HUGS 2012

Dualities and QCD

Josh Erlich

LECTURE 2

Outline

- The meaning of "duality" in physics (Example: The Ising model)
- Quark-Hadron duality (experimental and theoretical evidence)
- Electric-Magnetic Duality (monopole condensation and confinement)
- The AdS/CFT correspondence (gauge/gravity duality, holographic QCD)

Electric-Magnetic Duality

Strassler - "Confinement and Duality" lectures
Intriligator,Seiberg - hep-th/9509066
Seiberg,Witten - hep-th/9407087

Consider Maxwell's electrodynamics:

$$S = -\frac{1}{4e^2}\int d^4x\, F_{\mu\nu}F^{\mu\nu}$$

Source-free Maxwell's Equations:

$$\partial_\mu F^{\mu\nu} = 0$$ Euler-Lagrange eqs

$$\partial_\mu(\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}) = 0$$ Bianchi identities

Electric-Magnetic Duality

Source-free Maxwell's Equations:

$$\partial_\mu F^{\mu\nu} = 0 \quad \longleftarrow \quad \nabla \times \mathbf{B} + (-\dot{\mathbf{E}}) = 0, \quad \nabla \cdot (-\mathbf{E}) = 0$$

$$\partial_\mu (\epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}) = 0 \quad \longleftarrow \quad \nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0, \quad \nabla \cdot \mathbf{B} = 0$$

Define the dual field strength tensor:

$$\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma}$$

Exercise: $$\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = F_{\mu\nu} F^{\mu\nu}$$

Electric-Magnetic Duality

We can enforce the Bianchi identity by adding a Lagrange multiplier vector A_D

so that A_D would couple to monopoles with charge 1

$$S = -\frac{1}{4e^2}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} + \frac{e^2}{2\pi} A_{D\mu}\epsilon^{\mu\nu\lambda\sigma}\partial_\nu F_{\lambda\sigma} \right)$$

$$= -\frac{1}{4e^2}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} + \frac{e^2}{2\pi} A_{D\mu}\partial_\nu \tilde{F}^{\mu\nu} \right)$$

$$= -\frac{1}{4e^2}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} - \frac{e^2}{2\pi} (\partial_\nu A_{D\mu})\tilde{F}^{\mu\nu} \right)$$

$$= -\frac{1}{4e^2}\int d^4x \left(F_{\mu\nu}F^{\mu\nu} + \frac{e^2}{2\pi} \tilde{F}_{D\mu\nu}F^{\mu\nu} \right)$$

Electric-Magnetic Duality

$$S = -\frac{1}{4e^2}\int d^4x\,\left(F_{\mu\nu}F^{\mu\nu} + \frac{e^2}{2\pi}\tilde{F}_{D\mu\nu}F^{\mu\nu}\right)$$

Treat F as the physical field. The equation of motion for F gives:

$$F_{\mu\nu} = -\frac{e^2}{4\pi}\tilde{F}_{D\mu\nu}$$

$$S = -\frac{e^2}{64\pi^2}\int d^4x\,F_{D\mu\nu}F^{D\mu\nu}$$

Weak-Strong Duality!

This is the same as the Maxwell action in terms of F_D, but with $\boxed{e^2 \to \frac{16\pi^2}{e^2}}$.

Monopole Condensation and Confinement

't Hooft, Mandelstam -1976

In some theories electric-magnetic duality allows us to understand a relation between condensation of monopoles and confinement of (chromo)electric flux.

The basic insight comes from confinement of magnetic fluxes in superconductors.

Monopole Condensation and Confinement

In Type I and Type II superconductors Cooper pairs of electrons condense. A relativistic analogy of this is the Abelian Higgs model, where the Cooper pair field is the Higgs field ϕ.

