QCD and Models: Introduction
Theodore Wulf (1910)
Too Many Hadrons!
Quarks and the Eightfold Way
Quarks and the Eightfold Way

Three quarks for Muster Mark.

J. Joyce, *Finnegan’s Wake*

Der kleine Gott ... in jeden Quark begräbt er seine Nase.

Goethe, *Faust*

I was lucky enough to see and hear *Nycticorax Cyanocephalus Falklandicus*, commonly known as the Quark ... only to be found in these remote islands whose inhabitants told me they derive their names from the strangely beautiful call they emit on being disturbed.

R. Robinson, *Letter to the Times, 28.2.68*
recall that a symmetry implies a degeneracy in the spectrum of a Hamiltonian:

\[ [\vec{J}, H] = 0 \quad [J^+, H] = 0 \]

\[ H |jm\rangle = E_j |jm\rangle \quad J^+ H |jm\rangle = J^+ E_j |jm\rangle \]

\[ H J^+ |jm\rangle = E_j J^+ |jm\rangle \quad H |jm + 1\rangle = E_j |jm + 1\rangle \]
Quarks and the Eightfold Way

- ex: the `Threefold Way’ of spin

representation: $\frac{1}{2} \times \frac{1}{2} = 1 + 0$

dimension: $2 \times 2 = 3 + 1$

↑

a triplet of degenerate particles

representation: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{2} + \frac{1}{2} + \frac{1}{2}$

dimension: $2 \times 2 \times 2 = 4 + 2 + 2$

↑

a quartet of degenerate particles
Quarks and the Eightfold Way

Gell-Mann & Ne’eman: use SU(3) to categorize hadrons

ex: $3 \times 3 = 1 + 8$ (dimensions)

$\pi^+, \pi^0, \pi^-, \eta, K^+, K^0, \bar{K}^0, K^-$

140 135 140 547 494 498 498 494

$ud \quad d\bar{u} \quad u\bar{s} \quad d\bar{s} \quad s\bar{d} \quad s\bar{u}$

$u\bar{u} - d\bar{d} \quad u\bar{u} + d\bar{d}$
Quarks and the Eightfold Way

ex: $3 \times 3 \times 3 = 1 + 8 + 8 + 10$ (dimensions)
the Quark Model

assume reality of quarks u,d,s

proton (1964)
the Quark Model

\[ Q = I_z + \frac{1}{2}(B+S) \]

Gell-Mann--Nishijima relationship

ex:

\[
\begin{align*}
\text{p: } 1 &= \frac{1}{2} + \frac{1}{2}(1+0) \\
\pi^+: 1 &= 1 + \frac{1}{2}(0+0) \\
\Lambda^0: 0 &= 0 + \frac{1}{2}(1-1)
\end{align*}
\]
the Quark Model

But baryons = qqq

\[ u: \quad q = \frac{1}{2} + \frac{1}{2}(\frac{1}{3} + 0) = \frac{2}{3} \]
\[ d: \quad q = -\frac{1}{2} + \frac{1}{2}(\frac{1}{3} + 0) = -\frac{1}{3} \]
\[ s: \quad q = 0 + \frac{1}{2}(\frac{1}{3} - 1) = -\frac{1}{3} \]

a triplet of fractionally charged fermions!
I remember being very surprised by Figure 1 ... There was an enormous peak ... right at the edge of phase space. The fact that the $\varphi$ decayed predominantly into KK and not $\pi\rho$ was totally unintelligible. ... Only conservation laws suppress reactions. Here was a reaction that was allowed but did not proceed! I had thought that hadrons probably have constituents and this experiment convinced me that they do, and that they are real. ... This was a statement about dynamics which indicated that the constituents were not hypothetical objects carrying the symmetries of the theory, but real objects that moved in space-time from hadron to hadron.”

George Zweig
the Quark Model

problems:

(i) so where are they?

assumed very massive... superstrong forces

(ii) `statistics problem'

\[ \Delta^{±±} = \text{uuu (↑↑↑) } \psi \]

each is symmetric, yet the total wavefunction must be antisymmetric!
the Quark Model

a solution: assume that quarks have a new characteristic, or charge, of three different types

\[ \Delta^{+++} = uuu \ (\uparrow\uparrow\uparrow) \ \psi \ C \]

\[ C = \frac{1}{\sqrt{6}} (rgb - rbg - grb + brg - bgr + gbr) \]
the Quark Model

proton (1970)
the Quark Model

a new problem:

why haven’t we seen colour?

