Nuclear Physics with Electromagnetic Probes
Lectures 3 & 4

Lawrence Weinstein
Old Dominion University
Norfolk, VA USA

Hampton University Graduate School
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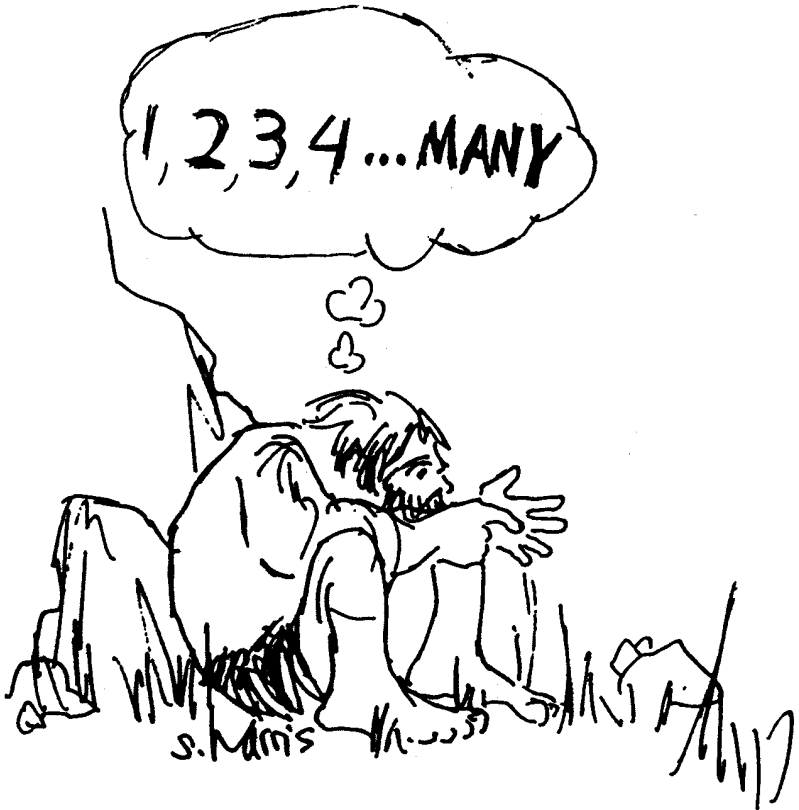
Course Outline

- Lecture 1: Beams and detectors
- Lecture 2: Elastic Scattering:
  - Charge and mass distributions
  - Deuteron form factors
- Lectures 3+4:
  - Single nucleon distributions in nuclei
    - Energy
    - Momentum
  - Correlated nucleon pairs.
- Lecture 5: Quarks in Nuclei
  - Nucleon modification in nuclei
  - Hadronization
  - Color transparency

Comprehensive Theory Overview

Nuclear Theory, circa 1980

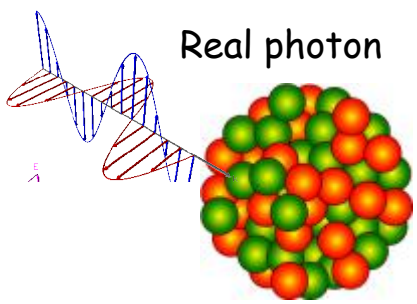


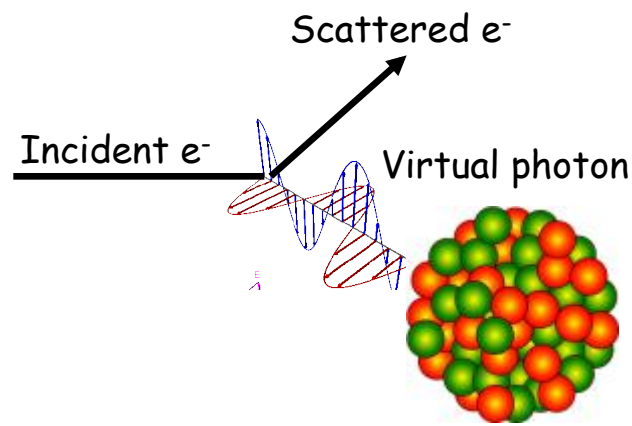
Nuclear Theory - circa 2000

Nuclear Theory - today: 1, 2, 3, ... 12, ... many

It's all photons!

- An electron interacts with a nucleus by exchanging a single* virtual photon.





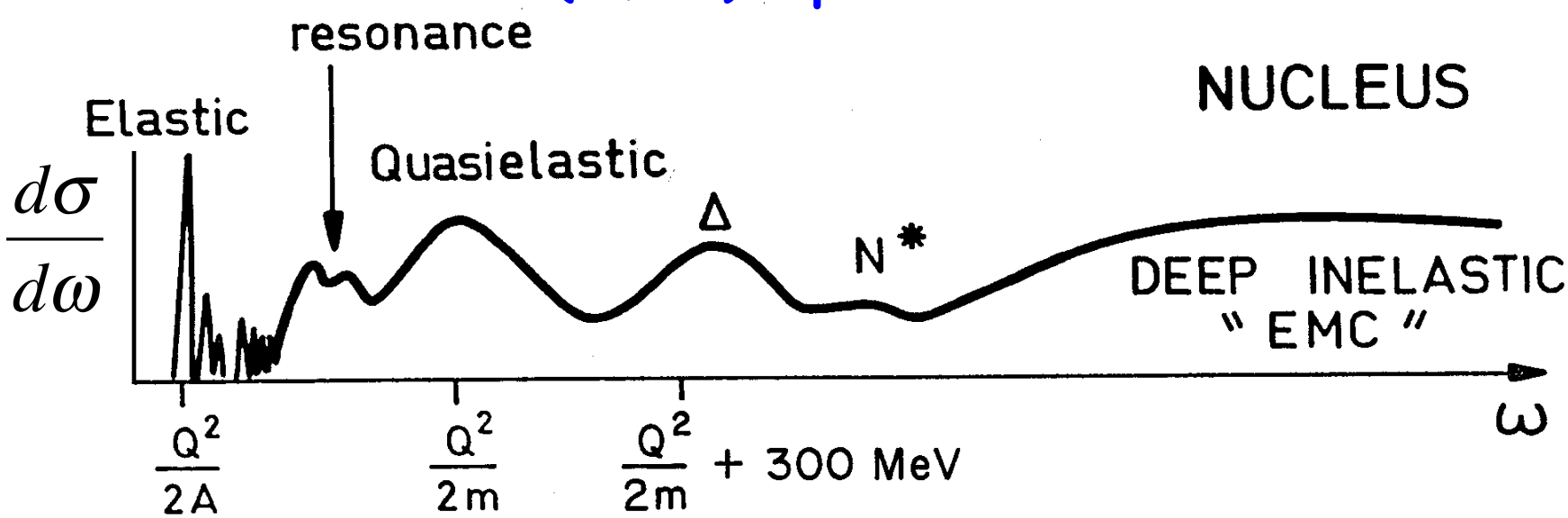
Real photon:
Momentum q = energy ν
Mass = $Q^2 = |\boldsymbol{q}|^2 - \nu^2 = 0$

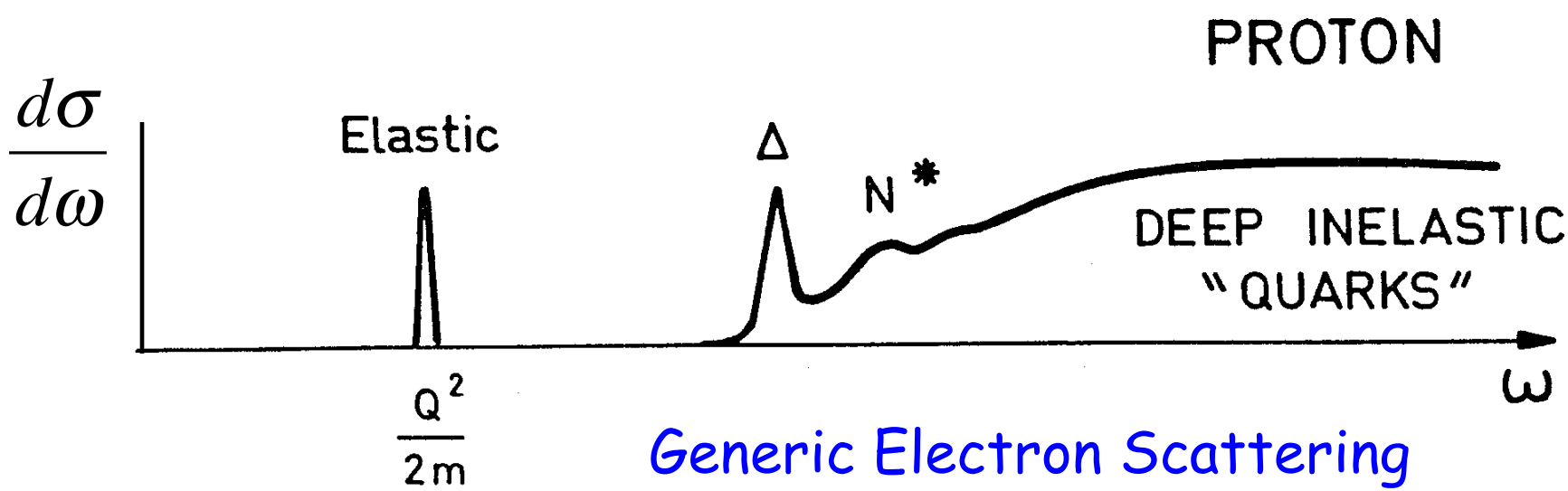
$$\lambda = \frac{\hbar}{|\vec{q}|}$$

Virtual photon:
Momentum $q >$ energy ν
$Q^2 = - q_\mu q^\mu = |\boldsymbol{q}|^2 - \nu^2 > 0$
Virtual photon has mass!

(ν and ω are both used for energy transfer)

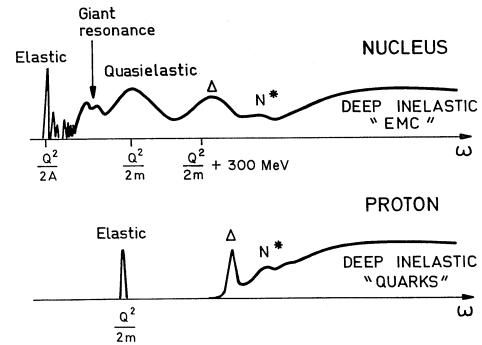
(e,e') spectrum





Generic Electron Scattering
at fixed momentum transfer

Experimental goals:



- Elastic scattering
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF
- Quasielastic (QE) scattering
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency
- Deep Inelastic Scattering (DIS)
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei

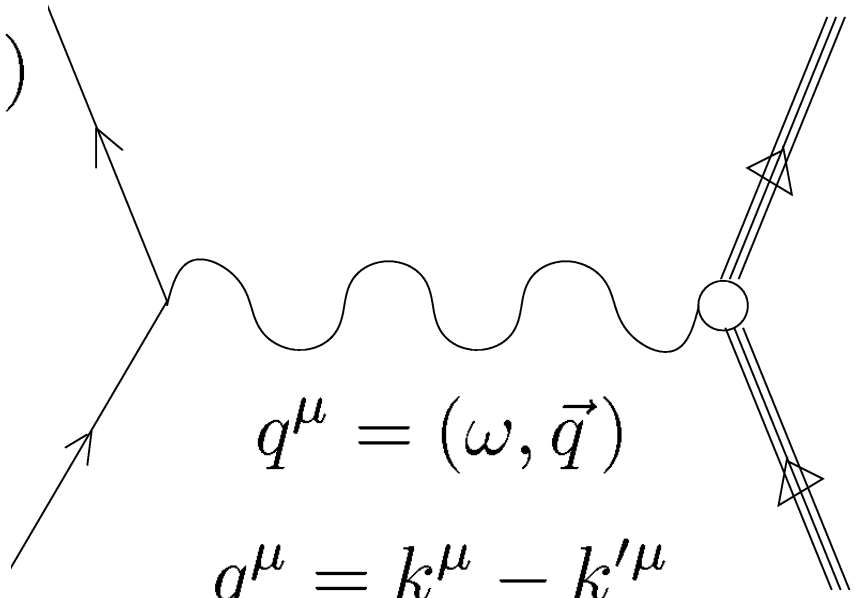
Inclusive electron scattering (e,e')

$k'^{\mu} = (E', \vec{k}\,')$

$p'^{\mu} = (E_p, \vec{p}_p)$

(not detected)

Lab frame kinematics



$q^{\mu} = (\omega, \vec{q})$

$k^{\mu} = (E, \vec{k}\,)$

$q^{\mu} = k^{\mu} - k'^{\mu}$

$p^{\mu} = (M, \vec{0}\,)$

Invariants:

$$p^{\mu} p_{\mu} = M^2 \qquad p_{\mu} q^{\mu} = M\omega$$

$$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2 \qquad W^2 = (q^{\mu} + p^{\mu})^2 = p'_{\mu} p'^{\mu}$$

(e,e′) Elastic cross section ($p'^2 = M^2$)

Recoil factor — Form factors

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left\{\left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\kappa^2 F_2^2(Q^2)\right] + \frac{Q^2}{2M^2}[F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right\}$$

$$= \sigma_M \frac{E'}{E}\left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau\tan^2\frac{\theta}{2} G_M^2(Q^2)\right]$$

$$= \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4} R_L(Q^2) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right) R_T(Q^2)\right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2\cos^2\left(\frac{\theta_e}{2}\right)}{4E^2\sin^4\left(\frac{\theta_e}{2}\right)}$$

For inelastic scattering:
$$R_L(Q^2) \rightarrow R_L(Q^2, \nu)$$

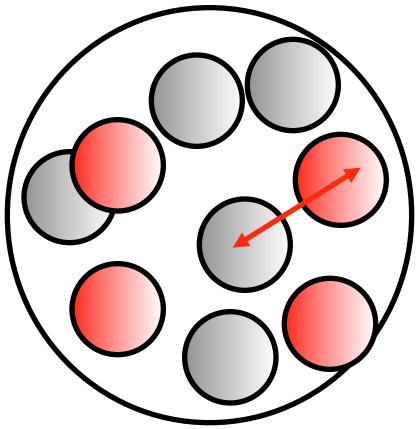
nucleons

- F_1, F_2: Dirac and Pauli form factors
- G_E, G_M: Sachs form factors (electric and magnetic)
  - $G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$ $\quad \tau = Q^2/4M^2$
  - $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$ $\quad$ (more standard definition of F_1 and F_2)
- R_L, R_T: Longitudinal and transverse response fn

Notes on form factors

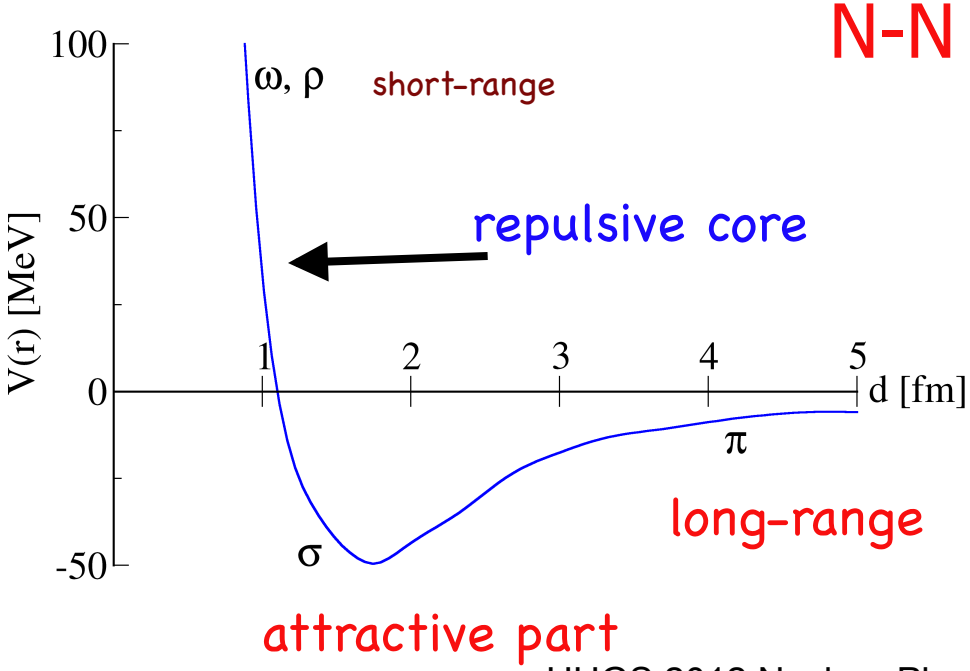
- G_E, G_M, F_1 and F_2 refer to nucleons
  - $F_1^p(0) = 1$, $F_2^p(0) = \kappa_p = 1.79$
  - $F_1^n(0) = 0$, $F_2^n(0) = \kappa_n = -1.91$
  - $G_E^p(0) = 1$, $G_M^p(0) = 1 + \kappa_p = 2.79$
  - $G_E^n(0) = 0$, $G_M^n(0) = \kappa_n = -1.91$
- R_L and R_T refer to nuclei

