Nucleon Spin Decomposition

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Outline

- Motivation
- Introduction
- Different results suggested / Latest Development
- Summary of results
Motivation

Spin of Nucleon (proton):

• Before 1980’s: quarks carry all of the nucleon spin
• Suggestion by European muon collaboration (EMC):
  – Spin carried by quarks insufficient to justify total spin of nucleon

- SMC at CERN, HERMES at DESY, COMPASS, Jlab etc. confirmed the original discovery of EMC
- only ~ 30% of nucleon spin is by quark spin.
- where remaining ~70% comes from?
- How is nucleon spin is carried by its constituents?

Spin crisis in nuclear physics !!!!
missing angular momentum !!!!
Introduction

Factors contributing for remaining 70% of nucleon spin:

- Quark Orbital Angular Momentum (OAM)
- from Quantum Chromodynamics (QCD): (exchange of gluon to bind quarks inside the nucleon)
  - Gluon spin
  - Gluon OAM

Spin sum rule

\[ \frac{1}{2} = \frac{1}{2} \Sigma_q + \Sigma_g + L_q + L_g. \]

- Quark spin- from polarized Deep Inelastic Collision (DIS)
- Gluon spin –measured by many experiments
- Quark and gluon OAM- Generalized Parton Distribution (GPD) through Deeply Virtual Compton Scattering (DVCS) in Jlab
Introduction

Summary of current status of nucleon spin:

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma^Q + \Delta g + \text{Orbital Angular Momenta} ? \]

\[ \Delta \Sigma^Q : \text{fairly precisely determined!} \quad (\sim 1/3) \]

\[ \Delta g : \text{likely to be small, but large uncertainties} \]

↓

What carries the remaining 2/3 of nucleon spin?

quark OAM?  gluon spin?  gluon OAM?
To decompose \( J \) (total angular momentum) into contributions from different constituents:

- Changing gauge may also shift angular momentum between various degrees of freedom.
- Decomposition depends on gauge and quantization scheme.
- Not necessarily be unique—like culture.
- What is the precise (QCD) definition of each term of the decomposition?
- How we extract each term by means of direct measurement?

Some Decomposers of Nucleon Spin:

- Jaffe Manohar Decomposition
- Ji Decomposition
- Chen, Sun and Leader et al.
- Decomposition by Wakamatsu
- OAM in QED / QCD
- Chen, Sun et al. decomposition
- Some latest updates

Big problem: Orbital Angular Momenta !!!!
Results Suggested for Nucleon Spin

Basic Gauge Principle: Observables must be gauge invariant

JM Decomposition of Nucleon Spin:

\[ \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \mathcal{L}^q + \frac{1}{2} \Delta G + \mathcal{L}^g \]

\[ M^{+xy} = \frac{1}{2} \sum_q \gamma_5 q_+ + \sum_q (\vec{r} \times \vec{\sigma})^2 q_+ + e^{-ij} TrF^i A^j + 2 TrF^j (\vec{r} \times \vec{\sigma})^2 A^j \]

where \( q_+ = \frac{1}{2} \gamma^- \gamma^+ q \) is the dynamical component of the quark field operators and \( A^+ \)

- not gauge invariant
- Quark spin -polarized DIS
- \( \Delta g \) from polarized DIS
- OAM on light-like hypersurface

Pizza quattro stagioni

R.L. Jaffe and A. Manohar, NPB 337, 1990 ~ 500 citations
Ji Decomposition of Nucleon Spin:

\[ \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L^z_q + J^z_g \]

\[ M^{0xy} = \frac{1}{2} \sum_q q^\dagger \Sigma^z q + \sum_q q^\dagger (\vec{r} \times i\vec{D})^z q + [\vec{r} \times (\vec{E} \times \vec{B})]^z \]

where \( i\vec{D} = i\vec{\partial} - gA. \)

- Each term is separately gauge invariant
- OAM on space–like hypersurface
- \( J_\alpha \) accessible through GPDs (X. Ji 1997)

\[ J^z_q = \frac{1}{2} \Delta q + L^z_q = \frac{1}{2} \int_0^1 dxx[H_q(x, 0, 0) + E_q(x, 0, 0)]. \]

- DVCS to probe \( J_q = S_q + L_q \)
- No further decomposition of \( J_g \)
- i.e. no identification of gluon spin/OAM

X. Ji PRL 78, 1997
Results Suggested for Nucleon Spin

Compare: Ji and JM Decomposition

\[ L_Q(JM) \sim \psi^\dagger x \times p \psi \]

\[ L_Q(Ji) \sim \psi^\dagger x \times (p - gA) \psi \]

canonical OAM

\( (p: \text{canonical momentum}) \)

dynamical OAM

\( (p - gA: \text{dynamical momentum}) \)

not gauge invariant!

