HUGS 2014

29th Annual Hampton University Graduate Studies Program

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Spin Sum Rules and 3D Nucleon Structure (3/6)

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Outline

Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz

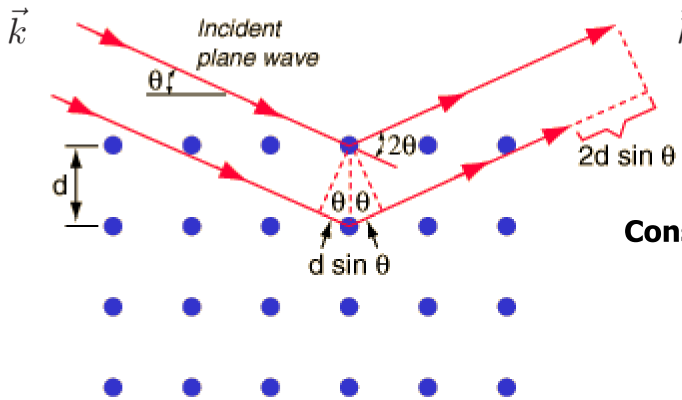
Lecture 2

- Photon point of view
- Galileo *vs* Lorentz : round 2
- Nucleon 1D picture

Lecture 3

- Nucleon 2D picture
- 1D+2D=3D
- Galileo *vs* Lorentz : round 3

Elastic scattering

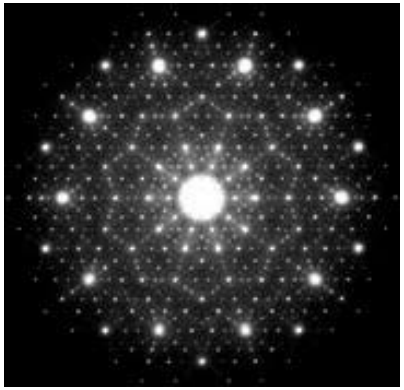


Crystal

Constructive interference (Bragg's law)

$$2d\,\sin\theta = n\lambda$$

Diffraction pattern



$\propto |A_{\mathrm{scatt}}|^2$

Scattered amplitude

$$A_{\mathrm{scatt}} \propto F(\vec{q}) = \int \mathrm{d}^3 r\, e^{i\vec{q}\cdot\vec{r}}\,\rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

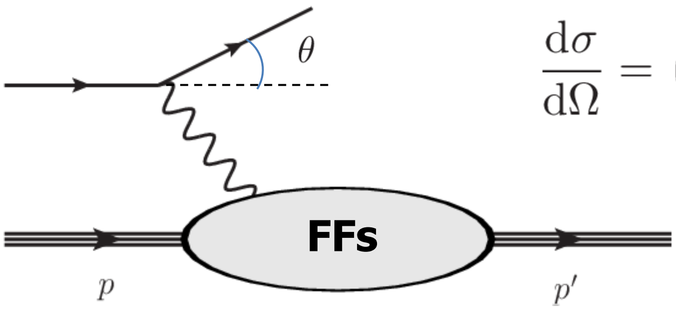
Form factor **Scatterer distribution**

Reconstructed charge distribution

$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}}\, F(\vec{q})$$

Let's replace the crystal by a nucleon !

Form factors (FFs)



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \underbrace{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}}}_{\textbf{Pointlike}} \underbrace{\left[F_1^2(Q^2) + \tau F_2^2(Q^2) + 2\tau\, G_M^2(Q^2)\,\tan^2\frac{\theta}{2}\right]}_{\textbf{Spatial structure}}$$

$$Q^2 = -\Delta^2$$

$$\tau = \frac{Q^2}{4M^2}$$

FF correlator

$$A^{[\Gamma]}_{\Lambda'\Lambda}(\Delta) = \frac{1}{2P^+}\,\langle p',\Lambda'|\overline{\psi}(0)\,\Gamma\,\psi(0)|p,\Lambda\rangle$$
$$= \int \mathrm{d}x\,\mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

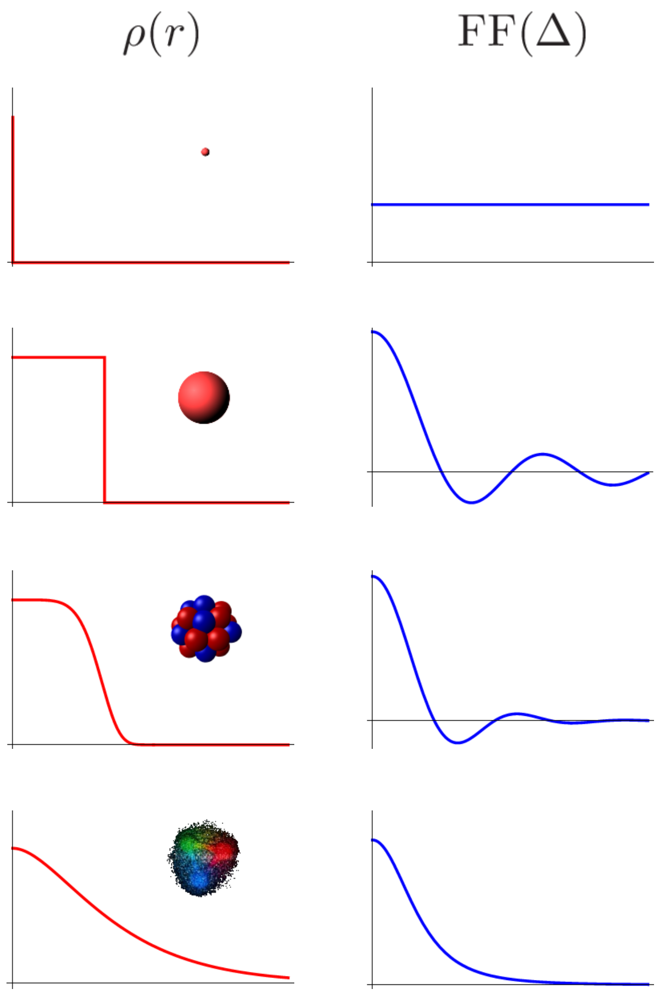
Parametrization (electromagnetic case)

