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Introduction to Partonic Hadron Structure

Lecture III

From hadrons to quarks

Hadron part formally:

we want to perform integration!

$$2M\,W_{\mu\nu}(P,q) = \frac{1}{2\pi}\sum_n \underbrace{\int \frac{d^3 P_n}{(2\pi)^3\, 2E_n}}\, \langle P|J^{+}_{\mu}(0)|P_n\rangle\langle P_n|J_\nu(0)|P\rangle$$

P_n here does not allow it

$$\delta^{(4)}(P+q-P_n)$$

Let us use this trick:

$$\delta^{(4)}(P+q-P_n) = \int d^4x\, e^{i(P+q-P_n)\cdot x}$$

$$2M\,W_{\mu\nu}(P,q) = \frac{1}{2\pi}\sum_n \int \frac{d^3 P_n}{(2\pi)^3\, 2E_n} \langle P|e^{iPx} J_\mu(0)\, e^{-iP_n x}|P_n\rangle$$

$$\times \langle P_n|J_\nu(0)|P\rangle\, e^{iqx}\, d^4x$$

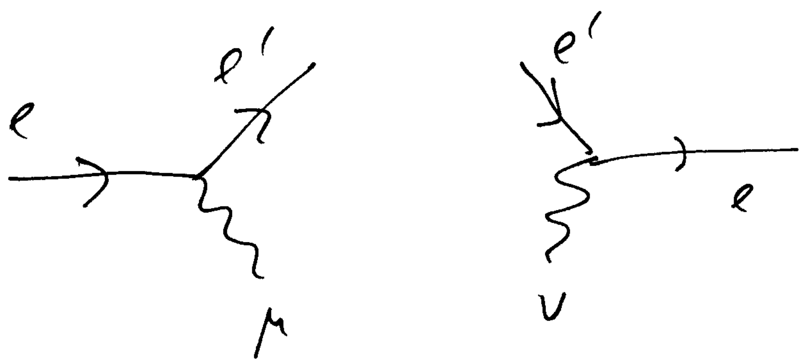
$$\left.\begin{aligned} \langle P|e^{iPx} &= \langle P|e^{i\hat{P}x} \\ e^{-iP_n x}|P_n\rangle &= e^{-i\hat{P}x}|P_n\rangle \end{aligned}\right\} \hat{P}\text{ - operator}$$

and $e^{i\hat{P}x} J_\mu(0)\, e^{-i\hat{P}x} = J_\mu(x)$ translation

$$\Rightarrow \int \frac{d^3 P_n}{(2\pi)^3\, 2E_n} |P_n\rangle\langle P_n| = \mathbb{1} \quad \text{Completeness, see lecture 1}$$

$$\Rightarrow 2M\,W_{\mu\nu}(P,q) = \frac{1}{2\pi} \underbrace{\int d^4x\, e^{iqx}}_{\text{Fourier transform}} \underbrace{\langle P|J_\mu(x)J_\nu(0)|P\rangle}_{\text{Some object to study}}$$

We will postpone discussion of hadron tensor till next lecture and concentrate on leptonic part.



called $L^{\mu\nu}$ – leptonic tensor.

How can we calculate it?

System is described by Lagrangian density

$$\mathcal{L}(\varphi, \partial_\mu \varphi, \ldots), \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \quad \varphi(x) \to \text{field}$$

that will become an operator in QFT.

Lagrangian is then $L = \int d^3x\, \mathcal{L}$

Action is $S = \int dt\, L = \int dt\, d^3x\, \mathcal{L} = \int d^4x\, \mathcal{L}(\varphi, \partial_\mu \varphi \ldots)$

Action is a Lorentz scalar (physics is Lorentz invariant)

Least action principle: S is stationary with respect to small perturbations:

$$S[\varphi + \delta\varphi] = S[\varphi] + O(\delta\varphi^2)$$

$$0 = \delta S = \int d^4x \left[\frac{\delta \mathcal{L}}{\delta \varphi} \delta\varphi + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \delta(\partial_\mu \varphi) \right] =$$

$$= (\delta \partial_\mu \varphi = \partial_\mu \delta\varphi) = \int d^4x \left[\frac{\delta \mathcal{L}}{\delta \varphi} \delta\varphi - \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \right) \delta\varphi \right] +$$

$+$ surface terms

$$\Rightarrow \quad \frac{\delta \mathcal{L}}{\delta \varphi} - \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta(\partial_\mu \varphi)} \right) = 0$$

Euler-Lagrange equations $\equiv$ equations of motion

We want to deal with <u>electrons</u>

Some useful formulae

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\psi$$

γ^μ - gamma matrices

$\partial_\mu \equiv \dfrac{\partial}{\partial x^\mu}$, $\bar{\psi} \equiv \psi^+ \gamma^0$

Global gauge transformations

$$\psi'(x) = e^{i\alpha}\psi(x)$$

$$\bar{\psi}'(x) = \bar{\psi}(x)\, e^{-i\alpha}$$

$$\mathcal{L} \to \mathcal{L}'$$

$\Rightarrow$ current

$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu \psi(x)$$

Ensure conservation laws

Local gauge transformations

$$\begin{cases} \psi'(x) = e^{i\alpha(x)}\psi(x) \\ \bar{\psi}'(x) = \bar{\psi}(x)\, e^{-i\alpha(x)} \end{cases}$$

Interaction <u>is</u> defined by local gauge invariance

$$\partial_\mu \psi(x) = e^{-i\alpha(x)}(\partial_\mu - i\partial_\mu \alpha(x))\psi'(x)$$

$\Rightarrow$

$$\mathcal{L} = \bar{\psi}'(x)\left(i\gamma^\mu(\partial_\mu - i\partial_\mu\alpha(x)) - m\right)\psi'(x)$$

$\mathcal{L} \nrightarrow \mathcal{L}'$ any more!