\[ \Delta g \quad + \quad \mathcal{L}^g \neq J^g \quad \text{gauge invariant!} \]

no sense to mix them

\( \Rightarrow J_q - \Delta g \) has no connection to OAM

Important question:

how significant is the difference between \( L_q \) and \( \mathcal{L}_q \), etc.?
Results Suggested for Nucleon Spin

OAM in Scalar Diquark Model by MB and BC:

- Toy model for nucleon-nucleon splits into quark and scalar ‘diquark’

\[ J^z = \frac{1}{2} \Delta \Sigma + \sum_q L_q + J_g \]

Jaffe: \[ J^z = \frac{1}{2} \Delta \Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g \]

\[ L_q = \int_0^1 dx \int \frac{d^2 k}{16 \pi^3} (1 - x) \left| \psi_{-\frac{1}{2}} \right|^2 \]

\[ J_q^z = \frac{1}{2} \Delta q + L_q^z = \frac{1}{2} \int_0^1 dx [H_q(x,0,0) + E_q(x,0,0)]. \]
Results Suggested for Nucleon Spin

Orbital Angular Momentum in Quantum electrodynamics (QED):
(Angular Momentum Decomposition in QED)

OAM of $e^-$ according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2k_\perp \left[ (1 - x) \left| \Psi_{1/2}^{+1}(x, k_\perp) \right|^2 - \left| \Psi_{1/2+1}(x, k_\perp) \right|^2 \right]$$

$e^-$ OAM according to Ji

$$L_e = \frac{1}{2} \int_0^1 dx \cdot x \left[ q(x) + E(x, 0, 0) \right] - \frac{1}{2} \Delta q$$

$$\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$$

Likewise, computing $J_\gamma$ from photon GPD, and $\Delta \gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and defining $\hat{L}_\gamma \equiv J_\gamma - \Delta \gamma$ yields

$$\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma; \ \alpha/4\pi \text{ is small.}$$

Similar calculation in QCD for quark and gluon:

Applying these results to a (massive) quark with $J^z = +\frac{1}{2}$ yields to $\mathcal{O}(\alpha_s)$

$$\mathcal{L}_q^z - L_q^z = \frac{\alpha_s}{3\pi},$$

i.e., for $\alpha_s \approx 0.5$ about 10% of the spin budget for this quark.

Coined new term “Vector potential “to contribute to nucleon spin
Results Suggested for Nucleon Spin

Important: so far, quest for gauge invariant decomposition of $J_g$

Gauge Invariant Decomposition by Chen, Sun et. all:

basic idea

$$A^\mu = A^\mu_{phys} + A^\mu_{pure}$$

which is a kind of generalization of the decomposition of photon field into the transverse and longitudinal components in QED:

$$A_{phys} \Leftrightarrow A_{\perp}, \quad A_{pure} \Leftrightarrow A_{\parallel}$$

$$A = A_{pure} + A_{phys} \quad \text{with} \quad \nabla \cdot A_{phys} = 0 \quad \nabla \times A_{pure} = 0$$

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left( \frac{1}{2} \Delta q + L'_q \right) + S'_g + L'_g$$ with $\Delta q$ as in JM/Ji

$$L'_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i \vec{D}_{pure} \right)^3 q(\vec{x}) | P, S \rangle$$

$$S'_g = \int d^3x \langle P, S | \left( \vec{E} \times \vec{A}_{phys} \right)^3 | P, S \rangle$$

$$L'_g = \int d^3x \langle P, S | E^i \left( \vec{x} \times \vec{\nabla} \right)^3 A^i_{phys} | P, S \rangle$$

Chen Sun Goldman PRL 103, 2009
Results Suggested for Nucleon Spin

\[ \bar{J}_{\text{QCD}} = \int d^3x \bar{\psi} \gamma^1 \bar{\Sigma} \psi + \int d^3x \bar{\psi} \gamma^1 \bar{D} \psi + \int d^3x \bar{E} \times \bar{A} + \int d^3x \bar{E} \times E_i \bar{D} \bar{A}_i \]

\[ \equiv \bar{S} + \bar{L}_q + \bar{S} + \bar{L}_q. \]

• \( S_g \) very small, but large uncertainties
• Reduces Jaffe-Manohar decomposition in a Gauge

\[ A_{\text{pure}} = 0, \quad A = A_{\text{phys}} \]

• OAM is canonical OAM operator
• \( S \) (gluon spin in coulomb Gauge) = (5/9) \( \Delta g \) (light cone gauge).
• Suggests that under proper identification, gluon spin to nucleon spin may be drastically smaller than conventional wisdom.

