Generalized Parton Distributions

A.V. Radyushkin

Physics Department, Old Dominion University
&
Theory Center, Jefferson Lab

Phenomenological Functions

DAs
GPDs
DVCS
DDs
Models

“Old” functions:

- Form Factors
- Usual Parton Densities
- Distribution Amplitudes

“New” functions:

Generalized Parton Distributions (GPDs)

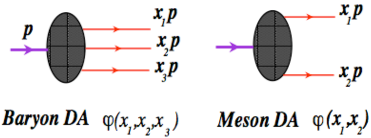
GPDs = Hybrids of

Form Factors, Parton Densities and Distribution Amplitudes

“Old” functions

are limiting cases of “new” functions

Distribution Amplitudes



DAs may be interpreted as

- LC wave functions integrated over transverse momentum
- Matrix elements $\langle 0|\mathcal{O}|p\rangle$ of LC operators

For pion (π^+):

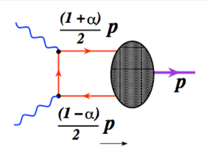
$$\langle 0 \,|\, \bar{\psi}_d(-z/2)\gamma_5\gamma^\mu\psi_u(z/2) \,|\, \pi^+(p)\,\rangle\big|_{z^2=0}$$
$$= ip^\mu f_\pi \int_{-1}^{1} e^{-i\alpha(pz)/2}\varphi_\pi(\alpha)\, d\alpha$$

with $\alpha = x_1 - x_2$ or $x_1 = (1+\alpha)/2,\ x_2 = (1-\alpha)/2$

Factorization Limit of Hard Exclusive Processes

- DAs describe hadrons in situations when pQCD hard scattering approach is applicable to exclusive processes
- Classic application: $\gamma^*\gamma \to \pi^0$ transition form factor

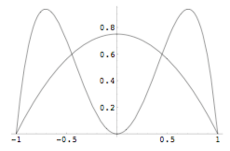
Handbag diagram



Amplitude is proportional to

$$I \equiv \int_{-1}^{1} \frac{\varphi_\pi(\alpha)}{1-\alpha^2}\, d\alpha$$

Two models: asymptotic and Chernyak-Zhitnitsky DA

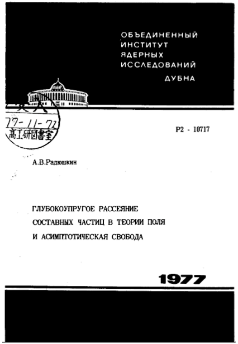


Functional Form:

$$\varphi_\pi^{as}(\alpha) = \tfrac{3}{4}(1-\alpha^2)$$
$$\varphi_\pi^{CZ}(\alpha) = \tfrac{15}{4}\alpha^2(1-\alpha^2)$$

Perturbative QCD for Pion EM Form Factor

- First application of pQCD to exclusive processes


ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

77-11-22
高工研図書室

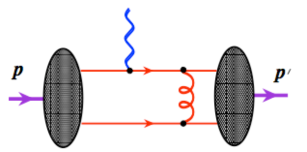
P2 - 10717

А.В.Радюшкин

ГЛУБОКОУПРУГОЕ РАССЕЯНИЕ
СОСТАВНЫХ ЧАСТИЦ В ТЕОРИИ ПОЛЯ
И АСИМПТОТИЧЕСКАЯ СВОБОДА

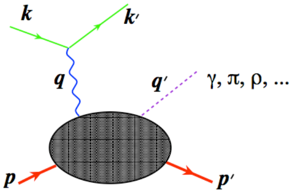
1977

Hard gluon exchange diagram



$$F_\pi^{\rm as}(Q^2) = \frac{8\pi f_\pi^2 \alpha_s(Q^2)}{Q^2}$$

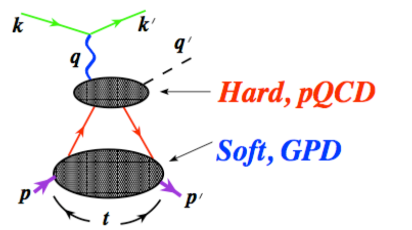
Hard Electroproduction Processes: Path to GPDs



Deeply Virtual Photon and Meson Electroproduction:

Attempt to use perturbative QCD to extract new information about hadronic structure

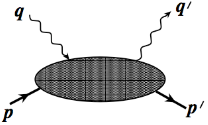
pQCD Factorization



Hard kinematics:

Q^2 is large
$s \equiv (p+q)^2$ is large
$Q^2/2(pq) \equiv x_{\rm Bj}$ is fixed
$t \equiv (p-p')^2$ is small

