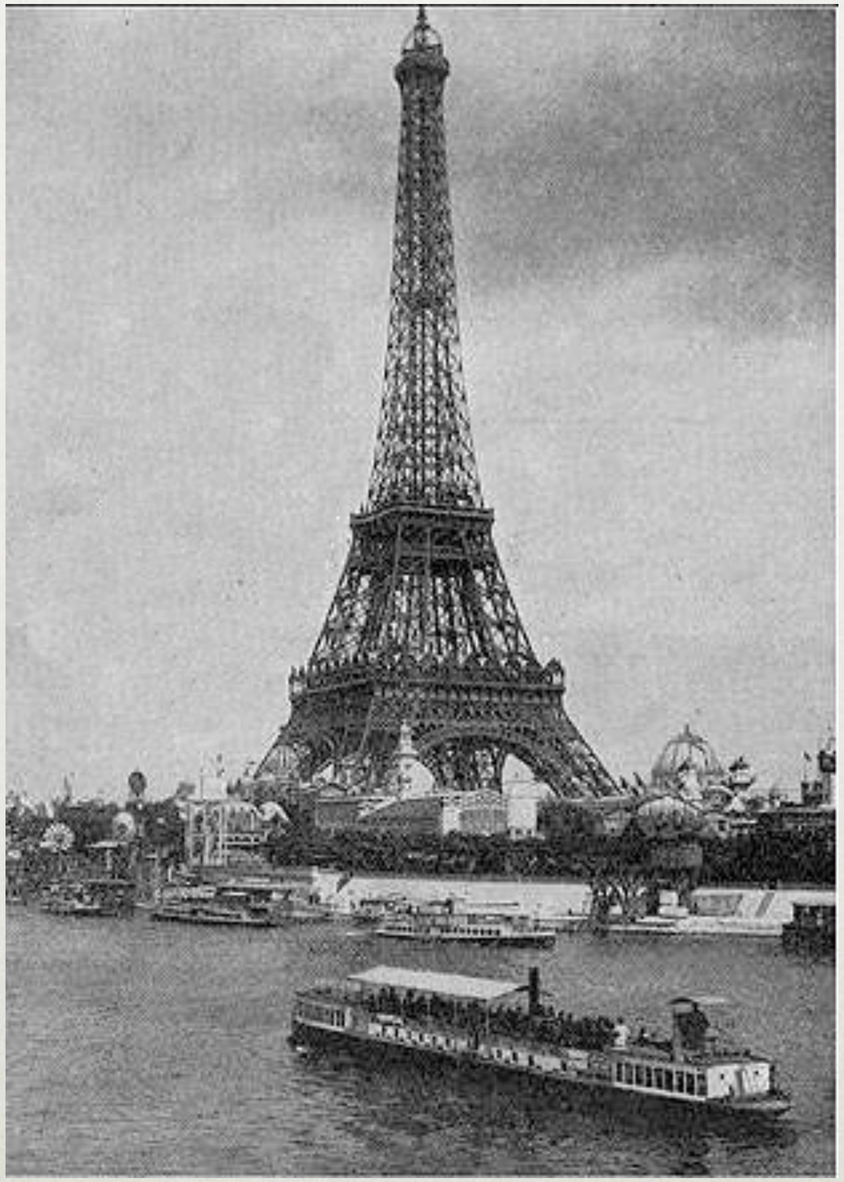
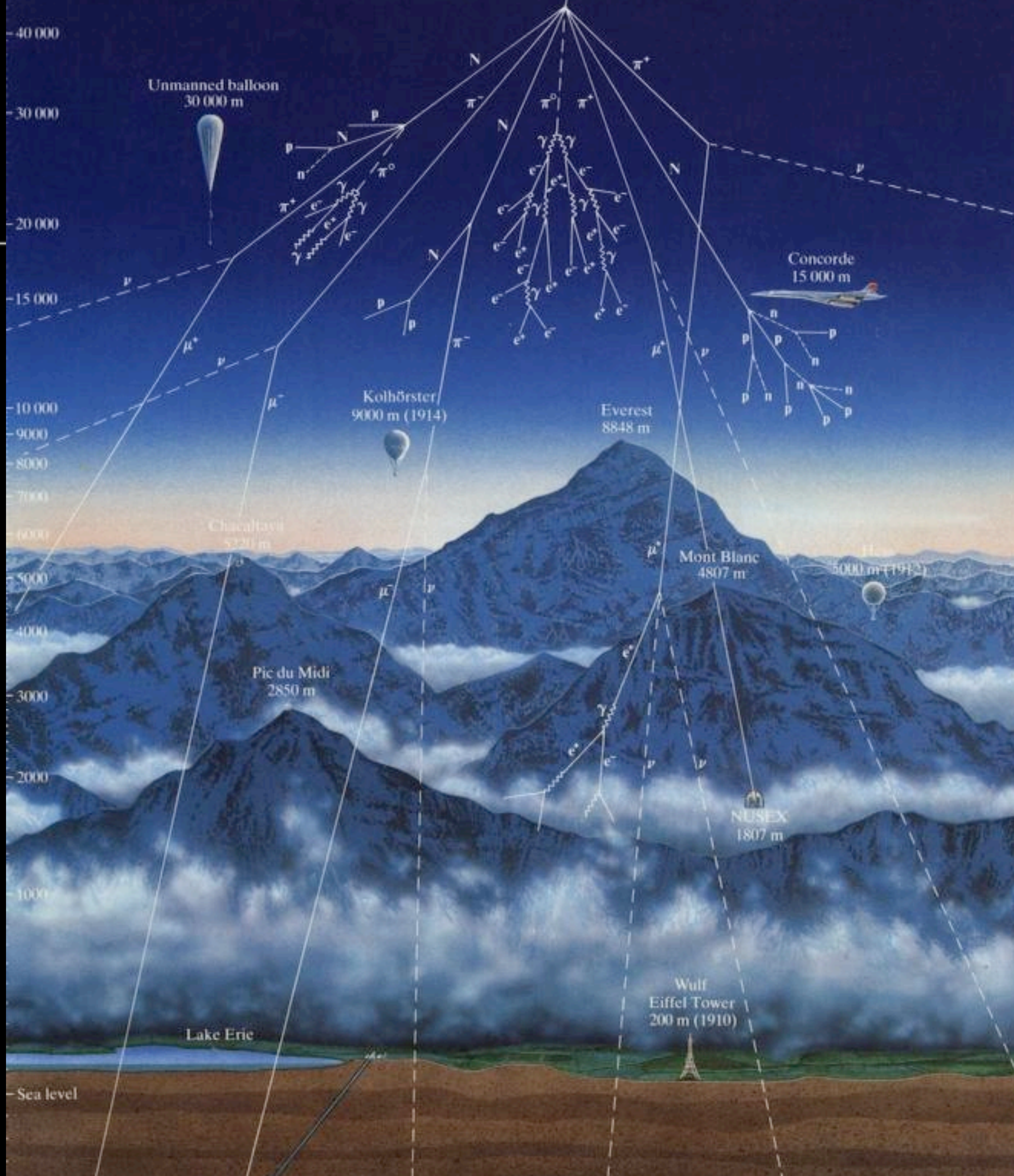
QCD and Models : introduction