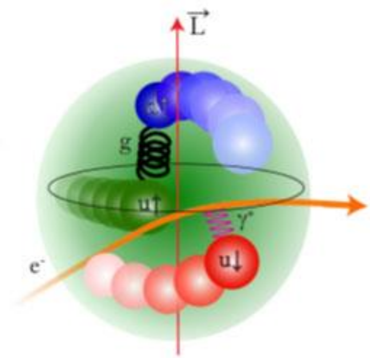
$$\langle p',\Lambda'|\overline{\psi}(0)\gamma^\mu\psi(0)|p,\Lambda\rangle = \overline{u}(p',\Lambda')\left[\gamma^\mu\, F_1(Q^2) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\, F_2(Q^2)\right] u(p,\Lambda)$$

Sachs FFs

$$G_E(Q^2) = F_1(Q^2) - \tau\, F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

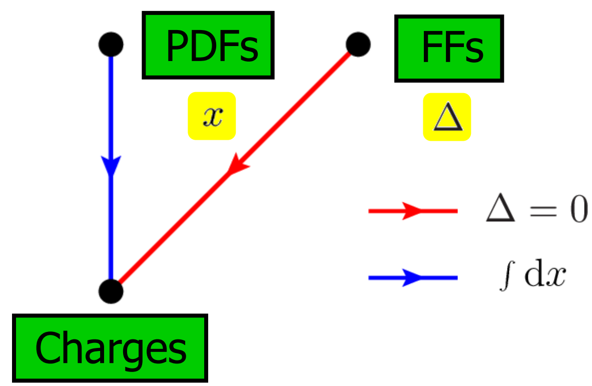
Form factors (FFs)





$F_1(0) = e$ **Electric charge**

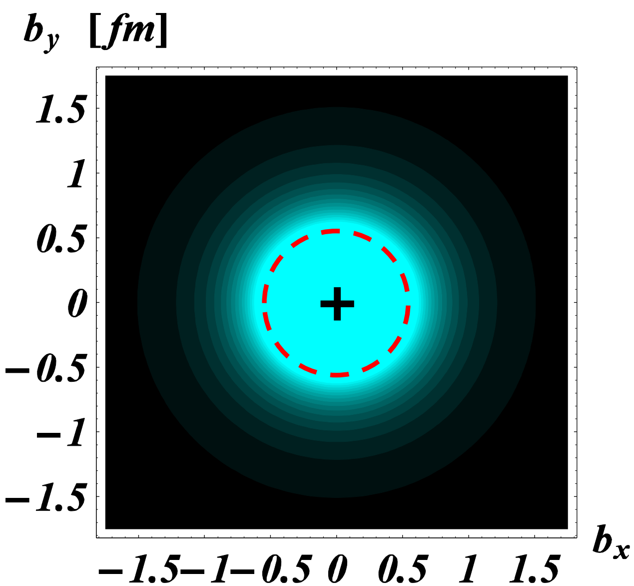
$F_2(0) = \kappa$ **Anomalous magnetic moment**



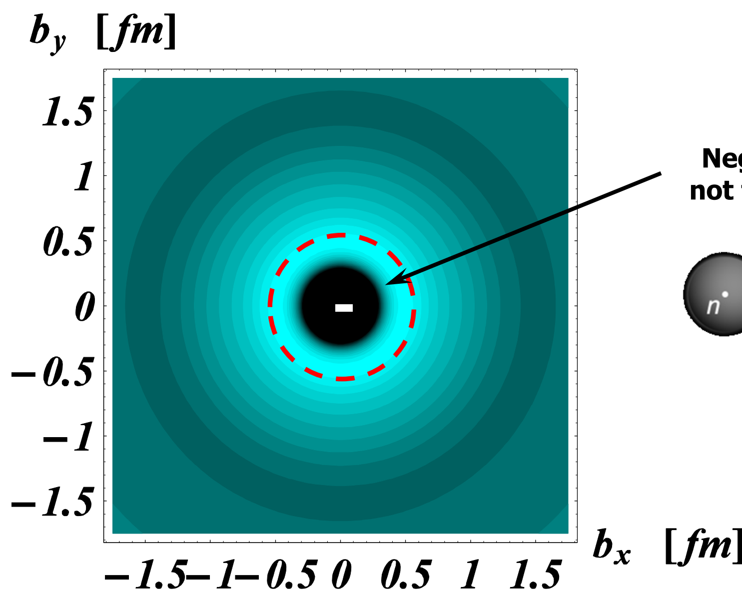
Longitudinally polarized nucleon

$$\rho(\vec{b}_\perp) = \underbrace{\int \frac{\mathrm{d}Q}{2\pi}\, Q\, J_0(Qb_\perp)\, F_1(Q^2)}_{\text{Monopole}}$$

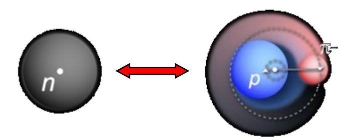
Proton



Neutron



Negative core does not fit naive picture !

