Nuclear Physics with Electromagnetic Probes

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Lecture 2
Hampton University Graduate School
2012
Course Outline

• Introduction to EM probes.
  - Electron and photon interactions with nuclei.
  - Beams and detectors.
• Nuclear Physics with electrons and photons.
  - Elastic electron-nucleus scattering.
  - Charge, matter and momentum distributions.
  - Single nucleons
  - Correlated nucleon pairs.
  - Nucleon modification in nuclei
  - Hadronization in nuclei
  - Nuclear transparency
Why use electrons and photons?

- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak ($\alpha = 1/137$)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

BUT:
- Cross sections are small
- Electrons radiate
It’s all photons!

• An electron interacts with a nucleus by exchanging a single* virtual photon.

Real photon:
Momentum $q = \text{energy } \nu$
Mass $= Q^2 = |q|^2 - \nu^2 = 0$

Virtual photon:
Momentum $q > \text{energy } \nu$
$Q^2 = -q_\mu q^\mu = |q|^2 - \nu^2 > 0$
Virtual photon has mass!

($\nu$ and $\omega$ are both used for energy transfer)
(e,e') spectrum

Generic Electron Scattering at fixed momentum transfer

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Experimental goals:

- **Elastic scattering**
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF

- **Quasielastic (QE) scattering**
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency

- **Deep Inelastic Scattering (DIS)**
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei
Program central to all of nuclear science

Nature of Confinement

Quark-Gluon Structure Of Nucleons and Nuclei

Correlations
n-radii: N ≠ Z Hypernuclei
Hadrons in-medium
Effective NN (+ HN) force

Precise few-nucleon calculations

Exotic mesons and baryons

...n-stars

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Energy vs length

Select spatial resolution and excitation energy independently
- Photon energy $\nu$ determines excitation energy
- Photon momentum $q$ determines spatial resolution: $\lambda \approx \frac{\hbar}{q}$

Three cases:
- **Low $q$**
  - Photon wavelength $\lambda$ larger than the nucleon size ($R_p$)
- **Medium $q$:** $0.2 < q < 1$ GeV/c
  - $\lambda \sim R_p$
  - Nucleons resolvable
- **High $q$:** $q > 1$ GeV/c
  - $\lambda < R_p$
  - Nucleon structure resolvable
Types of photon Polarisation

• Both real and virtual photons can have polarisation
  - Real photons: transverse pol only
  - Virtual photons: longitudinal or transverse

• Determining azimuthal distribution of reaction products around these polarisation directions gives powerful information (see later)

**Linear polarisation**
(Electric field vector oscillates in a plane)

**Circular polarisation**
(Electric field rotates clockwise or anticlockwise)

Virtual photons can also have a **longitudinal polarisation** component - related to coulomb (charge) scattering
Real and virtual photon coupling

Photon-nucleus EM interaction: Photon EM field 4-vector potential couples to the nuclear charge and current

\[ J_\mu A^\mu \]

- \( \rho \Phi \)
- (e/m)(p x A)
- \( \bar{s} x (q x A) \)

Coulomb
Convection
spin

Longitudinal
 Virtual photon only

Transverse
 Real and Virtual Photons
Inclusive electron scattering \((e,e')\)

\[ k'\mu = (E', \vec{k}') \]

\[ q^\mu = (\omega, \vec{q}) \]

\[ p^\mu = (M, \vec{0}) \]

Lab frame
kinematics

\[ k^\mu = (E, \vec{k}) \]

\[ q^\mu = k^\mu - k'^\mu \]

Invariants:

\[ p^\mu p_\mu = M^2 \]

\[ p_\mu q^\mu = M\omega \]

\[ Q^2 = -q^\mu q_\mu = |\vec{q}|^2 - \omega^2 \]

\[ W^2 = (q^\mu + p^\mu)^2 = p'_\mu p'^\mu \]

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Formalism

- Inclusive scattering:
  - measure scattering angle $\theta_e$ and energy $E'_e$ ($\omega = E_e - E'_e$) and the cross section $d\sigma / d\Omega_e d\omega$