Structure of the nucleus

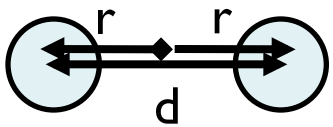


- nucleons are bound
  - energy (*E*) distribution
  - shell structure
- nucleons are not static
  - momentum (*k*) distribution

determined by the
N-N potential

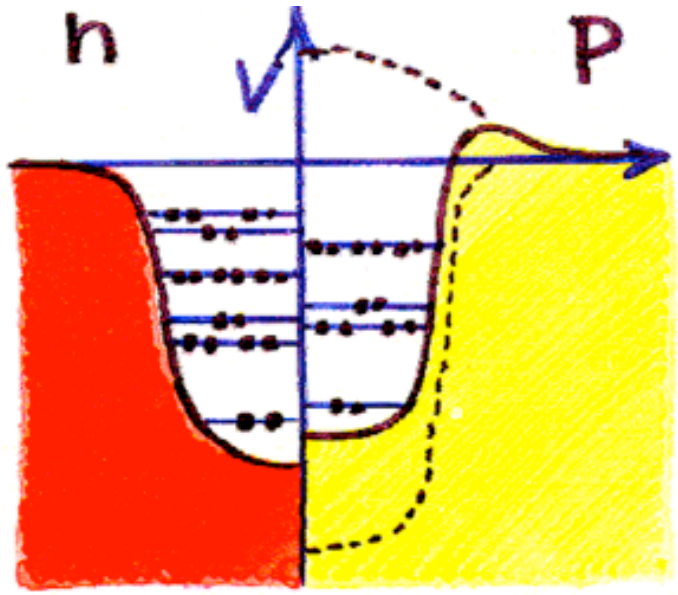


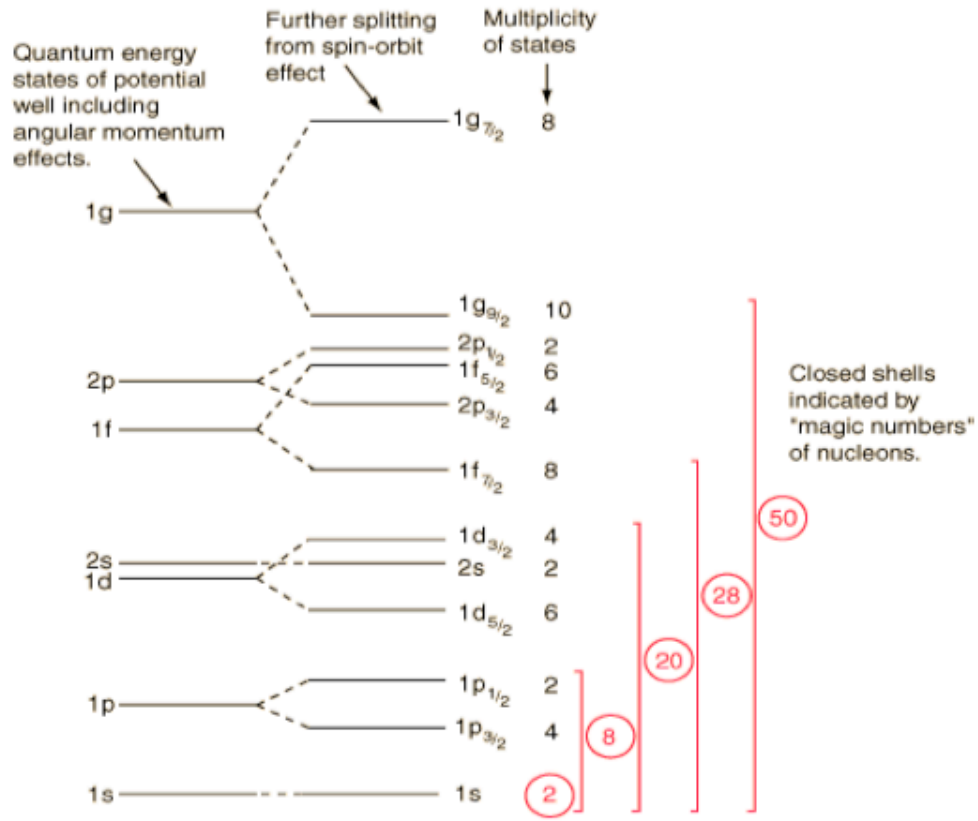
on average:
Net binding energy: ≈ 8 MeV
distance: ≈ 2 fm



Strong repulsion
⟹ NN correlations

Shell Structure (Maria Goeppert-Mayer, Jensen, 1949, Nobel Prize 1963)





nuclear density 10^{18} kg/m^3

With the enormous strong force acting between them and with so many nucleons to collide with, how can nucleons possibly complete whole orbits without interacting?

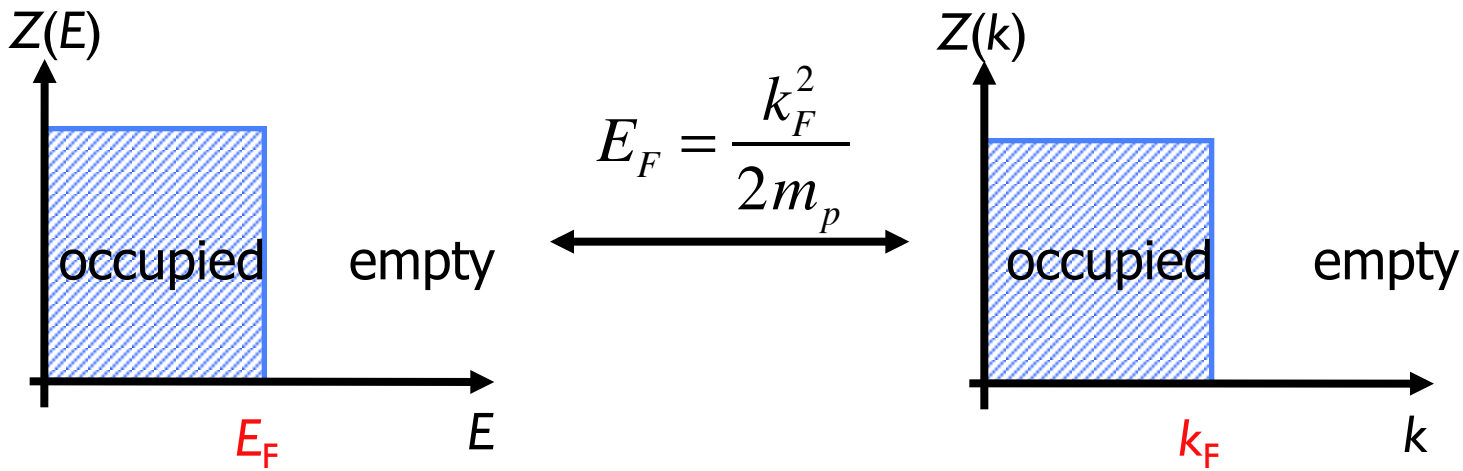
But: there is experimental evidence for shell structure

Pauli Exclusion Principle: → nucleons can not scatter into occupied levels: Suppression of collisions between nucleons

Independent Particle Shell model (IPSM)

- single particle approximation:
  nucleons move independently from each other
  in an average potential created by the other nucleons (mean field)
  spectral function $S(E,k)$:
  probability of finding a proton with initial momentum k and energy E in the nucleus
- factorizes into energy & momentum part

nuclear matter:

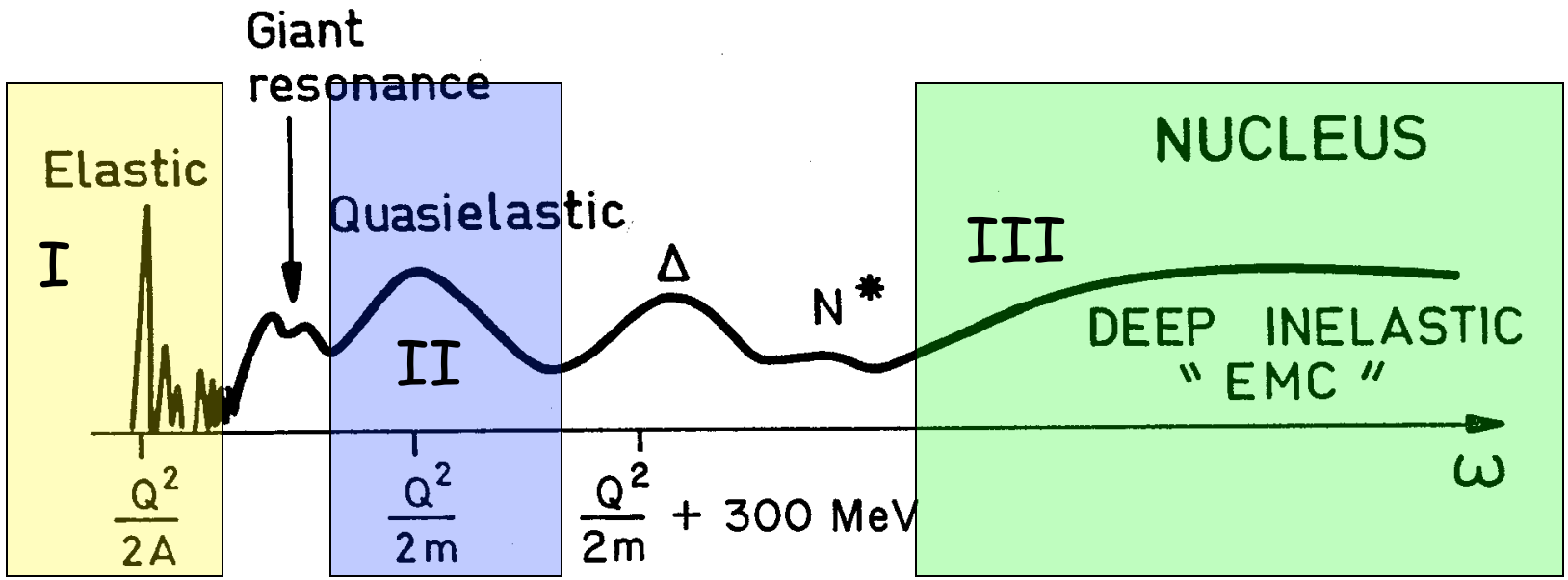


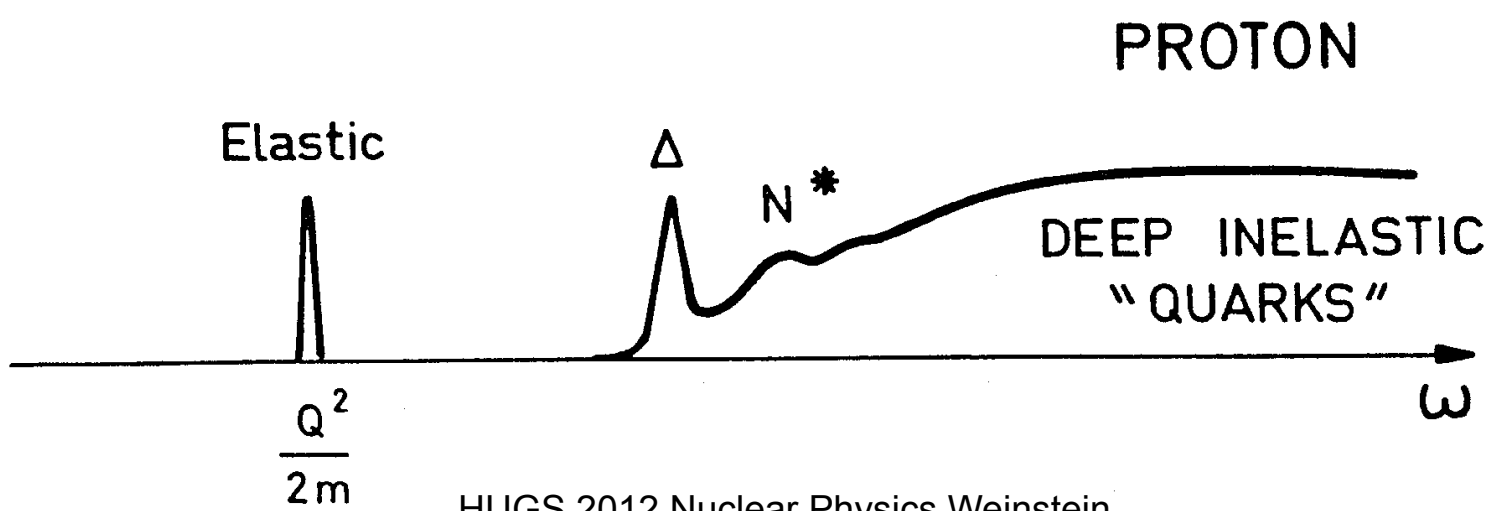
$$E_F = \frac{k_F^2}{2m_p}$$

nuclei: $$S(\vec{p},E) = \sum_i |\Phi_a(p)|^2 \, \delta(E+\epsilon_a)$$

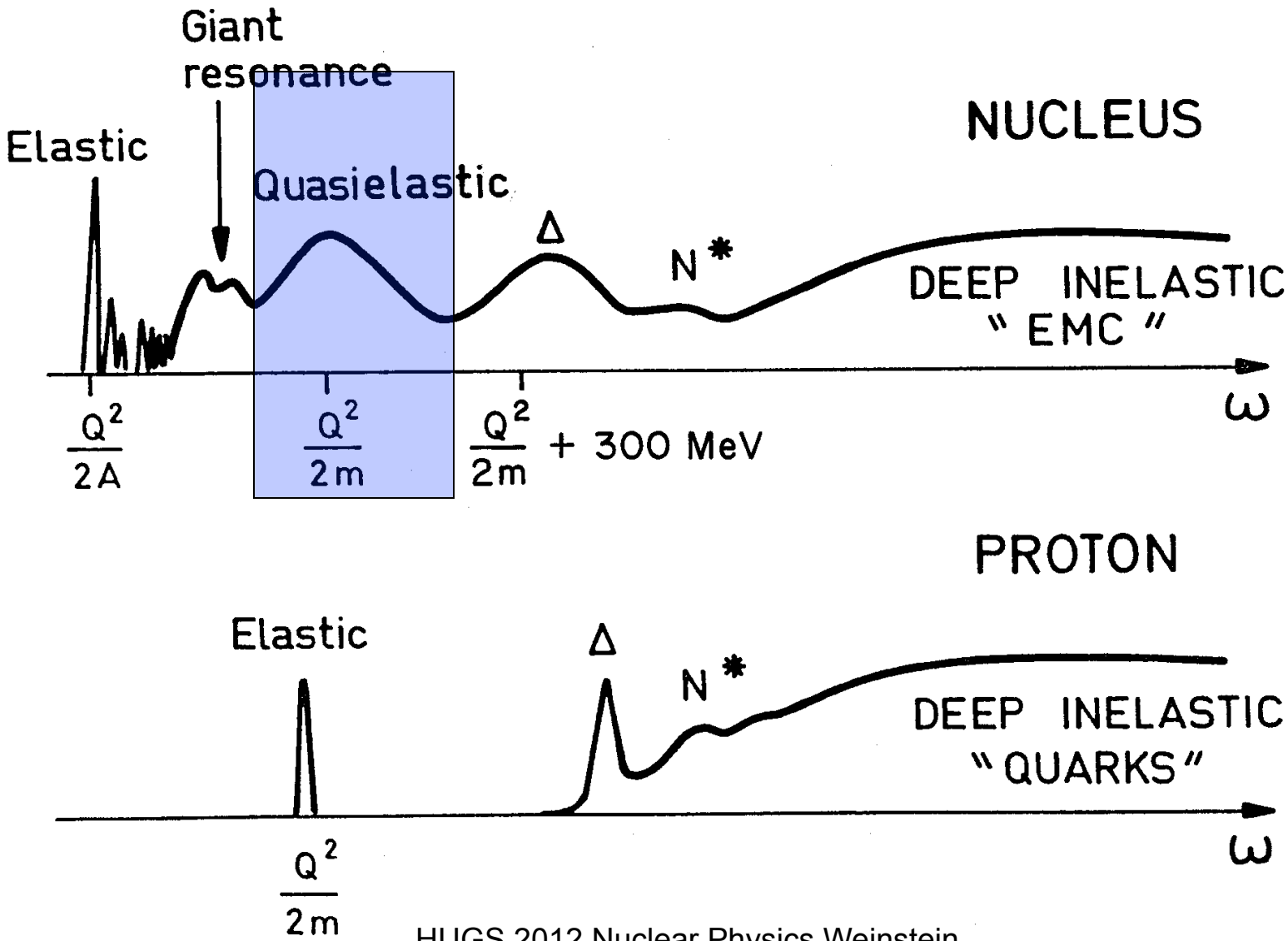
Not 100% accurate, but a good starting point

