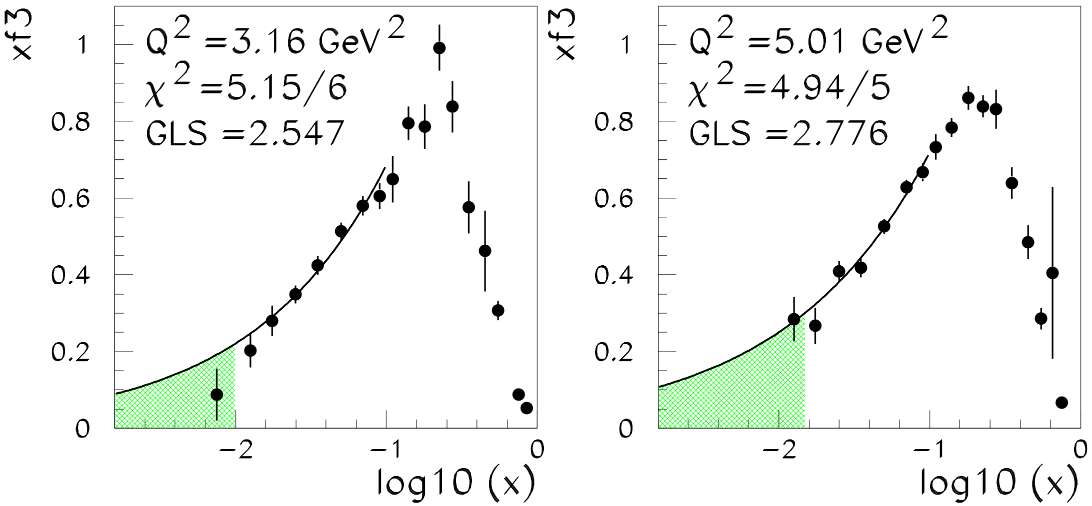
as the parton distributions actually diverge.

Adler Sum rule $\simeq N_u - N_d$

$$\int_0^1 \frac{dx}{x}(F_2^{\overline{\nu}p} - F_2^{\nu p}) \simeq 2\int_0^1 \frac{dx}{x}[u(x) + \overline{d}(x) + \overline{s}(x) - d(x) - \overline{u}(x) - \overline{c}(x)] \simeq 2$$

This has small corrections if one needs to worry about charm production thresholds. It is hard to measure as neutrino-proton scattering data is very scarce.

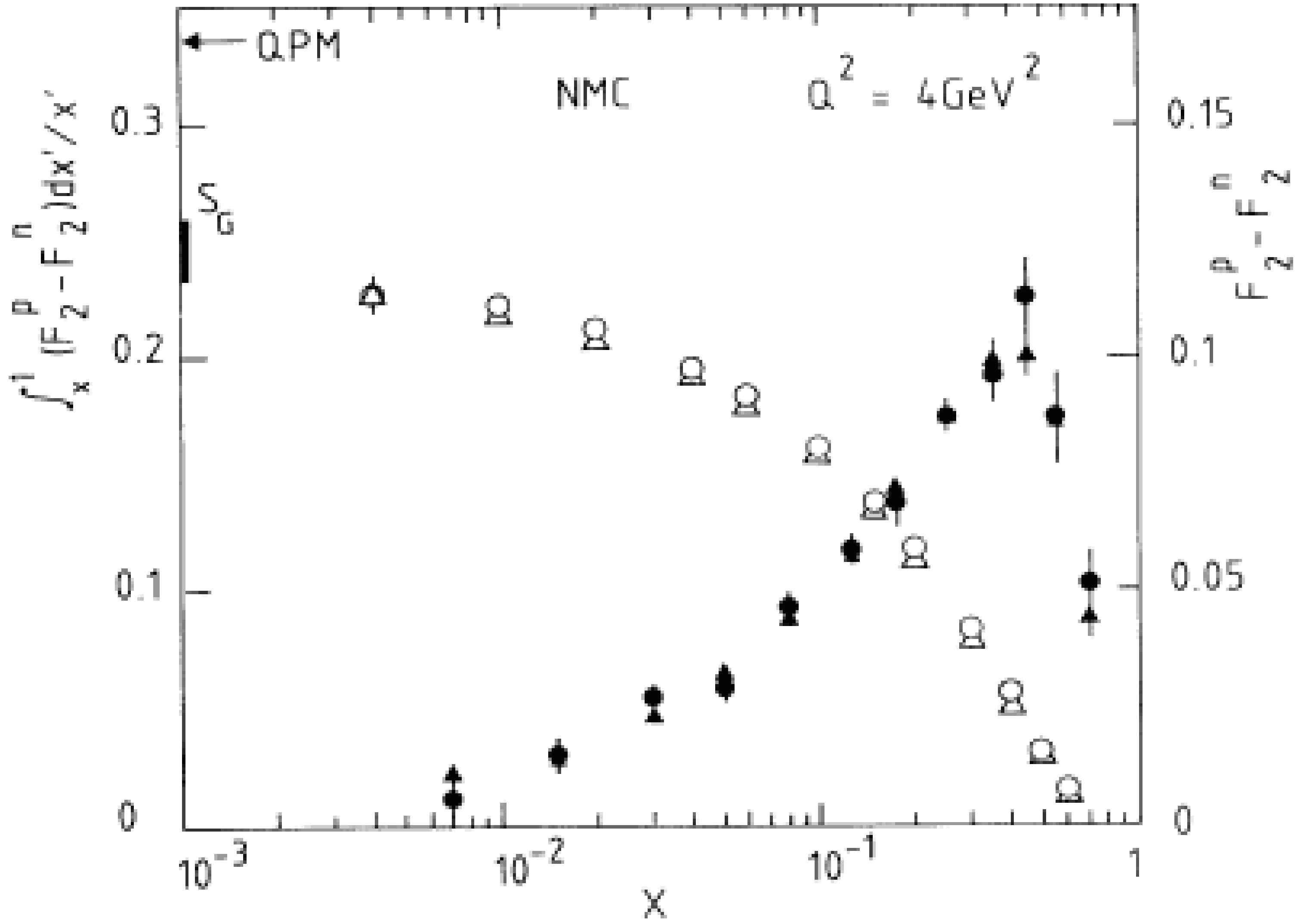
Gross-Llewellyn-Smith sum rule $\simeq N_q - N_{\overline{q}}$