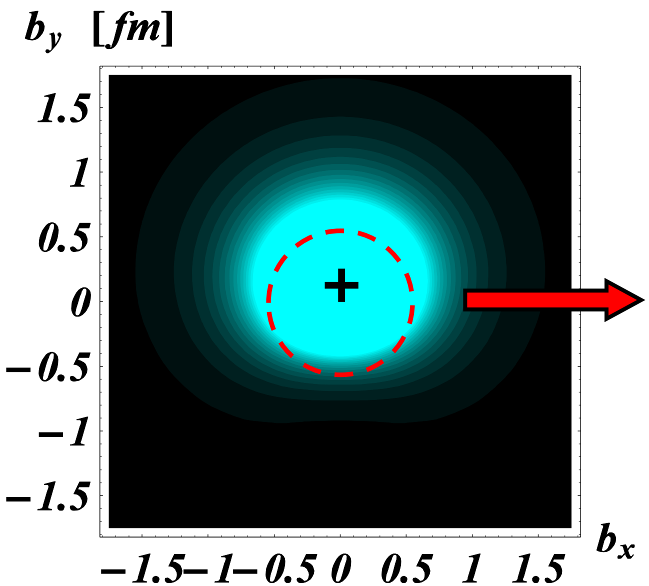


[Miller (2007)]

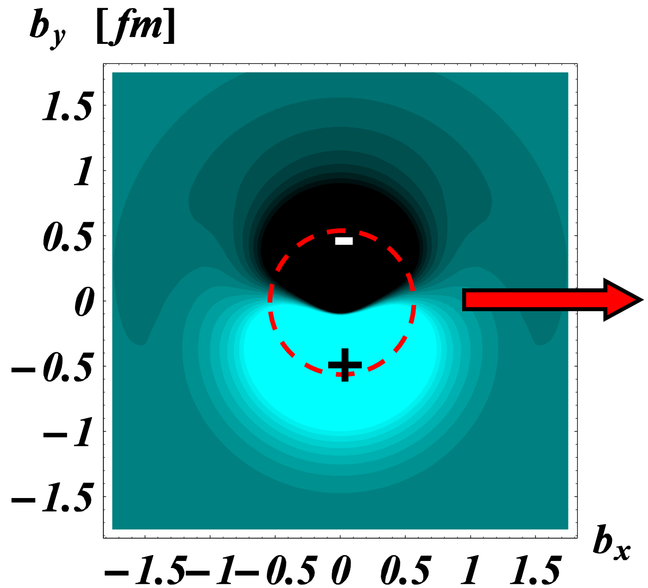
Transversely polarized nucleon

$$\rho_T(\vec{b}_\perp) = \underbrace{\rho(\vec{b}_\perp)}_{\text{Monopole}} + \underbrace{\sin(\phi_b - \phi_S) \int \frac{\mathrm{d}Q}{2\pi} \frac{Q^2}{2M} J_1(Qb_\perp) F_2(Q^2)}_{\text{Dipole}}$$

Proton

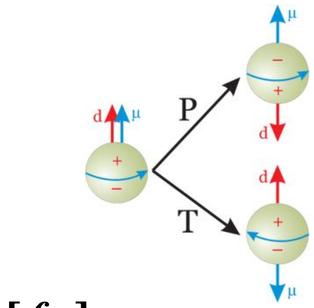


Neutron



Apparent electric dipole !

P & T violations ???



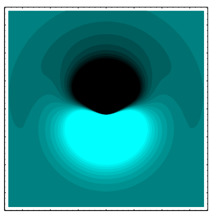
No !!!

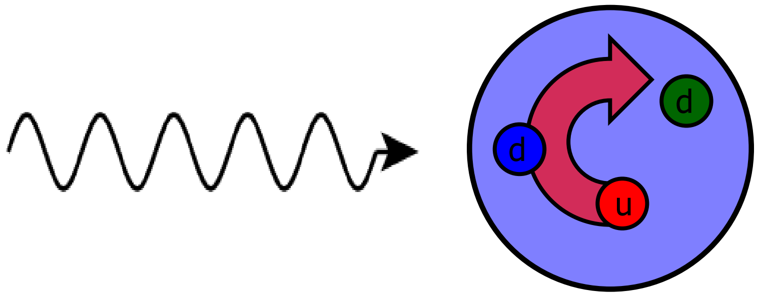
[Carlson, Vanderhaeghen (2008)]

Form factors (FFs)