Electron-nucleus interactions



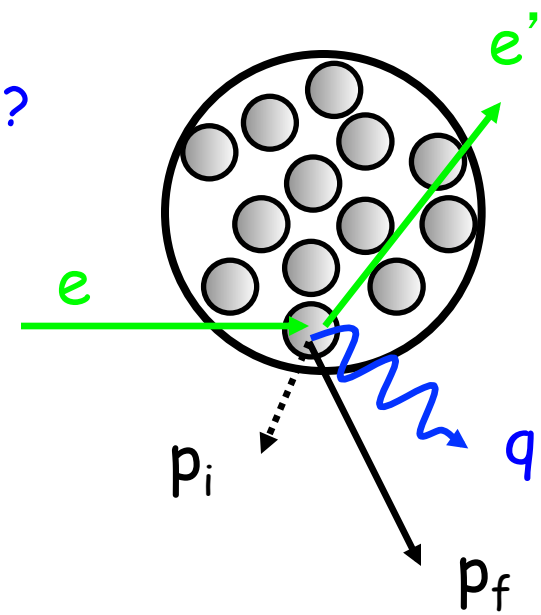


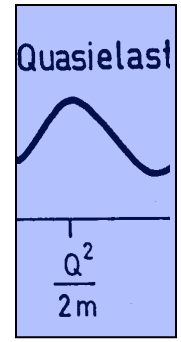
II. Quasielastic scattering



Fermi gas model:

how simple a model can you make ?





Initial nucleon energy: $KE_i = p_i^2 / 2m_p$

Final nucleon energy: $KE_f = p_f^2 / 2m_p = (\vec{q} + \vec{p}_i)^2 / 2m_p$

Energy transfer: $\nu = KE_f - KE_i = \frac{\vec{q}^2}{2m_p} + \frac{\vec{q} \cdot \vec{p}_i}{m_p}$

Expect:

- Peak centroid at $\nu = q^2/2m_p + \varepsilon$
- Peak width $2qp_{\text{fermi}}/m_p$
- Total peak cross section = $Z\sigma_{ep} + N\sigma_{en}$

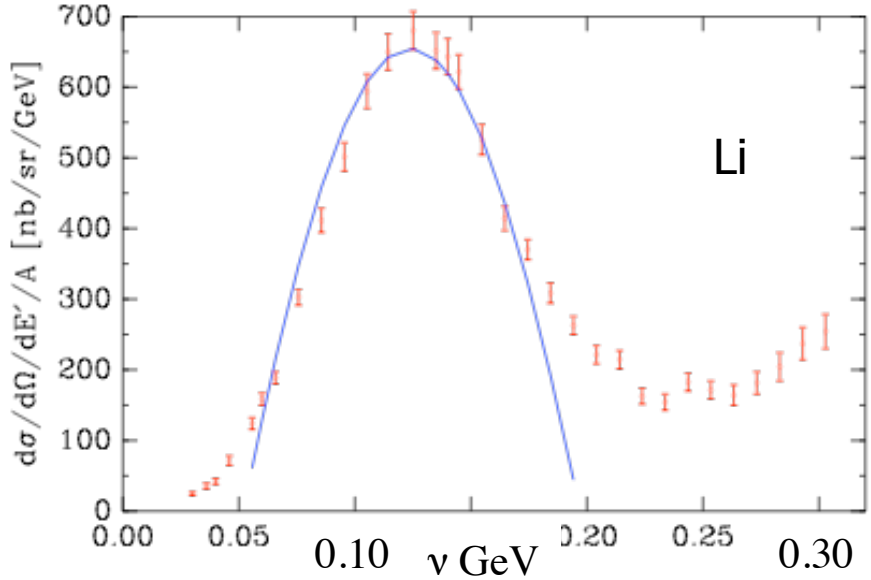
Early 1970's Quasielastic Data

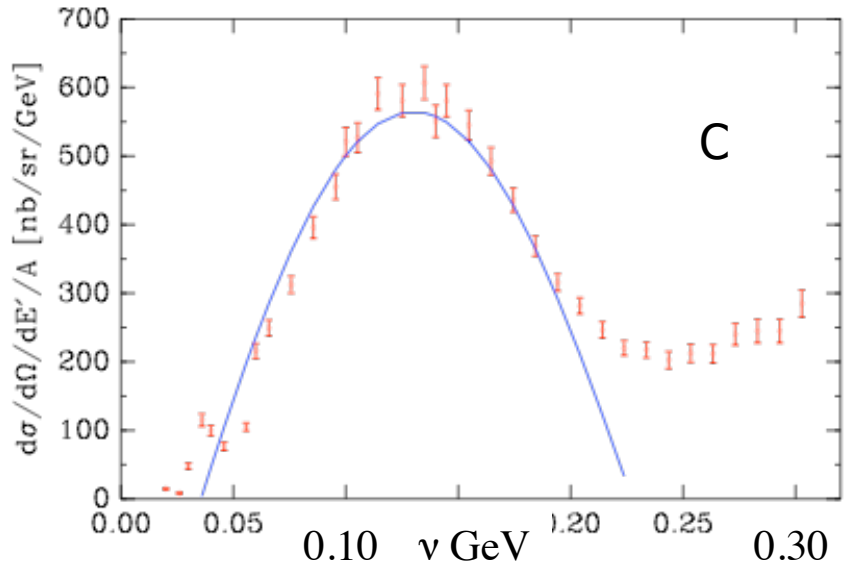
-> getting the bulk features

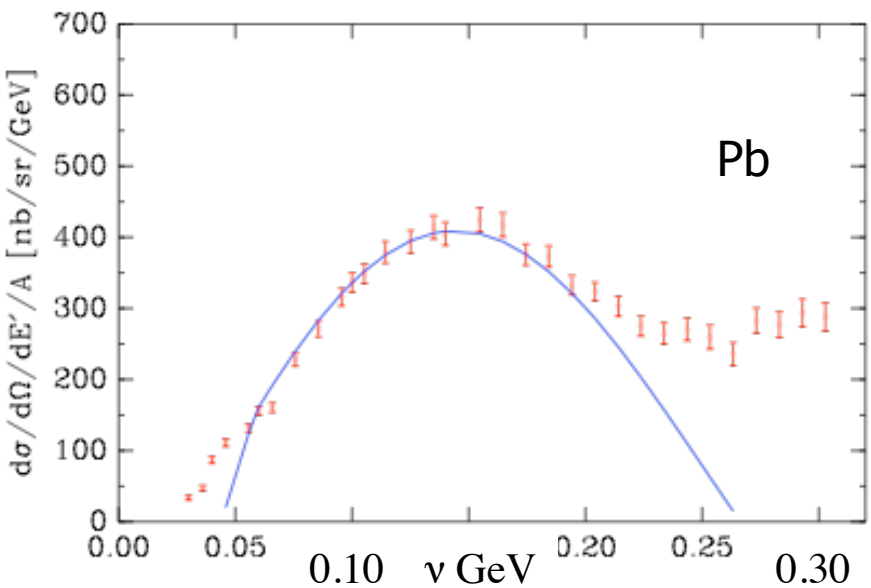
500 MeV, 60 degrees

$\vec{q} \simeq 500 MeV/c$

R.R. Whitney et al., PRC 9, 2230 (1974).



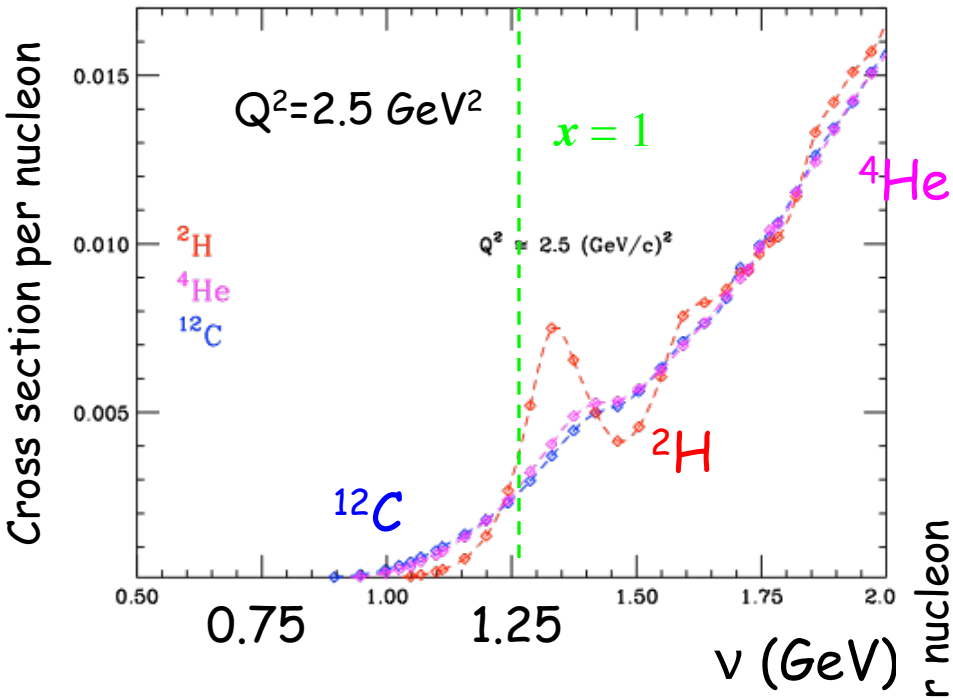




Nucleus	k_F MeV/c	$\bar{\epsilon}$ MeV
^{6}Li	169	17
^{12}C	221	25
^{24}Mg	235	32
^{40}Ca	251	28
^{nat}Ni	260	36
^{89}Y	254	39
^{nat}Sn	260	42
^{181}Ta	265	42
^{208}Pb	265	44

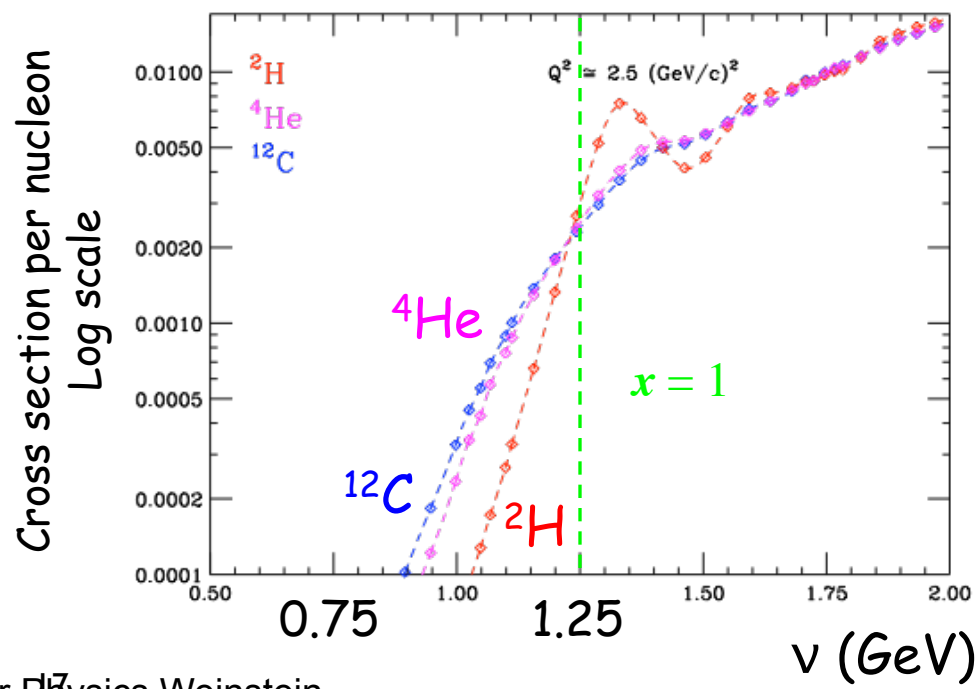
compared to Fermi model:fit parameter k_F and ε

Nuclear mass (A) dependence



Heavier nucleus
➔higher nucleon momenta
➔broadened peak

(same plot, log scale)

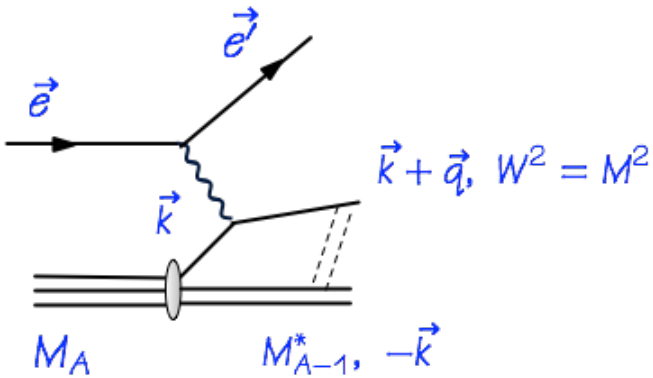


$$x = Q^2 / (2m_p \nu)$$

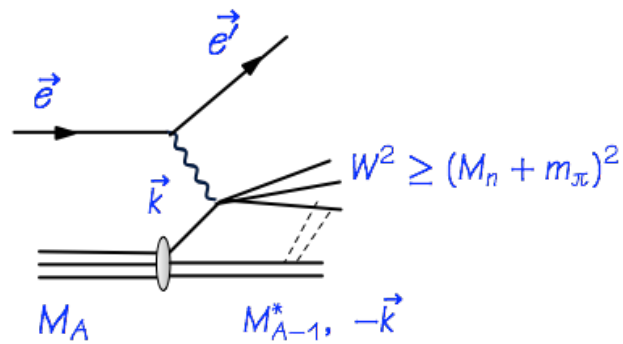
$$x = 1 \rightarrow \nu = Q^2 / (2m_p)$$