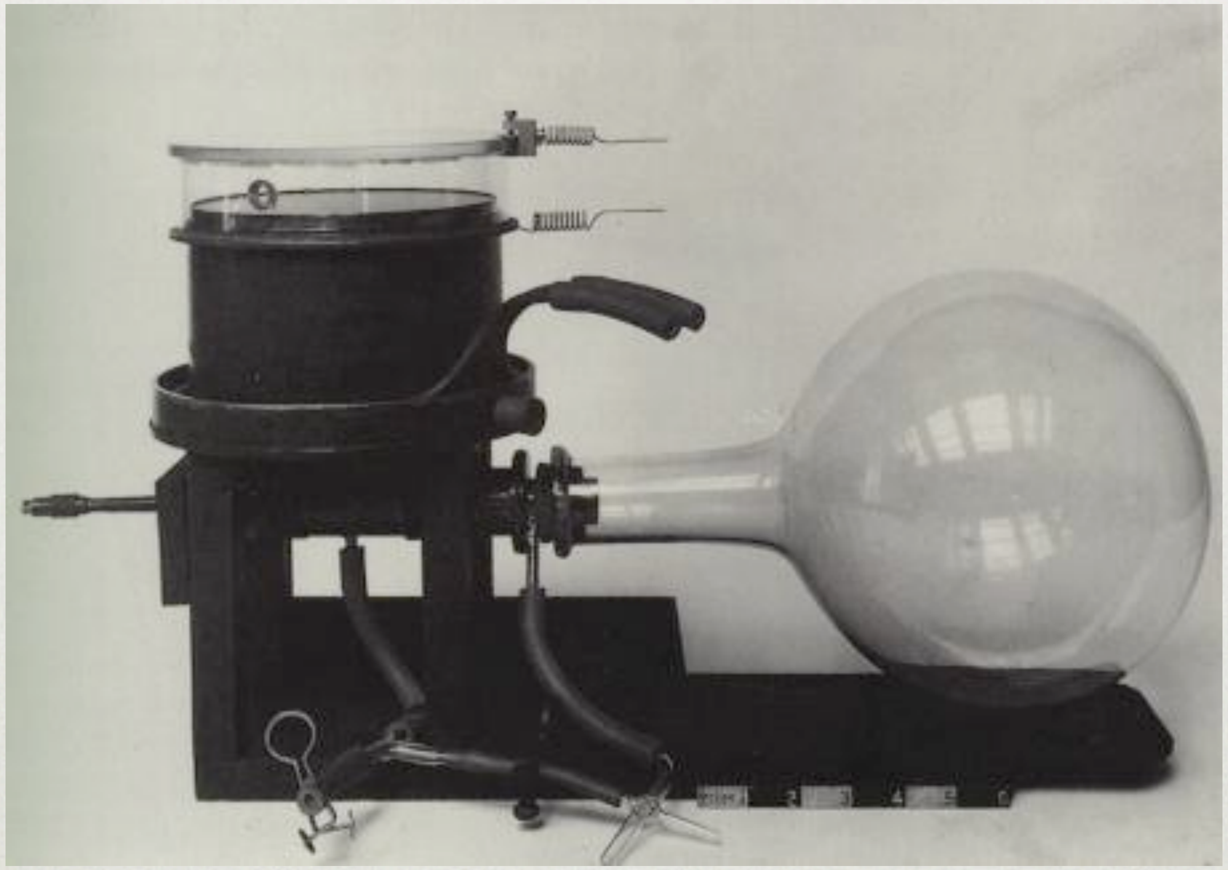
Eric Swanson

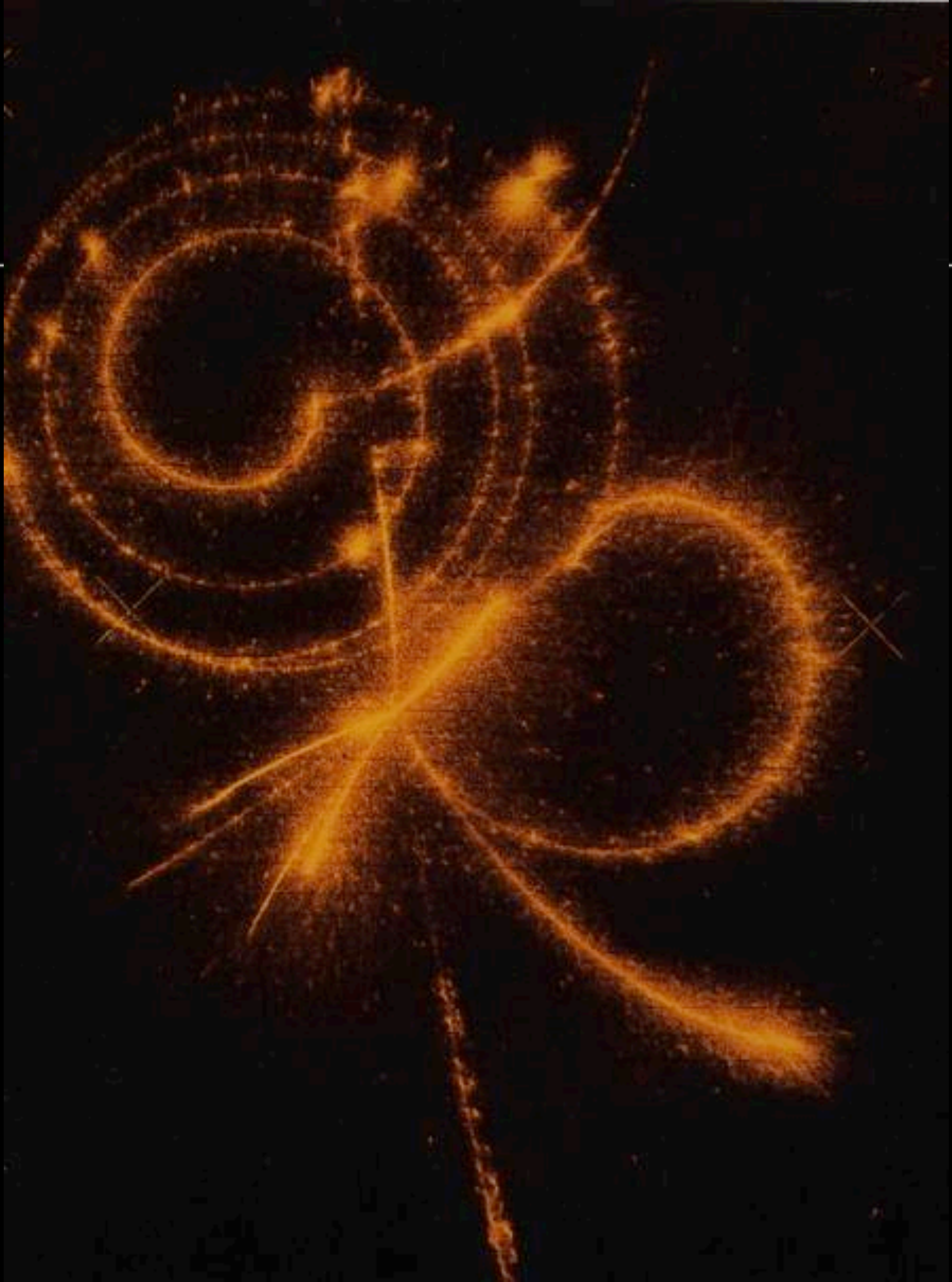
Theodore Wulf (1910)



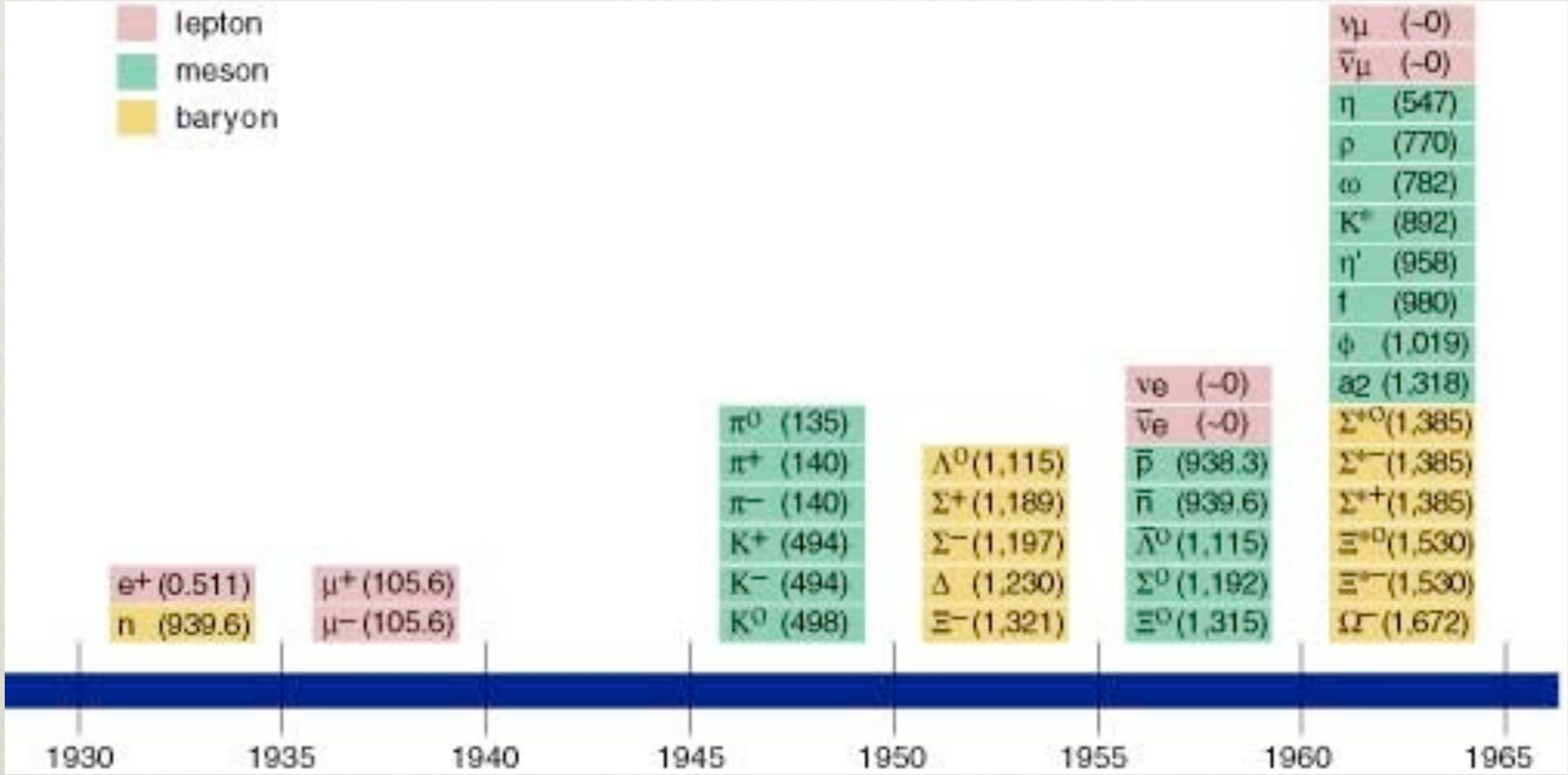


40 000
30 000
20 000
15 000
10 000
9000
8000
7000
6000
5000
4000
3000
2000
1000
Sea level
Unmanned balloon
30 000 m
Concorde
15 000 m
Kolhörster
9000 m (1914)
Everest
8848 m
Mont Blanc
4807 m
Pic du Midi
2850 m
NUSEX
1807 m
Lake Erie
Wulf
Eiffel Tower
200 m (1910)