Transversely polarized nucleon

Origin of EDM : light-front artifact





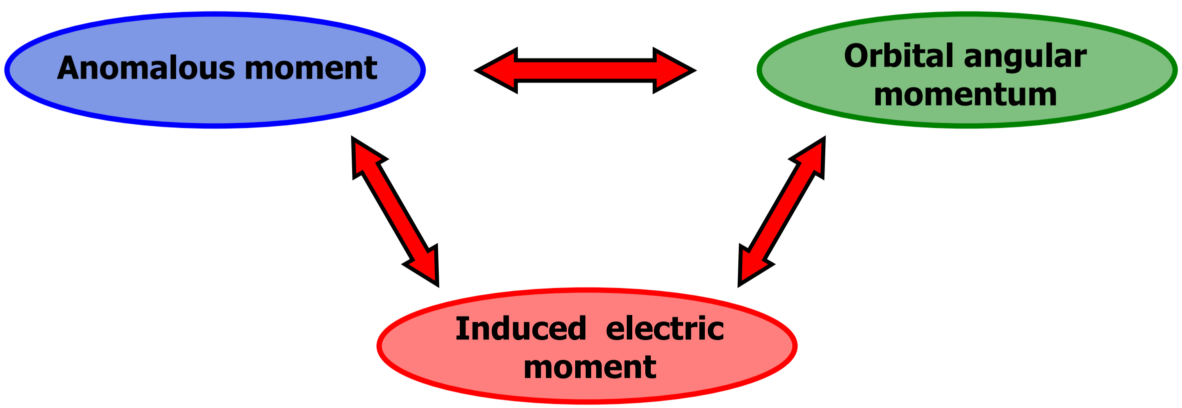
$j^3 < 0$

$j^3 > 0$

$$\boxed{j^+ = \frac{1}{\sqrt{2}}(j^0 + j^3)}$$

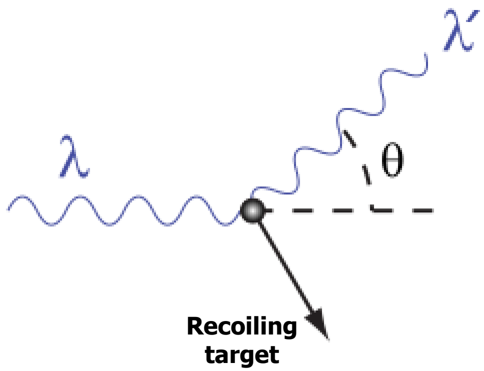
$\vec{S} \odot \quad \kappa_n = -1.91$

[Burkardt (2003)]



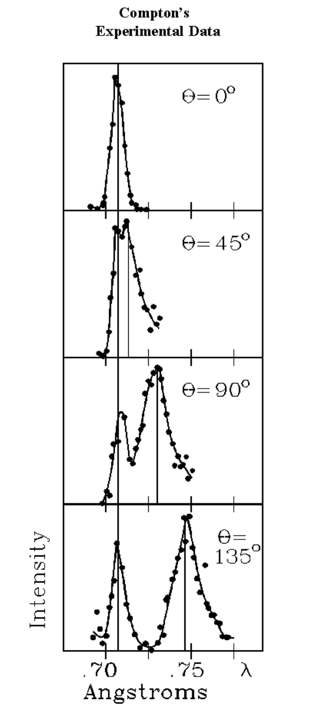
[Lorcé (2009)]

Real Compton Scattering



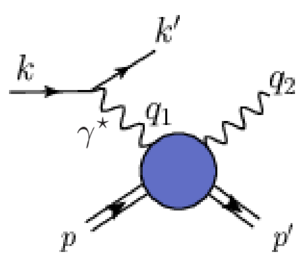
$$\Delta\lambda = \frac{2\pi}{m}(1-\cos\theta)$$

Compton wavelength



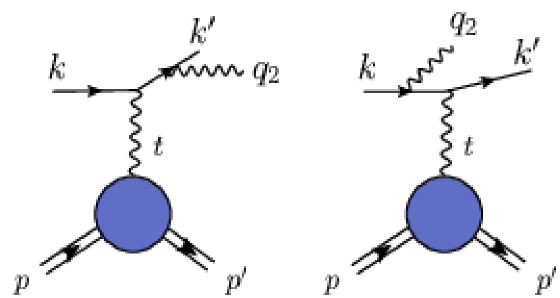
Question : why are there 2 peaks?

Virtual Compton scattering (VCS)



VCS

interferes with



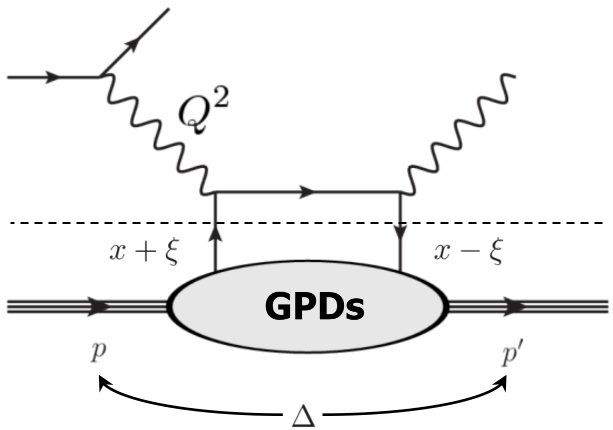
Bethe-Heitler

Deeply virtual Compton scattering (DVCS)