Inclusive Electron Scattering from Nuclei: Two processes

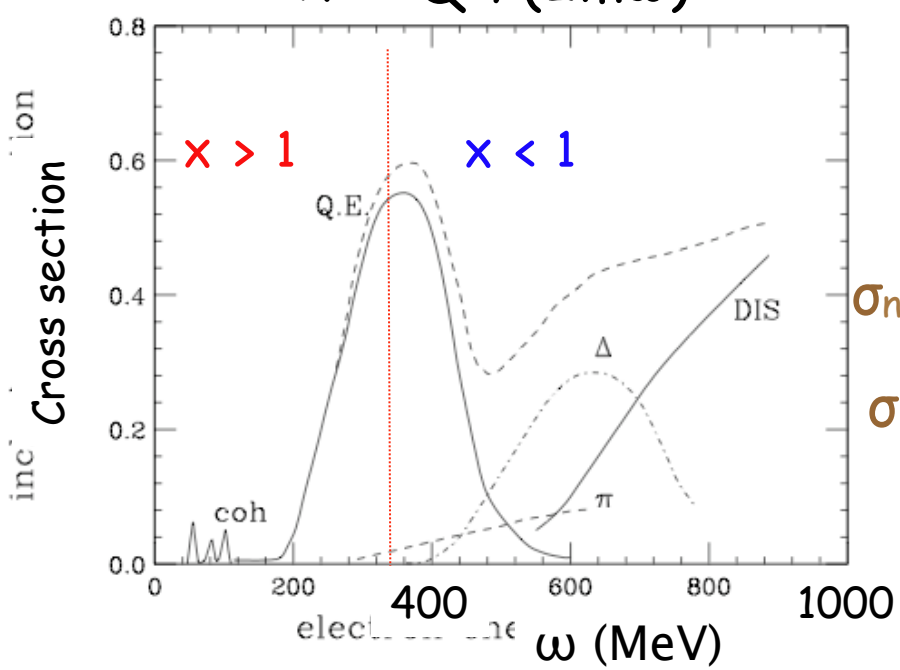
Quasielastic from nucleons



Inelastic from nucleons (including Deep Inelastic Scattering (DIS))



$x = Q^2/(2m\omega)$

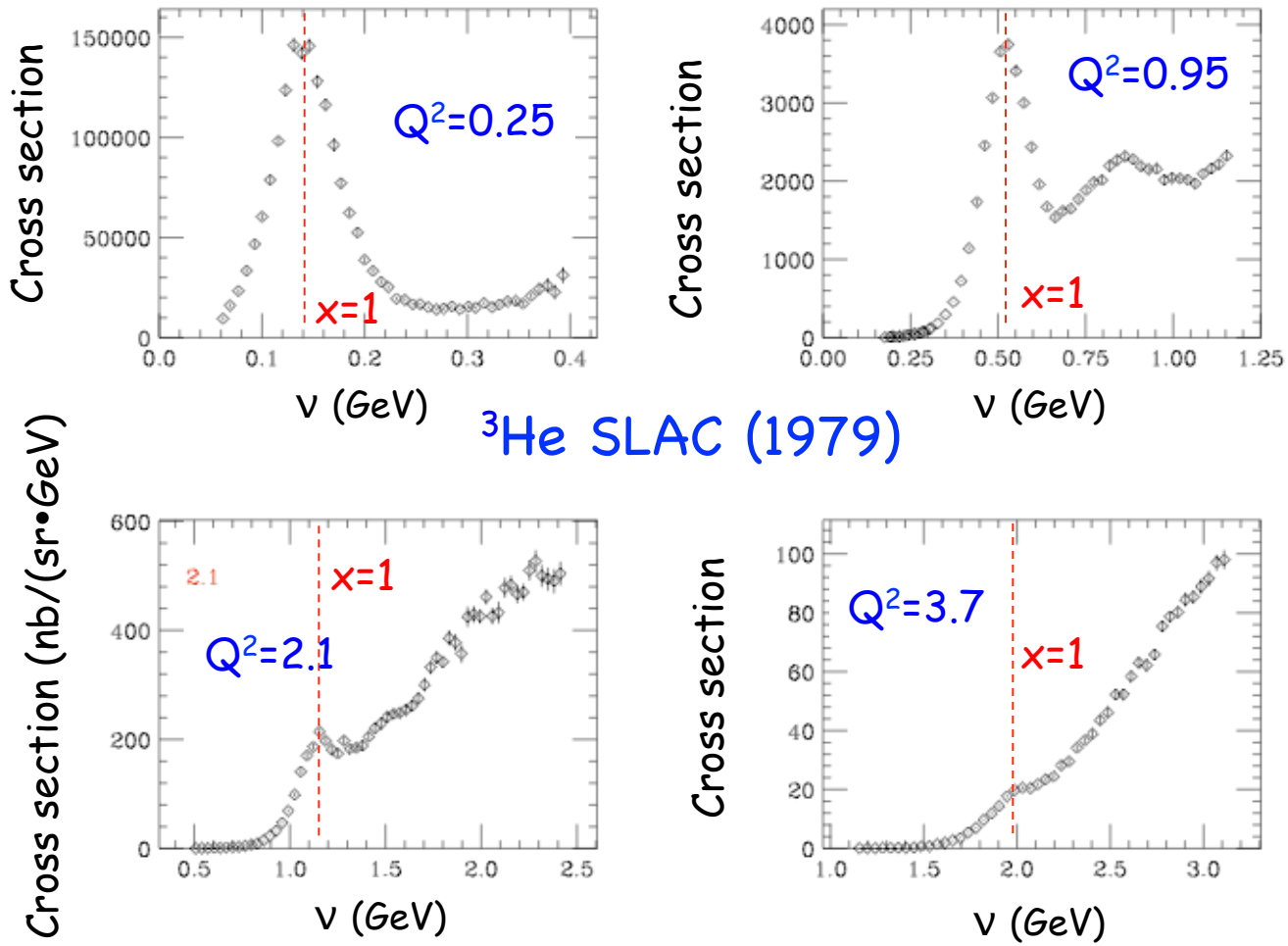


Inclusive final state means cannot separate two processes

Exploit their different Q^2 dependencies

$\sigma_{nucleon} \sim$ (nucleon elastic form factor)^2

$\sigma_{DIS} \sim \ln(Q^2)$ (at large Q^2)



- As $Q^2 \gg 1$ inelastic scattering from the nucleons begins to dominate
- Quasi Elastic scattering is still dominant at low energy loss (ν), even at high Q^2

Scaling

- The dependence of a cross section, in certain kinematic regions, on a single variable.
  - If the data scales, it validates the scaling assumption
  - Scale-breaking indicates new physics
- At moderate Q^2 and $x>1$ we expect to see evidence for y-scaling, indicating that the electrons are scattering from quasifree nucleons
  - y = minimum momentum of struck nucleon
- At high Q^2 we expect to see evidence for x-scaling, indicating that the electrons are scattering from quarks.
  - $x = Q^2/2m\nu$ = fraction of nucleon momentum carried by struck quark (in infinite momentum frame)

Classical Scaling

Galileo realized that that if one simply scaled up an animals size its weight would increase significantly faster than its strength, "....you can plainly see the impossibility of increasing the size of structures to vast dimensions...if his height be increased inordinately, he will fall and be crushed under his own weight"

$$\frac{\text{Strength}}{\text{Weight}} \propto \frac{\text{Area}}{\text{Volume}} \propto \frac{L^2}{L^3} \propto \frac{1}{\text{Weight}^{1/3}}$$

Neohipparion (small horse) Mastodon

Smaller animals appear stronger

Explains why small animals can leap as high as large one ...

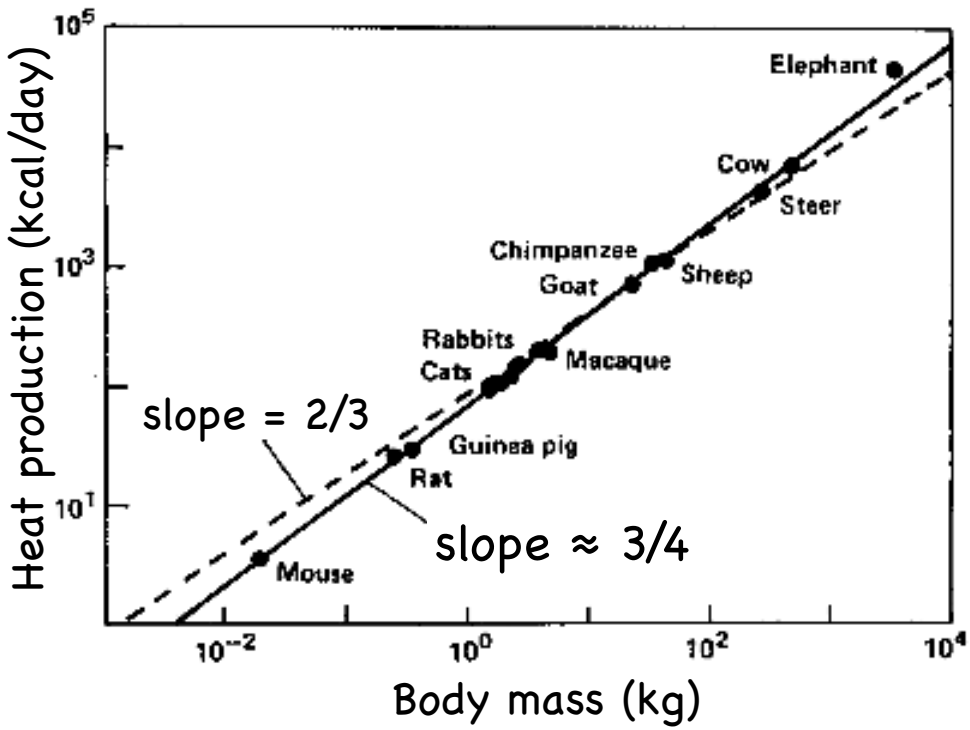
G. West, LANL report

Metabolism

Metabolic rate B: heat lost by a body in a steady inactive state

Should be dominated by sweating and radiation (proportional to surface area or weight$^{2/3}$)

$B \propto W^{2/3}$

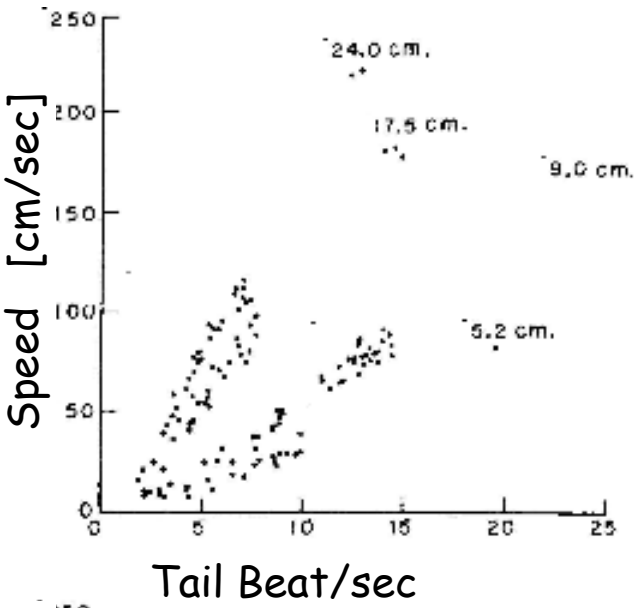


Best fit slope ≈ 3/4
Therefore not just pure geometry
- Different shape animals
- Different insulation (elephants have less fur)

Deviations from the geometrical or kinematic analysis reflects the dynamics of the system.

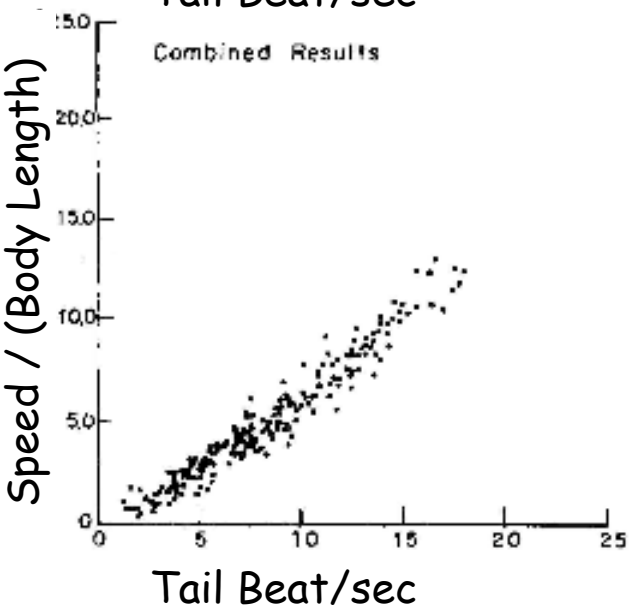
Deviations from naive scaling probe other features of the system

Scaling: Selecting the relevant variables



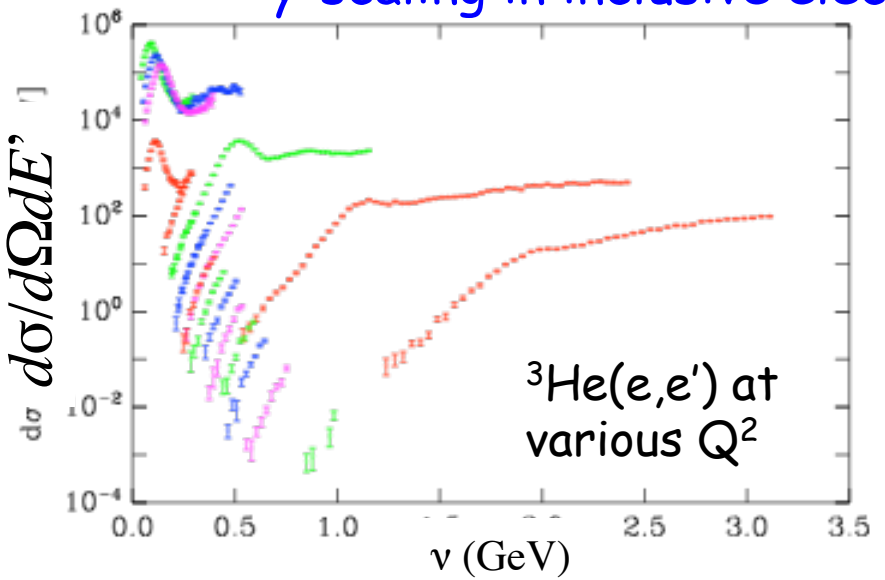
The Dace, a fresh water fish

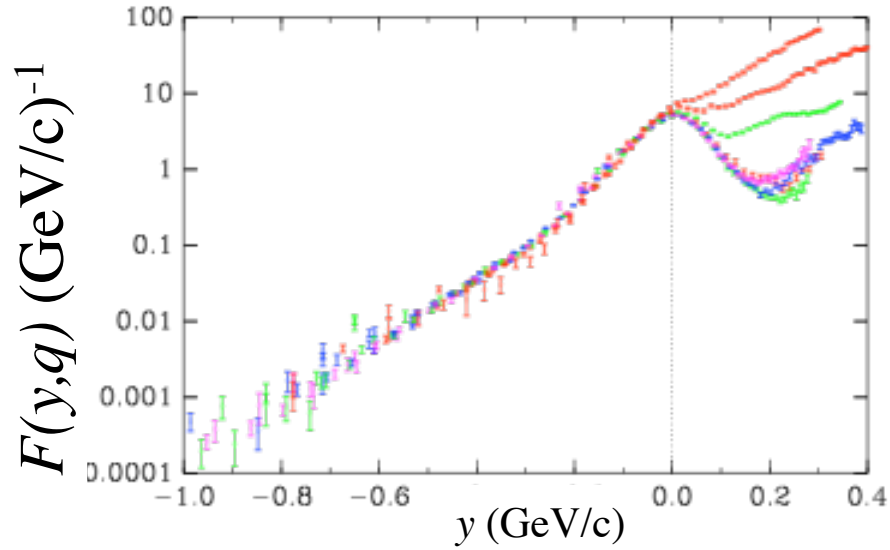
Scaling and scaling violations reveal information about the dynamics of the system



Knut Schmidt-Nielsen, from Scaling: Why is Animal Size So Important?