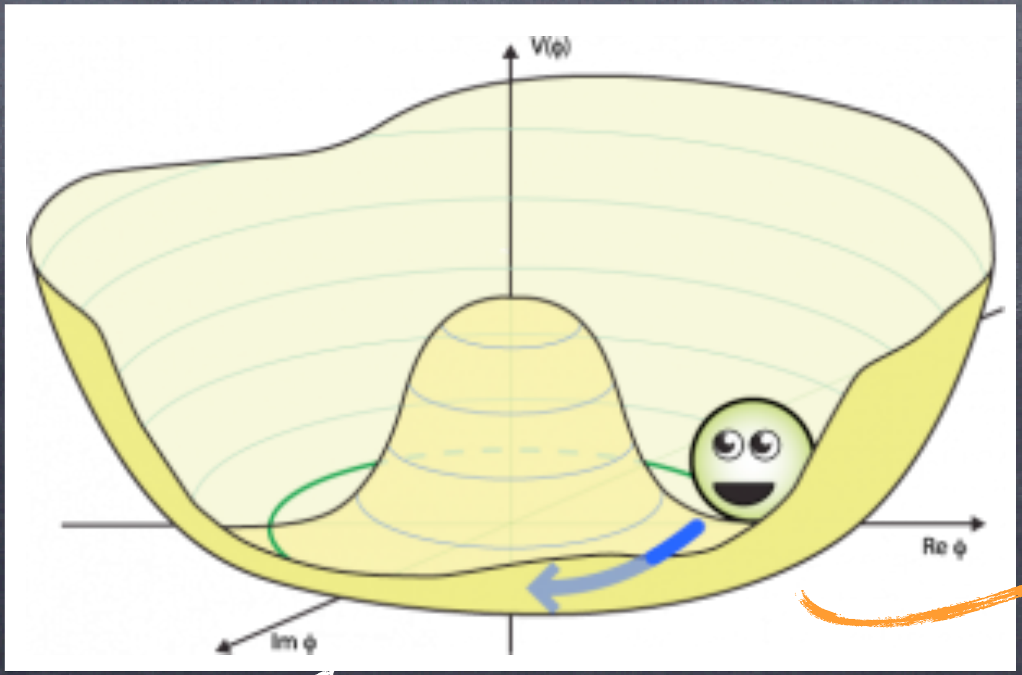

image from Quantum Diaries

Vacuum expectation value $|\phi| = v$

Monopole Condensation and Confinement

Expand the Lagrangian about the VEV $\phi = v$.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |(i\partial_\mu - 2eA_\mu)\phi|^2 - V(|\phi|^2)$$

Cooper pair has charge 2

$\supset 4e^2v^2\, A_\mu A^\mu \leftarrow$ Mass term for photon (Anderson-Higgs-... mechanism)

Higgs mass small compared to photon mass: Type I

Higgs mass large compared to photon mass: Type II

Monopole Condensation and Confinement

Equation of motion for A^0 with static electric point charge source:

$$(\nabla^2 + 8e^2v^2)A^0 = \delta(x)$$

$$A^0(r) \sim \frac{e^{-2\sqrt{2}ev\,r}}{r}$$

screening of electric field

Monopole Condensation and Confinement

In Type II superconductors magnetic flux can penetrate into the superconductor by creating localized Cooper pair currents called Abrikosov vortices