$P = \frac{p' + p}{2}$ $\quad x = \frac{k^+}{P^+}$

$\Delta = p' - p$ $\quad \xi = -\frac{\Delta^+}{2P^+}$

$t = \Delta^2$



Compton FF

$$T_{\text{DVCS}} \sim \boxed{\int_{-1}^{1} \mathrm{d}x \, \frac{\mathrm{GPD}(x,\xi,t)}{x \pm \xi - i\epsilon}} + \mathcal{O}(\tfrac{t}{Q^2})$$

$$\sim \mathrm{PV} \int_{-1}^{1} \mathrm{d}x \, \frac{\mathrm{GPD}(x,\xi,t)}{x \pm \xi} \pm i\pi \, \mathrm{GPD}(\mp\xi,\xi,t) + \mathcal{O}(\tfrac{t}{Q^2})$$

Factorization
Evolution @ NLO

[Collins, Freund (1999)]
[Belitsky *et al.* (2000)]

GPD correlator

$$F^{[\Gamma]}_{\Lambda'\Lambda}(x,\Delta) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\big|_{z^+=z_\perp=0}$$

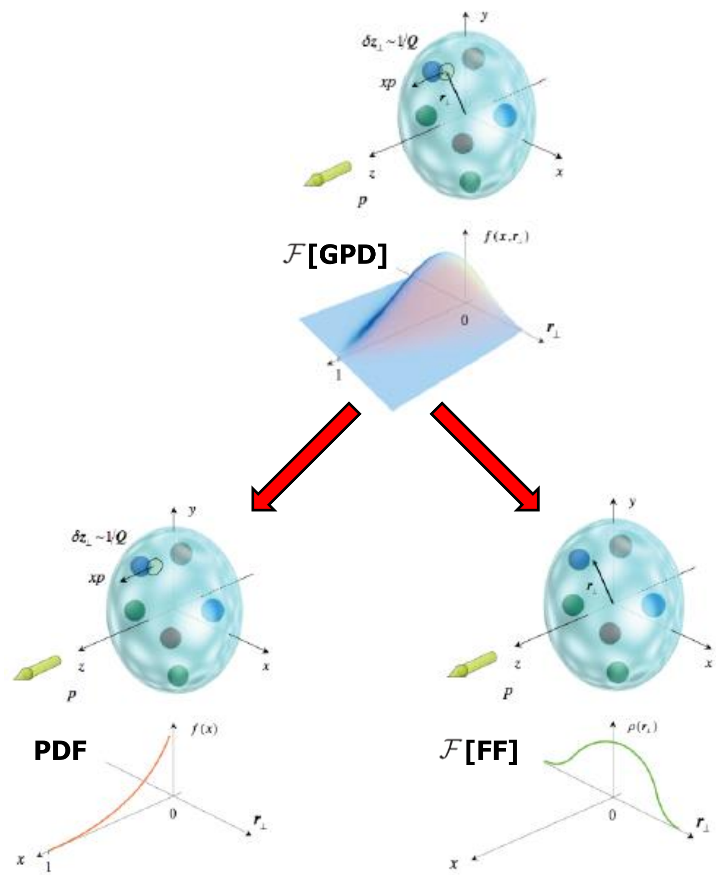
$$= \int \mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

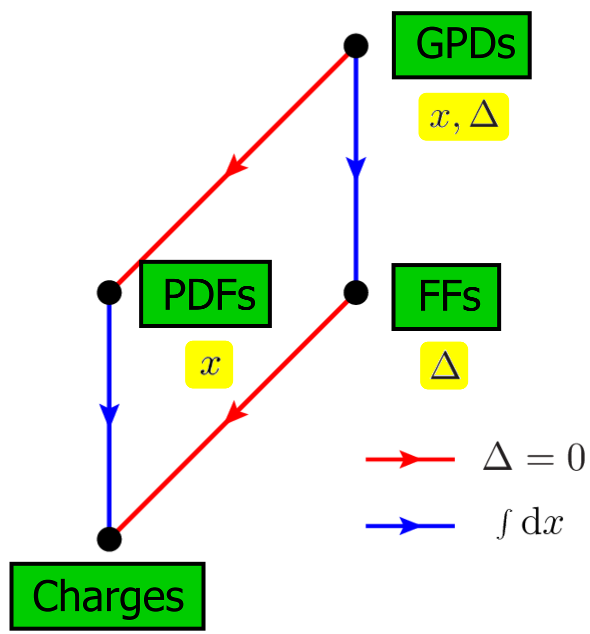
Reviews :

[Diehl (2003)]
[Belitsky, Radyushkin (2005)]
[Boffi, Pasquini (2008)]

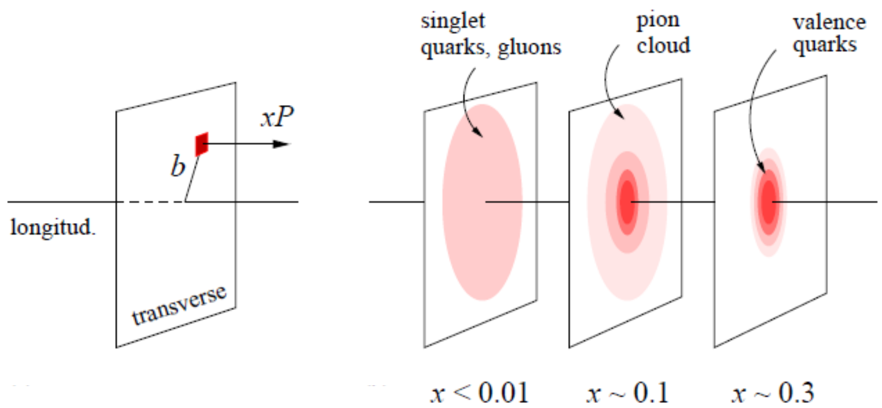
Generalized parton distributions (GPDs)

