HUGS 2012

Dualities and QCD

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LECTURE 5

Outline

- The meaning of "duality" in physics (Example: The Ising model)
- Quark-Hadron duality (experimental and theoretical evidence)
- Electric-Magnetic Duality (monopole condensation and confinement)
- The AdS/CFT correspondence (gauge/gravity duality, holographic QCD)

Building a Bottom-Up Model

Step 1: Choose 5D gauge group and geometry.

- Tower of vector mesons are identified with tower of Kaluza-Klein gauge bosons.

SU(2) isospin $\rightarrow$ 5D SU(2) gauge theory
Conformal in the UV $\rightarrow$ Anti-de Sitter space near its boundary

Can choose geometry by matching spectrum to Pade approx of SU(2) current-current correlator in deep Euclidean regime $-q^2 \gg m_\rho^2$.

Result: geometry = slice of AdS space
(Shifman; JE,Kribs,Low; Falkowski,Perez-Victoria).

Building a Bottom-Up Model

To include the full chiral symmetry, not just the vector subgroup,

SU(2)×SU(2) chiral symmetry → SU(2)×SU(2) 5D gauge group

Additional tower of gauge bosons → tower of axial-vector mesons.
(5D parity → 4D parity)

(Also describes pions after symmetry breaking)

Building a Bottom-Up Model

Step 2: Include pattern of chiral symmetry breaking

Hint from AdS/CFT: 4D operator $\rightarrow$ 5D field
$\overline{q}_i q_j \rightarrow$ Scalar fields X_{ij}, bifundamental under SU(2)×SU(2)

Background profile for X_{ij}:
Non-normalizable mode $\rightarrow$ source $\mathcal{L}_{4D} \supset m_{ij}\,\overline{q}_i q_j$
Normalizable mode $\rightarrow$ VEV $\langle \overline{q}_i q_j \rangle$

(Klebanov,Witten; Balasubramanian et al.)

The scalar field background explicitly and spontaneously breaks the chiral symmetry.

Building a Bottom-Up Model

For definiteness, we need to choose 5D mass of scalar field.

AdS/CFT: $\boxed{m_X^2 = \Delta_{\bar{q}q}(\Delta_{\bar{q}q} - 4)}$ in units of AdS curvature.

In the UV, $\Delta_{\bar{q}q} = 3$, so we choose $\boxed{m_X^2 = -3}$.

Note: This choice is made for definiteness, but is not necessary.

Building a Bottom-Up Model

In summary, the model is:

SU(2)×SU(2) gauge theory in slice of AdS_5 with background bifundamental scalar field.

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}(|D_MX|^2 - 3|X|^2)\right)$$

$$ds^2 = \frac{1}{z^2}\left(dx_\mu dx^\mu - dz^2\right), \quad \epsilon < z < z_{IR}$$

$$X_0(x,z) = \frac{m_q}{2}z + \frac{\langle\overline{q}q\rangle}{2}z^3$$

Model parameters: $\boxed{g_5,\ m_q,\ \langle\overline{q}q\rangle,\ z_{IR}}$

(JE,Katz,Son,Stephanov; DaRold,Pomarol)

Building a Bottom-Up Model

Vector: $V_\mu^a = \frac{L_\mu^a + R_\mu^a}{2}$

Axial-Vector: $A_\mu^a = \frac{L_\mu^a - R_\mu^a}{2}$

Pions: Mixture of longitudinal part of A_μ^a and scalar field X

Soft-Wall AdS/QCD

In the Hard Wall model $m_n^2 \sim n^2$

To obtain a linear Regge trajectory, the geometry can be modified while coupling to a dilaton background.
(Karch,Katz,Son,Stephanov '06)

$$S = \int d^5x \sqrt{g} e^{-\Phi(x,z)} \mathcal{L}$$

$$\Phi_0(z) \sim z^2, \quad g_{MN} = \mathrm{AdS_5\ Metric}$$

Low-energy predictions are similar to hard-wall model

Hard-Wall (5D tree level)