- One photon exchange:

$$\mathcal{M}_n = \frac{e^2}{q^2} \bar{u}(k') \gamma_\mu u(k) < n|J_\mu(0)|p, S >$$

$$d\sigma = \frac{1}{4MK} \sum_n |\mathcal{M}_n|^2 (2\pi)^4 \delta^4(p + q - p') \frac{d^3 k'}{(2\pi)^3 2E'}$$

$$= \frac{|k'|}{MEQ^4} \frac{\alpha^2}{L^{\mu\nu} H_{\mu\nu} d\omega d\Omega}$$

$L^{\mu\nu}$ and $H_{\mu\nu}$ are the lepton and hadron tensors
More Formalism

Lepton tensor known (QED):

\[ L_{\mu\nu} = \sum_{s'} (\bar{u}(k', s')\gamma_{\mu} u(k, s))^* (\bar{u}(k', s')\gamma_{\nu} u(k, s)) \]

\[ = 2(k_{\mu} k'_{\nu} + k_{\nu} k'_{\mu}) + q^2 g_{\mu\nu} + 2i m_1 \epsilon_{\mu\nu\alpha\beta} q^\alpha s_i^\beta \]

Spin dependent

Hadron tensor unknown:

\[ H_{\mu\nu} = \frac{1}{4\pi} \sum_n <p, S|\hat{J}_\mu(0)|n> <n|\hat{J}_\nu(0)|p, S> (2\pi)^4 \delta^4(p + q - p') \]

\[ = \frac{1}{4\pi} \int d^4\xi \exp(iq \cdot \xi) <p, S| [\hat{J}_\mu(\xi), \hat{J}_\nu(0)] |p, S> \]

Nuclear current operators

Two approaches:
1. Calculate \( J_{\mu} \) from models or
2. Create most general hadron tensor from tensors (bilinear Lorentz vectors) \( A_{\mu\nu}^i = a_{\mu} b_{\nu} \) and Lorentz scalars \( S_i = a_{\mu} b^\mu : H_{\mu\nu} = \sum A_{\mu\nu}^i F_i(S_1, S_2, ...) \)
Most general hadron tensor:

EM current conservation $\Rightarrow \quad q^\mu H_{\mu\nu} = H_{\mu\nu} q^\nu = 0$
parity conservation $\Rightarrow \quad H_{\mu\nu}(p, q) = H_{\mu\nu}(\tilde{p}, \tilde{q})$
Time reversal symmetry $\Rightarrow \quad H_{\mu\nu}(p, q) = H_{\mu\nu}^*(\tilde{p}, \tilde{q})$

Available independent four vectors for (e,e'):

- target momentum $p_\mu$
- photon momentum $q_\mu$
- target polarization $S_\mu$

\[ H^{\text{no spin}}_{\mu\nu}(p, q) = F_1 \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) + F_2 \frac{1}{p \cdot q} \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \]

\[ H^{\text{spin}}_{\mu\nu}(p, q) = \frac{M}{p \cdot q} \left[ \epsilon_{\mu\nu\alpha\beta} q^\alpha S^\beta (g_1 + g_2) - \epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta g_2 \right] \]

\text{Spin dependent}

Four independent structure functions

- Spin independent: $F_1(Q^2, x)$ and $F_2(Q^2, x)$
- Spin dependent: $g_1(Q^2, x)$ and $g_2(Q^2, x)$

(arguments also written as $(Q^2, \omega)$ or $(Q^2, \nu)$)

\[ x = \frac{Q^2}{2 p \cdot q} = \frac{Q^2}{2 M \nu} \]
Elastic cross section ($p'^2 = m^2$)