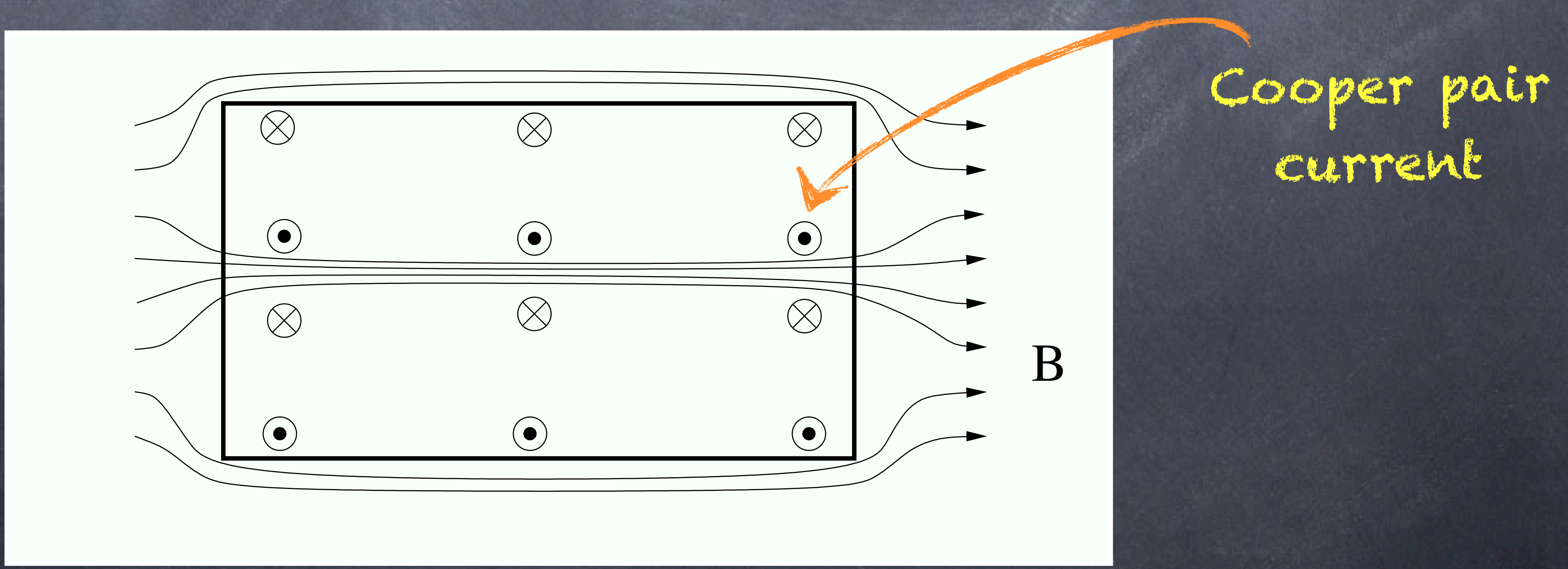

image from Strassler, "Confinement and Duality"

Monopole Condensation and Confinement

In the relativistic context, Vortex solutions are called Nielsen-Olesen vortices.

To have finite energy, vortex solutions must behave at large r as:

$$\phi(r) \sim v\, e^{is\theta}, \qquad A_\theta \sim \frac{s}{2e}, \qquad \text{where} \quad s \in \mathbb{Z}$$

quantized, confined magnetic flux!

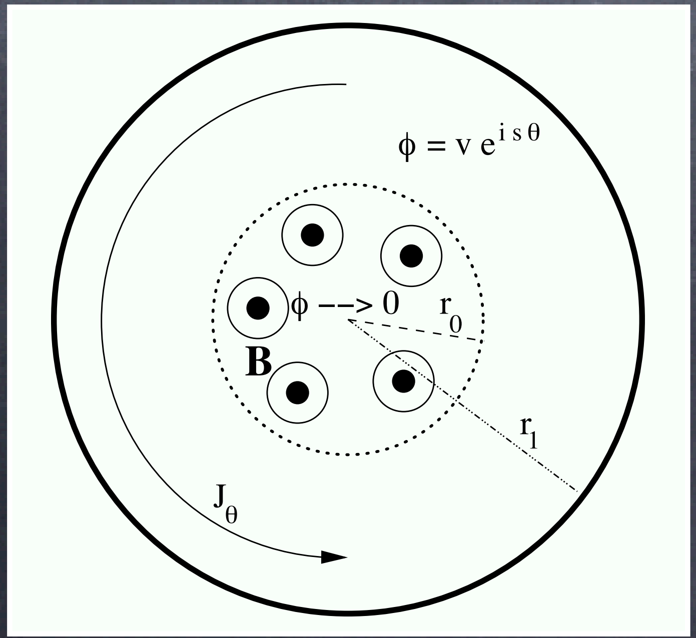

image from Strassler, "Confinement and Duality"

Monopole Condensation and Confinement

In vacuum the phase of ϕ becomes the helicity-0 component of the massive photon.

The magnitude of ϕ is massive. (Exercise)
Hence, this theory confines magnetic flux and has a mass gap.

If we could flip this story around, exchanging the electric and magnetic fields, we would have confined electric flux and a mass gap -- like QCD!