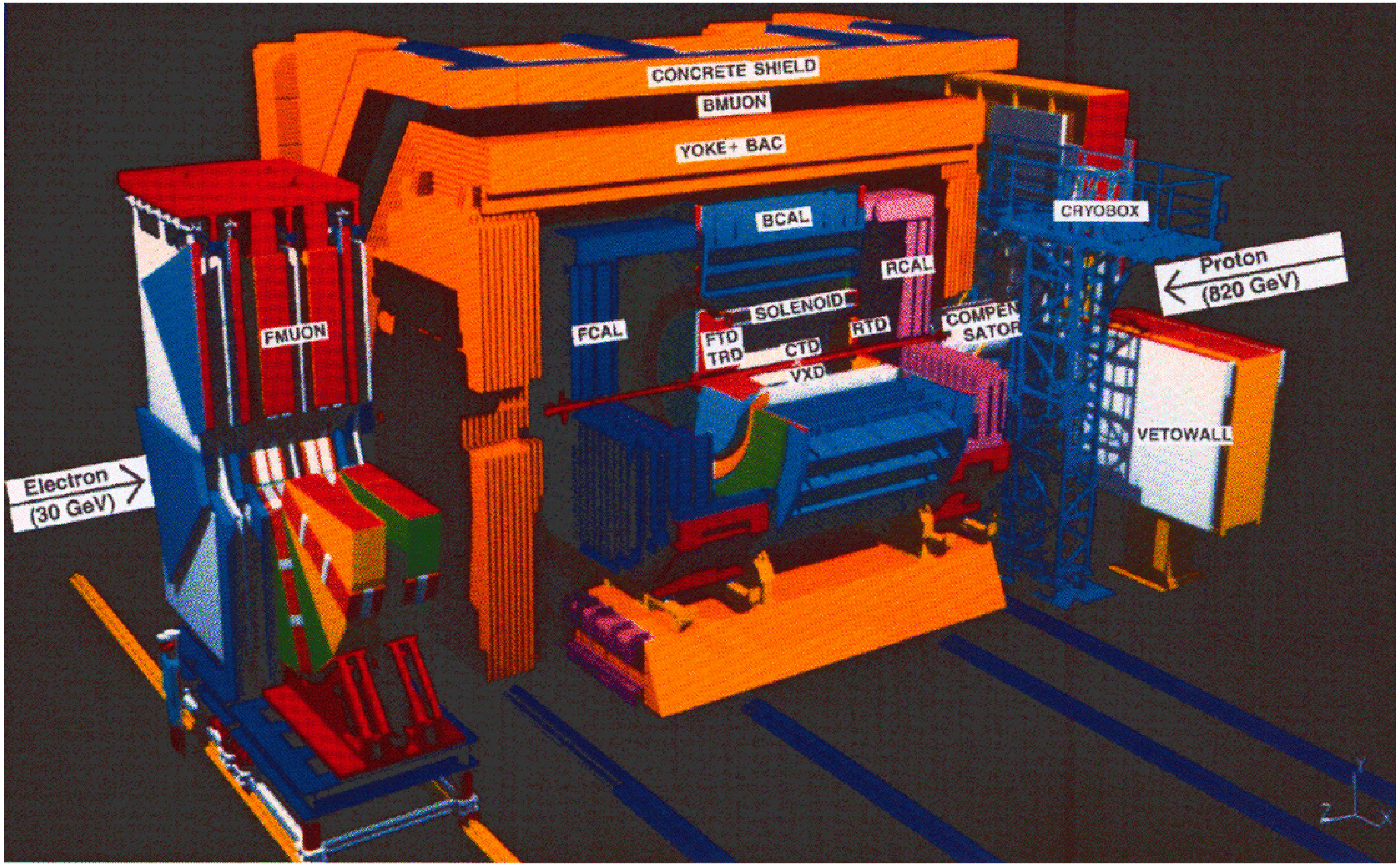
$$\frac{1}{2}\int_0^1 (F_3^{\overline{\nu}p} + F_3^{\nu p}) \simeq \int_0^1 dx[u(x)+d(x)+s(x)-\overline{u}(x)-\overline{d}(x)-\overline{s}(x)] \simeq u_v(x)+d_v(x) \simeq 3$$