Nucleon tomography/imaging

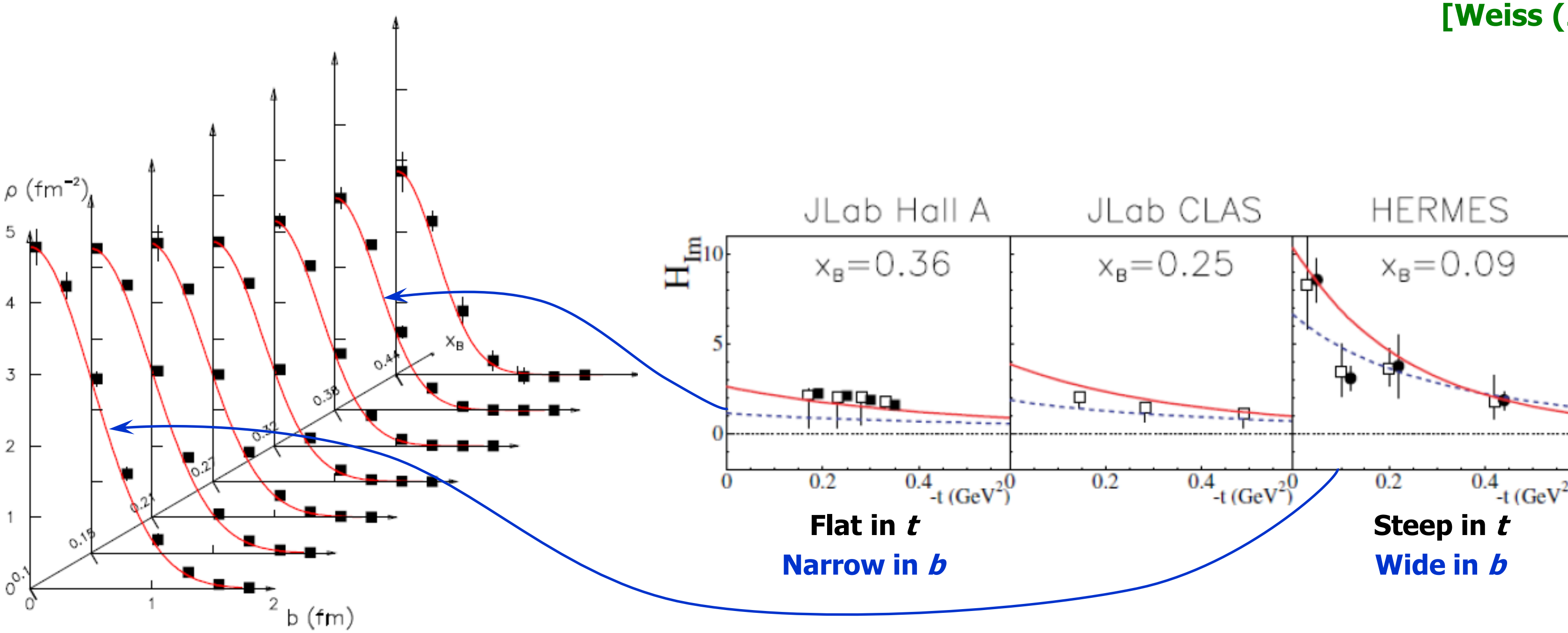




Generalized parton distributions (GPDs)



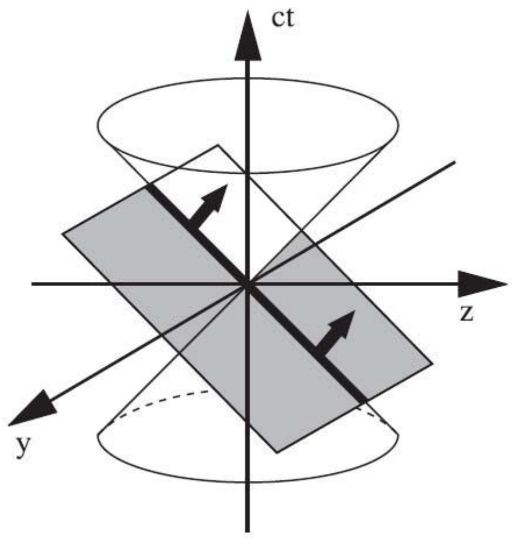
[Weiss (2009)]



[Guidal *et al.* (2013)]

Generalized parton distributions (GPDs)

Explicit Lorentz invariance is broken by the preferred light-front direction



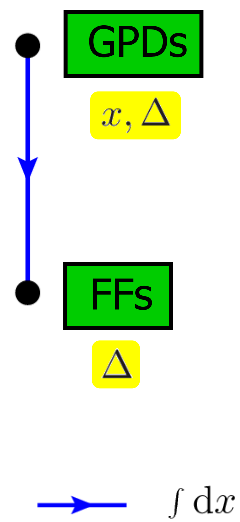
	Lorentz invariance
$t = \Delta^2$	✓
$x = \dfrac{k^+}{P^+}$	✗
$\xi = -\dfrac{\Delta^+}{2P^+}$	✗