Recoil factor

$$\frac{d\sigma}{d\Omega} = \sigma_M \left( \frac{E'}{E} \right) \left( \left[ F_1^2(Q^2) + \frac{Q^2}{4M^2} \kappa^2 F_2^2(Q^2) \right] + \frac{Q^2}{2M^2} [F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2 \frac{\theta}{2} \right)$$

$$= \sigma_M \left( \frac{E'}{E} \right) \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau \tan^2 \frac{\theta}{2} G_M^2(Q^2) \right]$$

$$= \sigma_M \left( \frac{E'}{E} \right) \left[ \frac{Q^4}{q^4} R_L(Q^2) + \left( \frac{Q^2}{2q'^2} + \tan^2 \frac{\theta}{2} \right) R_T(Q^2) \right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2 \cos^2 \left( \frac{\theta_x}{2} \right)}{4E^2 \sin^4 \left( \frac{\theta_x}{2} \right)}$$

$F_1, F_2$: Dirac and Pauli form factors

$G_E, G_M$: Sachs form factors (electric and magnetic)

$$G_E(Q^2) = F_1(Q^2) - \tau \kappa F_2(Q^2) \quad \tau = \frac{Q^2}{4M^2}$$

$$G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2) \quad \kappa = \text{anomalous magnetic moment}$$

$R_L, R_T$: Longitudinal and transverse response fn
Electron-nucleus interactions

I. Elastic

\[ \frac{Q^2}{2A} \]

II. Quasielastic

\[ \frac{Q^2}{2m} \]

\[ \frac{Q^2}{2m} + 300 \text{ MeV} \]

III. Nucleus

DEEP INELASTIC "EMC"

\( \omega \)

III. Nucleus

DEEP INELASTIC "QUARKS"

\( \omega \)

PROTON

Elastic

\[ \frac{Q^2}{2m} \]

\( \Delta \)

\( N^* \)
Electrons as Waves

Scattering process is quantum mechanical

De Broglie wavelength: \[ \lambda = \frac{h}{p} \]

Electron energy: \[ E_e \approx pc \]

\( \lambda \) resolving "scale": \[ \lambda = \frac{2\pi(197 \text{ MeV} \cdot \text{fm})}{E_e} \]
Analogy between elastic electron scattering and diffraction
Simple analogy for elastic electron scattering....

**Classical Fraunhofer Diffraction**

Amplitude of wave at screen:

\[ \Phi \propto \int_0^a \int_0^{2\pi} \exp(ibr \cos \phi) r d\phi dr \]
Classical Fraunhofer Diffraction

Intensity: \( \Phi^2 \propto \left( \frac{J_1((2\pi a / \lambda)\sin \theta)}{\sin \theta} \right)^2 \)

Minima occur at zeroes of Bessel function. 1\textsuperscript{st} zero: \( x = 3.8317 \)

...some algebra...

Hence \( 2a \approx \frac{1.22\lambda}{\sin \theta} \)
Excursion: Babinet’s principle

Screen with apertures

Complementary screen

Patterns appear the same
Example: $^{30}\text{Si}(e,e')$

$1^{\text{st}}$ minimum = 1.3 fm$^{-1}$
$\Rightarrow \theta = 32.8^\circ$

Electron energy = 454.3 MeV
$\Rightarrow \lambda = 2.73$ fm

Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm
(from fit to entire curve)

$(1 \text{ fm}^{-1} = 197 \text{ MeV/c})$
I. Elastic Electron Scattering from Nuclei (done formally)

Fermi’s Golden Rule

\[ \frac{d\sigma}{dQ} = \frac{2\pi}{\hbar} |M_{fi}|^2 D_f \]

- \( M_{fi} \): scattering amplitude
- \( D_f \): density of the final states (or phase factor)

\[ M_{fi} = \int \Psi_f^* V(x) \Psi_i d^3x \]

\[ = \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x \]

\[ = \int e^{i q \cdot x} V(x) d^3x \]

Plane wave approximation for incoming and outgoing electrons
Born approximation (interact only once)
I. Elastic Electron Scattering from (spin-0) Nuclei

Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, \( \rho(x) \),

\[
V(x) = -\frac{Ze^2}{4\pi\epsilon_0} \int \frac{\rho(x')}{|x-x'|} d^3x'
\]

\[
M_{fi} = -\frac{Ze^2}{4\pi\epsilon_0} \int e^{iq \cdot x} \int \frac{\rho(x')}{|x-x'|} d^3x' d^3x
\]

\[
= -\frac{Ze^2}{4\pi\epsilon_0} \int e^{iq \cdot x} \left[ \int \frac{\rho(x')}{R} d^3x' \right] d^3R
\]

\[
= -\frac{Ze^2}{4\pi\epsilon_0} \int \frac{e^{iq \cdot R}}{R} d^3R \int e^{iq \cdot x'} \rho(x') d^3x'
\]

\[
F(q) = \int e^{iq \cdot x'} \rho(x') d^3x'
\]

Charge form factor \( F(q) \) is the Fourier transform of the charge distribution \( \rho(x) \)
I. Elastic Electron Scattering from (spin-0) Nuclei

Form factor and cross section

- For point-like particle, $\rho(x') = \delta(x')$ and $F(q) = 1 \rightarrow$ Rutherford-like scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}$$

- Scattering from a charge distribution

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(q)|^2$$

$F=1$ for point particle

$1 \text{ fm}^{-1} = 197 \text{ MeV/c}$
I. Elastic (e,e’) Scattering ⇒ charge distributions

Elastic electron scattering measured for many nuclei over a wide range of Q2 (mainly at Saclay in the 1970s)

Measured charge distributions agree well with mean field theory calculations.
Lead $^{208}\text{Pb}$ Radius Experiment: PREX

$^{208}\text{Pb}(e,e')$ Elastic Scattering

Parity Violating Asymmetry

$E_{\text{beam}} = 1 \text{ GeV}$

$\theta_e = 5^\circ$

$I_{\text{beam}} = 60 \mu\text{A}$

Spokespersons

• Paul Souder
• Krishna Kumar
• Guido Urciuoli
• Robert Michaels

Slides courtesy of R. Michaels
Parity Violating Asymmetry

\[ A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx 10^{-6} \]

\[ \sigma \approx \gamma + Z^0 \]

Applications of PV at Jefferson Lab

- Nucleon Structure (strangeness) -- HAPPEX / G0
- Standard Model Tests (\( \sin^2 \theta_W \)) -- e.g. Qweak
- Nuclear Structure (neutron density) : PREX
Lead $^{208}$Pb Radius Experiment: PREX

Z$^0$ of Weak Interaction:
Clean Probe Couples Mainly to Neutrons

Left-Right Cross section asymmetry (simplified):

$$A = \left( \frac{d\sigma}{d\Omega} \right)_R - \left( \frac{d\sigma}{d\Omega} \right)_L = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}} \left[ 1 - 4\sin^2 \theta_w - \frac{F_W(Q^2)}{F_P(Q^2)} \right]$$

w/ Coulomb distortions:

$$\frac{dA}{A} = 3\% \rightarrow \frac{dR^n}{R_n} = 1\%$$

$R_n$ = neutron matter radius

$F_W(Q^2)$: $^{208}$Pb Weak Form Factor

$F_P(Q^2)$: $^{208}$Pb Charge Form Factor

(T.W. Donnelly, J. Dubach, I. Sick)

(C. Horowitz)

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<th>proton</th>
<th>neutron</th>
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<tr>
<td>Weak charge</td>
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PREX & Neutron Stars

(C.J. Horowitz, J. Piekarewicz)

- $R_n$ calibrates EOS of Neutron Rich Matter
  - Crust Thickness
  - Explain Glitches in Pulsar Frequency?

- Combine PREX $R_n$ with Obs. Neutron Star Radii
  - Phase Transition to “Exotic” Core?
  - Strange star? Quark Star?