y-scaling in inclusive electron scattering from ^{3}He





$$F(y) = \frac{\sigma^{\exp}}{(Z\tilde{\sigma}_p + N\tilde{\sigma}_n)} \cdot K$$

$$n(k) = -\frac{1}{2\pi y}\frac{dF(y)}{dy}$$

Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

y is the momentum of the struck nucleon parallel to the momentum transfer: $y \approx -q/2 + m\nu/q$ (nonrelativistically)

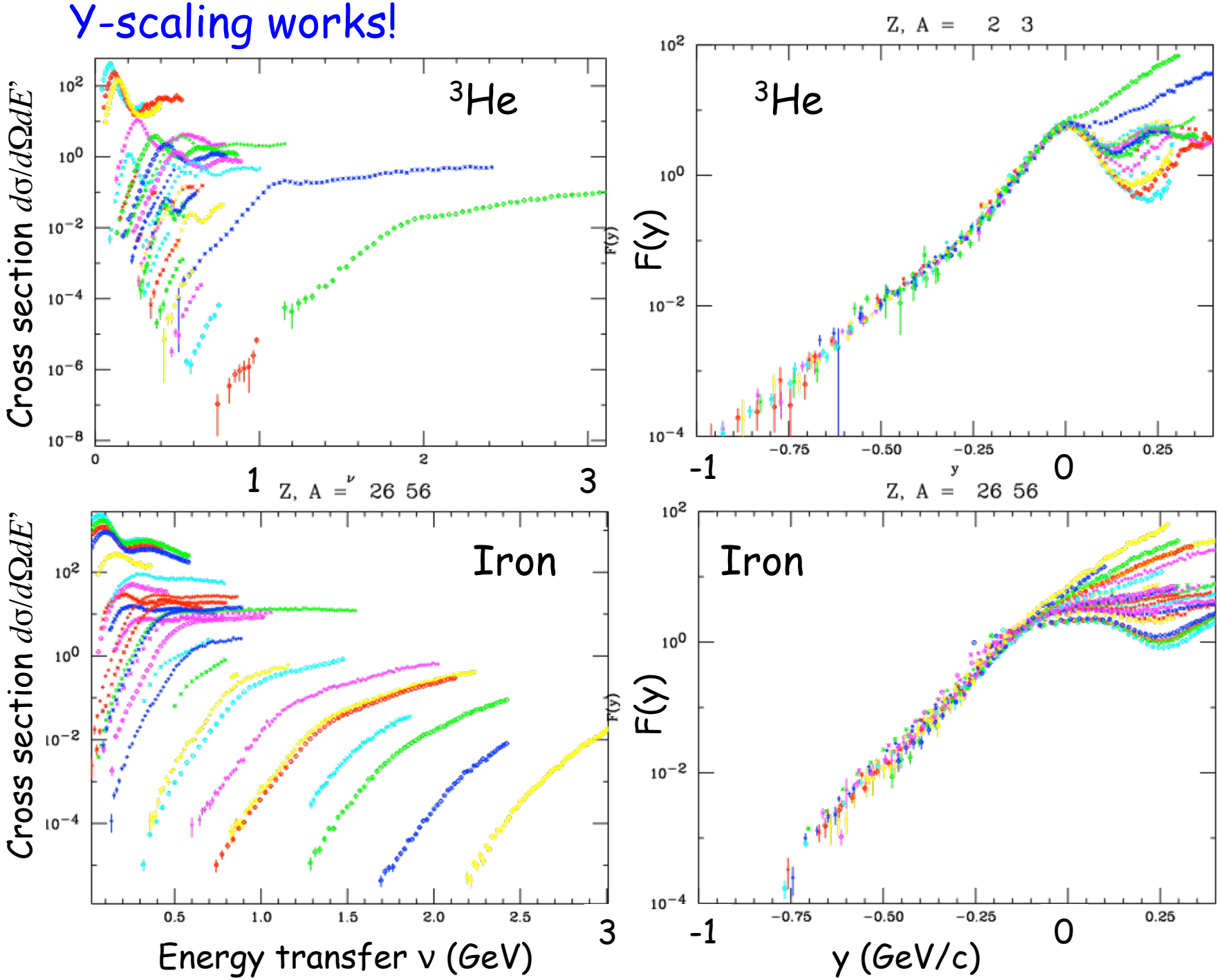
IF the scattering is quasifree, then $F(y)$ is the integral over all perpendicular nucleon momenta (nonrelativistically).

Goal: extract the momentum distribution $n(k)$ from $F(y)$.

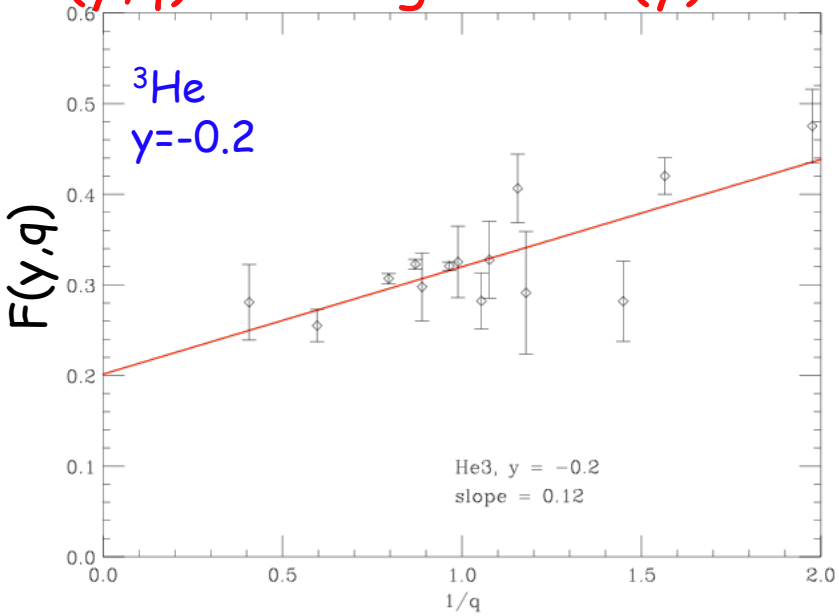
Assumptions & Potential Scale Breaking Mechanisms

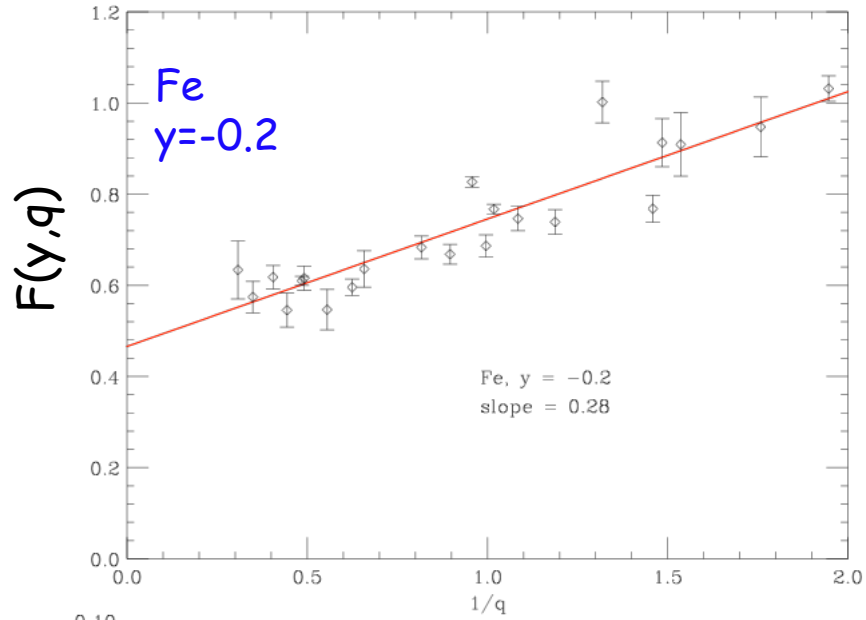
- No Final State Interactions (FSI)
- No internal excitation of (A-1)
- Full strength of Spectral function can be integrated over at finite *q*
- No inelastic processes (choose y<0)
- No medium modifications (discussed later)

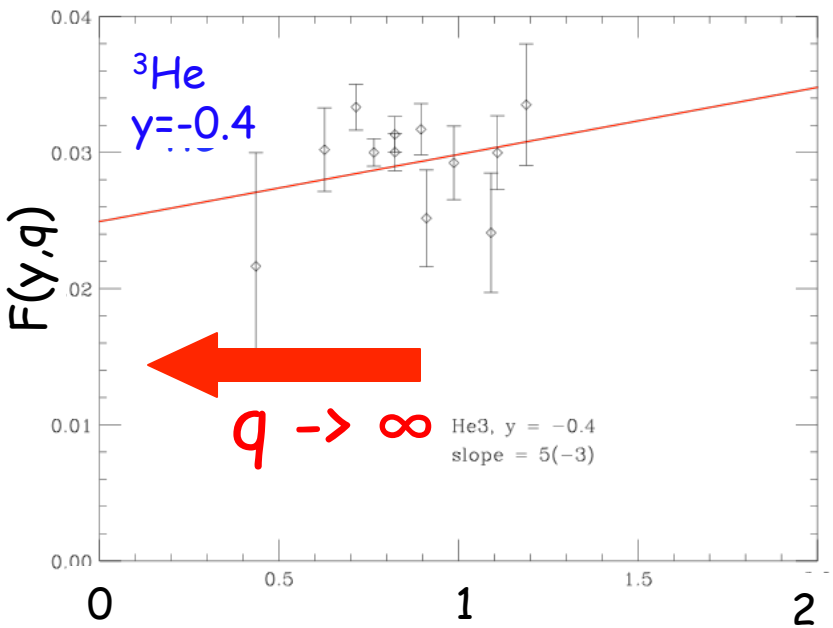
Y-scaling works!

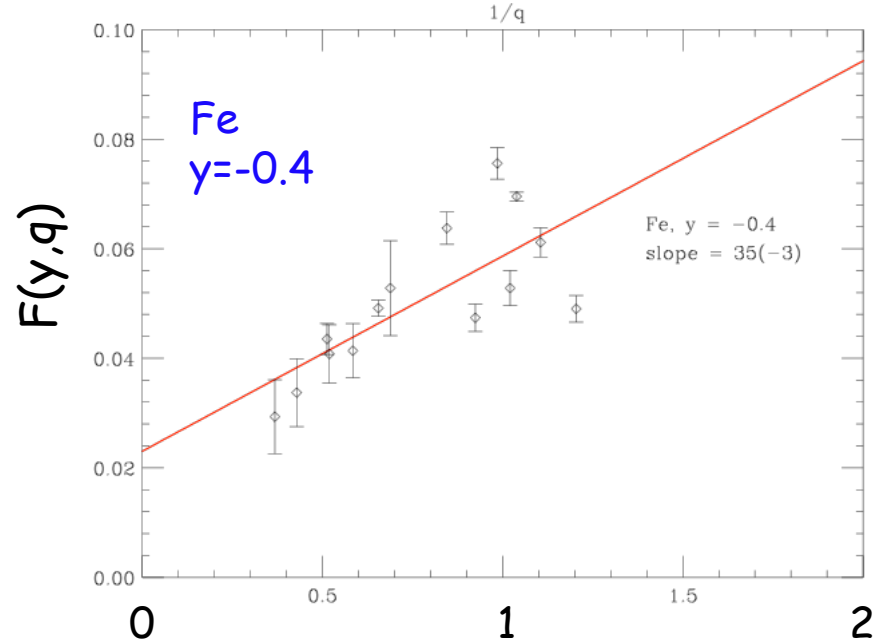


F(y,q) converges to F(y) at moderate momentum transfer





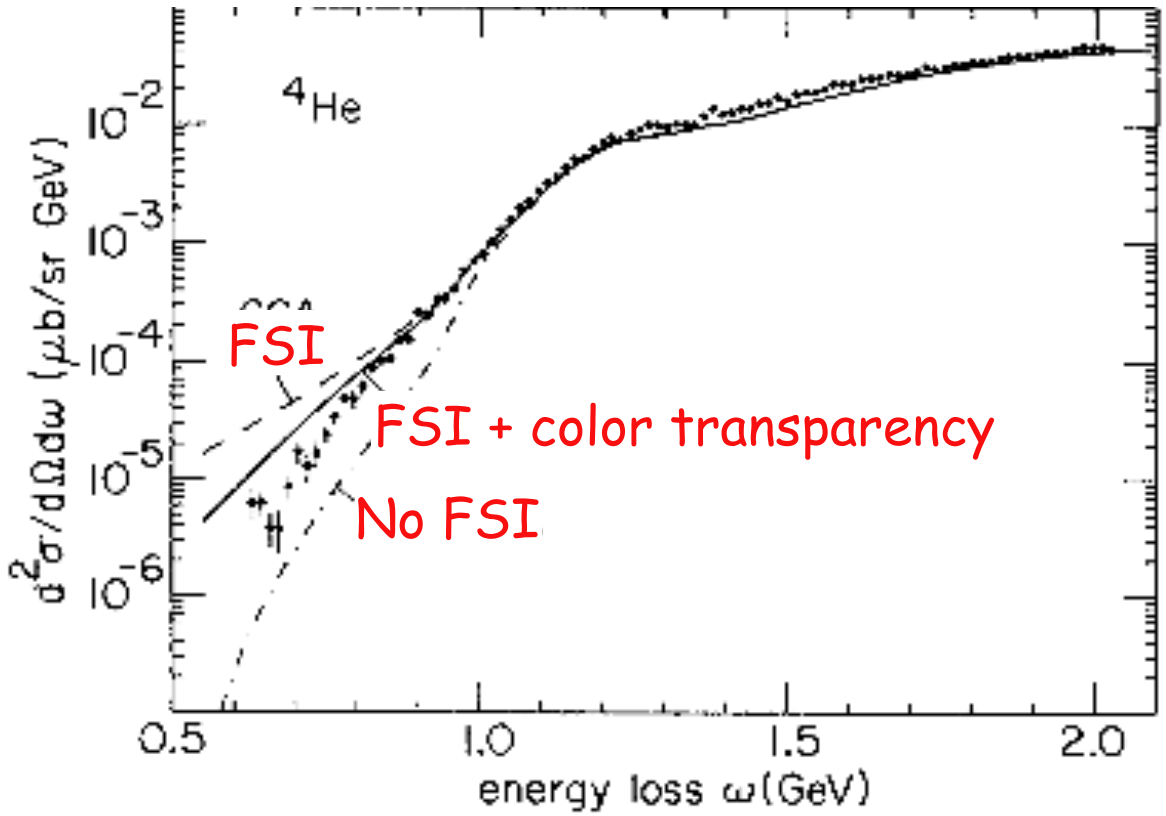




27

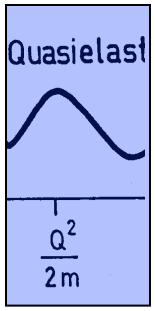
Final State Interactions (FSI) complicate this simple picture

^{4}He(e,e') at 3.595 GeV, 30°



Benhar et al. PRC 44, 2328
Benhar, Pandharipande, PRC 47, 2218
Benhar et al. PLB 3443, 47

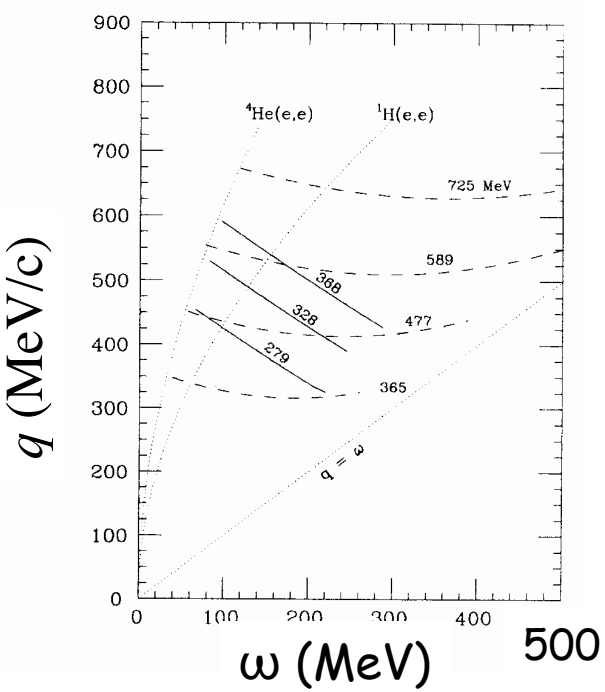
Now let's separate R_L (longitudinal) and R_T (transverse): ^{4}He(e,e')