Monopole Condensation and Confinement

Well, sort of.

The field ϕ transforms under a U(1) gauge invariance, $\phi \to e^{2ie\theta(x)}\phi$, not the SU(3) gauge invariance of QCD.

Unless the SU(3) is broken in an appropriate way, there are no analogous magnetic flux tubes. However, for a non-Abelian "electric" theory, the "magnetic" dual theory may have a different gauge group.

Montonen-Olive Duality

The first well understood extension of the electric-magnetic duality to non-Abelian theories was to N=4 supersymmetric gauge theories.

We will not describe SUSY in any detail during these lectures, but it is worthwhile discussing some features of SUSY dualities, as they are in some ways better understood than their non-SUSY counterparts.

Montonen-Olive Duality

N=4 SUSY gauge theory with gauge group G has a dual description in terms of gauge group G_D (whose weight lattice is the dual lattice to that of G), with "electric" and "magnetic" charges exchanged.

It is a weak-strong duality like the U(1) electric-magnetic duality described earlier.

$$G = SU(N)$$
$$G_D = SU(N)/\mathbf{Z}_N$$

Monopole Condensation and Confinement

$G = SU(N)$ ← No confined "magnetic" fluxes

$G_D = SU(N)/\mathbf{Z}_N$ ← Can have confined "electric" fluxes!

Could this help to explain confinement of the chromoelectric field in QCD?

Perhaps, but we would need effective scalar field(s) charged under dual gauge group G_D analogous to the Cooper pair field -- monopole condensation?

Monopole Condensation and Confinement

To examine the possibility of monopole condensation in QCD, let's first consider the simpler example of the SU(2) gauge theory with an adjoint Higgs field.

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + D_\mu\phi^a D^\mu\phi^a - V(\phi^a\phi^a)$$

$$D_\mu\phi^a = \partial_\mu\phi^a - e\epsilon^{abc}A^b_\mu\phi^c$$

We assume the potential has minima at

$$\phi^a\phi^a = v^2$$

which breaks SU(2)$\rightarrow$U(1)

Monopole Condensation and Confinement

If there are monopole solutions, to have finite energy at spatial infinity the Higgs field must approach vacuum solutions $\phi^a\phi^a = v^2$.

This means that the 2-sphere at infinity is mapped into vacua, which are labeled by elements of SU(2)/U(1).

These mappings are characterized by the homotopy group $\pi_2(SU(2)/U(1)) = \mathbb{Z}$

Monopole Condensation and Confinement

Explicit monopole solutions to the classical equations of motion are known. At large r one such solution (the 't Hooft-Polyakov "hedgehog") behaves as:

SU(2) gauge index

spatial index

$$n^i \equiv \frac{\phi^i}{v} \sim \frac{x^i}{r}$$

$$A_i^a \sim \epsilon_{iak} \frac{x^k}{r}$$

Monopole Condensation and Confinement

Define: $$F_{\mu\nu} \equiv n^a F^a_{\mu\nu}$$
$$B_\mu \equiv n^a A^a_\mu$$

Exercise: $$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \frac{1}{e}\epsilon_{ijk} n^i (\partial_\mu n^j \partial_\nu n^k)$$

If we choose a gauge so that n=(0,0,1) the last term vanishes, and F looks like the field strength of a U(1) gauge field. Monopoles carry charge under this "magnetic" U(1). The U(1) can be spontaneously broken by monopole condensation.

Monopole Condensation and Confinement

't Hooft argued that even without a Higgs field, one can define an analog to the Abelian projection and describe monopoles and their condensation.

Choose any operator O^a that transforms in the adjoint rep of the gauge group (like F^a_{12}).

Monopole Condensation and Confinement

The analogy of the Abelian projection is a gauge such that $O^a = (0, 0, O_3(x))$. A residual U(1) transformation leaves this form invariant.

At points where $O^a(x)=0$ the gauge transformation is singular. The singularity depends on mappings of the 2-sphere around the singular point to elements of SU(2)/U(1), as expected for a monopole!

Monopole Condensation and Confinement

A similar analysis goes through with gauge group SU(3).

