One of HUGS goals is to introduce students to “topics of interest in nuclear physics”. My lectures will attempt to describe what is interesting in the study of nuclei.

- Lecture 1: Search for the limits of nuclear binding and production of new isotopes
- Lecture 2: Attempts to model atomic nuclei I
- Lecture 3: Attempts to model atomic nuclei II
- Lecture 4: Nuclear Reactions
- Lecture 5: The origin of atoms – Nuclear Astrophysics I
- Lecture 6: The origin of atoms – Nuclear Astrophysics II

It is important for you to ask questions.
Benefits along the way to discovery

- Radioisotopes in medicine - Over 10,000 hospitals worldwide use radioisotopes in medicine, and about 90% of the procedures are for diagnosis. (World Nuclear Association)

- Pathologists have devised hundreds of tests to determine the constituents of blood, serum, urine, hormones, antigens and many drugs by means of associated radioisotopes; named Radioimmunoassay, RIA. (World Nuclear Association)

- Other fields of science - The most accurate means for estimating the Earth's age is a Pb/Pb isochron, which is derived from samples of the Earth and meteorites. This involves ratios of three isotopes of lead (206-Pb, 207-Pb, and either 208-Pb or 204-Pb). A plot is constructed of the ratio of 206-Pb/208-Pb versus 207-Pb/208-Pb for various samples. The slope depends on time since 207-Pb comes from 238-U and 206-Pb from 235-U.

- Nuclear science allow us to understand (mostly) how the Sun generates energy
Due to the difference in half-lives for the parents of 206-Pb and 207-Pb, they are produced at different rates.

Slope of the line gives the age.

http://www.talkorigins.org/faqs/isochron-dating.html
We want to model physical phenomena that are the result of the strong force.

This includes understanding atomic nuclei, hadrons, QGP, …

We have made remarkable progress in modeling hadrons – Nobel prize in 2004 Gross, Politzer, Wilczek; LQCD calculation of nucleon and meson masses (Dürr, Fodor, Lippert et al., Science 322 (2008))

There is room for significant progress in understanding atomic nuclei.

Illustration from David Dean.
The light hadron spectrum from Lattice QCD

Goal: Comprehensive Understanding of Nuclei

- How do we model atomic nuclei? QCD, but we need approximations.

- Nuclear interactions inside nuclei have even more complications since nucleons have structure and many-body forces are also important.

- Are protons and neutrons in medium modified from their free structure?

What is the nature of the “hard-core” repulsion in the nuclear force?

The standard assumption is that we can model nuclei as collections of neutrons and protons (Electron scattering: rms size 0.8 fm; rms distance 2 fm).

Electron elastic scattering experiments at JLAB and SLAC have shown that down to the level of 0.1 fm nuclei can be described with neutrons and protons as constituents.
Nuclear Quantum Numbers

- **Baryon Number**
  - Sum of the baryon numbers of the quarks
  - Usually called atomic mass number, A

- **Electric Charge**
  - Atomic number of a nucleus, Z

- **Angular Momentum**
  - \( j = l + s \)

- **Parity**
  - Often determined by the valence orbit (odd \( l \) – negative, even \( l \) – positive)

- **Isospin (T in nuclear structure, I in high energy)**
  - \( T = 3(N-Z) \), \( t_z \) is the projection
  - Proton has isospin \( T=1/2, t_z=-1/2 \), neutron \( T=1/2, t_z=1/2 \)

- In reactions of nuclei the quantities above must be conserved.
Hoyle State triple $\alpha$ process

See e.g. E. Epelbaum et al. PRL 106, 192501 (2011)

Energy [MeV]

http://www.tunl.duke.edu/nucldata/
Isospin Independence in Nuclei

- Nuclei with the same isospin, $T$, show nearly identical structure.
- Total energy is changes by the Coulomb force (+ other small differences).
- Nuclei with varying $t_z$ are called members of a multiplet.

Example: Nuclei

Example: Baryons
Excitation energy of first excited state (Calculated by Cakirli et al.)