Gottfried sum rule $\simeq N_u - N_d$

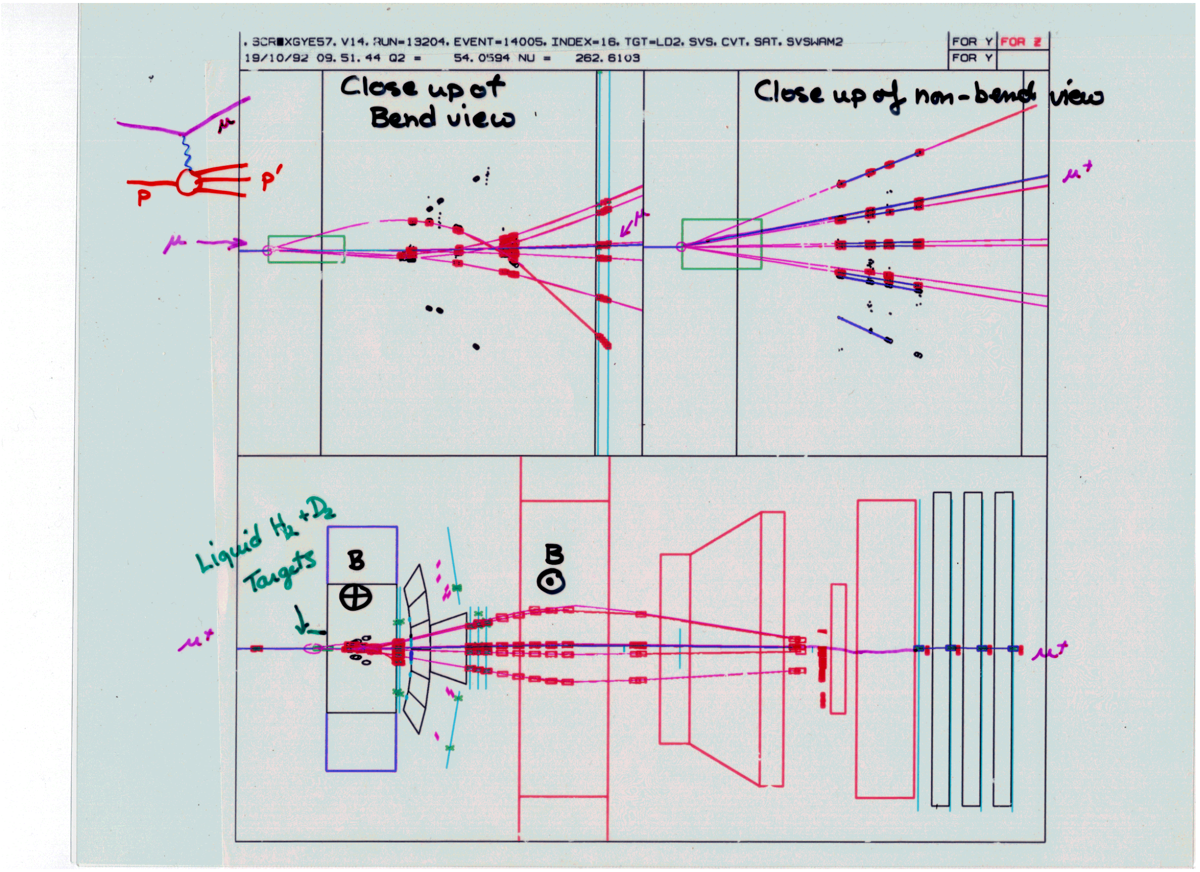
$$\begin{aligned}
\int_0^1 \frac{dx}{x}(F_2^{ep} - F_2^{en} &\simeq \int_0^1 \frac{dx}{x}[\frac{4}{9}(u(x) + \overline{u}(x) + \frac{1}{9}(d(x) + \overline{d}(x)) \\
&\quad -\frac{1}{9}(u(x) + \overline{u}(x)) - \frac{4}{9}(d(x) + \overline{d}(x))] \\
&\simeq \frac{1}{3}\int_0^1 \frac{dx}{x}[[u(x) - d(x) + \overline{u}(x) - \overline{d}(x)] \\
&\simeq \frac{1}{3} + \frac{1}{3}[\overline{u}(x) - \overline{d}(x)]
\end{aligned}$$

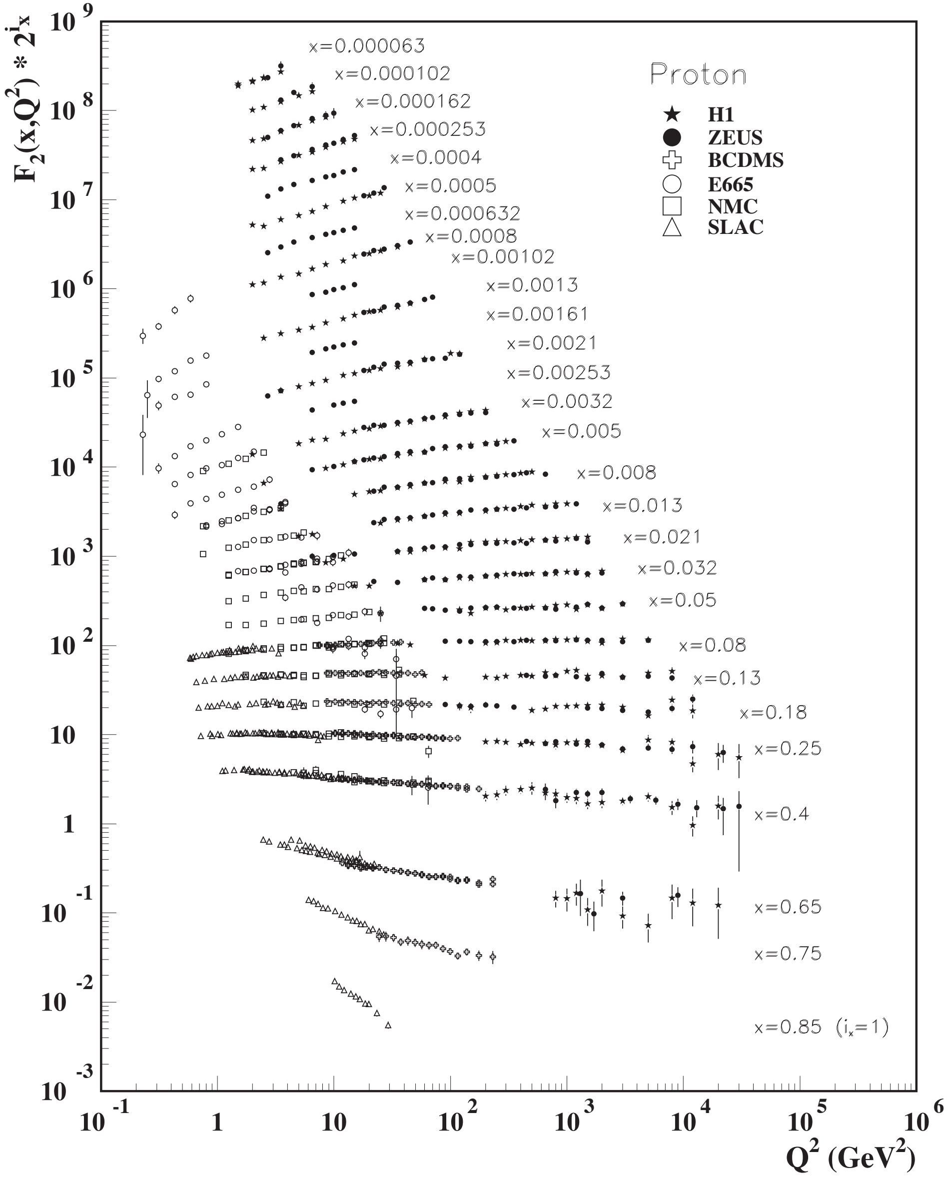

QPM
NMC
$Q^2 = 4GeV^2$
S_G
$\int_x^1 (F_2^p - F_2^n)dx'/x'$
$F_2^p - F_2^n$
0.3
0.2
0.1
0
0.15
0.1
0.05
0
10^{-3}
10^{-2}
10^{-1}
1
X