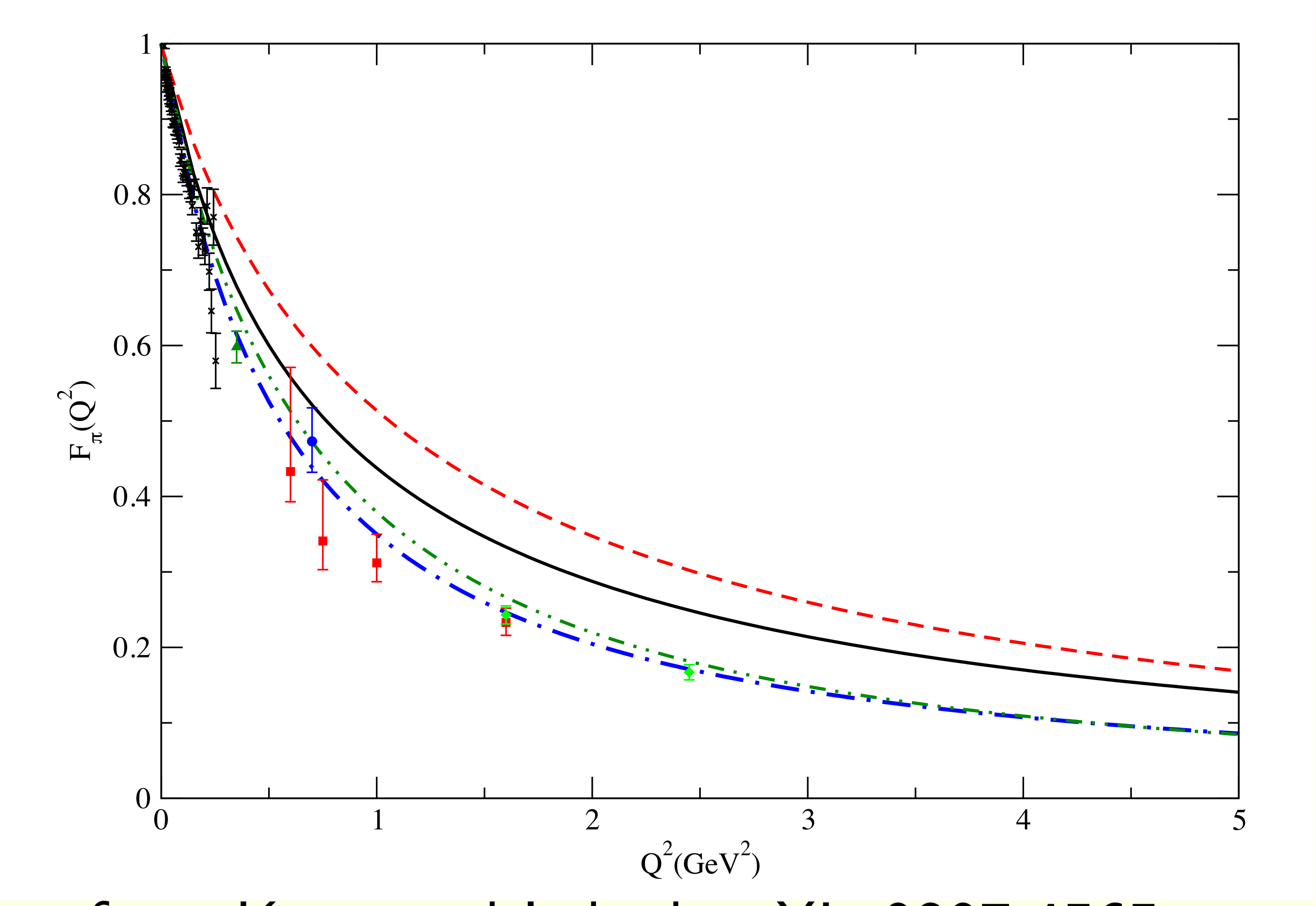
With strange quark mass parameter

Observable	Model A ($\sigma_s = \sigma_q$) (MeV)	Model B ($\sigma_s \neq \sigma_q$) (MeV)	Measured (MeV)
m_π	(fit)	134.3	139.6
f_π	(fit)	86.6	92.4
m_K	(fit)	513.8	495.7
f_K	104	101	113 ± 1.4
$m_{K_0^*}$	791	697	672
$f_{K_0^*}$	28.	36	
m_ρ	(fit)	788.8	775.5
$F_\rho^{1/2}$	329	335	345 ± 8
m_{K^*}	791	821	893.8
$F_{K^*}^{1/2}$	329	337	
m_{a_1}	1366	1267	1230 ± 40
$F_{a_1}^{1/2}$	489	453	433 ± 13
m_{K_1}	1458	1402	1272 ± 7
$F_{K_1}^{1/2}$	511	488	