Chen et al.’s papers arose quite a controversy on the feasibility of complete decomposition of nucleon spin.

• Y. Hatta, Phys. Rev. D84 (2011) 041701R.
Gauge Invariant Decomposition by Wakamatsu:

• Tried to give a clear picture of complete decomposition of nucleon spin:

\[
J_{QCD} = S^q + L^q + S^g + L^g
\]

\[
S^q = \int \psi \frac{1}{2} \sum \psi d^3x
\]

\[
L^q = \int \psi x \times (p - gA) \psi d^3x
\]

\[
S^g = \int E^a \times A_{phys}^a d^3x
\]

\[
L^g = \int E^{aj} (x \times \nabla) A_{phys}^{aj} d^3x + \int \rho^a (x \times A_{phys}^a) d^3x
\]

• Vector potential found in ‘MB and BC’

• Quark part is common to Ji decomposition

• Quark and gluon intrinsic spin part common to “Chen et. all” decomposition

quark OAM extracted from combined analysis of GPD
and polarized PDFs is \textbf{dynamical quark OAM}, not
the\textbf{ canonical OAM} as predicted by Chen et. all

A crucial difference with the Chen decomp. appears in the \textbf{orbital parts}

\[
L^q + L^g = L'^q + L'^g
\]

\[
L^g - L'^g = -(L^g - L'^q) = \int \rho^a (x \times A_{phys}^a) d^3x
\]
Results Suggested for Nucleon Spin

QED framework also supports his work: (Physical meaning of potential angular momentum)

It represents angular momentum associates with the longitudinal part of electric field generated by color-charged quarks!

Next:

Introduced the covariant generalization for all decomposition:

1. It is useful to find relations to high-energy DIS observables.
2. It is essential to prove Lorentz frame-independence of the decomposition.

\[
\Delta q = \int_{-1}^{1} \Delta q(x) \, dx, \quad \Delta g = \int_{-1}^{1} \Delta g(x) \, dx.
\]

\[
S_q = \frac{1}{2} \Delta q,
\]

\[
L_q = \frac{1}{2} \left[ A_{20}^q(0) + B_{20}^q(0) \right] - \frac{1}{2} \Delta q,
\]

\[
S_g = \Delta g,
\]

\[
L_g = \frac{1}{2} \left[ A_{20}^g(0) + B_{20}^g(0) \right] - \Delta g.
\]

\[
A_{20}^{q/g}(0) = \int_{-1}^{1} x \, H^{q/g}(x, 0, 0) \, dx,
\]

\[
B_{20}^{q/g}(0) = \int_{-1}^{1} x \, E^{q/g}(x, 0, 0) \, dx.
\]
Results Suggested for Nucleon Spin

we find two physically nonequivalent decompositions (I) and (II).

The great advantage of decomposition (I) over (II) is: Concrete connection between High Energy DIS (discussed by Wakamatsu)

**Decomposition (II)**

This decomposition reduces to any ones of Bashinsky-Jaffe, of Chen et al., and of Jaffe-Manohar, after an appropriate gauge-fixing in a suitable Lorentz frame, which reveals that these 3 decompositions are all gauge-equivalent!

These 3 are physically equivalent decompositions!
It was sometimes criticized that there are so many decompositions of nucleon spin.

\[
\frac{1}{2} = \frac{1}{2} \Sigma + L_Q + \Delta g + L_g \\
= \frac{1}{2} \Sigma' + L'_Q + \Delta g' + L'_g \\
= \frac{1}{2} \Sigma'' + L''_Q + \Delta g'' + L''_g \\
\vdots
\]

However, this is not true any more. One should recognize now that there are only two physically nonequivalent decompositions!

**Decomposition (I)**

extension of Ji's decomp.
including gluon part

**Decomposition (II)**

nontrivial gauge-invariant extension of Jaffe-Manohar's decomp.

**dynamical OAMs**

**“canonical” OAMs**

Since both decompositions are gauge-invariant, there is a possibility that they both correspond to observables!
Goldman argued that the nucleon spin decomposition is **frame-dependent**!


This is generally true, but our interest here is the longitudinal spin sum rule.

- The **longitudinal spin decomposition** is certainly **frame-independent**!

Leader recently proposed a sum rule for **transverse angular momentum**.

More you **read** more you get **confused** !!!!!!!

Thank you for your patience