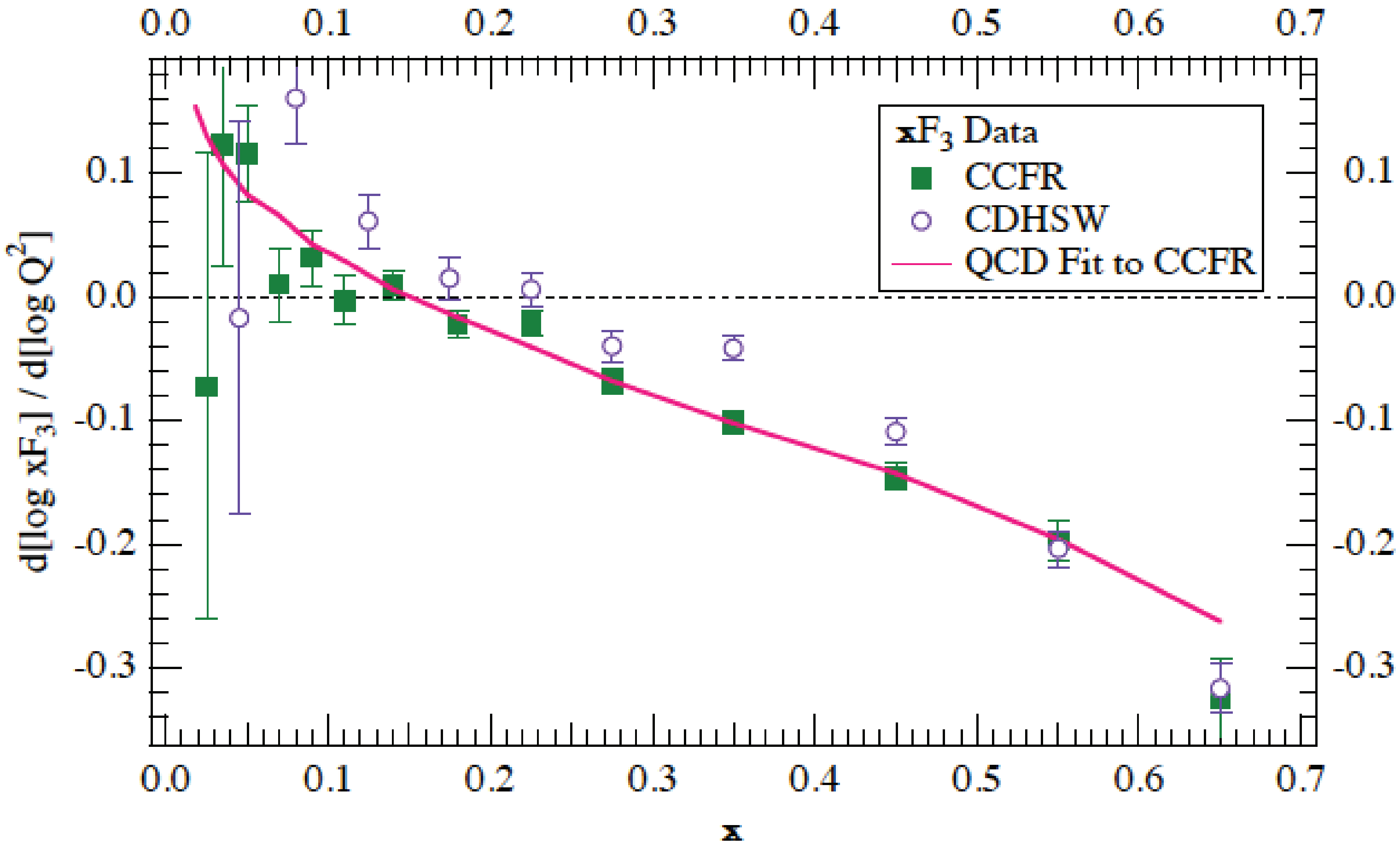
Zeus detector 30 GeV e + 820 GeV p



E665 detector 470 GeV μ + p, d, C, Ca, Pb at rest



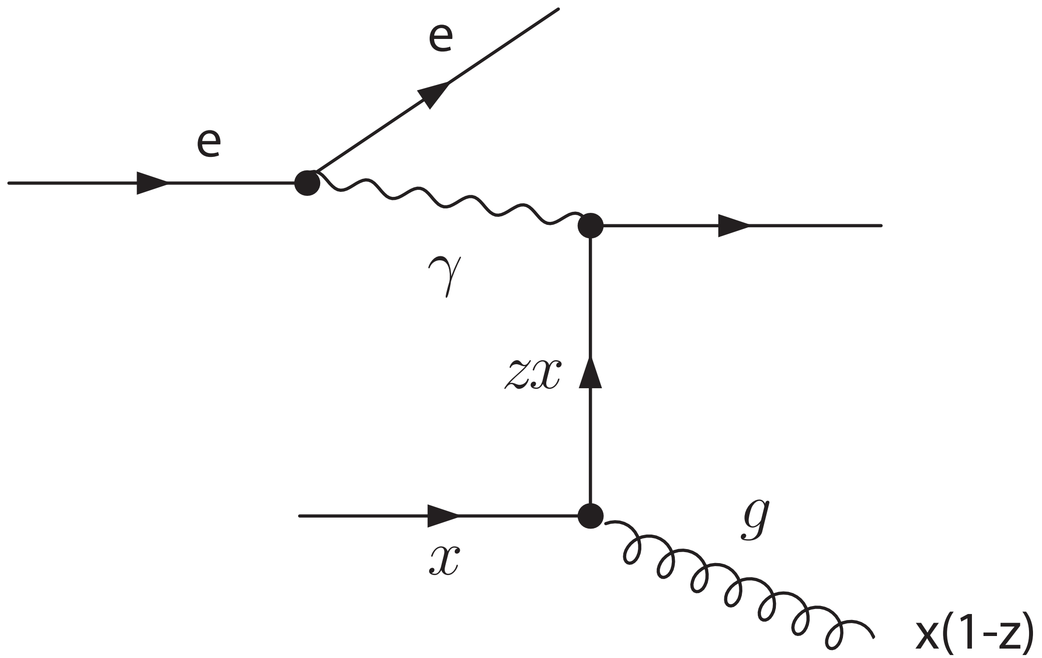

Proton
H1
ZEUS
BCDMS
E665
NMC
SLAC
$F_2(x,Q^2) * 2^{i_x}$
Q^2 (GeV^2)
x=0.000063
x=0.000102
x=0.000162
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (i_x=1)

