Topics on QCD and Spin Physics

(third lecture)

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Universidad de Buenos Aires

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Lecture 1: Introduction.
1.1 The origins of QCD: the quark model, Bjorken scaling and the parton model.
1.2 From QED to QCD: the QCD lagrangian and its Feynman diagrams.
1.3 The running coupling constant: renormalization and asymptotic freedom.

Lecture 2: DIS and PDFs.
2.1 DIS kinematics and cross sections.
2.2 Defining PDFs, evolution equations, LO, NLO and beyond.
2.3 PDFs extraction from data.

hadronization: a path to confinement
accessed by "jets"
fragmentation functions "FFs"

Lecture 3: Final states in QCD.
3.1 Factorization beyond DIS: fragmentation functions and jets.
3.2 SIDIS, e+e- -> hadrons, pp -> hadrons.
3.3 Combined global fits of fragmentation functions.
Looking for FFs:

\[ z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q} \]

\[ \frac{d\sigma}{dz}(e^+ e^- \rightarrow h X) = \sum_q \sigma(e^+ e^- \rightarrow q\bar{q}) \left[ D^h_q(z) + D^h_q(z) \right] \]

\[ \sum_h \int_0^1 z D^h_q(z) \, dz = 1 \quad \sum_q \int_0^1 \left[ D^h_q(z) + D^h_q(z) \right] \, dz = n_h \]

\[ D^h_q(z) \rightarrow D^h_q(z, Q^2) \]

\[ \frac{dD^h_q(z, Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_z^1 \frac{dy}{y} \left[ D^h_q(y, Q^2) P_{qq} \left( \frac{x}{y} \right) + D^h_q(y, Q^2) P_{gg} \left( \frac{x}{y} \right) \right] \]

\[ \frac{dD^h_q(x, Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[ \sum_q D^h_q(y, Q^2) P_{qq} \left( \frac{x}{y} \right) + D^h_q(y, Q^2) P_{gg} \left( \frac{x}{y} \right) \right]. \]
why does it work?
collinear emission?
A good example: \( \hat{\sigma}(e^+e^- \rightarrow q\bar{q}g) \)

\[
\begin{align*}
    z_q &\equiv \frac{2E_q}{Q} \\
    z_{\bar{q}} &\equiv \frac{2E_{\bar{q}}}{Q} \\
    z_g &\equiv \frac{2E_g}{Q}
\end{align*}
\]

\[z_q + z_{\bar{q}} + z_g = 2\]

\[
\frac{d\hat{\sigma}}{dz_q dz_{\bar{q}}} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{z_q^2 + z_{\bar{q}}^2}{(1-z_q)(1-z_{\bar{q}})}
\]
SIDIS: semi-inclusive deep inelastic scattering

"one-particle-inclusive"

\[ l + p \rightarrow l' + h + X \]

\[ d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D^h_j \]

- \( f_i \) parton density
- \( \hat{\sigma}_{ij} \) partonic cross section
- \( D^h_j \) fragmentation function

\[ 2F^h_1(x, z, Q^2) = \sum_{q, \bar{q}} e^2_q f_q(x, Q^2) D^h_q(z, Q^2) \]

LO
SIDIS: semi-inclusive deep inelastic scattering
“one-particle-inclusive”

\[ l + p \rightarrow l' + h + X \]

\[ d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h \]

- \( f_i \) parton density
- \( \hat{\sigma}_{ij} \) partonic cross section
- \( D_j^h \) fragmentation function

\[ 2F_1^h(x, z, Q^2) = \sum_{q, \bar{q}} e_q^2 f_q(x, Q^2) D_q^h(z, Q^2) \]

\[ + \frac{\alpha_s}{2\pi} \sum_{q, \bar{q}} e_q^2 \left[ f_q \otimes C_{qq} \otimes D_q^h + f_q \otimes C_{gq} \otimes D_g^h + f_q \otimes C_{qg} \otimes D_q^h \right] \]

NLO
Single-inclusive h production in p-p (p-pbar) collisions

\[ p + p \rightarrow h + X \]

\[ d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_{hk}^h \]

\[ \sim f_q \otimes f_\bar{q} \otimes \hat{\sigma}_{q\bar{q}g} \otimes D_{g}^h \]