Abdidin and Carlson '09

Hard/Soft-Wall (5D tree level)

Pion Form Factor



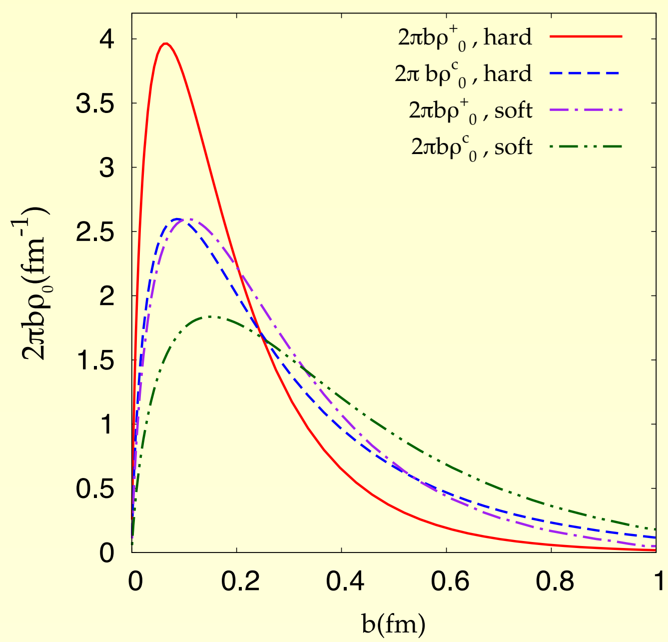
from Kwee and Lebed, arXiv:0807.4565

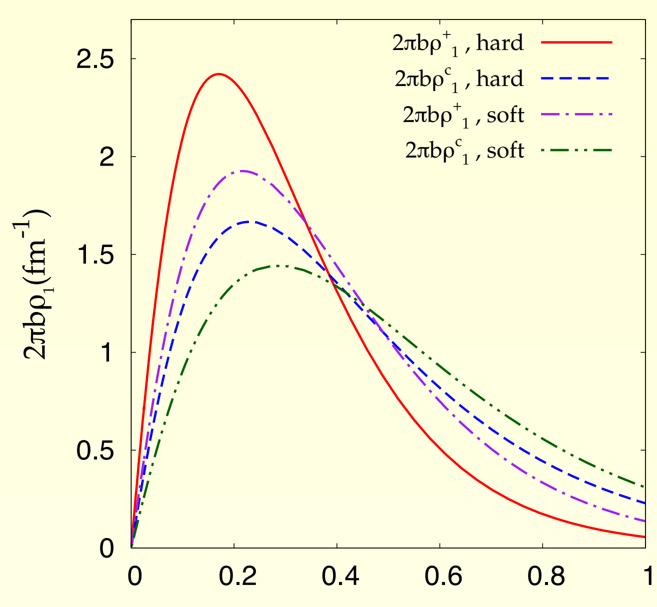
Solid black and blue curves: Hard wall model

Dotted red and green curves: Soft wall model

See also Grigoryan, Radyushkin '08

Hard/Soft-Wall (5D tree level)





Gravitational Form Factors and Generalized Parton Distributions

From Abidin and Carlson, arXiv:0801.3839

Top: p^+ and charge densities of Helicity-0 rho mesons in hard and soft wall models

Bottom: Same for Helicity-1 rho mesons

See also Lyubovitskij, Vega, ...

Hard-Wall (5D tree level)

Can determine meson radii from behavior of form factors near $q^2 = 0$.