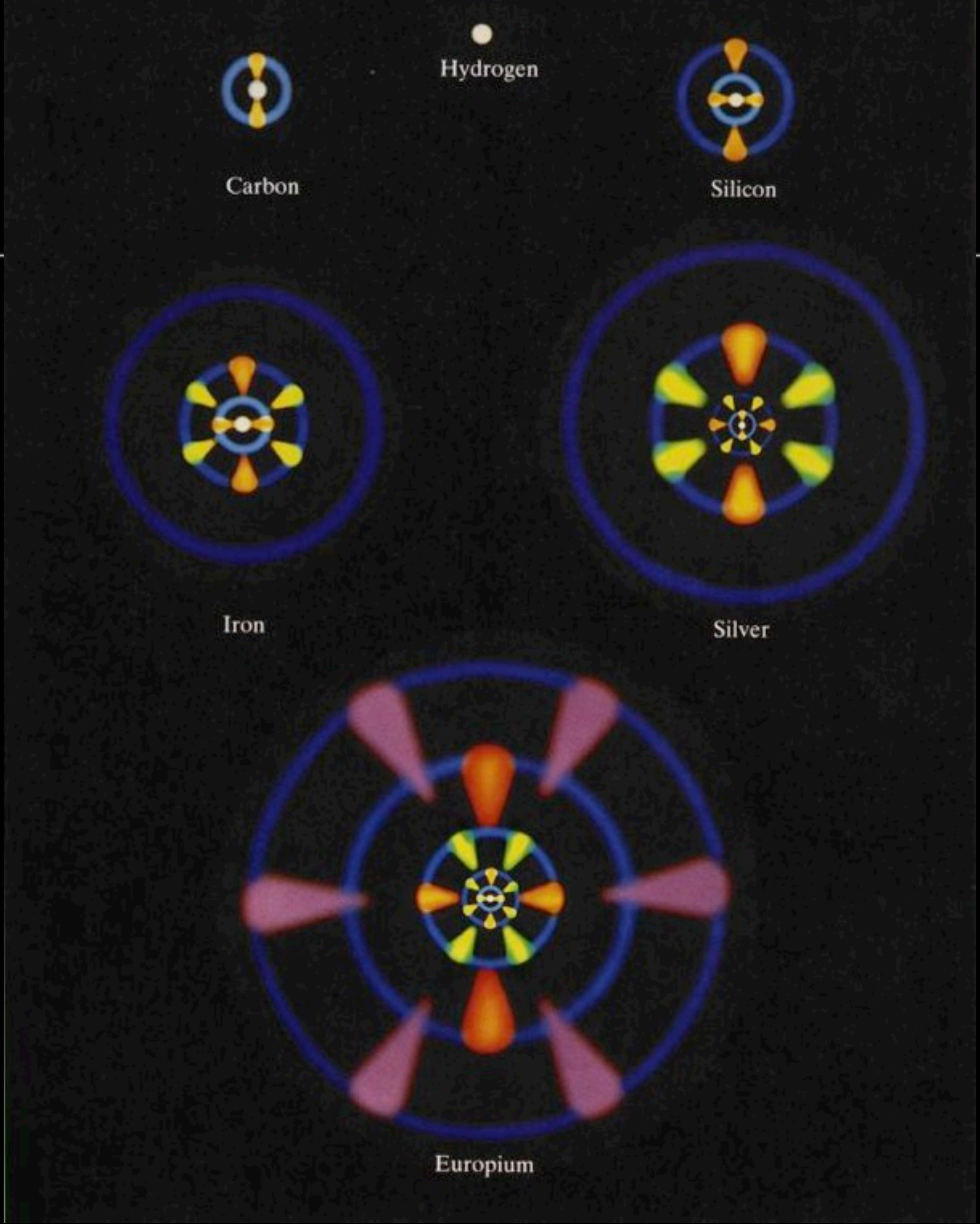


Too Many Hadrons!



Quarks and the Eightfold Way

Fig. 6.35 *Murray Gell-Mann (b.1929).*


Hydrogen
Carbon
Silicon
Iron
Silver
Europium

Quarks and the Eightfold Way

Three quarks for Muster Mark.

J. Joyce, *Finnegan's Wake*

Der kleine Gott ... in jeden Quark begräbt er seine Nase.

Goethe, *Faust*

I was lucky enough to see and hear Nycticorax Cyanocephalus Falklandicus, commonly known as the Quark ... only to be found in these remote islands whose inhabitants told me they derive their names from the strangely beautiful call they emit on being disturbed.

R. Robinson, *Letter to the Times, 28.2.68*

Quarks and the Eightfold Way

recall that a symmetry implies a degeneracy in the spectrum of a Hamiltonian:

$$[\vec{J}, H] = 0 \qquad [J^+, H] = 0$$

$$H|jm\rangle = E_j|jm\rangle \qquad J^+ H|jm\rangle = J^+ E_j|jm\rangle$$

$$HJ^+|jm\rangle = E_j J^+|jm\rangle \quad H|jm+1\rangle = E_j|jm+1\rangle$$

Quarks and the Eightfold Way

- ex: the `Threefold Way' of spin

representation: $1/2 \times 1/2 = 1 + 0$

dimension: $2 \times 2 = 3 + 1$

↑ a triplet of degenerate particles

representation: $1/2 \times 1/2 \times 1/2 = 3/2 + 1/2 + 1/2$

dimension: $2 \times 2 \times 2 = 4 + 2 + 2$

↑ a quartet of degenerate particles

Quarks and the Eightfold Way

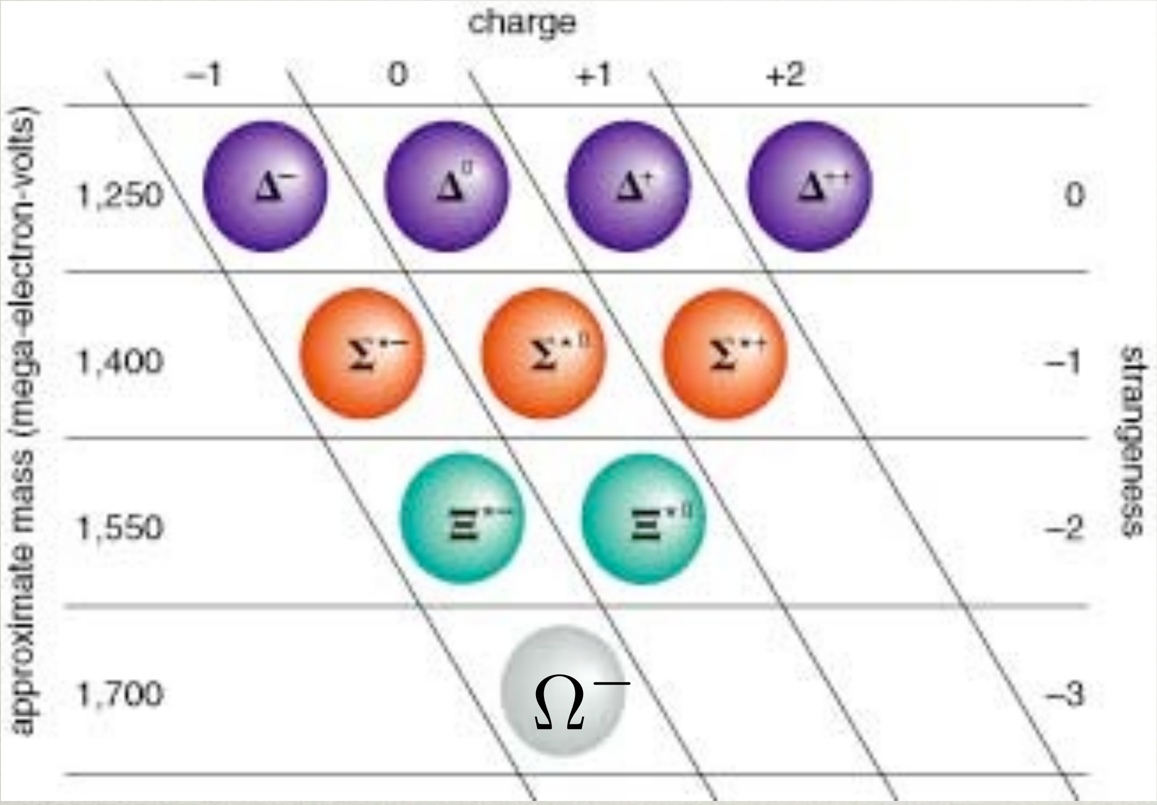
Gell-Mann &Ne'eman: use SU(3) to categorize hadrons

ex: $3 \times \bar{3} = 1 + 8$ (dimensions)

π^+,	π^0,	π^-,	η,	K^+,	K^0,	$\bar{K}^0$,	K^-
140	135	140	547	494	498	498	494
$u\bar{d}$		$d\bar{u}$		$u\bar{s}$	$d\bar{s}$	$s\bar{d}$	$s\bar{u}$
	$u\bar{u} - d\bar{d}$		$u\bar{u} + d\bar{d}$				

Quarks and the Eightfold Way

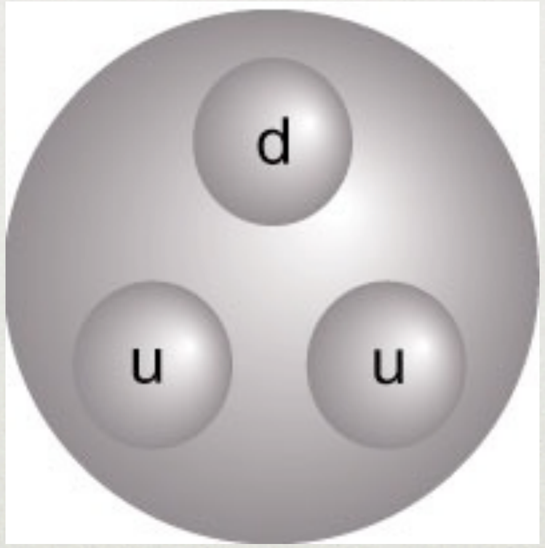
ex: 3x3x3= 1+8+8+10 (dimensions)



$$ddd \quad ddu \quad duu \quad uuu$$
$$sdd \quad sdu \quad suu$$
$$ssd \quad ssu$$
$$sss$$

the Quark Model

assume reality of quarks u,d,s



proton (1964)

the Quark Model

$$Q = I_z + 1/2(B+S)$$

Gell-Mann--Nishijima relationship

ex:

$$p: 1 = 1/2 + 1/2(1+0)$$

$$\pi^+: 1 = 1 + 1/2(0+0)$$

$$\Lambda^0: 0 = 0 + 1/2(1-1)$$

the Quark Model

But baryons = qqq

u: $q = 1/2 + 1/2(1/3+0) = 2/3$

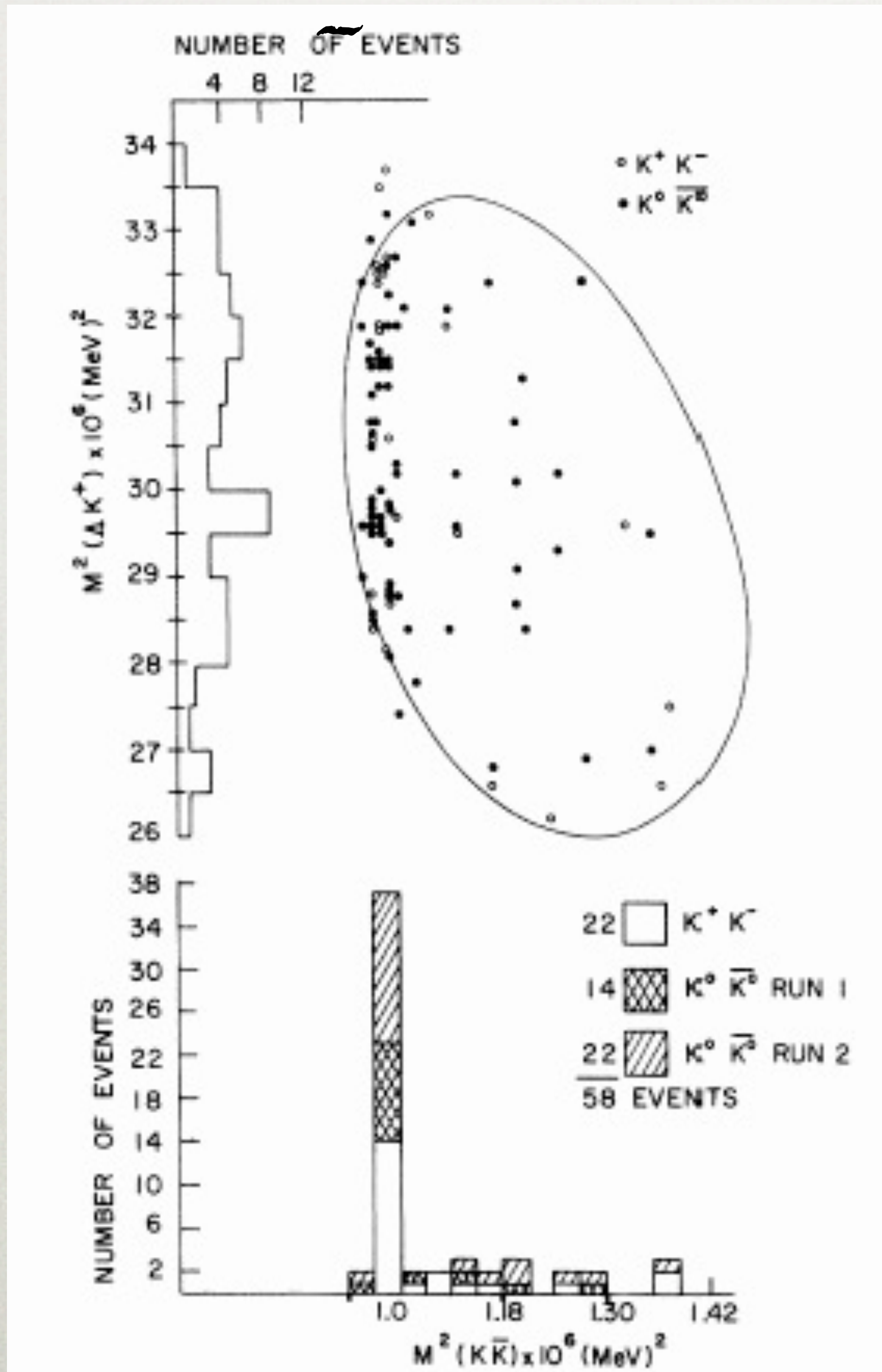
d: $q = -1/2 + 1/2(1/3+0) = -1/3$

s: $q = 0 + 1/2(1/3-1) = -1/3$

a triplet of fractionally charged fermions!

the Quark Model

Kp → Λ KK



I remember being very surprised by Figure 1 …There was an enormous peak … right at the edge of phase space. The fact that the φ decayed predominantly into KK and not πρ was totally unintelligible. … Only conservation laws suppress reactions. Here was a reaction that was allowed but did not proceed! I had thought that hadrons probably have constituents and this experiment convinced me that they do, and that they are real. … This was a statement about dynamics which indicated that the constituents were not hypothetical objects carrying the symmetries of the theory, but real objects that moved in space-time from hadron to hadron.”

George Zweig

P.L. Connelly et al., PRL**10**, 371 (1963)

the Quark Model

problems:

(i)so where are they?

assumed very massive... superstrong forces

(ii)`statistics problem'

$\Delta^{++} = uuu\ (\uparrow\uparrow\uparrow)\ \psi$

each is symmetric, yet the total wavefunction must be antisymmetric!

the Quark Model

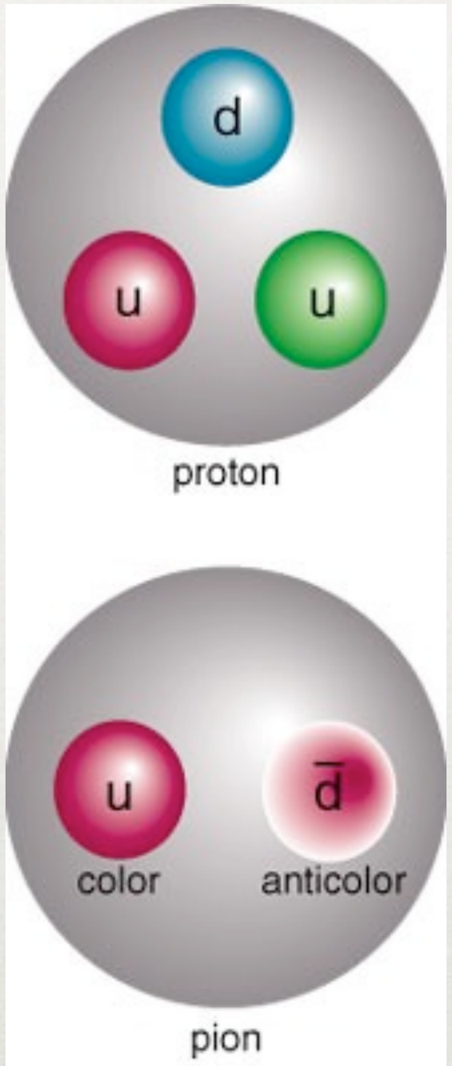
a solution: assume that quarks have a new characteristic, or charge, of three different types

$$\Delta^{++} = uuu\ (\uparrow\uparrow\uparrow)\ \psi\ C$$

$$C = \frac{1}{\sqrt{6}}(rgb - rbg - grb + brg - bgr + gbr)$$

the Quark Model

proton (1970)