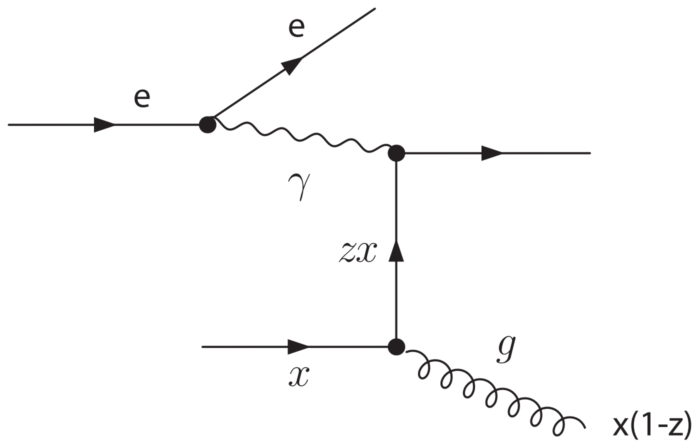


0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
-0.1
-0.2
-0.3
d[log xF3] / d[log Q2]
x
xF3 Data
CCFR
CDHSW
QCD Fit to CCFR

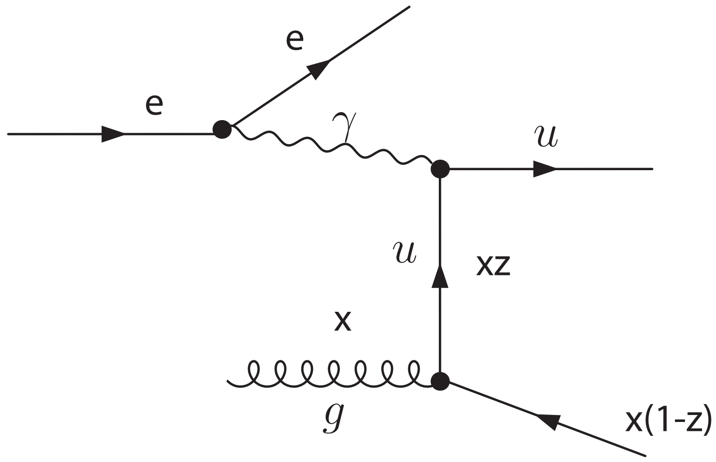
As you can see from these data, the structure functions F_2 and xF_3 do seem to have some Q^2 dependence. This can be explained by higher order QCD corrections to the basic scattering diagrams.

The Leading Order QCD corrections involve the emission of gluons from quarks and pair production of quarks from gluons.

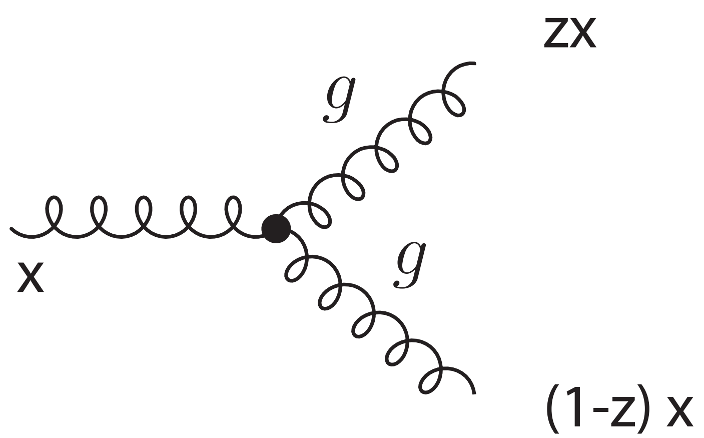




A quark which originally has momentum fraction x has a probability, $P_{qqG} \simeq \alpha_s(Q^2)P_{qq}(z)$, emit a gluon with momentum fraction $x(1-z)$, leaving a quark carrying a lower momentum fraction xz. Our lepton then scatters off of that quark. Since we find the "x" value from the photon quark vertex, we measure xz instead of x as the momentum fraction.



A gluon which originally has momentum fraction x has a probability, $P_{Gqq} \simeq \alpha_s(Q^2)P_{Gq}(z)$, emit an antiquark with momentum fraction $x(1-z)$, leaving a quark carrying a lower momentum fraction xz. Our lepton then scatters off of that quark. Since we find the "x" value from the photon quark vertex, we measure xz instead of x as the momentum fraction (and see a gluon as a quark.)