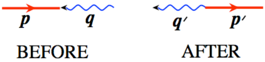
Deeply Virtual Compton Scattering



Kinematics

Total CM energy $s = (q+p)^2 = (q'+p')^2$
LARGE: Above resonance region
Initial photon virtuality $Q^2 = -q^2$
LARGE $(> 1\,\mathrm{GeV}^2)$
Invariant momentum transfer $t = \Delta^2 = (p - p')^2$
SMALL $(\ll 1\mathrm{GeV}^2)$

- Picture in $\gamma^* N$ CM frame

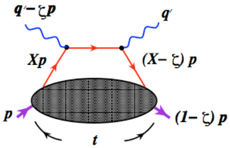


- Virtual photon momentum $q = q' - x_{Bj} p$ has component $-x_{Bj} p$ canceled by momentum transfer Δ
- $\Rightarrow$ Momentum transfer Δ has longitudinal component

$$\Delta^+ = x_{Bj} p^+ \quad , \quad x_{Bj} = \frac{Q^2}{2(pq)}$$

- "Skewed" Kinematics: $\Delta^+ = \zeta p^+$, with $\zeta = x_{Bj}$ for DVCS

Parton Picture for DVCS



Nonforward parton distribution

$\mathcal{F}_\zeta(X;t)$ depends on
X : fraction of p^+
ζ : skeweness
t : momentum transfer

- In forward $\Delta = 0$ limit

$$\mathcal{F}^a_{\zeta=0}(X, t=0) = f_a(X)$$

- Note: $\mathcal{F}^a_{\zeta=0}(X, t=0)$ comes from Exclusive DVCS Amplitude, while $f_a(X)$ comes from Inclusive DIS Cross Section
- Zero skeweness $\zeta = 0$ limit for nonzero t corresponds to nonforward parton densities

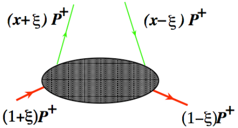
$$\mathcal{F}^a_{\zeta=0}(X, t) = \mathcal{F}^a(X, t)$$

- Local limit: relation to form factors

$$(1-\zeta/2)\int_0^1 \mathcal{F}^a_\zeta(X,t)\,dX = F^a_1(t)$$

Off-forward Parton Distributions

Momentum fractions taken wrt average momentum $P = (p + p')/2$



4 functions of x, ξ, t:

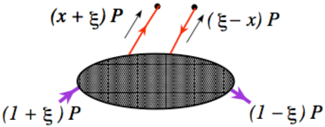
$H, E, \widetilde{H}, \widetilde{E}$

wrt hadron/parton helicity flip

$+/+, -/+, +/-, -/-$

- Skeweness $\xi \equiv \Delta^+/2P^+$ is $\xi = x_{Bj}/(2 - x_{Bj})$
- 3 regions:

$$
\begin{array}{rl}
\xi < x < 1 & \sim \text{ quark distribution} \\
-1 < x < -\xi & \sim \text{ antiquark distribution} \\
-\xi < x < \xi & \sim \text{ distribution amplitude for } N \to \bar{q}qN'
\end{array}
$$



Relation to Form Factors

GPDs, part II

DAs
GPDs
DVCS
DDs
Models

- Nucleon helicity non-flip distributions:

$$\sum_a q_a \int_{-1}^{1} H^a(x,\xi;t)\,dx = F_1(t)$$

- Helicity flip distributions $E^a(x,\xi;t)$ are related to $F_2(t)$ form factor:

$$\sum_a q_a \int_{-1}^{1} E^a(x,\xi;t)\,dx = F_2(t)$$

- E comes with Δ_μ factor: it is invisible in DIS (then $\Delta = 0$)
- BUT: $t = 0, \xi = 0$ limit of E exists: $E^a(x, \xi = 0; t = 0) \equiv e^a(x)$
- Proton anomalous magnetic moment κ_p is given by

$$\sum_a q_a \int_{-1}^{1} e^a(x)\,dx = \kappa_p$$

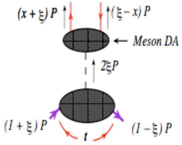
- Total orbital momentum contribution into the proton spin involves both $f(x)$ and $e(x)$

$$L_q = \frac{1}{2} \sum_a \int_{-1}^{1} x\,[f^a(x) + e^a(x)]\,dx$$

- NB: Only valence quarks contribute to κ_N (non-singlet operator)
  L involves also sea quarks (singlet operator)

Two approaches are used:

- Direct calculation in specific dynamical models: bag model, chiral soliton model, light-cone formalism , etc.
- Phenomenological construction based on relation of GPDs to usual parton densities $f_a(x), \Delta f_a(x)$ and form factors $F_1(t), F_2(t), G_A(t), G_P(t)$
- Formalism of Double Distributions is often used to get self-consistent phenomenological models