Perhaps the monopoles described by an appropriate O^a (evidence shows that it doesn't work for all O^a) condense and confine the chromoelectric flux?

Seiberg-Witten Duality

Review: Alvarez-Gaume,Hassan - hep-th/9701069

N=2 SUSY SU(N) gauge theory contains monopoles and dyons, as in N=4 SUSY.

When the N=2 theory is deformed to N=1 by making certain fields massive, monopoles condense and the theory confines.

We will summarize some of the features of the theory and its analysis.

Seiberg-Witten Duality

N=2 SUSY SU(N) gauge theory: Contains

Vector field A^a_μ
Fermions $\lambda^a \quad \tilde{\psi}^a$
Scalar field ϕ^a

All fields are in the adjoint rep of the SU(N) gauge group.

Seiberg-Witten Duality

If $\langle\phi^a\rangle \neq 0$ then SU(N) is generically broken to $U(1)^N$.

Consider the SU(2) theory. The theory has a continuum of physically inequivalent vacua.

The vacuum can be parametrized by $u \equiv \langle\phi^a\phi^a\rangle$.

Seiberg-Witten Duality

The low-energy U(1) theory is weakly coupled if $u/\Lambda_{QCD} \gg 1$.

The coupling (and theta parameter) depends on $\ln(u/\Lambda_{QCD})$ and changes as a loop is made in complex u space:

$$u \to u\, e^{2\pi i}$$

Then there must be singularities at small u consistent with the monodromy at large u.

Seiberg-Witten Duality

What singularities?

Massless monopoles or dyons might exist for specific values of u, and would modify the beta function of the magnetic dual coupling in a calculable way.

Seiberg and Witten argued that there are precisely two values of u at which a monopole or dyon becomes massless.

Seiberg-Witten Duality

In terms of the magnetic dual description with massless monopoles, N=2 SUSY requires a scalar potential including

$$V = 2|\phi_D^a M|^2 + 2|\phi_D^a \tilde{M}|^2 + 2|M\tilde{M}|^2$$

VEV of dual adjoint scalar

Scalar monopole fields

Seiberg-Witten Duality

$$V = 2|\phi^a_D M|^2 + 2|\phi^a_D \tilde{M}|^2 + 2|M\tilde{M}|^2$$

dual adjoint scalar

Scalar monopole fields

Now add a small mass term for ϕ^a, which is thought to result in a confining theory:

$$\mathcal{L} = m^2\,\phi^a\phi^a = m^2 u(\phi^a_D)$$

Seiberg-Witten Duality

To maintain N=1 SUSY we have to also add a few more terms to the scalar part of the Lagrangian, with the result:

$$V = 2|\phi_D^a M|^2 + 2|\phi_D^a \tilde{M}|^2 + 2\left| M\tilde{M} + \frac{m}{\sqrt{2}}\frac{du}{d\phi_D^a}\right|^2$$

The potential is a sum of squares, so in the ground state(s) each term in the sum vanishes.

Seiberg-Witten Duality

$$M\tilde{M} + \frac{m}{\sqrt{2}}\frac{du}{d\phi_D^a} = 0 \qquad \rightarrow M\tilde{M} \neq 0$$

$$\phi_D^a M = \phi_D^a \tilde{M} = 0 \qquad \rightarrow \phi_D^a = 0$$

The fact that the monopole fields are nonvanishing in the ground state is tantamount to monopole condensation.

In this theory we see explicitly how monopole condensation occurs.

Seiberg Duality

The subject of dualities in N=1 supersymmetric gauge theories with matter is quite rich, and we dont have time to do the subject justice.

One important aspect of the electric-magnetic duality is that the magnetic dual gauge group depends of both the number of flavors and the number of colors, $SU(N_c) \to SU(N_f - N_c)$

Dualities Lecture 2 Summary

Maxwell's source-free electrodynamics has a strong-weak duality which exchanges the electric and magnetic fields.

Confinement in QCD may be related to condensation of magnetic monopoles, a dual to confinement of magnetic flux by condensation of electrically charged particles.

Supersymmetric models allow for an explicit realization of electric-magnetic duality and monopole condensation in non-Abelian gauge theories.