Gluon fragmentation at lowest order

\[ \sim f_g \otimes f_g \otimes \hat{\sigma}_{ggg} \otimes D_{g}^h \]
Target fragmentation: fracture functions

initial state partons carry no $p_T$

final state hadrons produced collinearly

in $\gamma - p$ c.m. frame (lowest order)

(only “backward” hadrons in a collider)

(order $\alpha_s$)

(“forward” hadrons suppressed)
no $\mathcal{O}(\alpha_s^0)$ contribution for "forward" processes
no consistent factorization of (forward) divergencies

current fragmentation

target fragmentation
\[ M^h_i(x, z, Q^2) \text{ fracture functions} \]

\[ D^h_i(z, Q^2) f_i(x, Q^2) \]

\[
\frac{\partial M^h_i(x, z, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u} P_{i \leftarrow j}(u) M^h_j \left( \frac{x}{u}, z, Q^2 \right)
\]

\[
+ \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{x} \int \frac{du}{u} \int \frac{dv}{v} P_{ki \leftarrow j}(u, v) f_j \left( \frac{x}{u}, Q^2 \right) D^h_k \left( \frac{z}{x v}, Q^2 \right)
\]

"non-homogeneous evolution"
Global fits FFs:

increasing interest in accurate FFs:
precise one particle inclusive measurements
full NLO framework available

recent NLO analyses:

KRE(2000)
HKNS(2007)

\[
D_h^{\pm} \quad D_{\pi q}^{\pm} \quad D_{K q}^{\pm} \quad D_{p q}^{\pm} \\
D_{\pi q}^{+} \quad D_{\pi q}^{-} \quad D_{K q}^{+} \quad D_{K q}^{-} \quad \ldots
\]

uncertainties

L. Bourhis et al.,
S. Albino et al.,
S. Kretzer et al.,
M. Hirai et al.,
Standard FFs analyses:

\[ e^+e^- \rightarrow (\gamma, Z) \rightarrow H \]

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^H}{dz} = \frac{\sigma_0}{\sum_q \hat{e}_q^2} \left[ 2 F_1^H(z, Q^2) + F_L^H(z, Q^2) \right]
\]

\[
2F_1^H(z, Q^2) = \sum_q \hat{e}_q^2 \left\{ \left[ D_q^H(z, Q^2) + D_{\bar{q}}^H(z, Q^2) \right] + \frac{\alpha_s(Q^2)}{2\pi} \left[ C_{q}^{1} \otimes (D_{q}^{H} + D_{\bar{q}}^{H}) + C_{g}^{1} \otimes D_{g}^{H} \right] (z, Q^2) \right\}
\]

SIA can only give information on the sum \( D_q^H(z, Q^2) + D_{\bar{q}}^H(z, Q^2) \)!

gluon fragmentation strongly suppressed \( \sim \frac{\alpha_s(M_Z^2)}{2\pi} \)
ansatz needed if only SIA or charge averaged data used,  

“linear suppression”

$$D_{q}^{h^+}(z, Q^2) = (1 - z) D_{q}^{h^+}(z, Q^2)$$

Further disadvantages:

SIA data dominated by precise LEP/SLD measurements at $M_Z$
mostly determine “singlet”
distribution:

$$\Sigma = D_u + D_{\bar{u}} + D_d + D_{\bar{d}} + D_s + D_{\bar{s}} + D_c + D_{\bar{c}} + D_b + D_{\bar{b}}$$

bad resolution for $g$ fragmentation

not precise at large $z$ (relevant for pp collisions)
DSS set of FFs:

\[ D^H_{q+\bar{q}}(z, Q^2) \]

\[ D^H_q(z, Q^2) \quad D^H_{\bar{q}}(z, Q^2) \]

\[ D^H_g(z, Q^2) \]

charge & flavor separation

D.de Florian, R.S., M. Stratmann 2007

gluon fragmentation
DSS set of FFs:

Flexible parametrization:

\[ D_i^H(z, Q_0^2) = N_i z^{\alpha_i} (1 - z)^{\beta_i} \left[ 1 + \gamma_i (1 - z)^{\delta_i} \right] \]

at initial scale

\[ Q_0^2 = 1 \text{ GeV}^2 \quad u, d, s, g \]

\[ Q_0^2 = m_Q^2 \quad c, b \]

with \( \alpha_s \) and \( \Lambda_{QCD} \) from MRST

Try to avoid Isospin symmetry assumptions:

breaking of SU(3) in the sea and SU(2) in “favored” FFs (unless data can not discriminate)

for “unfavored” fragmentations

\[ D_{d+\bar{d}}^{\pi^+} = N D_{u+\bar{u}}^{\pi^+} \]

\[ D_{s}^{\pi^+} = D_{\bar{s}}^{\pi^+} = N' D_{\bar{u}}^{\pi^+} \]

\[ D_{\bar{u}}^{\pi^+} = D_{d}^{\pi^+} \]

\[ D_{u}^{K^+} = D_{s}^{K^+} = D_{d}^{K^+} = D_{\bar{d}}^{K^+} \]
Is it possible to still have a good fit to SIA data?

KRE: S. Kretzer 2000
AKK: S. Albino 2005

large errors at $z > 0.5$
Real challenge: SIDIS “charge discriminated” data

ad-hoc charge separation from Kretzer fails

large z covered

HERMES preliminary, A. Hillenbrand PhD Thesis

crucial data!
Real challenge: RHIC “charge discriminated” data

\[ \mu_F = \mu_R = p_T \]

\[ E \frac{d^3 \sigma}{dp^3} \] [mb / GeV^2]

BRAHMS data
\[ \eta = 2.95 \]

\[ \frac{\text{(data - theory)}}{\text{theory}} \]

Charged pions (at large z)
\( D_{q+\bar{q}}^h(z, Q^2) \) should work for \( \pi^0 \) :

**Neutral pions at PHENIX**

**Neutral pions at STAR**
Cross checks:

charged pions at STAR

not included

STAR data

not included
FFs (pions)

\[ zD_{1}^{\pi^{+}}(z) \]
\[ Q^2 = 10 \text{ GeV}^2 \]

\[ zD_{1}^{\pi^{+}}(z) \]
\[ Q^2 = 10 \text{ GeV}^2 \]
FFs (Kaons)
Uncertainties in PDFs/FFs

How accurate are they?

black box: data → PDFs

data uncertainties → ??
th. uncertainties → ??

fit assumptions → ??
Uncertainties: lagrange multipliers

\[ \Phi(\lambda, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i O_i(\{a_j\}) \]

See how fit to data deteriorates when PDFs/FFs forced (artificially) to give different prediction for \( O(\{a\}) \)

We study truncated moments:

\[ \int_{0.2}^{1} z \, D_i^H(z, Q = 5 \text{ GeV}) \, dz \]
Uncertainties: pions as an example

profiles for the different flavors

Individual profiles by experiment

\[ \chi^2 \] profiles

Tension

Complementarity

Precision

\[ \Delta \chi^2 = 15 (\simeq 2\%) \]

\( u \sim 2\% \)

\( s \sim 10\% \)

Uncertainties: pions as an example
Not so nice:

NLO

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<th>rel. norm.</th>
<th>data points</th>
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Not so nice: 20%!
Protons, anti-protons, and charged hadrons

\[ D_i^p \quad D_i^{\overline{p}} \]

Graph showing the ratio of STAR data to theory against \( p_T \) for different fits.
Protons, anti-protons, and charged hadrons

\[ D_p^+, D_p^-, D_h^+, D_h^- \]

\[ h^\pm \equiv \pi^\pm + K^\pm + p^\pm + \text{res} \]
Protons, anti-protons, and charged hadrons

\[ D_i^p \quad D_i^{\bar{p}} \quad D_i^{h^+} \quad D_i^{h^-} \]

\[ h^\pm \equiv \pi^\pm + K^\pm + p^\pm + res \]
Protons, anti-protons, and charged hadrons

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