Hard wall model:

$$\langle r_\pi^2 \rangle_{charge} = 0.33 \ \mathrm{fm}^2$$

$$\langle r_\pi^2 \rangle_{grav} = 0.13 \ \mathrm{fm}^2$$

$$\langle r_\rho^2 \rangle_{charge} = 0.53 \ \mathrm{fm}^2$$

$$\langle r_\rho^2 \rangle_{grav} = 0.21 \ \mathrm{fm}^2$$

$$\langle r_{a_1}^2 \rangle_{charge} = 0.39 \ \mathrm{fm}^2$$

$$\langle r_{a_1}^2 \rangle_{grav} = 0.15 \ \mathrm{fm}^2$$

H. Grigoryan and A. Radyushkin; Z. Abidin and C. Carlson '07,'08

Universality in AdS/QCD?

Some observables are truly universal, *i.e.* independent of details of model.

Famous Example: Viscosity to Entropy Density η/s

Finite temperature $\rightarrow$ spacetime horizon

Prediction, independent of details of spacetime geometry:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun, Son and Starinets '04

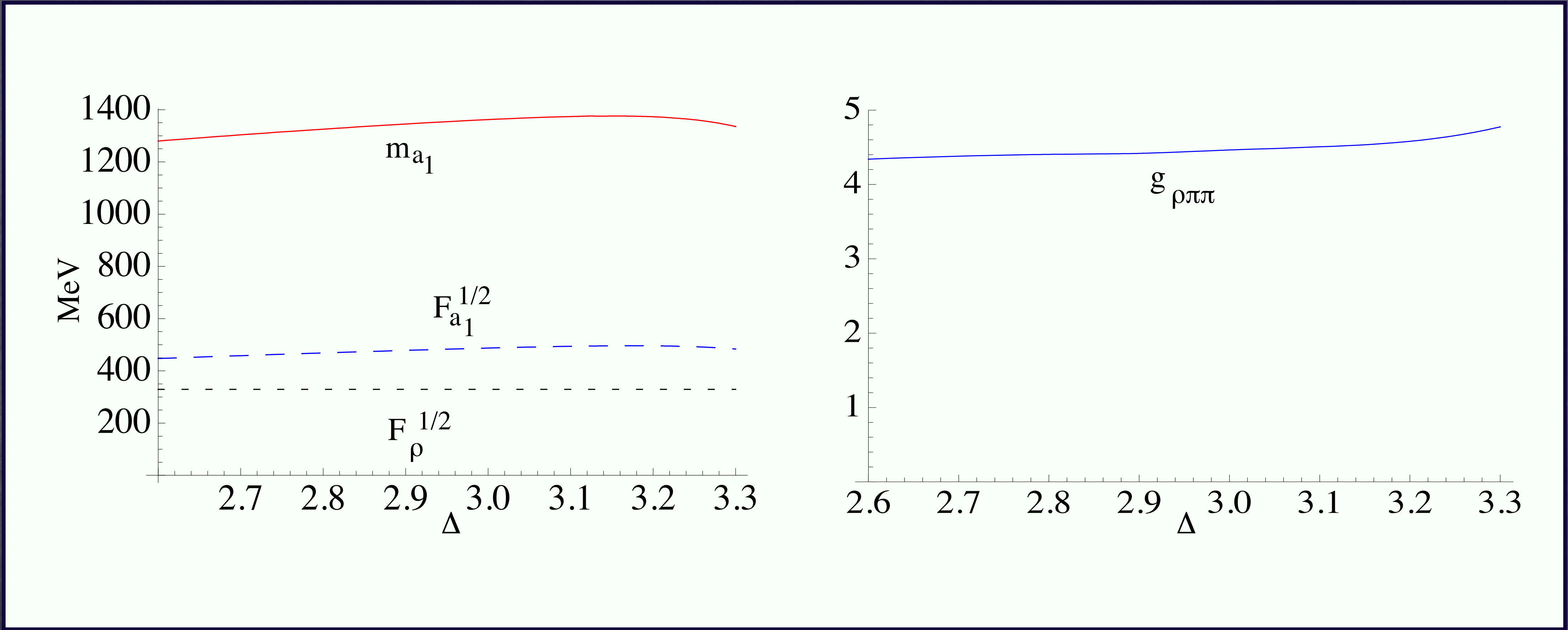
Another example: Electrical Conductivity to Charge Susceptibility σ/χ

$$\frac{\sigma}{\chi} = \frac{1}{4\pi T}\frac{d}{d-2}$$

Kovtun and Ritz '08

Universality?