the Quark Model

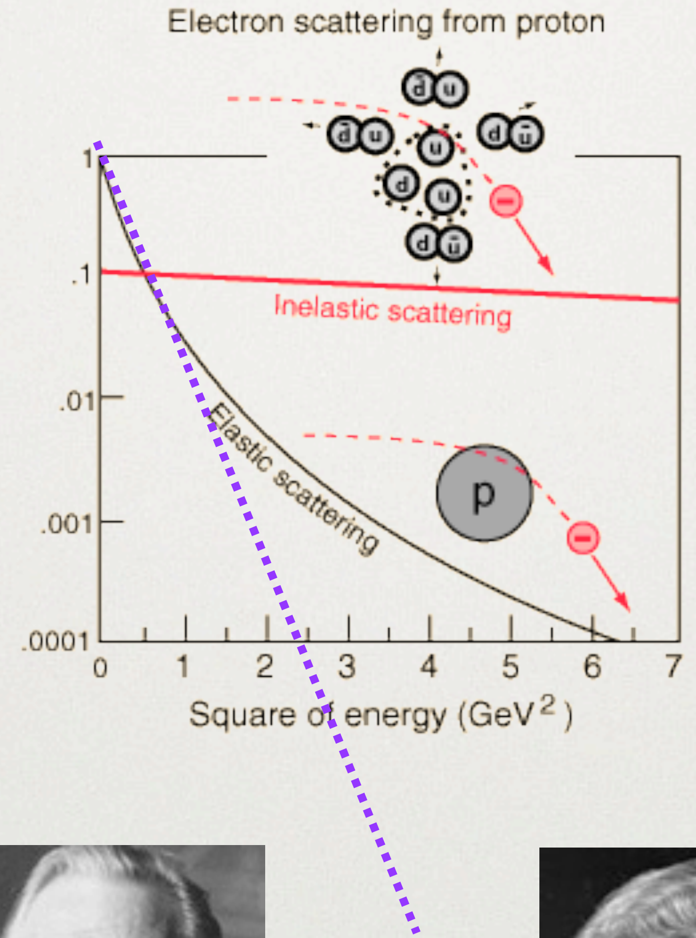
a new problem:

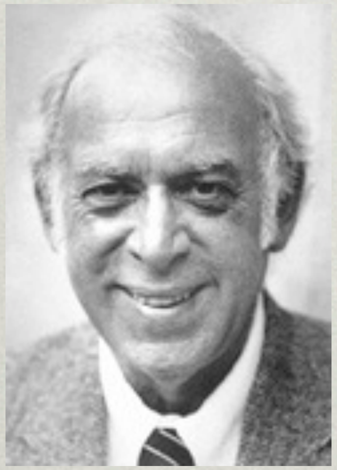
why haven’t we seen colour?

Combine with our left over old problem:

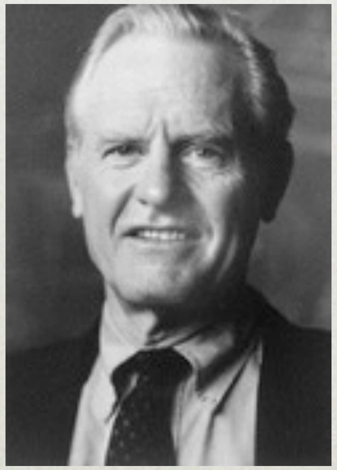
the colour confinement hypothesis:
colour nonsinglet hadrons do not exist

the Quark Model





Jerome Friedman
(1930-)



Henry Kendall
(1926-1999)

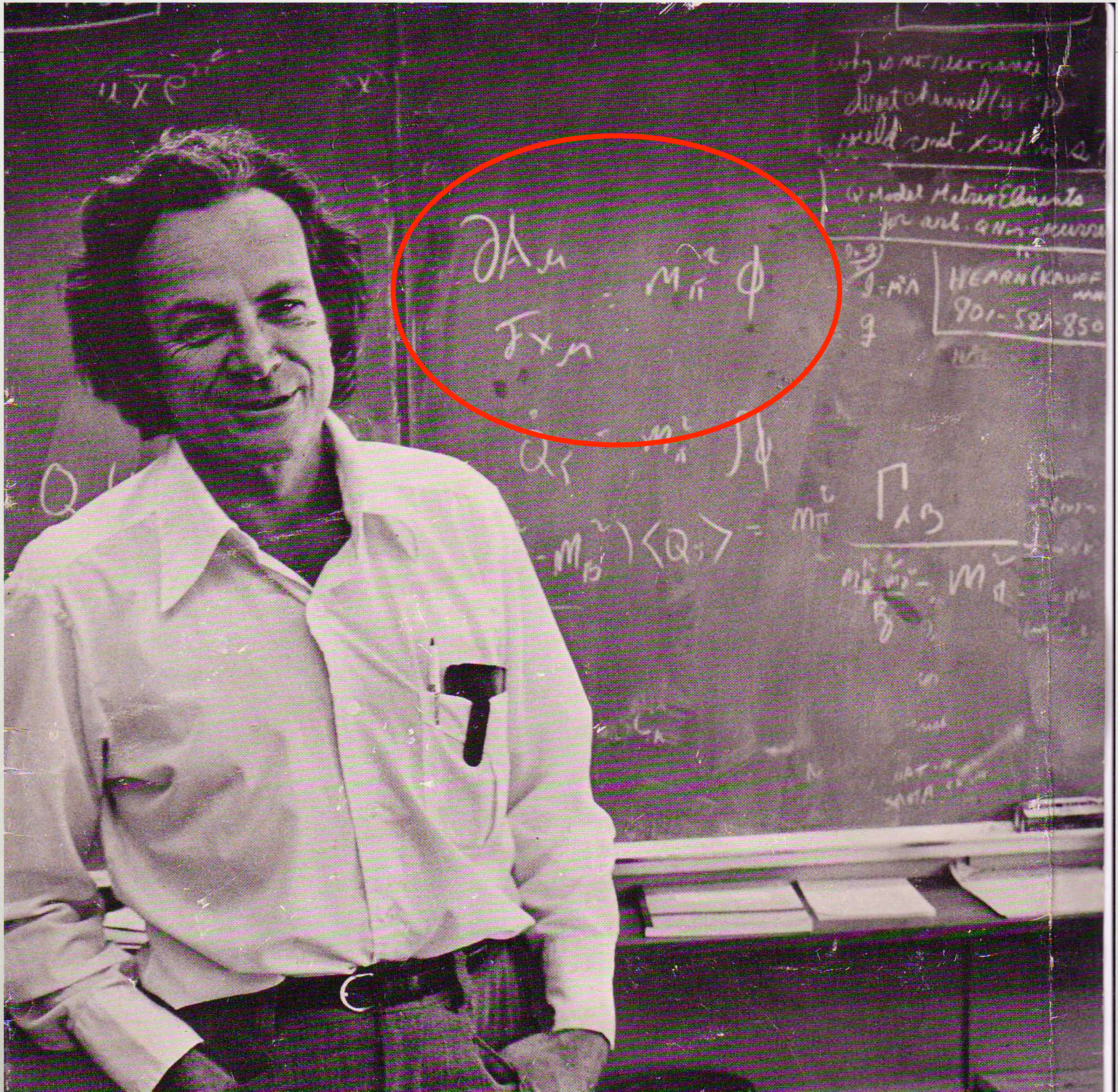
Richard Taylor
(1929-)