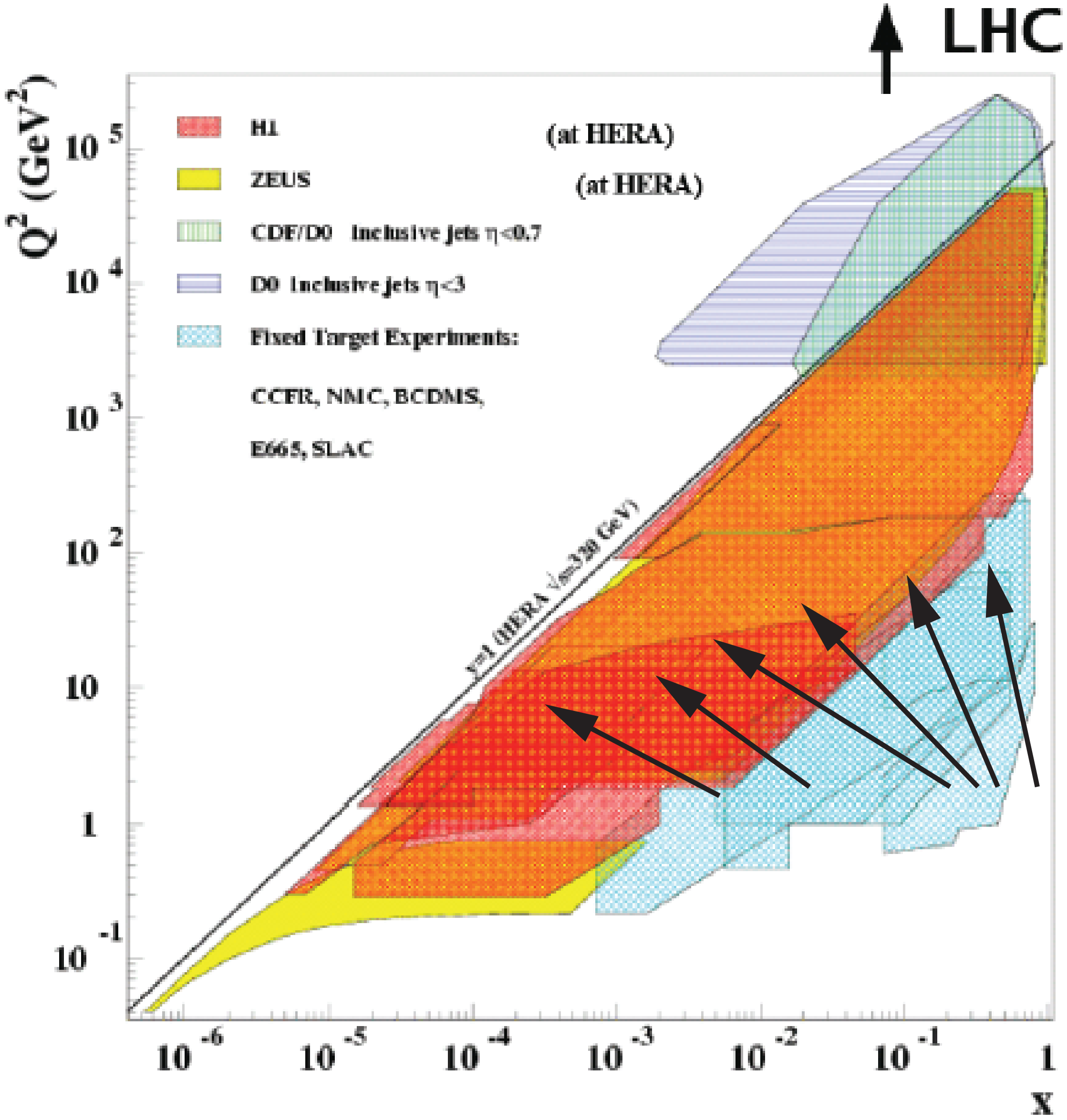
A gluon can also split into 2 gluons with momentum fraction xz and $x(1-x)$ with probability $\simeq \alpha_s(Q^2)P+GG(z)$

You can calculate these splitting probabilities using QCD:

$$
\begin{aligned}
P_{qq}(z) &= \frac{4}{3}\frac{1+z^2}{1-z} \\
P_{qG}(z) &= \frac{1}{2}[z^2+(1-z)^2] \\
P_{Gq}(z) &= \frac{4}{3}\frac{1+(1-z)^2}{z} \\
P_{GG}(z) &= 6[\frac{z}{(1-z)}+\frac{1-z}{z}+z(1-z)]
\end{aligned}
$$

for $z < 1$.

The PDF's we measure are thus produced by PDF's at higher x radiating down to the value we measure.


LHC
H1
ZEUS
CDF/D0 Inclusive jets η<0.7
D0 Inclusive jets η<3
Fixed Target Experiments:
CCFR, NMC, BCDMS,
E665, SLAC
(at HERA)
(at HERA)
y=1 (HERA √s=320 GeV)
Q² (GeV²)
x
10⁻⁶ 10⁻⁵ 10⁻⁴ 10⁻³ 10⁻² 10⁻¹ 1
10⁻¹ 1 10 10² 10³ 10⁴ 10⁵

They lead to the following evolution equations [a] for the parton distribution functions:

$$\frac{dq_i(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^y \frac{dy}{y}\sum_j q_j(y,Q^2)P_{q^i,q^j}(\frac{x}{y}) + G(y,Q^2)P_{q^i,G}(\frac{x}{y})$$

$$\frac{dG(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^y \frac{dy}{y}\sum_j q_j(y,Q^2)P_{G,q^j}(\frac{x}{y}) + G(y,Q^2)P_{G,G}(\frac{x}{y})$$

[a] DGLAP for Yu. L. Dokshitzer, V.N. Gribov, L.N. Lipatov, G. Altarelli and G. Parisi

Show evolution slides

Longitudinal Structure Function F_L

The general structure function formulation in terms of $2xF_1$, F_2 and xF_3 allows for a longitudinal structure function

$$F_L = F_2 - 2xF_1$$

. F_L can be extracted by measuring the y dependence of the cross section.

$$F_L(x,Q^2) = \int_x^1 \frac{dy}{y} f_q^{(1)}(y) F_2\left[\frac{x}{y}, Q^2\right]$$

$$+ \int_x^1 \frac{dy}{y} f_g^{(1)}(y) \frac{x}{y} G\left(\frac{x}{y}, Q^2\right),$$