- Some Neutron Stars seem too Cold
  - Cooling by neutrino emission (URCA)
  - $R_n - R_p > 0.2$ fm $\rightarrow$ URCA probable, else not
PREX in Hall A at JLab

Spectrometers

Lead Foil Target

Hall A

Pol. Source

CEBAF

Position Monitors
Intensity Monitors
Luminosity Monitors
Detectors
Modulation Coils
High Resolution Spectrometers

Spectrometer Concept:
 Resolve Elastic

Pb 1st excited state at 2.6 MeV

target
Dipole
Quad
Inelastic
Elastic
detector

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Lead Target - melts easily

Diamond Backing:
- High Thermal Conductivity
- Negligible Systematics

Liquid Helium Coolant

Beam, rastered 4 x 4 mm
Target Assembly
PRex Results

\[ APV = 656 \pm 60 \text{ (stat)} \pm 14 \text{ (syst)} \text{ ppb} \]

Look for PRex II, coming soon to a lab near you!
Deuteron Elastic Scattering

- Only bound 2 nucleon system
- Prime testing ground for models: where is the limit for the description in terms of nucleons and mesons
- Questions:
  - How does the photon interact with a BOUND nucleon?
  - Effects of meson interactions?
  - Final State Interactions?
  - Deuteron wave function at small distance?
  - Where do quarks come in?

Review Articles:
Elastic $D(e,e')$

\[ \frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \left[ A(Q^2) + B(Q^2) \tan^2(\theta/2) \right] \]

\[ A = G_C^2 + \frac{2}{3} \eta G_M^2 + \frac{8}{9} \eta^2 G_Q^2 \]

\[ B = \frac{4}{3} \eta(1 + \eta)G_M^2 \]

\[ G_C \quad \text{Charge form factor} \]

\[ G_M \quad \text{Magnetic form factor} \]

\[ G_Q \quad \text{Quadrupole form factor} \]

\[ T_{20} = -\frac{8}{9} \eta^2 G_Q^2 + \frac{8}{3} \eta G_C G_Q + \frac{2}{3} \eta G_M^2 \left[ \frac{1}{2} + (1 + \eta) \tan^2(\theta/2) \right] \frac{1}{\sqrt{2} [A + B \tan^2(\theta/2)]} \]

\[ \eta = Q^2/4M_D^2 \]

Note: $A$ and $B$ are linear combinations of $F_1$ and $F_2$ or of $R_L$ and $R_T$. 

To measure the tensor polarization $T_{20}$, use: $\bar{d}(e,e')d$

\[ d(e,e'd) \]
Calculation: H. Arenhövel et al. PRC 61, 034002

\[ 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \]

\[ Q^2 \]

\[ T_{20} \]

Experiments:
- Hall C: D. Abbott et al. PRL 82 (1999) 1379
- Hall C: D. Abbott et al. PRL 84 (2000) 5053
5.2. Quark-gluon approaches to the N-N interaction and the deuteron

The issue of quark-gluon vs. hadronic degrees of freedom was discussed in Sec. 2. In this section we focus on the high-momentum transfer NN interaction, and the high-momentum structure of the deuteron. It is generally accepted for these reactions that only the leading \(qq\) Fock state of the nucleon needs to be considered. As shown in [10], high energy hadron-hadron reactions which can proceed via quark exchange have cross sections an order of magnitude larger than reactions which proceed via gluon exchange or quark-antiquark annihilation. This leads to the conclusion that the high-energy NN reaction is dominated by quark interchange diagrams, such as that no MEC contributions with MEC

Rel. Calculations in Hamiltonian dynamics:

IMII and IM+EII

Y.Huang and W.N.Polyzou
PRC80 (2009) 025503

Rel. Calculations in propagator dynamics:

Need \(\rho\pi\gamma\) diagram - not well constrained

D.R.Phillips et al. PRC72 (2005) 014006
Deuteron Elastic Scattering Summary:

- Non-Relativistic models fail at the highest $Q^2$
- Relativistic models successfully describe Deuteron form factors
- MEC contributions are very important
- $\rho\pi\gamma$ exchange current important and not well constrained

Photon couples to nucleon

Meson Exchange Current

$\rho\pi\gamma$ Meson Exchange Current
Elastic Scattering Summary

• Beautiful measurements of the nuclear charge distribution
  • And first measurements of the nuclear matter radius
• Beautiful measurements of the deuteron
  • A, B and Tensor polarization
  • Allows separation of charge, magnetic and quadrupole form factors

\[ m=0 \quad \text{and} \quad m=\pm 1 \]