Observables are roughly independent of X mass



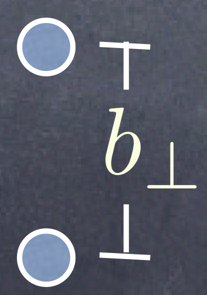
From JE,Westenberger '09

$m_X^2 = \Delta(\Delta - 4)$.
$\Delta = 3$ is the value set by matching to the UV.

Light-Front AdS/QCD

Brodsky and De Teramond have discovered some remarkable similarities between wave equations in AdS and the light-front wave equation for multi-parton states.

Example: Two-parton state



Longitudinal momentum fraction $\boxed{x = k^+/P^+}$

Define light-front kinematical variable

$$\boxed{\zeta \equiv x(1-x)b_\perp^2}$$

Wavefunction: $\boxed{\psi(x,\zeta,\varphi) = e^{iM\varphi}\, X(x)\, \phi(\zeta)/\sqrt{2\pi\zeta}}$

Light-Front AdS/QCD

Light-front wave equation $\boxed{P_\mu P^\mu |\psi\rangle = \mathcal{M}^2 |\psi\rangle}$

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\,\phi(\zeta)}$$

Confining interaction potential

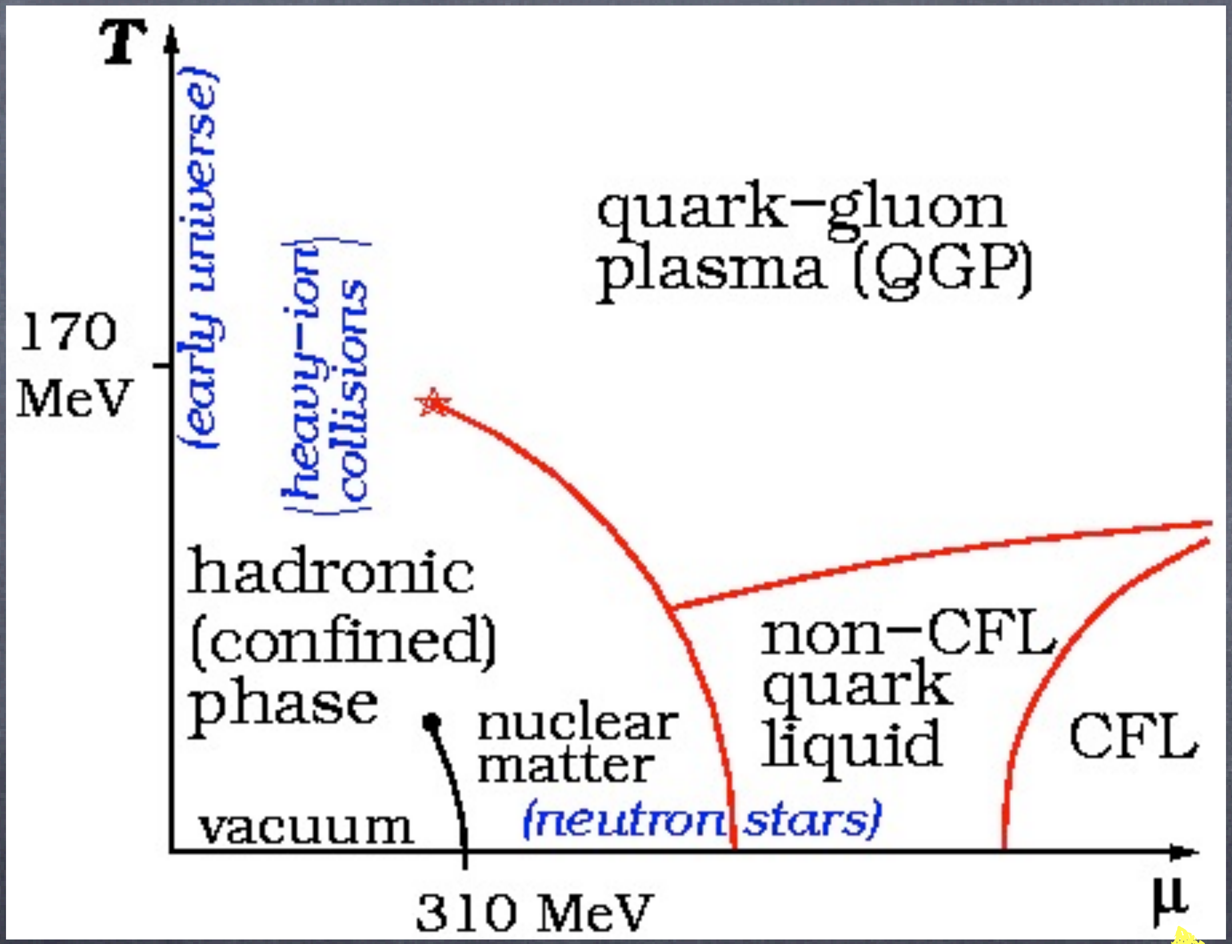
Scalar wave equation in soft-wall model:

$$\boxed{\left[-\frac{z^3}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^3}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\psi(z) = \mathcal{M}^2\psi(z)}$$

Identifying these equations gives the AdS coordinate z a kinematical interpretation

Phases of QCD

According to Wikipedia



Baryon chemical potential

More of the QCD Phase Diagram

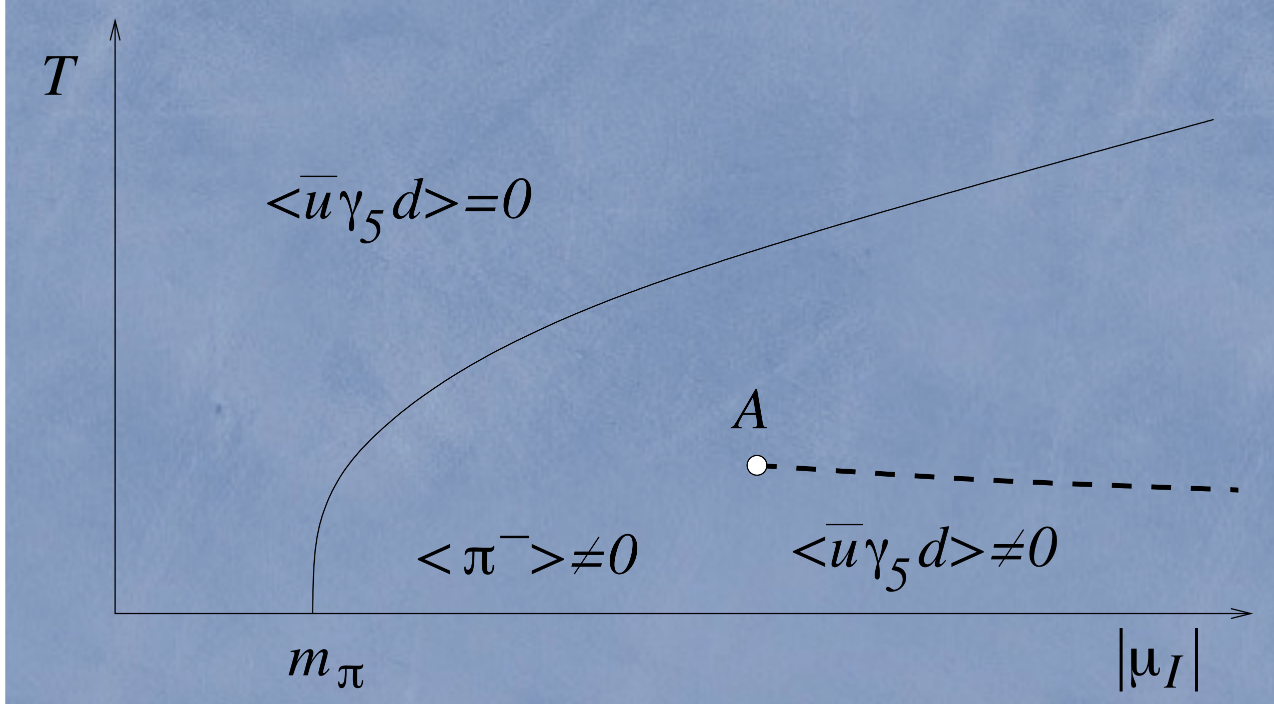


FIG. 1. Phase diagram of QCD at finite isospin density.