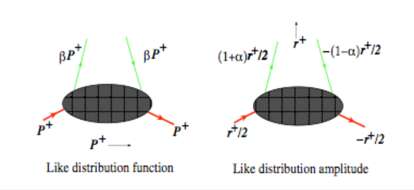


Meson exchange contribution

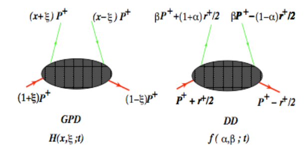
- GPD $\widetilde{E}(x,\xi;t)$ is related to pseudoscalar form factor $G_P(t)$ and is dominated for small t by pion pole term $1/(t-m_\pi^2)$
- Dependence of $\widetilde{E}(x,\xi;t)$ on x is given by pion distribution amplitude $\varphi_\pi(\alpha)$ taken at $\alpha = x/\xi$

Double Distributions

“Superposition” of P^+ and r^+ momentum fluxes



Connection with OFPDs



Basic relation between fractions

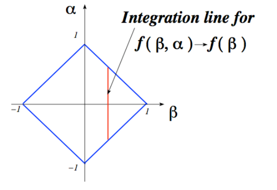
$$x = \beta + \xi\alpha$$

- Forward limit $\xi = 0, t = 0$ gives usual parton densities

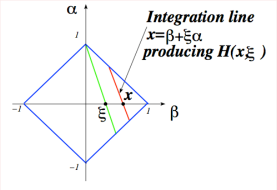
$$\int_{-1+|\beta|}^{1-|\beta|} f_a(\beta, \alpha; t = 0)\, d\alpha = f_a(\beta)$$

Getting GPDs from DDs

DDs live on rhombus $|\alpha| + |\beta| \leq 1$



Converting DDs into GPDs



"Munich" symmetry:

$$f_a(\beta, \alpha; t) = f_a(\beta, -\alpha; t)$$

GPDs $H(x, \xi)$ are obtained from DDs $f(\beta, \alpha)$

by scanning DDs
at ξ-dependent angles

$\Rightarrow$ DD-tomography

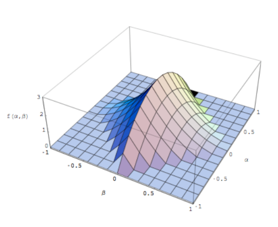
Illustration of DD→GPD conversion

Factorized model for DDs:

($\sim$ usual parton density in β-direction) $\otimes$ ($\sim$ distribution amplitude in α-direction)

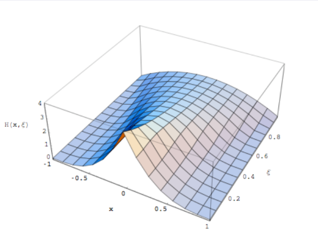
Toy model for double distribution

$$f(\beta,\alpha) = 3[(1-|\beta|)^2 - \alpha^2]\,\theta(|\alpha|+|\beta| \le 1)$$



- Corresponds to toy "forward" distribution $f(\beta) = (1-|\beta|)^3$

GPD $H(x,\xi)$ resulting from toy DD

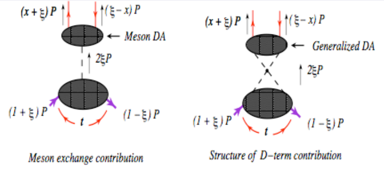


- For $\xi = 0$ reduces to usual parton density
- For $\xi = 1$ has shape like meson distribution amplitude

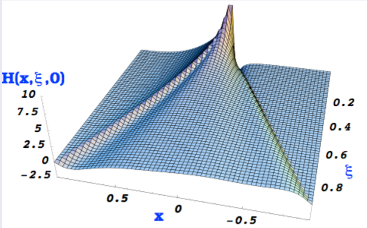
Realistic Model for GPDs based on DDs

- DD modeling misses terms invisible in the forward limit:
  - Meson exchange contributions
  - D-term, which can be interpreted as σ exchange
- Inclusion of D-term induces nontrivial behavior in $|x| < \xi$ region

Meson and D-term terms



DD + D-term model



- Profile model for DDs: $f_a(\beta,\alpha) = f_a(\beta) h_a(\beta,\alpha)$

Normalization

$$\int_{-1}^{1} d\alpha\, h(\beta,\alpha) = 1$$

Guarantees forward limit

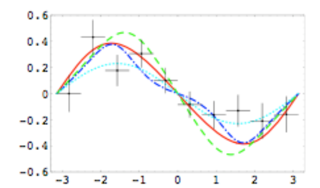
$$\int_{-1}^{1} d\alpha\, f(\beta,\alpha) = f(\beta)$$