We can restore gauge invariance if we use

$$(\partial_\mu + ieA_\mu(x))\psi(x)$$

$$(\partial_\mu + ieA_\mu(x))\psi(x) = e^{-i\alpha(x)}(\partial_\mu + ieA'_\mu(x))\psi'(x)$$

where

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x)$$

$\partial_\mu + ieA_\mu(x) \rightarrow$ covariant derivative. "D_μ"

$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^\mu(\partial_\mu + ieA_\mu(x)) - m\right)\psi(x)$$

$\uparrow$

invariant also under local gauge transformation.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{Interaction}}$$

$$\mathcal{L}_I = -e\, j^\mu A_\mu \quad \text{where} \quad j^\mu = \underbrace{\bar{\psi}(x)\gamma^\mu\psi(x)}_{\text{current.}}$$

charge

Construction $\rightarrow$ local gauge invariance

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \equiv \{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$$

$$\gamma^0 (\gamma^\mu)^\dagger \gamma^0 = \gamma^\mu \Rightarrow (\gamma^0)^\dagger = \gamma^0, \; (\gamma^k)^\dagger = -\gamma^k$$

Independent fields $\psi(x)$ & $\bar{\psi}(x) = \psi^\dagger_{(x)} \gamma^0$

$$\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} = 0 \quad \Big\}$$ Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\psi}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = (i \gamma^\mu \partial_\mu - m)\psi, \quad \frac{\partial \mathcal{L}}{\partial \partial_\mu \bar{\psi}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \psi} = -m \bar{\psi}, \quad \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} = i \bar{\psi} \gamma^\mu$$

$$\Rightarrow \begin{cases} (i \gamma^\mu \partial_\mu - m) \psi(x) = 0 \\ i \partial_\mu \bar{\psi}(x) \gamma^\mu + m \bar{\psi}(x) = 0 \end{cases}$$

Solutions $\to 4$, 2 with $p_0 > 0$, 2 with $p_0 < 0$

Lets consider only positive energy:

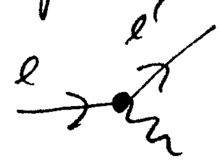
$$\psi(x) = u(p,s)\, e^{-ip\cdot x}, \quad p^2 = m^2, \; p_0 > 0$$

We have

$$(i\gamma^\mu \partial_\mu - m)\,\Psi(x) = 0$$

$$\Rightarrow \quad (\gamma^\mu p_\mu - m)\, u(p) = 0 \quad , \quad \gamma^\mu p_\mu = \not{p}$$

$(\not{p} - m)\, u(p) = 0$, $u(p)$ is called spinor



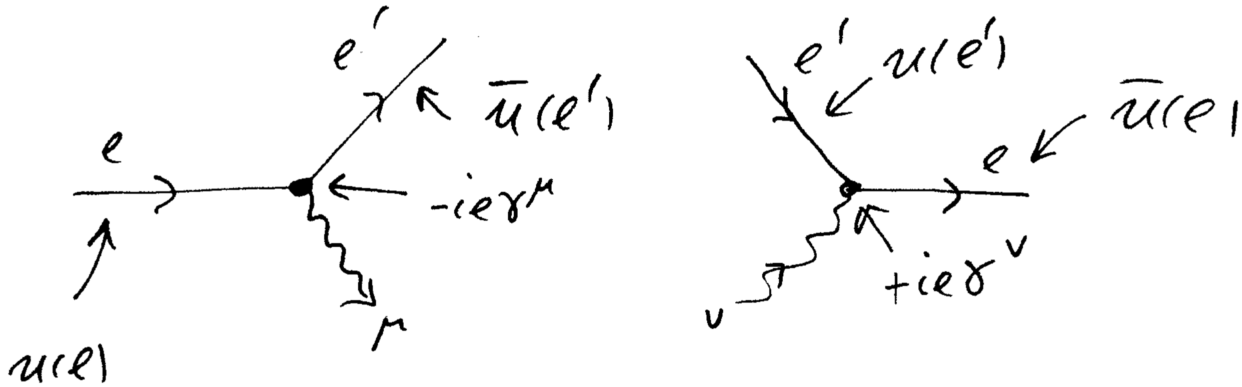
$q = e - e'$

$$\bar{u}(p)\,(\not{p} - m) = 0$$

Current conservation: $\partial_\mu j^\mu = 0 \Rightarrow j^\mu = \bar{u}(e')\gamma^\mu u(e)\, e^{-i(e-e')x}$

$$\partial_\mu j^\mu = -i q_\mu j^\mu = 0 \Rightarrow \boxed{q_\mu j^\mu_{(x)} = 0}$$

Now we have all ingredients to construct Feynman diagrams. For example $L^{\mu\nu}$:



(note that we do not insert here $\epsilon^{(\lambda)*}_\mu(q)$ and $\epsilon^{(\lambda)}_\nu(q)$ as this part was already extracted in „photon propagators" $\propto 1/q^4$)

$$L_{p}^{\mu\nu} = \frac{1}{2s+1} \sum_{s'} \bar{u}_\alpha(\ell,s)(-ie\gamma^\nu)_{\alpha\beta} u_\beta(\ell',s')\, \bar{u}_a(\ell',s')(+ie\gamma^\mu)_{ab} u_b(\ell,s)$$