How can we understand this regularity from microscopic origins – simplicity in complex systems
Dynamical Symmetries in Nuclei – Interacting Boson Model

- IBM - Iachello and Arima
- Casten – Bonner Prize 2010
- What is the microscopic origin of the simple patterns and symmetries that nuclei exhibit?
- Dynamical Symmetries, Phase Transitions, Critical Point Symmetries

R. Casten
However…Are Nucleons Modified in the Nuclear Medium? Maybe Yes

- EMC “European Muon Collaboration” Effect circa 1983, CERN
Short Range Correlations Show a Preference for NP vs PP Pairs

Size of SRC Grows in Heavier Nuclei

Observation: EMC Effect is Correlated with SRC

- N. Formin et al., PRL 108 (2012) 092502
The Liquid Drop Model

Nuclei have the features of a drop of liquid.

\[ \text{BE}(N, Z) = \alpha_1 A - \alpha_2 A^{2/3} - \frac{\alpha_3 Z^2}{A^{1/3}} - \frac{\alpha_4 (N - Z)^2}{A} \]

- **Binding Energy**
  - Volume term
  - Surface term
  - Coulomb
  - Asymmetry

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**Graph:**
- **Experiment**
- **Liquid Drop**

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**FRIIB**
Facility for Rare Isotope Beams
U.S. Department of Energy Office of Science
Michigan State University

HUGS Sherrill Lecture 2, Slide 19
Variants on the Liquid Drop Model are Still Used

- Benchmark that all nuclear models can be measured against – Example drip lines
- Drip lines are sensitive to aspects of the nuclear force (see right)
- Along the drip lines the structure of nuclei is qualitatively different (Haloes and Skins – next lecture)
- The drip line is determined by terms in the liquid-drop formula. Parameters can be related to the nuclear matter equation of state
- Terms depending on neutron number are related to neutron stars

Macroscopic model diff ( high/low DD symmetry energy coefficient, L)

\[ w = w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + \left[ S_0 + \frac{L}{3n_0}(n - n_0) \right] \alpha^2 \]
Nuclear Charge Density

- Can be accurately determined by electron scattering
  \[ \int \rho_{\text{ch}}(r) \, d\tau = 4\pi \int \rho_{\text{ch}}(r) r^2 \, dr = Z \]

- Charge probability
  \[ P_{\text{ch}}(r) = \frac{4\pi}{Z} r^2 \rho_{\text{ch}}(r) \]

- Neutrons and protons more or less fill the same volume
  \[ A = a_0 r^{1/3} \]
The same idea used for determining the charge radius can be used to measure the neutron radius by scattering off the weak charge.

- Weak charge proton = 0.048, Weak charge neutron = -1

- Hall A experiment – weak scattering violates parity

Neutron Skin = $R_N - R_P = 0.33 \pm 0.16 - 0.18$ fm

PREX I
PRL 108 (2012) 112502
PRC 85 (2012) 03250(R)
Conceptual Picture of a Nucleus

12-Carbon Nucleus

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/shell.html
Nuclear Mean Field – Shell Model

Atomic Nuclei

The Nobel Prize in Physics 1963
Eugene Wigner, Maria Goeppert Mayer, J. Hans D. Jensen

http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html
The interaction of nuclei create a “mean field”. We think of nucleon moving in this potential.

- Shell Model is the most common in nuclear science
- Solve the equation $H\Psi = E\Psi$

$$H_A = T_{rel} + V = \frac{1}{A} \sum_{i<j=1}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i<j=1}^{A} V_{NN} \left( + \sum_{i<j<k}^{A} V_{ijk}^{3b} \right)$$

- Introduce a basis (usually harmonic oscillator) and solve the matrix equation
- Can assume an inert closed core (e.g. $N=Z=20$)
- No core shell model does not make this assumption
- All shell models use effective operators (interactions depend on model space

$$V(r) = \frac{V_0}{[1 + \exp(r - R)/a]}$$

$R$ – nuclear radius

$a$ - diffuseness
Shell Model

http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/shell.html

Coulomb barrier

Quantum energy states of potential well including angular momentum effects.