Sensitivity to DD Profile Width

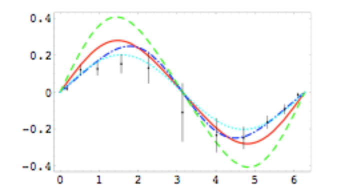
- General form of model profile $h(\beta,\alpha) = \frac{\Gamma(2+2b)}{2^{2b+1}\Gamma^2(1+b)} \frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$
- Power b is parameter of the model
- $b=\infty$ gives $h(\beta,\alpha)=\delta(\alpha)$ and $H(x,\xi)=f(x)$
- Single-Spin Asymmetry

$$A_{LU}(\varphi) = \frac{d\sigma^{\uparrow}-d\sigma^{\downarrow}}{d\sigma^{\uparrow}+d\sigma^{\downarrow}}$$

HERMES Data



JLab CLAS Data



- Models:

Red:	$b_{\text{val}} = 1$	$b_{\text{sea}} = \infty$
Green:	$b_{\text{val}} = 1$	$b_{\text{sea}} = 1$
Blue:	$b_{\text{val}} = \infty$	$b_{\text{sea}} = \infty$

Models with Regge behavior of $f(\beta)$

- Szczepaniak et al: constructed model equivalent to

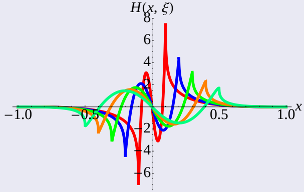
$$H(x,\xi) = x \int_\Omega d\beta \, \frac{f(\beta)}{\beta(1-|\beta|)} \, \delta(x-\beta-\xi\alpha)$$

- Corresponds to $b=0$ flat profile $h(\beta,\alpha) = \frac{1}{2(1-|\beta|)}$
- Regge ansatz $f(\beta) \sim |\beta|^{-a}$ gives singularity at border point $x=\xi$

$$H(x,\xi)|_{x\sim\xi} \sim \left|\frac{x-\xi}{1-\xi}\right|^{-a} \quad \text{Bad}: \; A_{\text{DVCS}} \sim \int_{-1}^{1} \frac{dx}{x-\xi+i\epsilon} H(x,\xi)$$

- Flat profile follows from hard $1/k_i^2$ behavior of parton-hadron amplitude $T(p_1,p_2;k_1,k_2)$
- Changing to faster $(1/k_i^2)^{b+1}$ fall-off gives b-profile
- No singularities with $b \geq a$

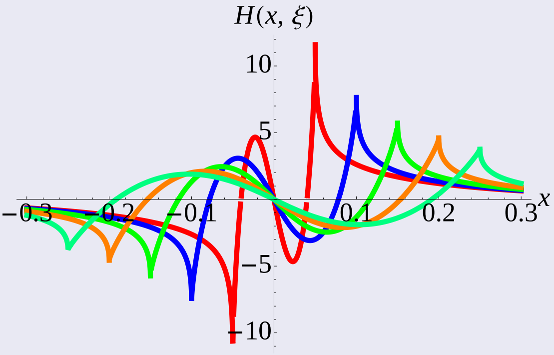
b=1 DD with $a=0.5$ Regge PDFs



$\xi = 0.1, 0.2, 0.3, 0.4, 0.5$

Results for $n = 1$ profile $\sim (1-\beta)^2 - \alpha^2$

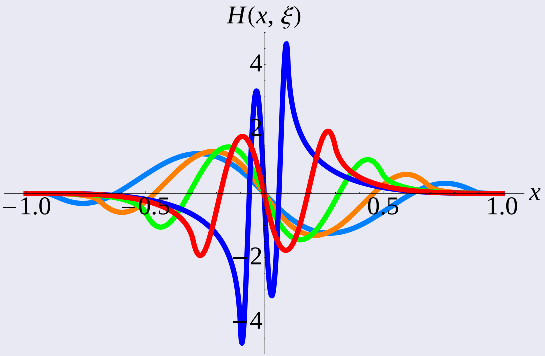
$H(x,\xi);\ \xi = 0.05, 0.1, 0.15, 0.2, 0.25$



Results for $n = 2$ profile $\sim [(1-\beta)^2 - \alpha^2]^2$

DAs
GPDs
DVCS
DDs
Models

$H(x, \xi)$; $\xi = 0.05, 0.1, 0.15, 0.2, 0.25$



Summary

1. Distribution Amplitudes
2. Generalized Parton Distributions
3. Deeply Virtual Compton Scattering
4. Double Distributions
5. Models