Spin projector

$$\sum_{s'} u_\beta(\ell',s')\bar{u}_a(\ell',s') = (\not{\ell}' + m)_{\beta a}$$

$$u_b(\ell,s)\bar{u}_\alpha(\ell,s) = \left[\frac{(\not{\ell}+m)(1+\gamma_5 \not{s})}{2}\right]_{b\alpha}$$

where $\gamma_5 = +i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma_5^\dagger = \gamma_5$, $(\gamma_5)^2 = 1$, $\{\gamma_5, \gamma^\mu\} = 0$

Let's sum over s and obtain symmetric part of $L^{\mu\nu}$:

$$L^{\mu\nu} = \frac{e^2}{2} \underbrace{(\not{\ell}+m)_{b\alpha}(\gamma^\nu)_{\alpha\beta}(\not{\ell}'+m)_{\beta a}(\gamma^\mu)_{ab}}_{\text{Trace}}$$

neglect m and

$$L^{\mu\nu} = \frac{e^2}{2} \mathrm{Tr}(\not{\ell}\gamma^\nu\not{\ell}'\gamma^\mu)$$

Traces:

$$\mathrm{Tr}(\text{odd } \# \, \gamma) = 0$$

$$\mathrm{Tr}(\not{a}\not{b}) = 4\, a\cdot b$$

$$\mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) = 4\left[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(c\cdot d)\right]$$

$$\mathrm{Tr}(\gamma^a\gamma^b\gamma^c\gamma^d) = 4\left[g^{ab}g^{cd} - g^{ac}g^{bd} + g^{ad}g^{cd}\right]$$

Thus we get

$$L^{\mu\nu} = 2e^2\left(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}(\ell\cdot\ell')\right)$$

Now we go from leptonic to hadronic tensor

γ matrices can have different representations,
for example Weyl representation

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}$$

σ^i – Pauli matrices

Of course in order to obtain complete lagrangian of QED we should also add lagrangian for spin-1 (photon) field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad , \quad A_\mu \text{ -vector field}$$

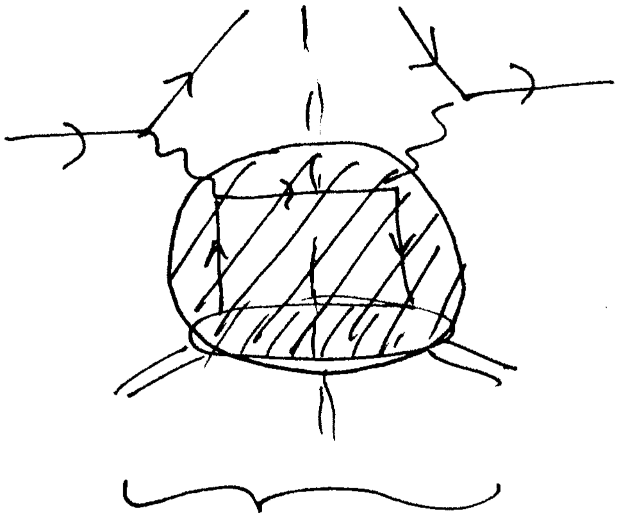
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

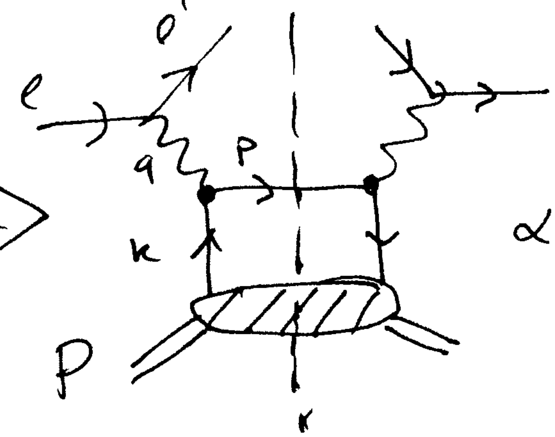
and then

$$\mathcal{L}_{QED} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu + i e A_\mu \quad - \text{ covariant derivative}$$

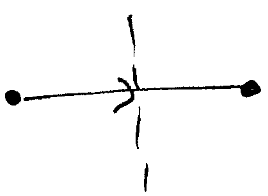
What about hadron?





$\propto (e_q)^2$

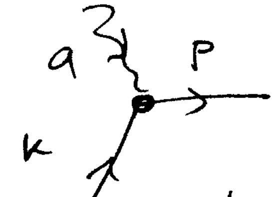
Square of the amplitude,
but also imaginary part
„optical theorem"



$$= \mathrm{Im}\,\frac{1}{p^2+i\epsilon} \propto \delta(p^2)$$

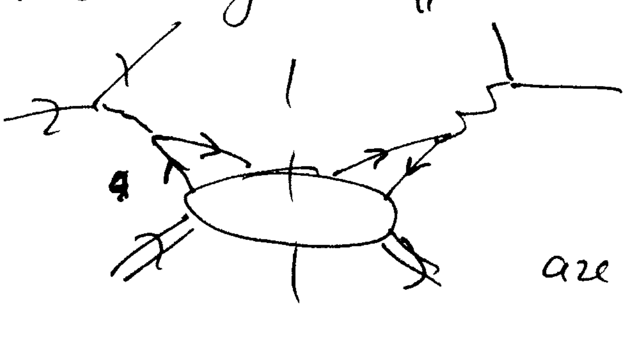
Indeed: $\frac{1}{x \pm i\epsilon} = PV\left(\frac{1}{x}\right) \mp i\pi\,\delta(x)$