→ x **and** ξ **dependences constrained by Lorentz invariance !**

Generalized parton distributions (GPDs)

Lowest moment

$$
\begin{aligned}
&P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z^-}{2}) \\
&= \cancel{P}^+ \int \frac{\mathrm{d}z^-}{2\pi}\, 2\pi\,\delta(\cancel{P}^+ z^-)\,\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z^-}{2}) \\
&= \overline{\psi}(0)\,\Gamma\,\psi(0)
\end{aligned}
$$



Generic moment in LF gauge $A^+ = 0 \Longrightarrow \mathcal{W} = \mathbb{1}$ $\quad \mathcal{W}_{ba} = \mathcal{P}\left[e^{ig\int_a^b \mathrm{d}x^- A^+}\right]$

$$
\begin{aligned}
&(P^+)^{n+1} \int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi} \left[\left(-i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n e^{ixP^+z^-}\right] \overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left(i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n \left[\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,\psi(\tfrac{z^-}{2})\right] \\
&= \cancel{P}^+ \int \frac{\mathrm{d}z^-}{2\pi}\, 2\pi\, \delta(\cancel{P}^+ z^-) \left[\overline{\psi}(-\tfrac{z^-}{2})\,\Gamma\,(\tfrac{i}{2}\overleftrightarrow{\partial}^+)^n\,\psi(\tfrac{z^-}{2})\right] \\
&= \overline{\psi}(0)\,\Gamma\,(\tfrac{i}{2}\overleftrightarrow{\partial}^+)^n\,\psi(0)
\end{aligned}
$$

Generic moment

$$
\begin{aligned}
&(P^+)^{n+1} \int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi} \left[\left(-i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n e^{ixP^+z^-} \right] \overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \\
&= P^+ \int \mathrm{d}x \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left(i\tfrac{\mathrm{d}}{\mathrm{d}z^-}\right)^n \left[\overline{\psi}(-\tfrac{z^-}{2})\, \Gamma\, \mathcal{W}\, \psi(\tfrac{z^-}{2}) \right] \\
&= \cancel{P}^+ \int \frac{\mathrm{d}z^-}{2\pi}\, 2\pi\, \delta(\cancel{P}^+ z^-) \left[\overline{\psi}(-\tfrac{z^-}{2})\, \Gamma \left(\mathcal{W}\, \tfrac{i}{2}\overrightarrow{D}^+ - \tfrac{i}{2}\overleftarrow{D}^+\, \mathcal{W} \right)^n \psi(\tfrac{z^-}{2}) \right] \\
&= \overline{\psi}(0)\, \Gamma\, (\tfrac{i}{2}\overleftrightarrow{D}^+)^n\, \psi(0)
\end{aligned}
$$

using

$$
\tfrac{\mathrm{d}}{\mathrm{d}z^-} \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}} = -\frac{ig}{2} \left[A^+(-\tfrac{z^-}{2})\, \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}} + \mathcal{W}_{-\frac{z^-}{2}\,\frac{z^-}{2}}\, A^+(\tfrac{z^-}{2}) \right]
$$

Derivative of Wilson line

$$
\begin{aligned}
\overrightarrow{D}^+ &= \overrightarrow{\partial}^+ - ig\, A^+(\tfrac{z^-}{2}) \\
\overleftarrow{D}^+ &= \overleftarrow{\partial}^+ + ig\, A^+(-\tfrac{z^-}{2})
\end{aligned}
$$

Covariant derivatives

Generic local operator

$$\hat{O}_\Gamma^{\mu_1\cdots\mu_n} = \mathbf{S}\left[\overline{\psi}(0)\,\Gamma\,\tfrac{i}{2}\overleftrightarrow{D}^{\mu_1}\cdots\,\tfrac{i}{2}\overleftrightarrow{D}^{\mu_n}\,\psi(0)\right]$$

Symmetrization
+ trace removal

Generic FFs

Constrained by Lorentz and discrete space-time symmetries

$$\langle p'|\hat{O}_\Gamma^{\mu_1\cdots\mu_n}|p\rangle = \sum_{\Gamma'} \overline{u}(p')\,\Gamma'\,u(p)\sum_{i=0}^{n} \mathrm{FF}_{n+1,i}^{\Gamma\Gamma'}(t)\,\Delta^{\mu_1}\cdots\Delta^{\mu_i}\,P^{\mu_{i+1}}\cdots P^{\mu_n}$$

Polynomiality of GPDs

$$\boxed{\int_{-1}^{1}\mathrm{d}x\,x^n\,\mathrm{GPD}(x,\xi,t) = \sum_{i=1}^{n}(-2\xi)^{c_i}\,\mathrm{FF}_{n+1,i}(t)}$$

Generalized parton distributions (GPDs)

How to ensure polynomiality property?