James Bjorken
(1934-)

Finding Quarks

flavour	charge	mass	discovery
up	2/3	5 MeV	1911
down	-1/3	10 MeV	1932
charm	2/3	1600	1974
strange	-1/3	150 MeV	1947
top	2/3	174 GeV	1995
bottom	-1/3	5 GeV	1977

Chiral Symmetry



Chiral Symmetry

Current Algebra (1950s)

Partially Conserved Axial Current
Hypothesis:

$$\langle 0|A_\mu^a(0)|\pi^b(p)\rangle = i\delta^{ab} f_\pi p_\mu$$

$\Rightarrow$ low energy pion theorems which were (and are) understood as a result of spontaneous symmetry breaking in an effective field theory (more later).

requirements for a theory of the strong interactions

- partonic interactions
- colour confinement
- PCAC/ spontaneous chiral symmetry breaking
- renormalisable
- approximate $SU_f(3)$ symmetry

QCD

gauge $SU_c(3)$

local gauge invariance (QED):

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla\Lambda \qquad \phi \rightarrow \phi - \dot{\Lambda}$$

impose local gauge symmetry:

$$\psi(x) \rightarrow \mathrm{e}^{-i\Lambda(x)}\psi(x)$$

and get an interacting field theory:

$$\mathcal{L} = \int \bar{\psi}\gamma^\mu\partial_\mu\psi \rightarrow \int \bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi \qquad A_\mu \rightarrow A_\mu + \partial_\mu\Lambda$$

QCD

local gauge invariance (QCD):

impose local gauge symmetry: $\psi(x)_a \to U_{ab}\psi(x)_b$

for invariance of L:

$$\mathcal{L} = \int \bar{\psi}_a \delta^{ab}\gamma^\mu \partial_\mu \psi_b \to \int \bar{\psi}_a \left(\delta^{ab}\gamma^\mu \partial_\mu + ig\gamma^\mu (A_\mu)_{ab}\right)\psi_b$$

$$A_\mu \to U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger$$

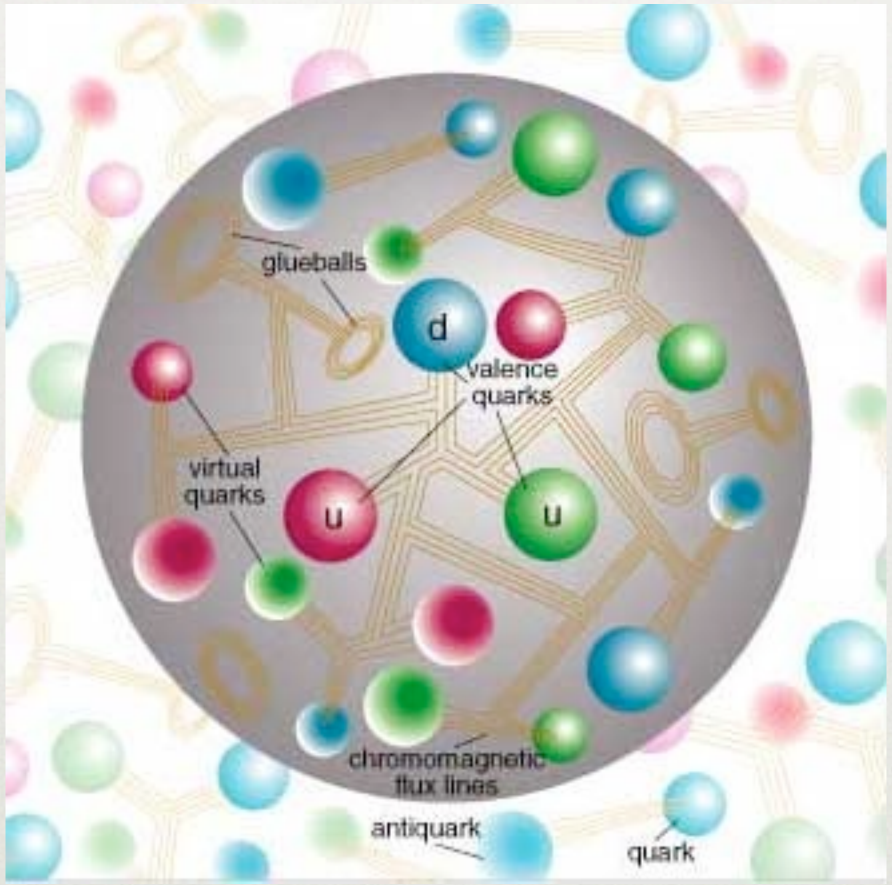
$$F_{\mu\nu} \propto [D_\mu, D_\nu] = ig(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2 [A_\mu, A_\nu]$$

QCD

$$\begin{aligned}\mathcal{L}_{QCD} &= \sum_f^{n_f} \bar{q}_f[i\gamma_\mu(\partial^\mu + igA^\mu) - m_f]q_f - \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \\ A_\mu &= A_\mu^a \frac{\lambda^a}{2} \\ [\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] &= if^{abc}\frac{\lambda^c}{2} \\ \mathrm{Tr}(\lambda^a\lambda^b) &= 2\delta^{ab}\end{aligned}$$

flavour, colour, Dirac indices

$$\mathcal{L}_\theta = \theta \frac{g^2}{64\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$$


glueballs
d
valence
quarks
virtual
quarks
u
u
chromomagnetic
flux lines
antiquark
quark

proton (1973+)

OPEN PROBLEMS

- confinement: solved but not proved
- strong CP problem
- emergent properties: nuclear physics, exotics, decays, extreme conditions, multi-scales. *Right now we can reliably compute almost no properties of hadrons.* Q: what if they made a theory that no one could compute with?