Conservation



$$\delta^{(4)}(k+q-p) \Rightarrow \underline{\underline{p = k+q}}$$

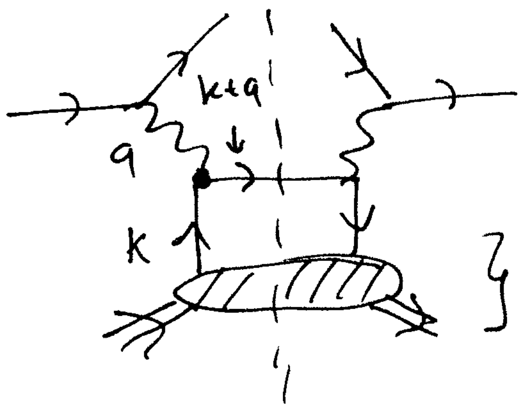
Other diagrams „CATs EARS"



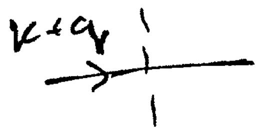
$\propto e_{q_1} e_{q_2}$

are suppressed by $\left(\frac{1}{Q^2}\right)^2$

Why x_{Bj} ?



$\Big\}$ $f(x)$ probability to find a quark $\underline{\underline{k = xP}}$.



$\equiv \quad \delta\left((k+q)^2\right)$

$$(k+q)^2 = k^2 + q^2 + 2kq = \emptyset - Q^2 + 2k \cdot q = \emptyset$$

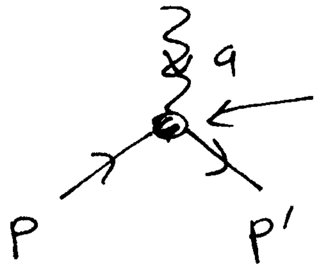
Let us assume that $k = xP$ when $x \in (-\infty, \infty)$

now we get

$$-Q^2 + 2x\, P \cdot q = 0 \Rightarrow \quad x = \frac{Q^2}{2P \cdot q} = x_{Bj}$$

thus $x = x_{Bj}$ – interaction is important!

Form factors vs distributions



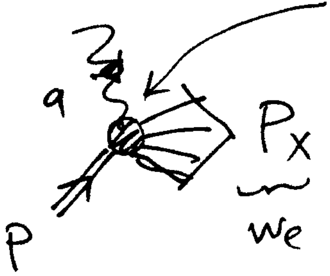
$\delta^{(4)}(p+q-p') \Rightarrow \underbrace{q = p'-p}$

$$q^2 = -Q^2 = (p'-p)^2 = p'^2 + p^2 - 2p'\cdot p = 2M^2 - 2p'p \to -2P'\cdot P$$

$$P\cdot q = P(P'-P) = P\cdot P' - M^2 \to P\cdot P'$$

$$\Rightarrow \quad x = \frac{Q^2}{2P\cdot q} = 1$$ not an independent variable

$\delta^{(4)}(p+q-P_X)$



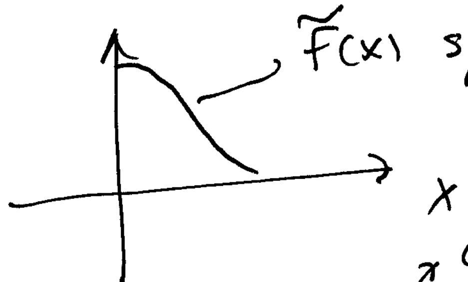
$\underbrace{P_X}$ we integrate over P_X !

$$\left.\begin{array}{l} q^2 = -Q^2 \to \infty \\ 2P\cdot q \to \infty \end{array}\right\} \text{independently}$$

$$x_{Bj} \equiv \frac{Q^2}{2P\cdot q}$$

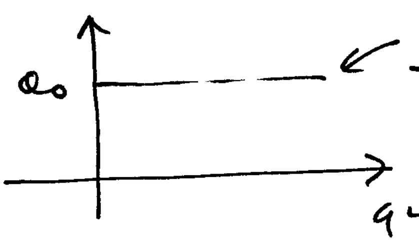
Form factors allow to study coordinate form of objects

$$F(q^2) = \int d^3x \, e^{iqx} \, \tilde{F}(x)$$

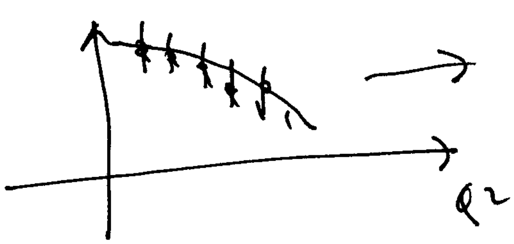


Suppose that $\tilde{F}(x) = Q_0 \, \delta^{(3)}(x)$ - point like
(charge: Q_0)

$$\Rightarrow \quad F(q^2) = \int d^3x \, e^{iqx} \, Q_0 \, \delta^{(3)}(x) = Q_0$$