where

$$F_2(x,Q^2) = \sum_{i=1}^{n_f} e_i^2 x [q_i(x,Q^2) + \bar{q}_i(x,Q^2)]$$

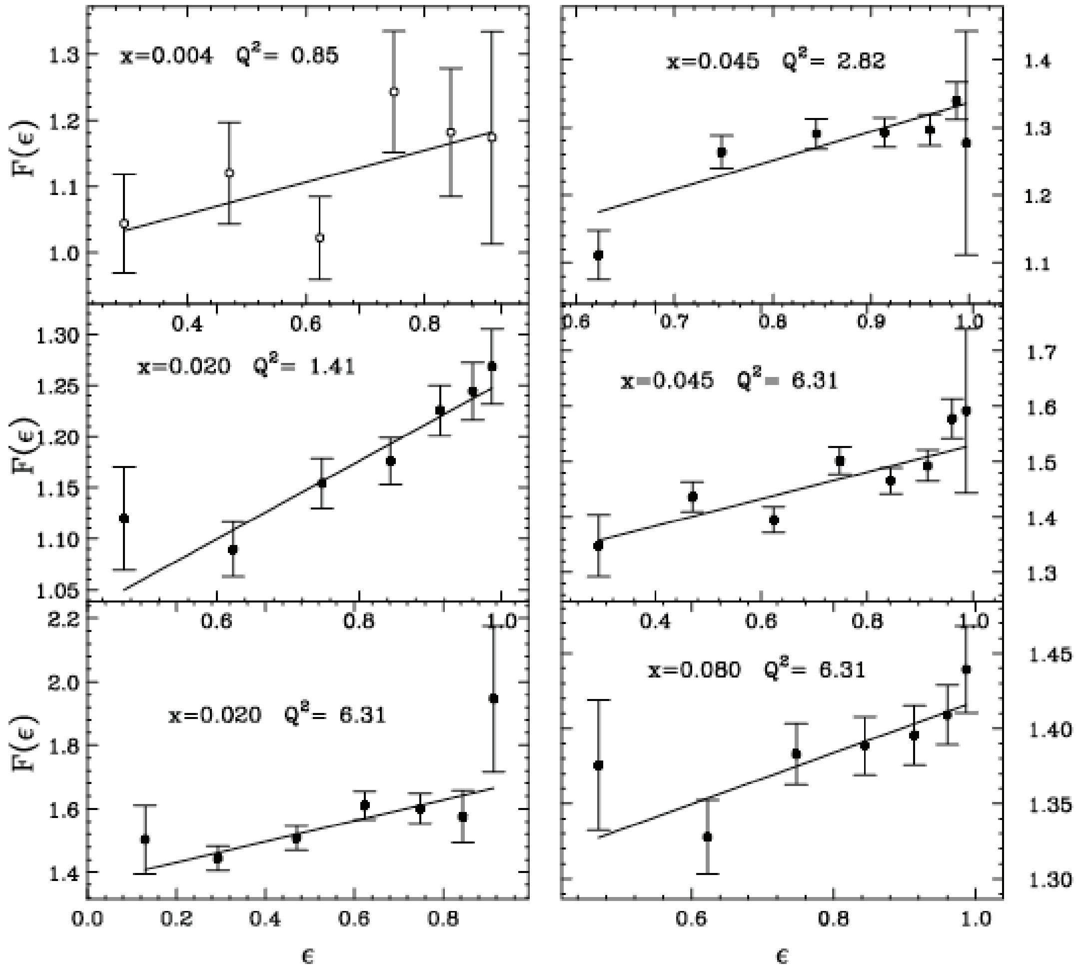
and

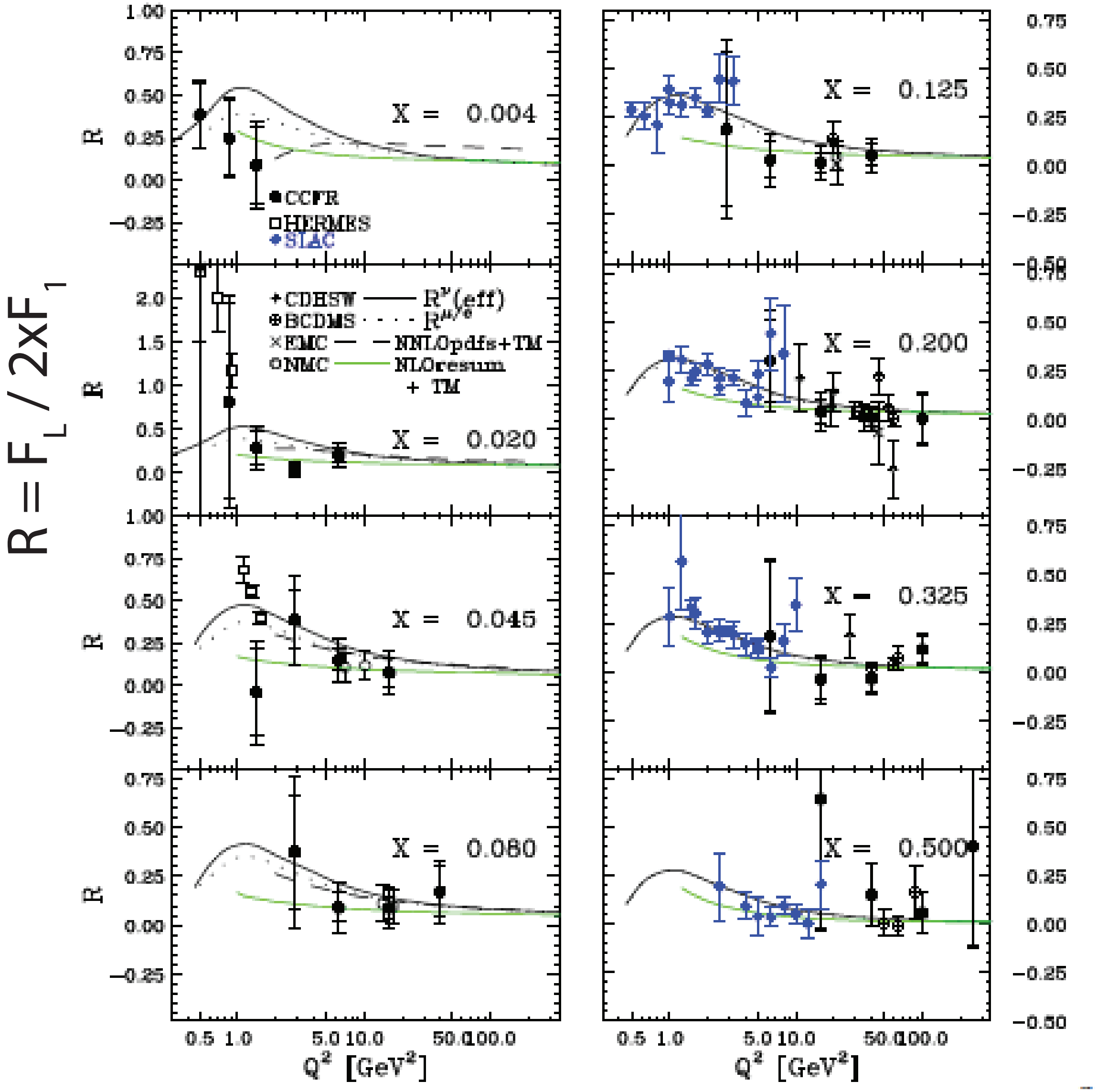
$$f_q^{(1)}(x) = \frac{\alpha_s}{4\pi} 4 C_F x^2,$$

$$f_g^{(1)}(x) = \frac{\alpha_s}{4\pi} \left(\sum_{i=1}^{n_f} e_i^2\right) 8 x^2 (1-x).$$