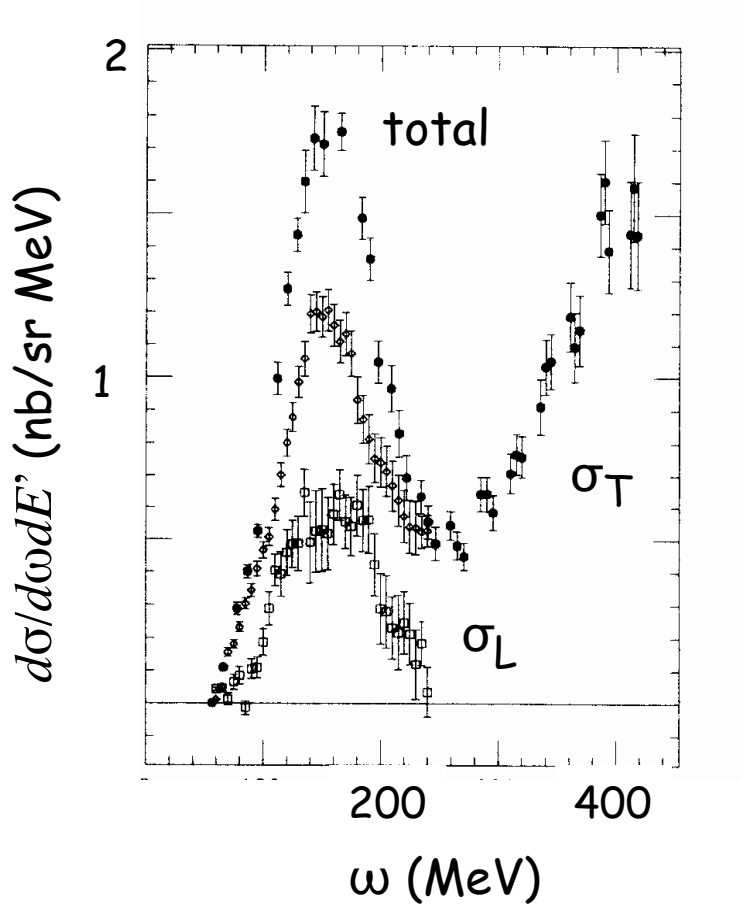


$$\frac{d\sigma}{d\Omega dE} = \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4}R_L(Q^2,\omega) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right)R_T(Q^2,\omega)\right]$$

Fix Q^2 and ω

1. Measure $d\sigma/d\omega dE'$ at large E_e and small θ
2. Measure $d\sigma/d\omega dE'$ at small E_e and large θ
3. Take linear combination to extract R_L, R_T

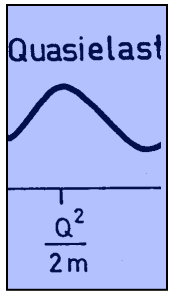


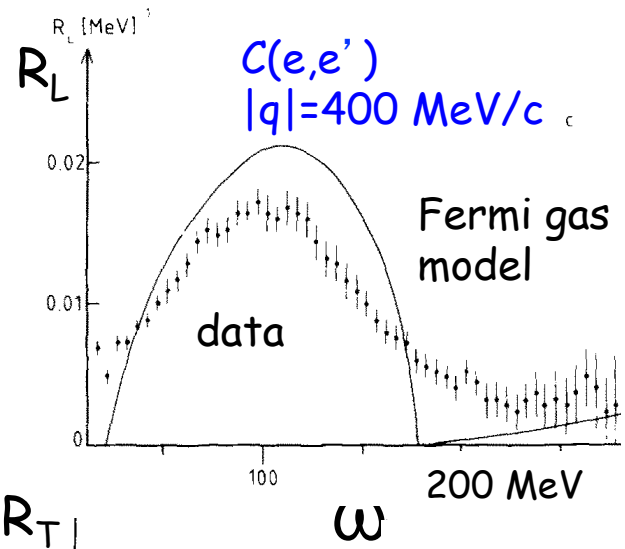


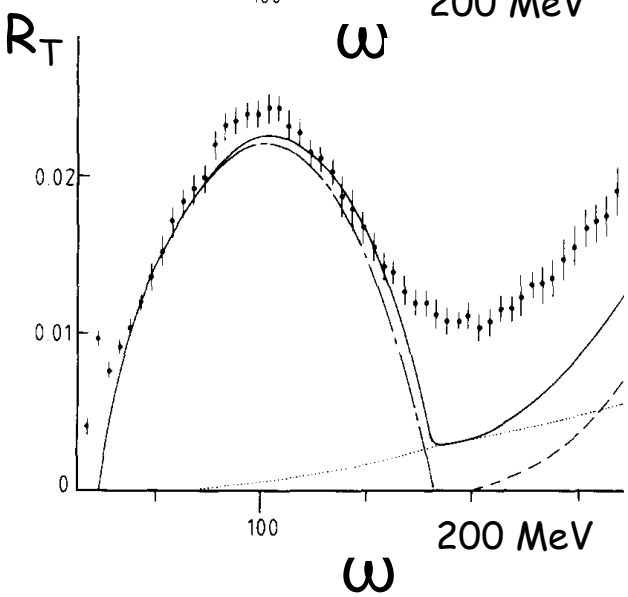
Von Reden et al, PRC 41, 1084 (1990)

Fermi Gas Model: Too good to be true?

$$\frac{d\sigma}{d\Omega dE} = \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4}R_L(Q^2,\omega) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right)R_T(Q^2,\omega)\right]$$







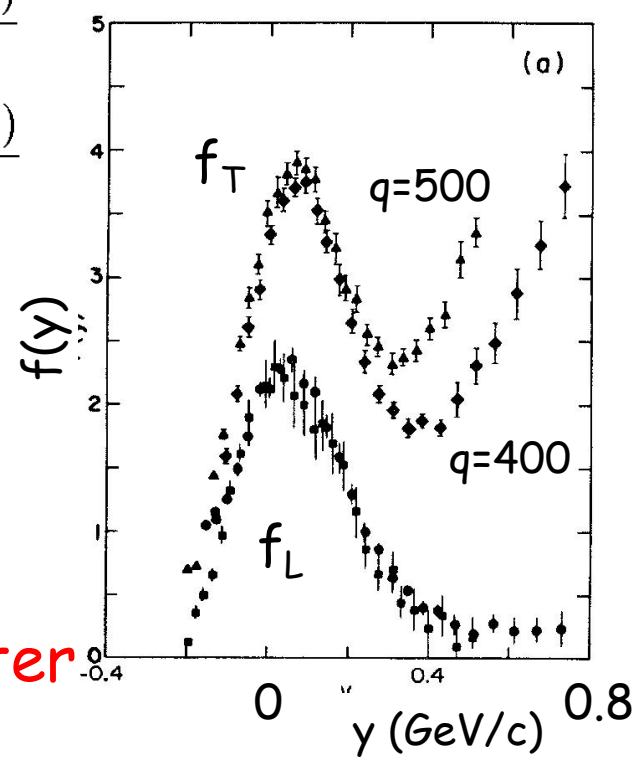
y = minimum initial nucleon momentum
$= m\omega/q - q/2$ (nonrelativistic only!)
f = reduced response function

$$f_L(Q^2,\omega) \propto \frac{R_L(Q^2,\omega)}{\tilde{G}_E^2(Q^2)}$$

$$f_T(Q^2,\omega) \propto \frac{R_T(Q^2,\omega)}{\tilde{G}_M^2(Q^2)}$$

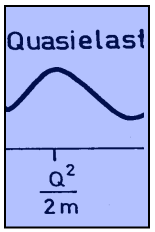
- L scales
- T scales
- T≠L!!

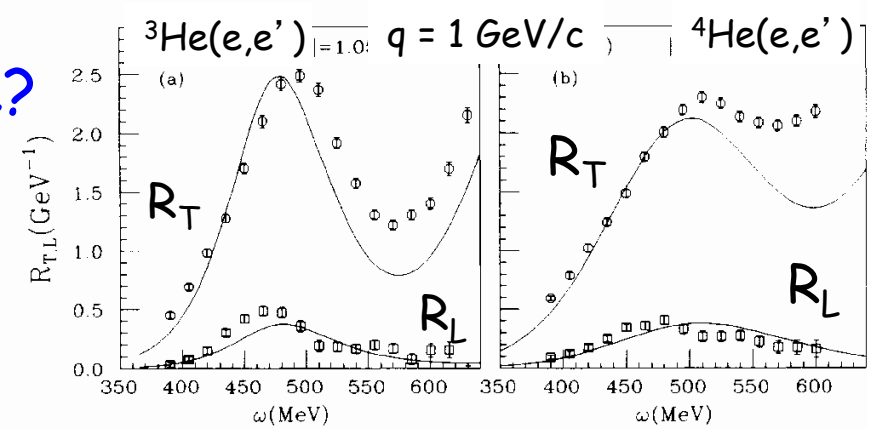
To be explained later

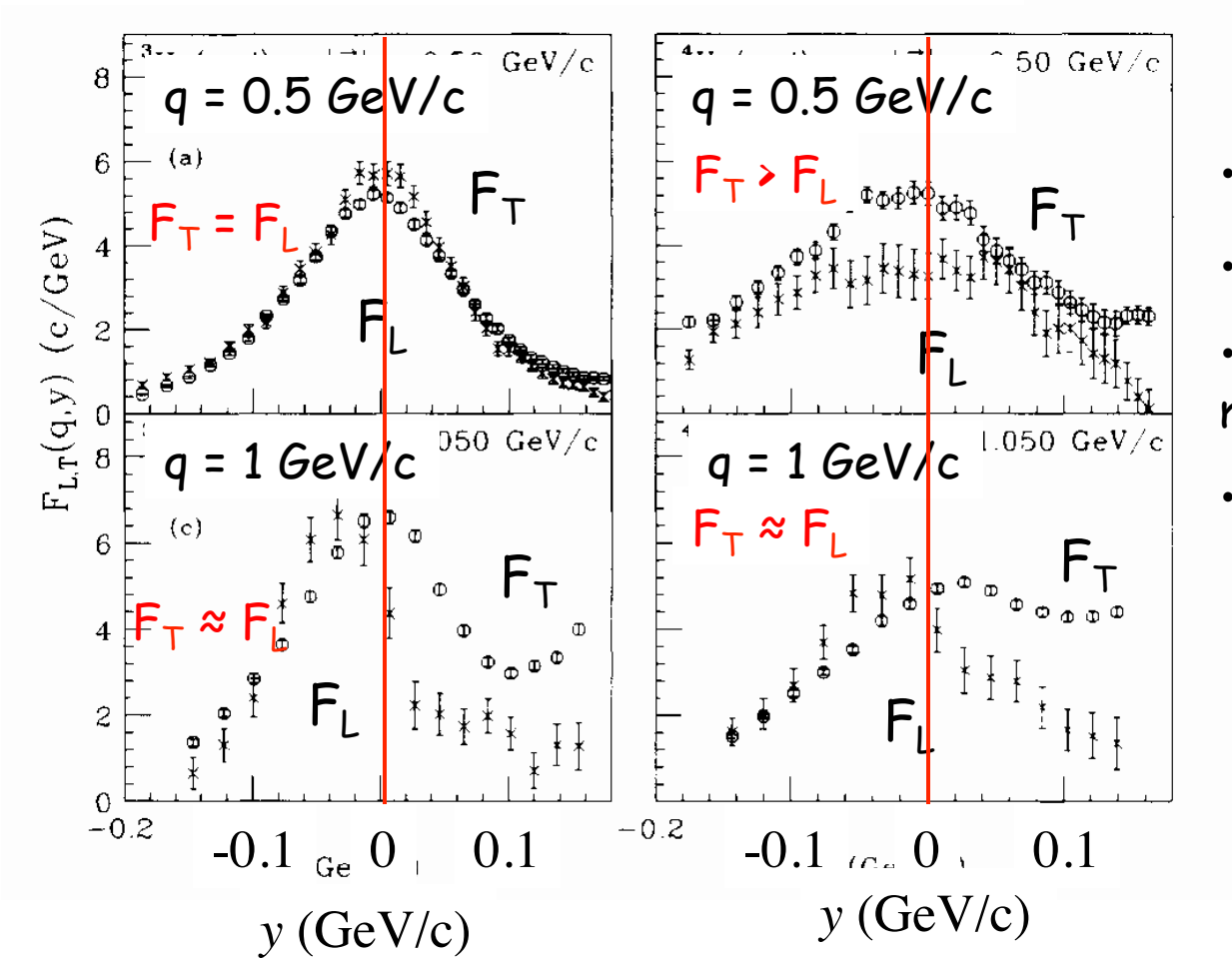


P. Barreau et al, NPA 402, 515 (1983)
Finn et al, PRC 29, 2230 (1984)

What causes the T/L difference?







- ^{3}He: $F_T = F_L$ (at y < 0)
- ^{4}He and C: $F_T > F_L$
- Extra transverse reaction mechanism in dense nuclei!
- Gets smaller at higher q

What is it?
To be continued …

Meziani et al, PRL 69 41 (1992)

(e,e') summary

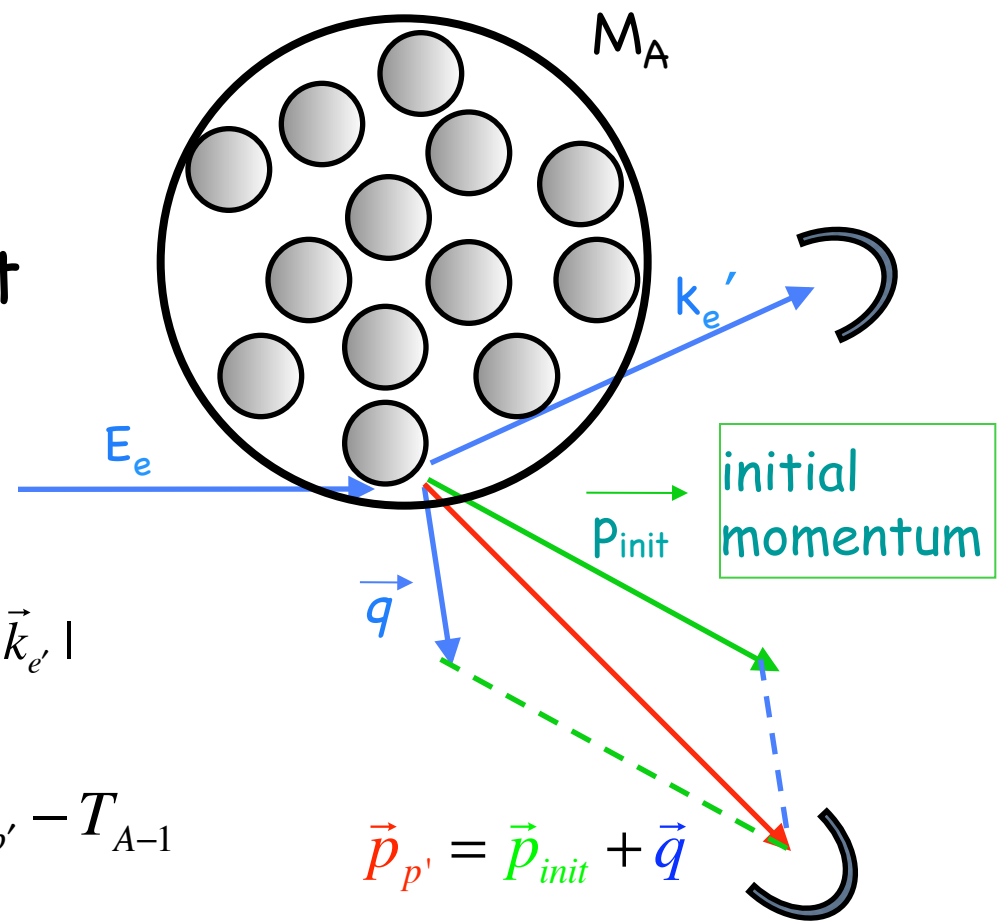
- Go to low ω side of QE peak ($y<0$ or $x>1$)
- Scaling ⟹ knockout is quasifree
- Measure momentum distribution of nucleons in nuclei
- But there are some complications