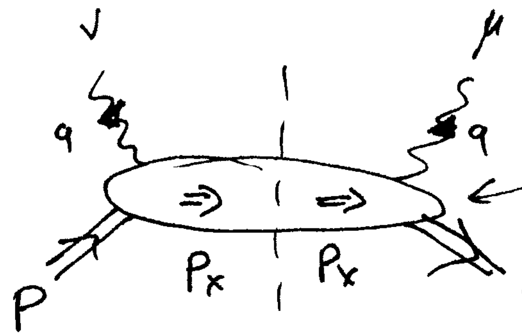


Unlike form factor of a proton



$\longrightarrow$ proton is an object of ~1 fm size

Hadronic tensor again



$\delta^{(4)}(P_X - P - q)$

Let's use

$$\delta^{(4)}(k) = \int \frac{d^4\xi}{\cancel{(2\pi)^4}}\, e^{+ik\cdot\xi}$$

$$\sum\!\!\!\!\!\!\!\int_X \equiv \int \frac{d^3P_X}{2E_X(2\pi)^3} = \int \frac{d^4P_X}{(2\pi)^4}\,\theta(E_X)\,\delta(P_X^2 - M_X^2)$$

$$2M W^{\mu\nu} = \frac{1}{2\pi}\sum\!\!\!\!\!\!\!\int_X \cancel{(2\pi)^4}\,\delta^{(4)}(-P_X + P + q)\, \langle P|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P\rangle$$

$$= \frac{1}{2\pi}\sum\!\!\!\!\!\!\!\int_X \int d^4\xi\, e^{+i(-P_X+P+q)\cdot\xi} \langle P|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P\rangle =$$

$$= \frac{1}{2\pi}\sum\!\!\!\!\!\!\!\int_X \int d^4\xi\, e^{iq\xi}\, \underbrace{\langle P|e^{iP\cdot\xi}}_{\langle P|e^{i\hat{P}\cdot\xi}} J^\mu(0)\underbrace{e^{-iP_X\cdot\xi}|X\rangle}_{e^{-i\hat{P}\cdot\xi}|X\rangle}\langle X|J^\nu(0)|P\rangle$$

momentum operator

$$e^{i\hat{P}\cdot z} J^\mu(0)\, e^{-i\hat{P}\cdot z} = J^\mu(z)$$ translation of fields

$$= \frac{1}{2\pi} \sum_X \int \frac{d^4 z}{\cancel{(2\pi)^4}} e^{iq\cdot z} \langle P| J^\mu(z) |X\rangle\langle X| J^\nu(0) |P\rangle$$

now we use $\sum_X |X\rangle\langle X| = \mathbb{1}$ completeness of states

and obtain

$$2MW^{\mu\nu} = \int \frac{d^4 z}{(2\pi)^4} e^{iq\cdot z} \underbrace{\langle P| J^\mu(z) J^\nu(0) |P\rangle}_{\text{coordinate space.}}$$

Again Bjorken limit $\quad P = (M, \vec{0})$

$q = (\nu, 0, 0, \sqrt{\nu^2 + Q^2})$

$$x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\nu}, \quad \underline{\underline{Q^2 \to \infty, \ \nu \to \infty}}$$

$$q\cdot z = q^0 \cdot z^0 - \vec{q}\cdot\vec{z} = \frac{(q^0+q^3)}{\sqrt{2}}\frac{(z^0-z^3)}{\sqrt{2}} + \frac{(q^0-q^3)}{\sqrt{2}}\frac{(z^0+z^3)}{\sqrt{2}} - q_T \cdot z_T$$

By the way $\quad A^\pm \equiv \dfrac{A^0 \pm A^3}{\sqrt{2}}$ light-cone coordinates

$$A\cdot B = A^+B^- + A^-B^+ - \vec{A}_T\cdot\vec{B}_T, \quad \vec{A}_T = (A^1, A^2)$$