Combine with our left over old problem:

the colour confinement hypothesis: colour nonsinglet hadrons do not exist
the Quark Model

Electron scattering from proton

Square of energy (GeV²)

Jerome Friedman (1930-)
Henry Kendall (1926-1999)
Richard Taylor (1929-)
James Bjorken (1934-)
Finding Quarks

<table>
<thead>
<tr>
<th>flavour</th>
<th>charge</th>
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<th>discovery</th>
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<tr>
<td>up</td>
<td>2/3</td>
<td>5 MeV</td>
<td>1911</td>
</tr>
<tr>
<td>down</td>
<td>-1/3</td>
<td>10 MeV</td>
<td>1932</td>
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<td>1600</td>
<td>1974</td>
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<td>150 MeV</td>
<td>1947</td>
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<td>2/3</td>
<td>174 GeV</td>
<td>1995</td>
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<tr>
<td>bottom</td>
<td>-1/3</td>
<td>5 GeV</td>
<td>1977</td>
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Chiral Symmetry
Chiral Symmetry

Current Algebra (1950s)

Partially Conserved Axial Current Hypothesis:

$$\langle 0 | A^a_\mu (0) | \pi^b (p) \rangle = i \delta^{ab} f_\pi p_\mu$$

⇒ low energy pion theorems which were (and are) understood as a result of spontaneous symmetry breaking in an effective field theory (more later).
requirements for a theory of the strong interactions

• partonic interactions
• colour confinement
• PCAC/ spontaneous chiral symmetry breaking
• renormalisable
• approximate SU$_f$(3) symmetry
QCD

gauge $\text{SU}_c(3)$

local gauge invariance (QED):

$$A \rightarrow A + \nabla \Lambda \quad \phi \rightarrow \phi - \dot{\Lambda}$$

impose local gauge symmetry:

$$\psi(x) \rightarrow e^{-i\Lambda(x)} \psi(x)$$

and get an interacting field theory:

$$\mathcal{L} = \int \bar{\psi} \gamma^\mu \partial_\mu \psi \rightarrow \int \bar{\psi} \gamma^\mu (\partial_\mu + ieA_\mu) \psi \quad A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$
local gauge invariance (QCD):

impose local gauge symmetry: \[ \psi(x)_a \rightarrow U_{ab} \psi(x)_b \]

for invariance of \( L \):

\[
\mathcal{L} = \int \bar{\psi}_a \delta^{ab} \gamma^\mu \partial_\mu \psi_b \rightarrow \int \bar{\psi}_a (\delta^{ab} \gamma^\mu \partial_\mu + ig \gamma^\mu (A_\mu)_{ab}) \psi_b
\]

\[ A_\mu \rightarrow U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger \]

\[
F_{\mu\nu} \propto [D_\mu, D_\nu] = ig(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2 [A_\mu, A_\nu]
\]
QCD

\[ \mathcal{L}_{QCD} = \sum_{f} n_f \bar{q}_f [i \gamma_\mu (\partial^\mu + i g A^\mu) - m_f] q_f - \frac{1}{2} \text{Tr}(F^{\mu \nu} F^{\mu \nu}) \]

\[ F^{\mu \nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig [A^\mu, A^\nu] \]

\[ A^\mu = A^a_\mu \lambda^a \]

\[ [\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = i f^{abc} \lambda^c / 2 \]

\[ \text{Tr}(\lambda^a \lambda^b) = 2 \delta^{ab} \]

\[ \mathcal{L}_\theta = \theta \frac{g^2}{64\pi^2} F^{\mu \nu} \tilde{F}^{\mu \nu} \]
proton (1973+)
OPEN PROBLEMS

• confinement: solved but not proved
• strong CP problem
• emergent properties: nuclear physics, exotics, decays, extreme conditions, multi-scales. Right now we can reliably compute almost no properties of hadrons. Q: what if they made a theory that no one could compute with?