Get more information:
Detect the knocked out nucleon (e,e'p)

coincidence experiment

measure: momentum, angles

electron energy: E_e
proton: $\vec{p}_{p'}$
scattered electron: $\vec{k}_{e'}$ $\quad E_{e'} = |\vec{k}_{e'}|$



reconstructed quantites:

missing energy: $E_m = \nu - T_{p'} - T_{A-1}$

missing momentum: $\vec{p}_m = \vec{q} - \vec{p}_{p'}$

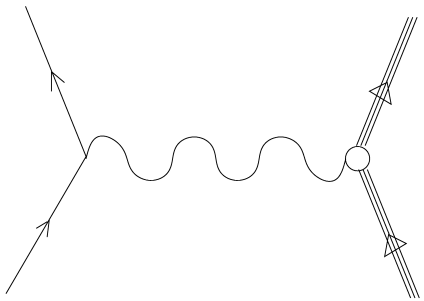
in Plane Wave Impulse Approximation (PWIA):
direct relation between measured quantities and theory:

$$|E| = E_m \qquad \vec{p}_{init} = -\vec{p}_m$$

Formalism (repeat from previous lecture)

- Inclusive scattering:
  - measure scattering angle θ_e and energy E'_e ($\nu = E_e - E'_e$) and the cross section $d\sigma/d\Omega d\nu$
- One photon exchange:

$$\mathcal{M}_n = \frac{e^2}{q^2}\bar{u}(k')\gamma_\mu u(k) < n|J_\mu(0)|p, S >$$



$$\begin{aligned} d\sigma &= \frac{1}{4M\vec{k}}\sum_n |\mathcal{M}_n|^2(2\pi)^4\delta^4(p+q-p')\frac{d^3k'}{(2\pi)^3 2E'} \\ &= \frac{|\vec{k}'|}{ME}\frac{\alpha^2}{Q^4}L^{\mu\nu}H_{\mu\nu}d\omega d\Omega \end{aligned}$$

$L^{\mu\nu}$ and $H_{\mu\nu}$ are the lepton and hadron tensors

Formalism Extension to (e,e'p)

Lepton tensor known (QED):

$$L_{\mu\nu} = \sum_{s'} (\bar{u}(k',s')\gamma_\mu u(k,s))^* (\bar{u}(k',s')\gamma_\nu u(k,s))$$
$$= 2(k_\mu k'_\nu + k_\nu k'_\mu) + q^2 g_{\mu\nu} + 2im_l \epsilon_{\mu\nu\alpha\beta} q^\alpha s_l^\beta$$

Spin dependent

Hadron tensor unknown:

$$H_{\mu\nu} = \frac{1}{4\pi} \sum_n < p,S|\hat{J}_\mu(0)|n >< n|\hat{J}_\nu(0)|p,S > (2\pi)^4 \delta^4(p+q-p')$$
$$= \frac{1}{4\pi} \int d^4\xi \exp(iq\cdot\xi) < p,S|[\hat{J}_\mu(\xi), \hat{J}_\nu(0)]\,|p,S >$$

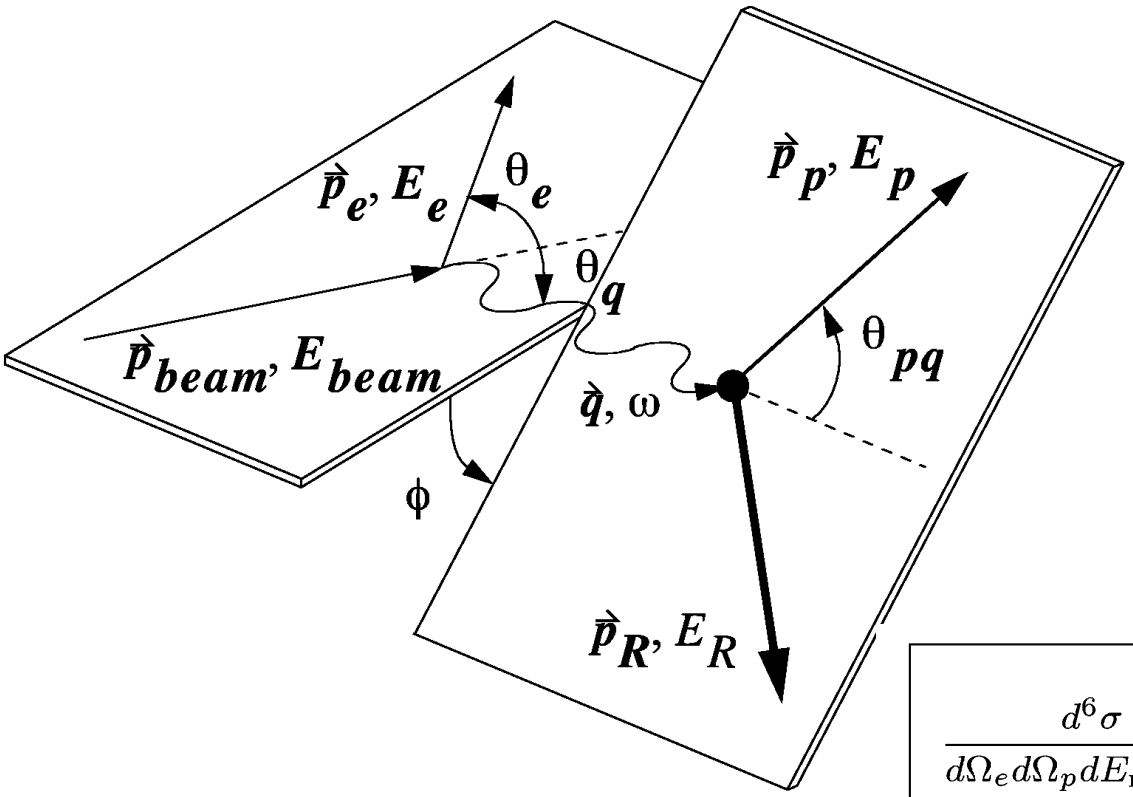
Nuclear current operators

Unchanged

Now we have another 4-vector (p') to make our Lorentz scalars and tensors from.

Available independent four vectors for (e,e'p):

- target momentum p_μ
- photon momentum q_μ
- proton momentum p'_μ (new for (e,e'p))



$$\frac{d^6\sigma}{d\Omega_e d\Omega_p dE_{\rm miss} d\omega} = K\sigma_{\rm Mott}\,[\, v_L \mathbf{R_L} + v_T \mathbf{R_T} + v_{LT}\mathbf{R_{LT}}\cos(\phi) + v_{TT}\mathbf{R_{TT}}\cos(2\phi)\,]$$

And then there were four (response functions, that is)

where

K = (phase space)

$\sigma_{\rm Mott}$ = (relativistic Rutherford scattering)

$v = v\ (q,\omega)$ (electron kinematics)

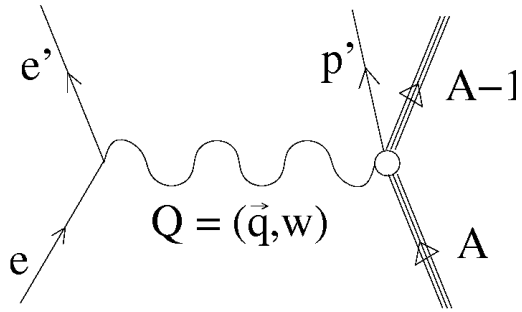
Each R now depends on more variables

$R = R(q, \omega, p_{miss}, E_{miss})$

(When you include electron and proton spin, there are 18. Yikes!)

(And if you scatter from a polarized spin-1 target, there are 41. Double Yikes!!)

(e,e'p) Plane Wave Impulse Approximation (PWIA)



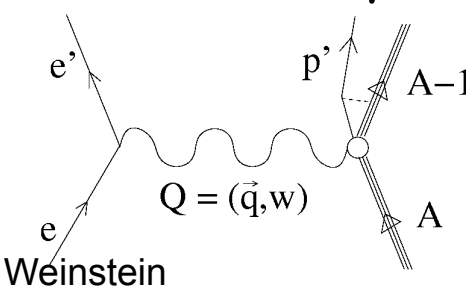
1. Only one nucleon absorbs the virtual photon
2. That nucleon does not interact further
3. That nucleon is detected

Cross section factorizes:

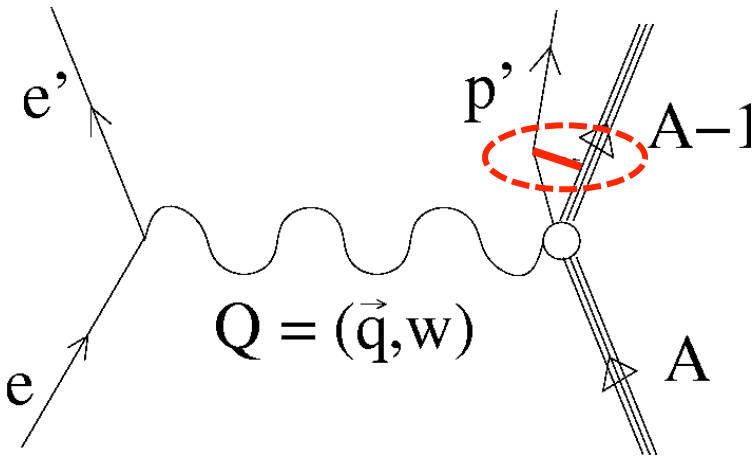
$$\frac{d\sigma^{fi}}{dE_1 d\Omega_1 dE_2 d\Omega_2} = KS(\vec{k}, E)\frac{d\sigma^{free}}{d\Omega}$$

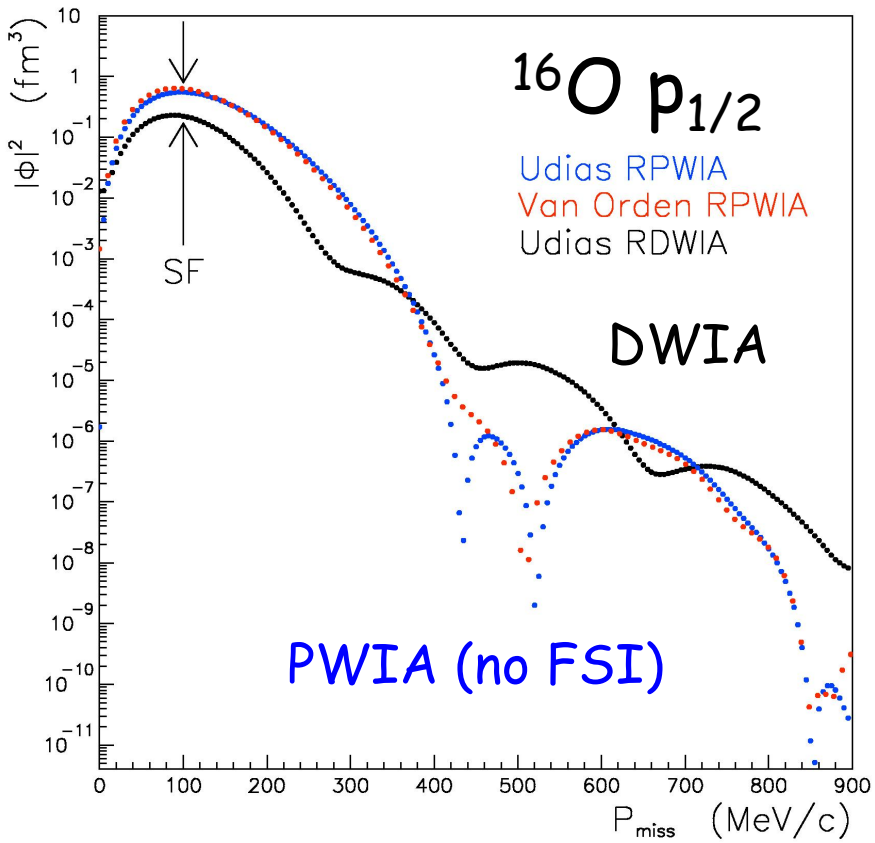
Single nucleon pickup reactions [eg: (p,d), (d,^{3}He) …] are also sensitive to S(p,E) but only sensitive to surface nucleons due to strong absorption in the nucleus

DWIA: If the struck nucleon interacts with the rest of the nucleus, then the cross section still factorizes (usually) but we measure a distorted spectral function.



(e,e'p) Distorted Wave Impulse Approximation (DWIA)