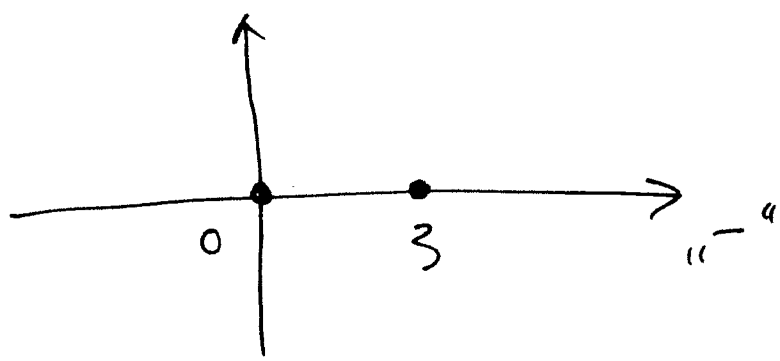
Bjorken limit

$$q^0 + q^3 \simeq 2\nu$$

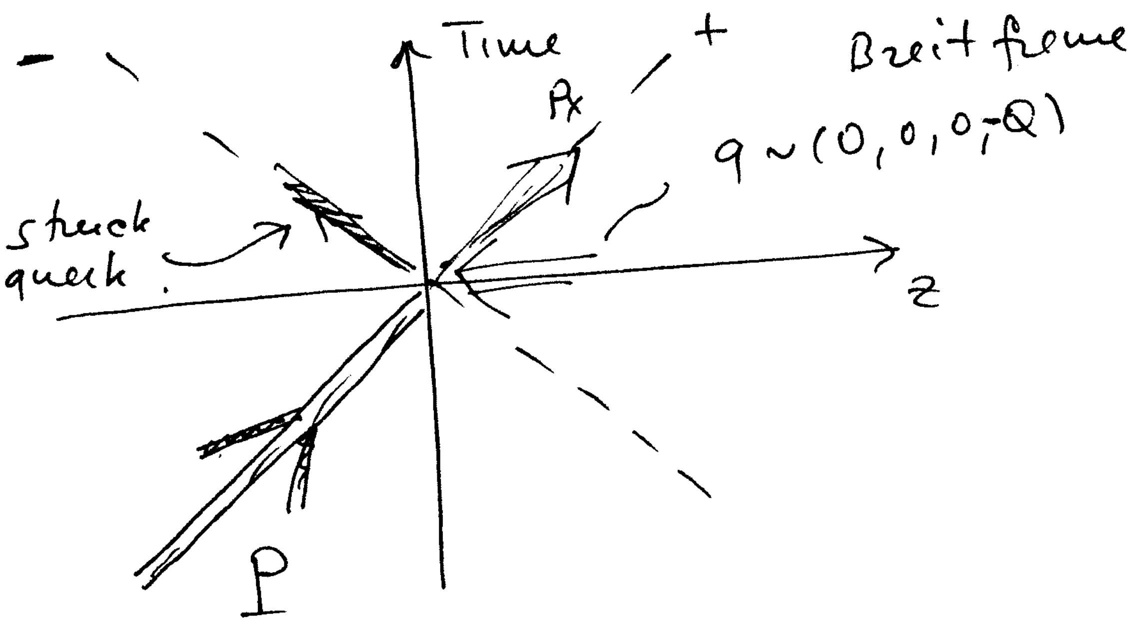
$$q^0 - q^3 \simeq \frac{Q^2}{2\nu}$$

main part of $e^{iq\cdot z}$ comes from region of less rapid oscillations $\Rightarrow$ $q\cdot z = O(1)$

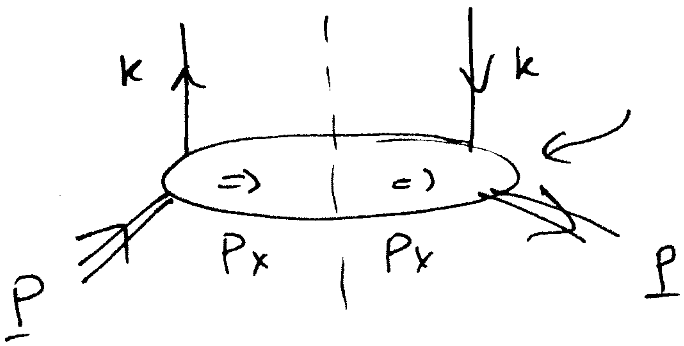
$\Rightarrow$ $\underbrace{z^0+z^3}_{z^+} \sim O(1/\nu)$, $\underbrace{z^0-z^3}_{z^-} \sim O(1/xM)$



DIS $\Longleftrightarrow$ Light cone behaviour



What about quarks?



$\delta^{(4)}(-P_X + k + P)$

$$\Phi(k, P) = \sum_X \delta^{(4)}(-P_X + k + P) \langle P | \bar{\Psi}(0) | X \rangle \langle X | \Psi(0) | P \rangle$$

$$= \int d^4 z \, e^{-ikz} \underbrace{\langle P | \bar{\Psi}(z) \Psi(0) | P \rangle}_{\text{contain all distributions}}$$

What is k? $\quad n_+^\mu = (1^+, 0^-, 0_\perp)$, $n_-^\mu = (0^+, 1^-, 0_\perp)$

$$k^\mu = k^+ n_+^\mu + k^- n_-^\mu + \vec{k}_\perp^\mu$$

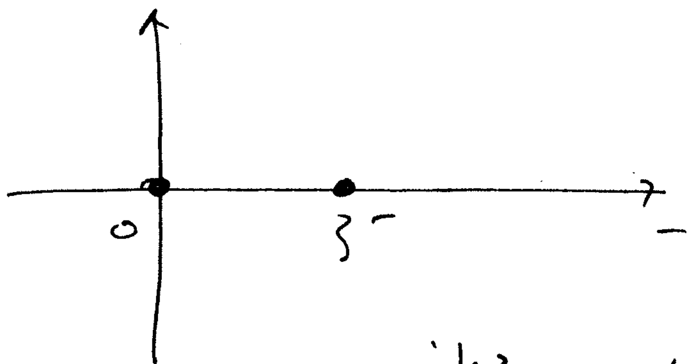
light-cone fraction

quark goes with the proton $\quad \underbrace{k^+ = x P^+}$

$$k^\mu = x P^+ n_+^\mu + \underbrace{\frac{+\vec{k}_\perp^2 + k^2}{2 x P^+}}_{\text{small}} n_-^\mu + \underbrace{\vec{k}_\perp^\mu}_{\text{may not be small}}$$

if we neglect k^- & $\vec{k}_\perp$ then $\quad k^\mu \simeq x P^+ n_+^\mu$

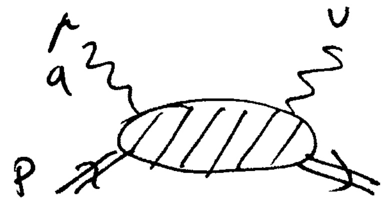
and we recover collinear picture



Why? We have $e^{-ik\xi}$ in the Fourier transform and $k \simeq (\underline{k}^+, 0^-, 0_\perp)$

$\Rightarrow k\xi = k^+\xi^-$ // only "$-$" direction matters

Hadronic tensor



can be parametrized using vector q^μ and vector P^μ

$$W^{\mu\nu} = -\left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2}\right) W_1(x, Q^2) + \left(P^\mu + \frac{q^\mu}{2x}\right)\left(P^\nu + \frac{q^\nu}{2x}\right) W_2(x, Q^2) \qquad (*)$$

(here we write only symmetric part of the tensor and do not take into account spin of the hadron)

Home work:

one can write
$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} P^\mu P^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2}\left(P^\mu q^\nu + q^\mu P^\nu\right) \qquad (**)$$

show that $(*)$ can be derived from $(**)$ using current conservation $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$.