Further splitting from spin-orbit effect

Multiplicity of states
Computational Challenge

Stability of Magic Nuclei

2\(^{+}\) levels in neutron-rich nuclei

Energy (MeV)

Neutron number

Ca
S
Si

Harder to excite
Stability of Magic Nuclei

$2^+$ levels in neutron-rich nuclei

- Ca (20 protons)
- S (16 protons)
- Si (14 protons)

Energy (MeV) vs Neutron number

Harder to excite
Surprise: Changing Magic Numbers

2\(^+\) levels in neutron-rich nuclei

Reason: A tensor force that depends on angular momentum and isospin (Otsuka et al.)
Step 1: Use *ab initio* theory and study of exotic rare isotopes to determine the interactions of nucleons in light nuclei and connect these to QCD by comparison to lattice calculations of NN and NNN forces.

Step 2: For mid-mass nuclei use configuration interaction models. The degrees of freedom and interactions must be determined from exotic nuclei.

Step 3: Use density functional theory to connect to heavy nuclei. Exotic nuclei help determine the form and parameters of the DFT. Example: Estimate of the number of isotopes for known elements Erler, Birge, Kortelainen, Nazarewicz, Olsen, Stoitsov, Nature 486, 509 (2012)

Step 3 is the one that is likely to answer the question about what are the heaviest elements.
“Ab Initio” start with NN forces

- Approach: Construct NN potentials based on neutron and proton scattering data and properties of light nuclei (Bonn, Reid, Illinois AV18, Nijmegen, etc.)
- More recent approach is to construct the potentials some more fundamental theory
  - QCD Inspired EFT
  - String Theory Inspired – Hashimoto et al
  - Lattice QCD

Constraints from nuclear matter

- Nuclear matter: infinite system governed by strong interaction
- Different energy scales (QGP versus atomic nuclei) = different degrees of freedom (quarks and gluons versus neutrons and protons)
- Used in nuclear structure to benchmark nuclear models: Saturation point of binding energy as function of density should be at $\rho = \rho_0 \approx 0.16 \text{ fm}^{-3}$


Comparison of Calculated and Measured Binding Energies with NN models

- Greens Function Monte Carlo techniques allow up to mass number 12 to be calculated
- Example blue 2-body forces $V_{18}$
- S. Pieper, B. Wiringa, et al.
New information from exotic isotopes

- Neutron rich nuclei were key in determining the isospin dependence of 3-body forces and the development of IL-2R from UIX.
- New data on exotic nuclei continues to lead to refinements in the interactions.

Properties of exotic isotopes are essential in determining NN and NNN potentials.
Current status of the GFMC calculations

Carlson, Pieper, Wiringa, et al.

Argonne v18 with Illinois-7 GFMC Calculations

- IL7: 4 parameters fit to 23 states
- 600 keV rms error, 51 states
- ~60 isobaric analogs also computed
Application of GFMC technique to reactions of nuclei

- Resonance states in $^5\text{He (n}^+\text{He)}$

K. Nollett, et al, PRL 2007; motivated by BBN modeling
Influence of 3N forces

- Big Bang Nucleosynthesis: Calculate all key reactions
- Neutron star masses

Gandolfi et al., PRC85, 032801 (2012)
A Gerzelis, Guelph

- Half-life of $^{14}$C (Maris, Navratil et al. PRL), structure of stable calcium isotopes, etc.
Another approach that people call \textit{ab initio} is to use the interactions in a shell model.

Start with a realistic interaction and then and diagonalizable in a large basis of many-body states.

From A. Poves, International School on Exotic Beams, Santiago de Compostela, September 4-11 2010 (see also J Vary, etc.)
Cool Questions

- How good is the approximation of neutrons and protons in the nucleus?
- How much are neutrons and protons modified in the nucleus and how is this reflected in nuclear structure?
- What are the interactions at play in a nucleus and how do we understand them from the underlying QCD?
- Is there a standard model for nuclear structure and what is it? Are there forces and interactions beyond this nuclear standard model?
References


- Overview of nuclear structure - http://folk.uio.no/mhjensen/phy981/alex.pdf


- Nuclear Theory Road Map – http://fribusers.org/8_THEORY/3_DOCUMENTS/Blue_Book_FINAL.pdf

- Effective Field theory for nuclei - Evgeny Epelbaum, arXiv: 1302.3241v1 [nucl-th]