Son,Stephanov '00

Positive fermion determinant allows for lattice study

Alford,Kapustin,Wilczek; Kogut,Sinclair; de Forcrand, Stephanov,Wenger; Detmold et al.

Systems with Isospin

- Neutron Stars (Low temp, Large isospin)
- Quark-Gluon Plasma at RHIC,LHC (Higher temp, Smaller isospin)

QCD w/ Isospin Chemical Potential

Small μ_I, T : Can use Chiral Lagrangian
(Son&Stephanov, 2000)

$T = 0$:

Chemical potential couples to isospin number density.

$$J_0^{(3)} = \overline{\psi}\gamma^0\tau^3\psi$$

$$\mathcal{L}_{4D} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\nabla_\nu\Sigma\,\nabla^\nu\Sigma^\dagger\right) + \frac{m_\pi^2 f_\pi^2}{4}\mathrm{Tr}\left(\Sigma + \Sigma^\dagger\right)$$

$$\Sigma = \exp(2i\pi^a\tau^a/f_\pi)$$

$$\nabla_0\Sigma = \partial_0\Sigma - \frac{\mu_I}{2}(\tau_3\Sigma - \Sigma\tau_3).$$

QCD w/ Isospin Chemical Potential

Son-Stephanov ansatz:

$$\overline{\Sigma} = \cos\alpha + i(\tau_1 \cos\phi + \tau_2 \sin\phi)\sin\alpha$$

Results: $\overline{\Sigma} \neq 0$ if $\mu_I > m_\pi$

Phase transition is second order

$c_s^2 > 1/3$

speed of sound

Isospin Chemical Potential in the Hard Wall Model

$$S = \int d^5x \sqrt{-g}\, \mathrm{Tr}\left\{ |DX|^2 + 3\,|X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) \right\}$$

Scalar field background

$$X_0(z) = \frac{1}{2}\left(m_q z + \sigma z^3\right) \equiv \frac{1}{2} v$$

Vector combination of gauge fields

$$V_M^a = 1/2(L_M^a + R_M^a)$$

Gauge choice

$$L_z^a = R_z^a = 0$$

Linearized equation of Motion

$$\partial_z \left(\frac{1}{z} \partial_z V_\mu^a \right) - \frac{1}{z} \partial_\alpha \partial^\alpha V_\mu^a = 0$$

Vector field background

$$V_0^3(z) = c_1 + \frac{c_2}{2} z^2$$

Source for $J_0^{(3)}$ $\longrightarrow$ $\boxed{c_1 = \mu}$

Isospin Chemical Potential in the Hard Wall Model

Can calculate all the properties of the isospin condensate phase discussed earlier; results agree with other approaches.

(Albrecht,JE)

Other Observations

Baryons are solitons in extra dimensions - like Skyrmions
(Piljin Yi's talk;Sakai,Sugimoto; Nawa et al.; Pomarol, Wulzer)

HoloQCD may address qualitative questions like chiral symmetry restoration
(D.K.Hong et al.; Shifman,Vainshtein; Klempt)

Can add higher dimension 5D operators or modify geometry to match power corrections in Operator Product Expansions
(Hirn,Sanz)

Dualities Lecture 5 Summary

QCD	Holographic QCD
Towers of bound states identified	
	
Turkey	**Tofu**

Dualities Lectures Final Summary

Dualities are alternative descriptions of the same physical systems.

Sometimes one description is easier to analyze than another (e.g. strong-weak dualities)

Dualities also motivate new models of strongly interacting physics, as in holographic QCD.