Instead $W_{1,2}$ one usually introduces:

$$\begin{cases} F_1(x,Q^2) = W_1(x,Q^2) \\ F_2(x,Q^2) = \nu W_2(x,Q^2) \\ F_L(x,Q^2) = F_2 - 2x F_1 \end{cases}$$

Proton structure functions

Note that Z exchange & neutrino DIS is not included here!

Let us try to calculate those functions in parton model

We introduce vectors:

$$p^\mu = (P, 0, 0, P)$$

$$n^\mu = \left(\frac{1}{2P}, 0, 0, -\frac{1}{2P}\right)$$

$$p^2 = n^2 = 0$$

$$p \cdot n = 1$$

Let's choose the frame such that

$$q^\mu = q_\perp^\mu + \nu n^\mu$$

such that $P \cdot q = \nu$, $(q)^2 = (q_\perp^\mu)^2 = -\vec{q}_\perp^{\,2} = -Q^2$

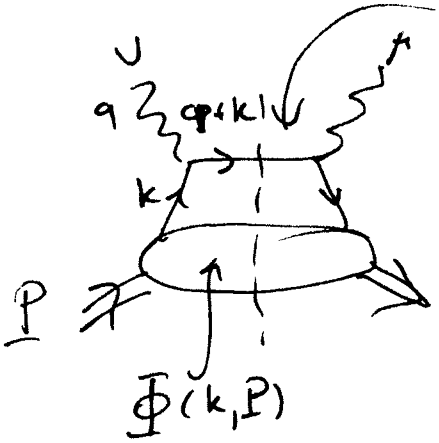
Then

$$p^\mu p^\nu W_{\mu\nu} = -\frac{\nu^2}{Q^2} W_1 + \frac{\nu^2}{4x^2} W_2 = \frac{\nu}{4x^2} F_L$$

$$\underbrace{n^\mu n^\nu W_{\mu\nu}}_{\text{projections}} = W_2 = \frac{1}{\nu} F_2$$

$$\Rightarrow \begin{cases} F_2 = \nu\, n^\mu n^\nu W_{\mu\nu} \\ F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu} \end{cases}$$

from quark propagator



$$W^{\mu\nu} = e_q^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\gamma^\mu (\not q + \not k)\gamma^\nu \phi(k,P)\right) \times \delta((k+q)^2)$$

$$k^\mu = x\, p^\mu + \frac{k^2 + k_\perp^2}{2x} n^\mu + k_\perp^\mu$$ (note that here I use different set of light cone vectors with respect to page 16.)

$$\delta((k+q)^2) = \delta(k^2 - Q^2 + 2x\nu - 2\vec{k}_\perp \cdot \vec{q}_\perp)$$

$$\approx \delta(2x\nu - Q^2) = \frac{1}{2\nu}\delta(x - x_{Bj})$$

$$F_2 = \nu\, n^\mu n^\nu W_{\mu\nu} = \frac{1}{2} e_q^2 \int \frac{d^4k}{(2\pi)^4} T_2 \Big(\not{n} \underbrace{\not{k} \not{n}}_{\substack{-\not{n}\not{k} + 2n\cdot k}} \phi(k,P) \Big) \delta(x - x_{Bj})$$

$$\underbrace{\qquad\qquad}_{2x\, T_2(\not{n}\, \phi)}$$

We can define:

$$\underbrace{f(x) = \int \frac{d^4k}{(2\pi)^4} T_2 \big(\not{n}\, \phi(k,P) \big) \delta(x - x_{Bj})}_{\text{Parton distribution}}$$

$\Rightarrow \quad F_2(x) = e_q^2\, x\, f(x)$ or summing over quarks

$$F_2(x) = \sum_q e_q^2\, x\, (f(x) + \bar{f}(x))$$

$F_2(x)$ depends <u>only</u> on x $\rightarrow$ Bjorken scaling!

now lets calculate

$$F_L = \frac{4x^2}{\nu} p^\mu p^\nu W_{\mu\nu} =$$

$$= \frac{4x^2}{2\nu^2} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\not{p} \underbrace{(\not{k}+\not{q}) \not{p}}_{-\not{p}(\not{k}+\not{q}) + 2\underbrace{p\cdot(k+q)}_{\frac{k^2+k_\perp^2}{x} + 2\nu}} \phi(k,P)\right) \delta(x - x_{Bj})$$

$$\simeq \frac{4x^2}{\nu} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left(\not{p}\, \phi(k,P)\right) \delta(x - x_{Bj})$$

$\nu \to \infty \Rightarrow F_L$ is suppressed! $\Rightarrow$

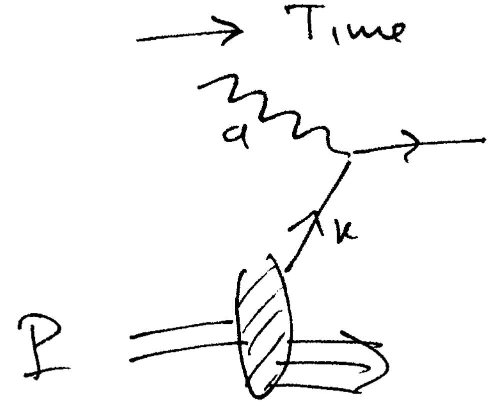
$$\boxed{F_2 = 2x F_1}$$ Callan-Gross relation:

tells us that quarks are spin-½ fermions.