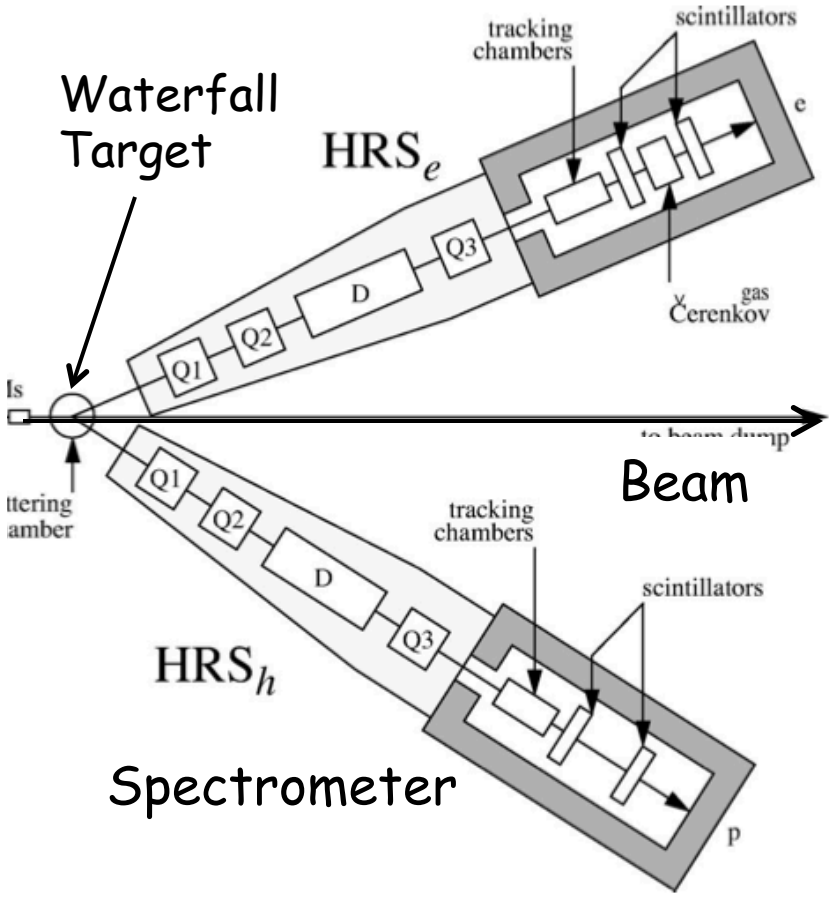


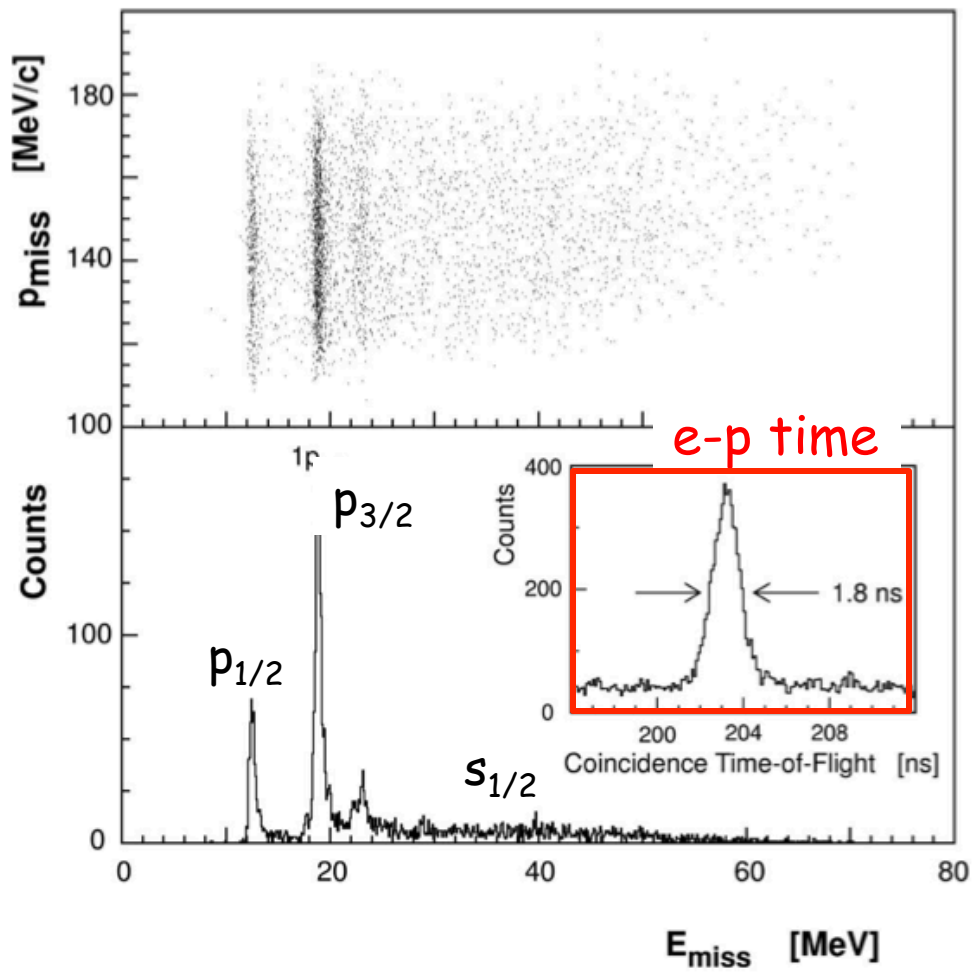
A. Pickelsimer and J.W. Van Orden, Phys. Rev. C. **40**, 290 (1989)
J. M. Udías et al., Phys. Rev. C. **64**, 024614 (2001)

- Low momentum (p < 0.5 GeV/c): use optical potential
- High momentum (p > 1 GeV/c): use Glauber approximation

Distortions (FSI) make it harder to measure the nucleon initial momentum distributions, especially at high momenta

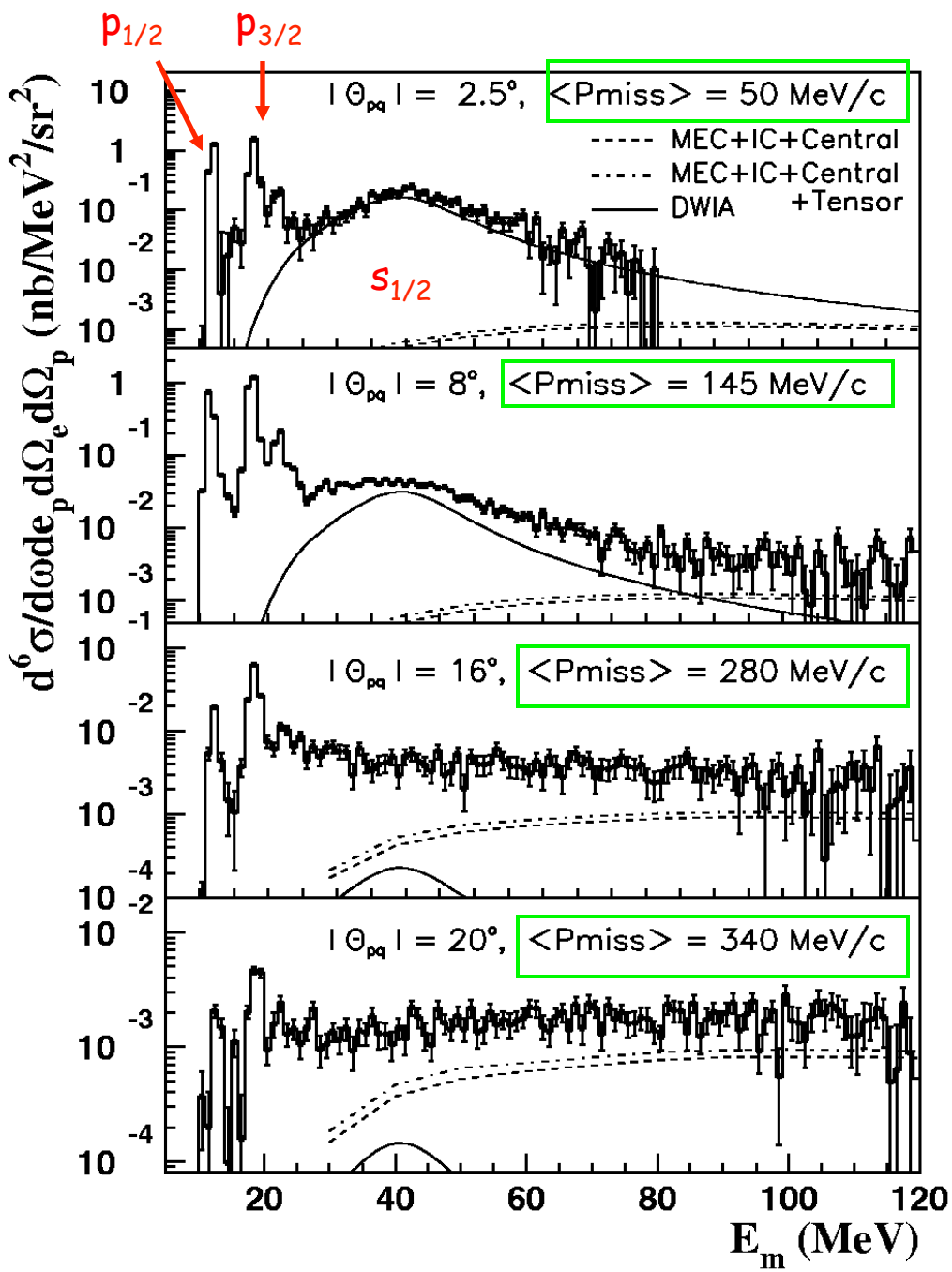
Measuring O(e,e'p) in Hall A

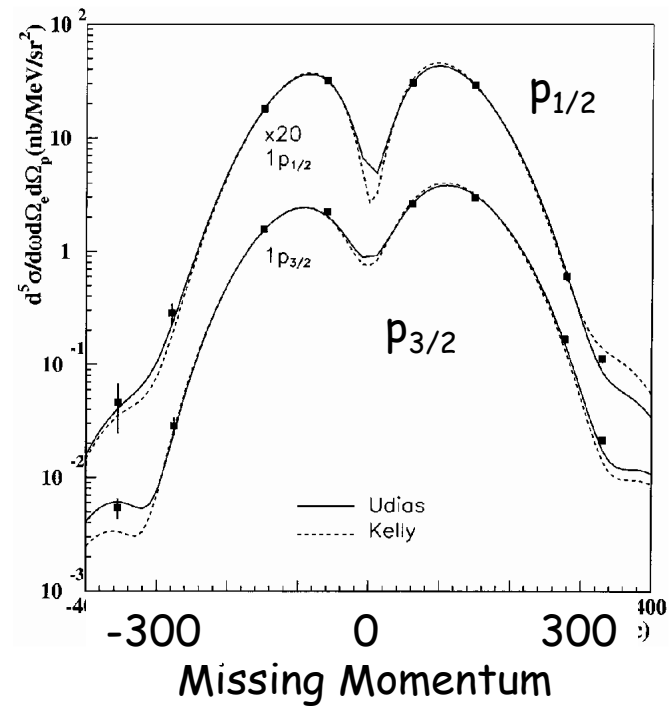




Fissum et al, PRC 70, (2004) 034606

O(e,e'p) and shell structure



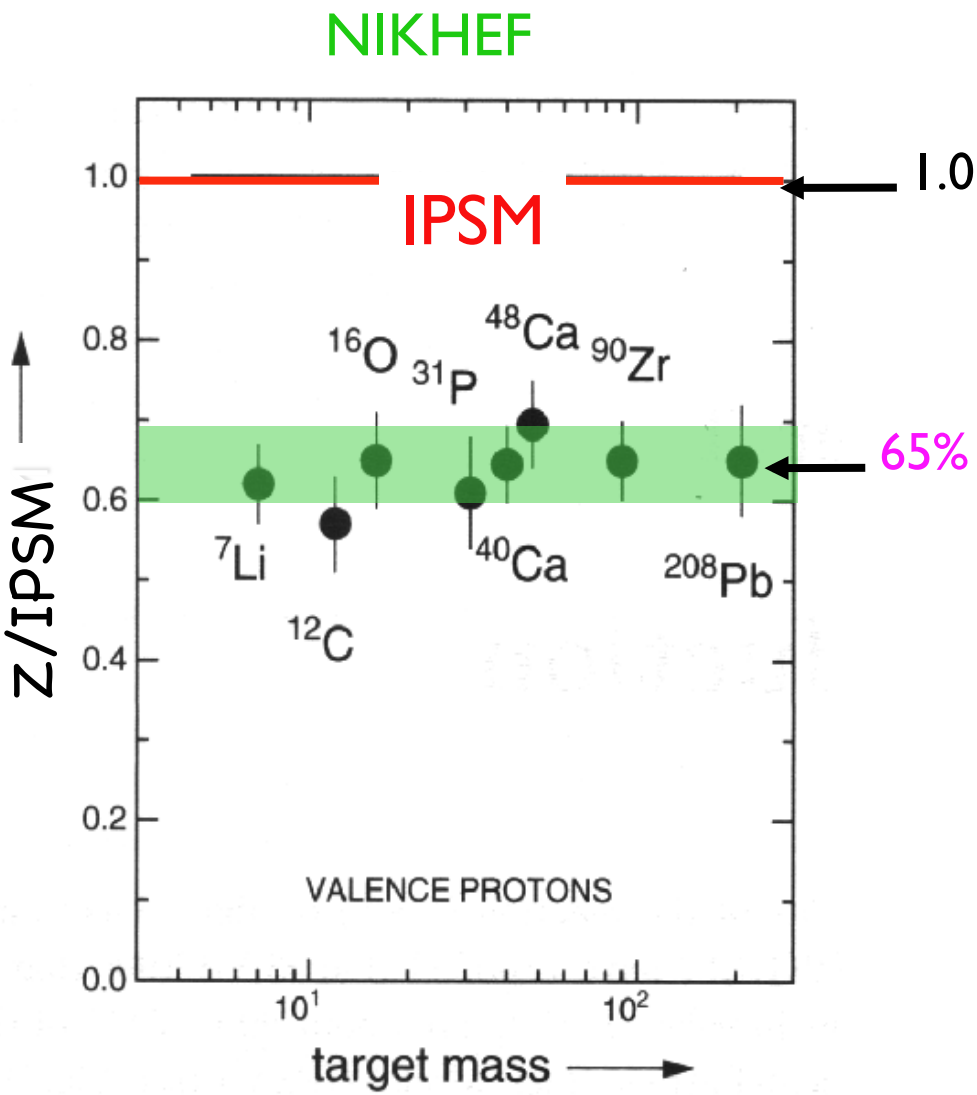


$1p_{1/2}$, $1p_{3/2}$ and $1s_{1/2}$ shells visible

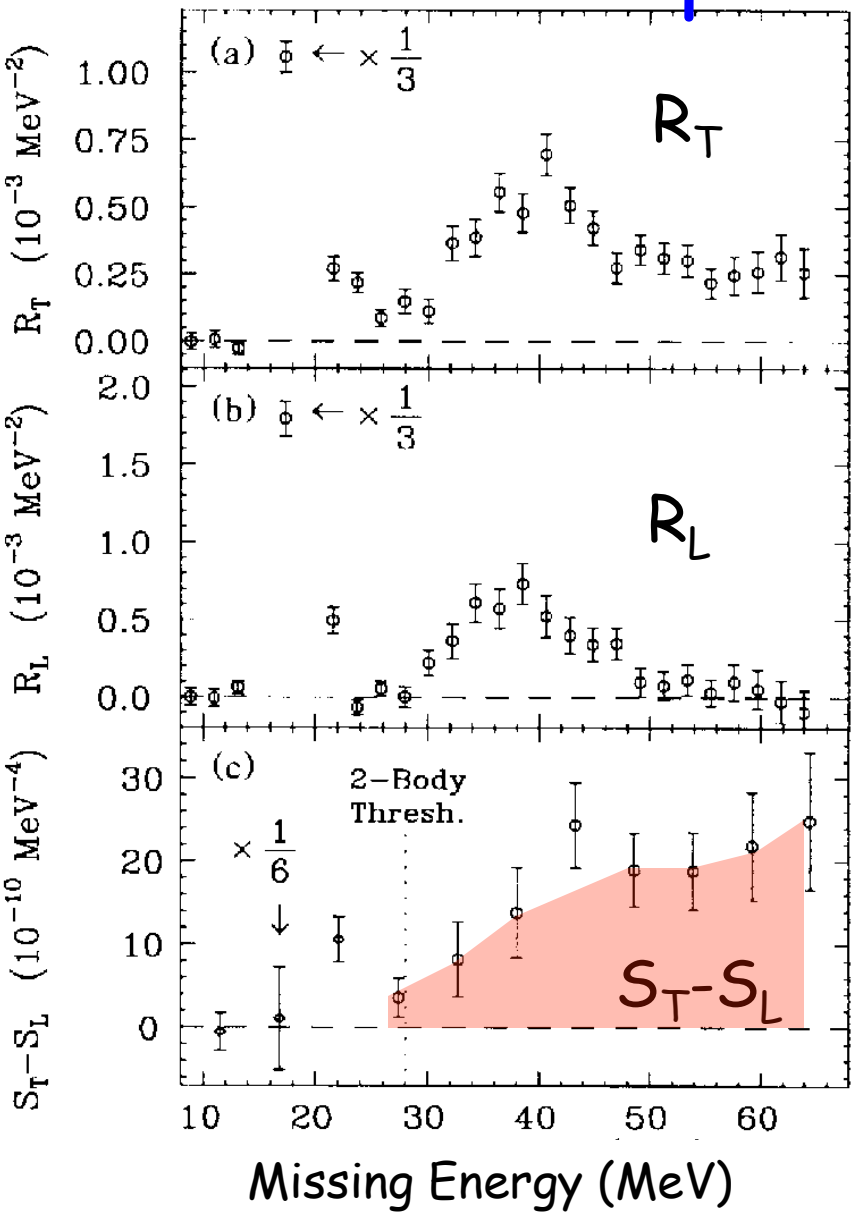
Momentum distribution as expected for $l = 0, 1$

Fissum et al, PRC 70, 034606 (2003)

But we do not see enough protons!



Now separate R_L and R_T



$^{12}C(e,e'p)$
$q = 0.4$ GeV and $x = 1$

(S_T and S_L are just scaled versions of R_T and R_L.)

There is extra transverse strength starting at the two-nucleon knockout threshold

Ulmer et al, PRL 59, 2259 (1987); Dutta et al, PRC 61, 061602 (2000)

(e,e'p) summary

- Measure shell structure directly
- Measure nucleon momentum distributions
- Extra transverse strength seen in (e,e') due to:
  - Two nucleon knockout via
  - Meson exchange currents and correlations
- But:
  - Not enough nucleons seen!