$\varepsilon = 2(1-y)/(1 + (1-y)^2)$

F_L ~ the slope





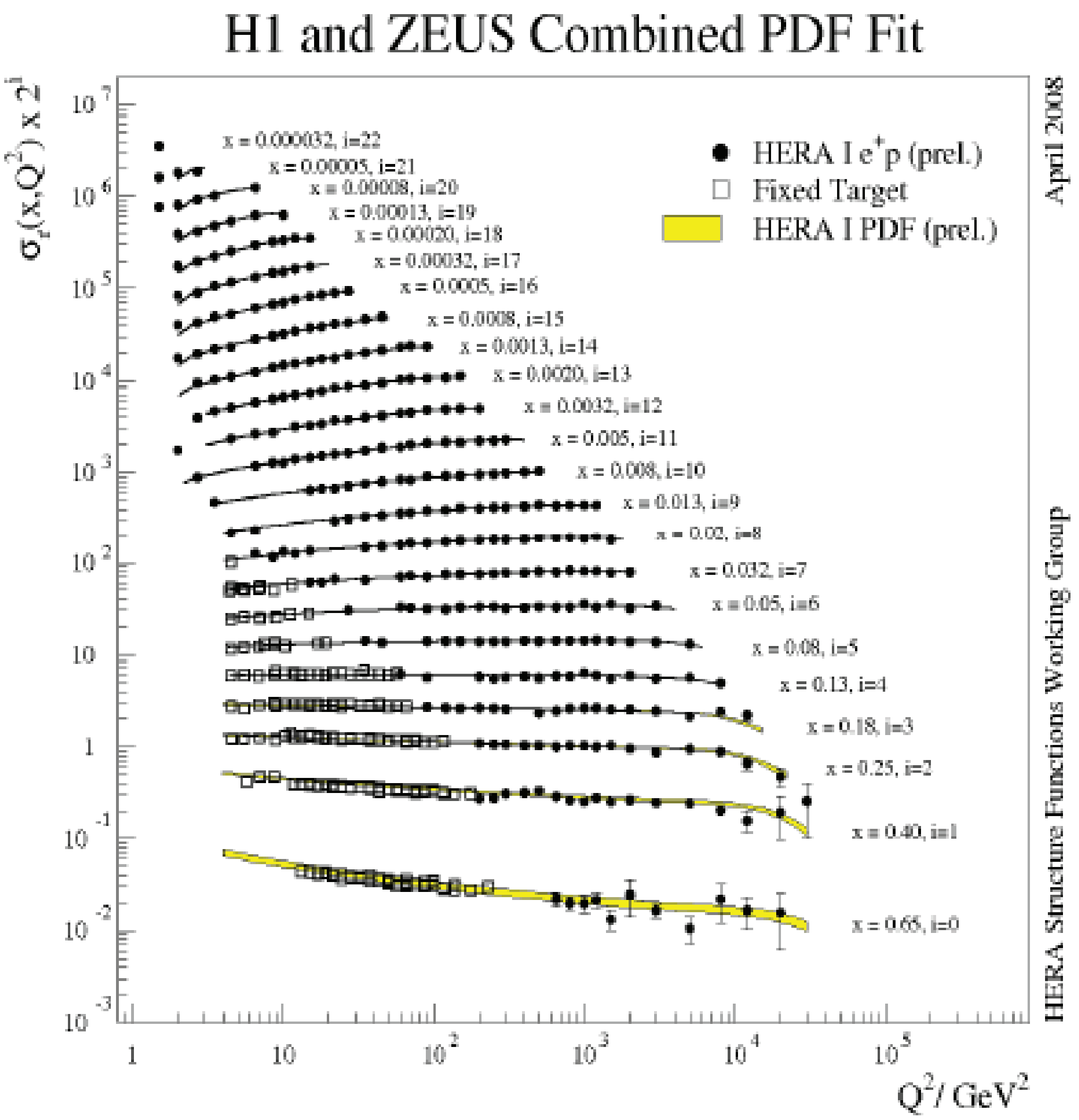
R = F_L / 2xF_1
X = 0.004
X = 0.125
X = 0.020
X = 0.200
X = 0.045
X = 0.325
X = 0.080
X = 0.500
CCFR
HERMES
SLAC
CDHSW
BCDMS
EMC
NMC
R^ν(eff)
R^{μ/e}
NNLOpdfs+TM
NLOresum + TM
R
Q² [GeV²]

The state of the art is now NNLO or NNNLO for the parton distribution functions so the equations become more complicated.

One can no longer interpret the structure functions as sums over quarks once one gets above LO, so the mathematics gets complicated very quickly. But you can still define parton distribution functions and use them.

QCD measurements with deep inelastic scattering

The Q^2 behavior depends on the strong coupling constant and you can measure $\alpha_S(Q^2)$ by measuring the Q^2 dependence. This requires good calibration and understanding of your detector!.


H1 and ZEUS Combined PDF Fit
HERA I e+p (prel.)
Fixed Target
HERA I PDF (prel.)
April 2008
HERA Structure Functions Working Group
x = 0.000032, i=22
x = 0.00005, i=21
x = 0.00008, i=20
x = 0.00013, i=19
x = 0.00020, i=18
x = 0.00032, i=17
x = 0.0005, i=16
x = 0.0008, i=15
x = 0.0013, i=14
x = 0.0020, i=13
x = 0.0032, i=12
x = 0.005, i=11
x = 0.008, i=10
x = 0.013, i=9
x = 0.02, i=8
x = 0.032, i=7
x = 0.05, i=6
x = 0.08, i=5
x = 0.13, i=4
x = 0.18, i=3
x = 0.25, i=2
x = 0.40, i=1
x = 0.65, i=0
σr(x,Q²) x 2^i
Q²/ GeV²

$\alpha_S(Q^2)$ from HERA