<u>Home work</u>: Derive Callan-Gross relations for <u>scalar quarks</u>

Hint: $\mathrm{Tr}(\gamma^\mu(\not{q}+\not{k})\gamma^\nu \phi(k,P)) \to \mathrm{Tr}(\gamma^\mu\gamma^\nu\phi(k,P))$

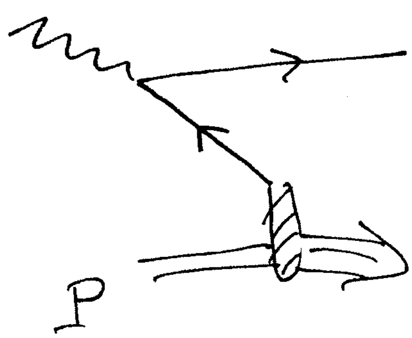
What do we actually measure in experiments?



Structure of the proton

?

or



Structure of vacuum fluctuations

Let's consider a plane wave fns

$$e^{-ik\cdot\xi}, \quad k=(k^0, 0, 0, k_z)$$

$$k\cdot\xi = k^0\xi^0 - \vec{k}\cdot\vec{\xi} = k^0\xi^0 - k_z\xi^3 =$$

$$= \frac{(k^0+k^3)(\xi^0-\xi^3)}{2} + \frac{(k^0-k^3)(\xi^0+\xi^3)}{2}$$

$$k \cdot z = k^+ z^- + k^- z^+$$

if we define z^+ as our new time then

if $k^- \simeq 0$ then the "time" is "frozen" for this wave.

z^- → new spacial coordinate.

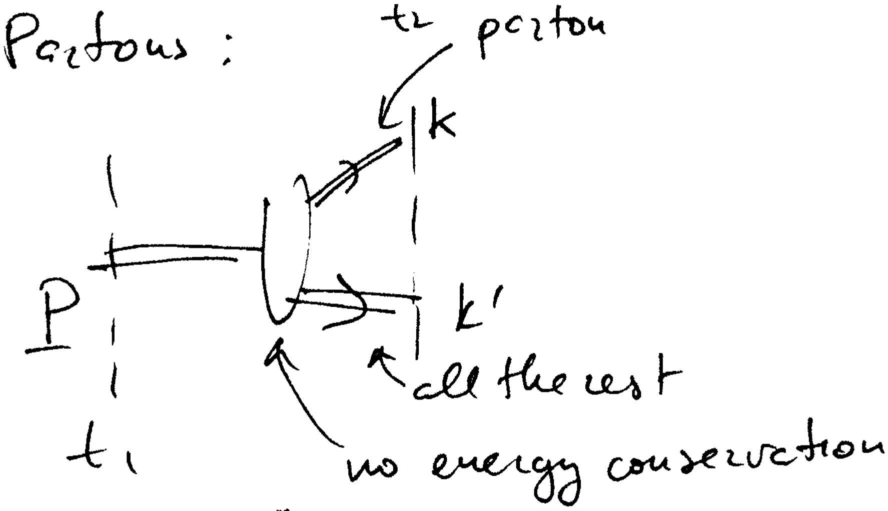
What is the advantage of Infinite Momentum Frame?

$$P^\mu = \left(\sqrt{P_z^2 + M^2}, 0, 0, P_z\right) \quad , P_z \to \infty$$

$$\Rightarrow \quad P^\mu \simeq (P_z, 0, 0, P_z) \quad , \quad \underline{\underline{P^- = 0}}.$$

Let us evaluate characteristic times for vacuum fluctuations and partons in this frame

Partons :



$$k_3 = x P_3 \quad , \quad P_3 \approx P_0$$

$$k'_3 = (1-x) P_3 ,$$

$$k_0 = \sqrt{k_\perp^2 + (k_3)^2} \simeq x P_3 \left(1 + \frac{1}{2} \frac{k_\perp^2}{x^2 P_3^2} \right)$$

$$k'_0 = \sqrt{k_\perp^2 + (k'_3)^2} \approx (1-x) P_3 \left(1 + \frac{1}{2} \frac{k_\perp^2}{(1-x)^2 P_3^2} \right)$$

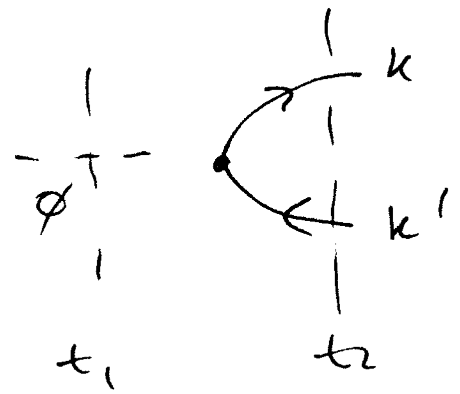
Energy at t_1 : $E_1 = P_0$

Energy at t_2 : $E_2 = k_0 + k'_0 = P_0 + \dfrac{k_\perp^2}{(1-x) x P_0}$

$$\Delta E = E_2 - E_1 = \frac{k_\perp^2}{(1-x) x P_0}$$

$$\Delta t \sim \frac{1}{\Delta E} = \frac{(1-x) x P_0}{k_\perp^2} \to \infty \text{ if } P_0 \to \infty$$

Vacuum fluctuation:



$$k_3 = x P_3 \Rightarrow k_0 \simeq x P_3 \approx x P_0$$

Energy at t_1: $\quad E_1 = 0$

Energy at t_2: $\quad E_2 > k_0 = x P_0$

$$\Delta E = E_2 - E_1 = x P_0$$

$$\Rightarrow \Delta t \approx \frac{1}{\Delta E} = \frac{1}{x P_0} \to 0 \text{ if } P_0 \to \infty$$

$\Rightarrow$ quantum fluctuations are suppressed and we study structure of the proton!