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\gamma^+\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^+=z_\perp=0} \quad \Longrightarrow \quad \langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^2=0}$$

« Covariantization »

Lorentz-invariant variables

$$P^\mu \quad \Delta^\mu \quad z^\mu \quad \Longrightarrow \quad \begin{array}{ccc} P\cdot\Delta = 0 & \boxed{P\cdot z} & \boxed{\Delta\cdot z} \\ P^2 = M^2 - \frac{t}{4} & \boxed{\Delta^2 = t} & z^2 = 0 \end{array}$$

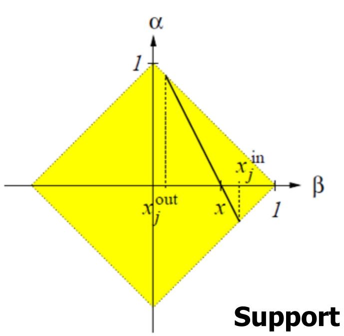
GPD parametrization

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^2=0} = \overline{u}(p')\,\not{z}\,u(p)\int \mathrm{d}x\, e^{-ixP\cdot z}\boxed{H^q(x,\xi,t)}$$
$$+\,\overline{u}(p')\,\frac{i\sigma^{\mu\nu}z_\mu\Delta_\nu}{2M}\,u(p)\int \mathrm{d}x\, e^{-ixP\cdot z}\boxed{E^q(x,\xi,t)}$$

Constraint : $\xi = -\frac{\Delta\cdot z}{2P\cdot z}$

Double distribution (DD) parametrization

$$\langle p'|\overline{\psi}(-\tfrac{z}{2})\,\not{z}\,\mathcal{W}\,\psi(\tfrac{z}{2})|p\rangle\big|_{z^2=0} = \overline{u}(p')\,\not{z}\,u(p)\int \mathrm{d}\beta\,\mathrm{d}\alpha\, e^{-i(\beta P-\alpha\frac{\Delta}{2})\cdot z}\boxed{f^q(\beta,\alpha,t)}$$
$$+\,\overline{u}(p')\,\frac{i\sigma^{\mu\nu}z_\mu\Delta_\nu}{2M}\,u(p)\int \mathrm{d}\beta\,\mathrm{d}\alpha\, e^{-i(\beta P-\alpha\frac{\Delta}{2})\cdot z}\boxed{k^q(\beta,\alpha,t)}$$
$$-\,\overline{u}(p')\,\frac{\Delta\cdot z}{2M}\,u(p)\int \mathrm{d}\alpha\, e^{i\alpha\frac{\Delta}{2}\cdot z}\boxed{D^q(\alpha,t)}$$



Support : $|\beta|+|\alpha|\leq 1$

[Müller *et al.* (1994)]
[Radyushkin (1999)]
[Polyakov, Weiss (1999)]

Generalized parton distributions (GPDs)

Relation between GPDs and DDs

$$H^q(x,\xi,t) = \int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)$$

$$E^q(x,\xi,t) = \int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,k^q(\beta,\alpha,t) - \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)$$

Polynomiality property

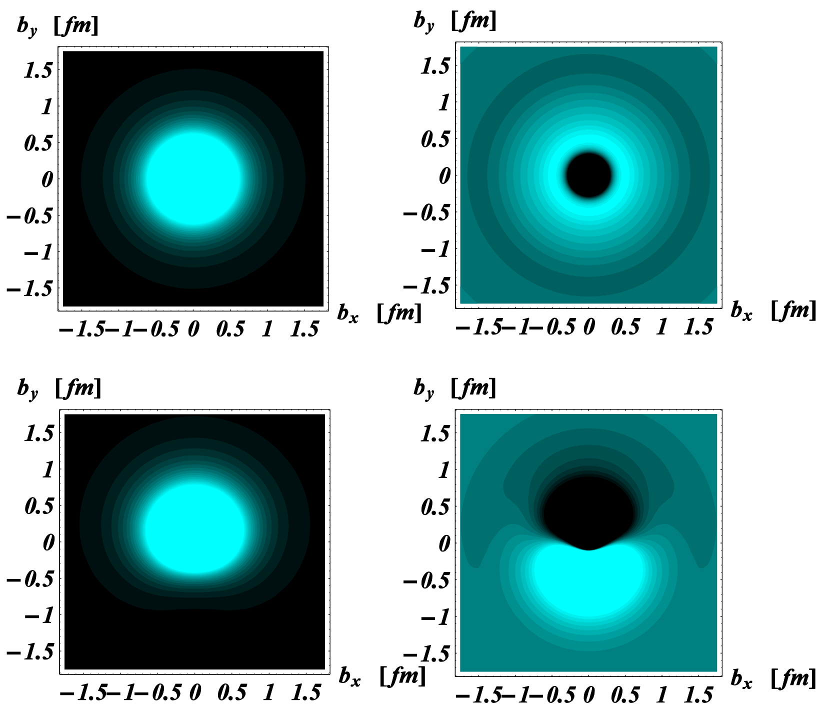
$$\begin{aligned}\int \mathrm{d}x\,x^n\,H^q(x,\xi,t) &= \int \mathrm{d}x\,x^n\left[\int \mathrm{d}\beta\,\mathrm{d}\alpha\,\delta(x-\beta-\xi\alpha)\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,D^q(\tfrac{x}{\xi},t)\right]\\ &= \int \mathrm{d}\beta\,\mathrm{d}\alpha\,(\beta+\xi\alpha)^n\,f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi)\,\xi^{n+1}\int \mathrm{d}y\,y^n\,D^q(y,t)\end{aligned}$$

$\Rightarrow$ α **and** β **dependences not constrained by Lorentz invariance !**

[Müller *et al.* (1994)]
[Radyushkin (1999)]
[Polyakov, Weiss (1999)]

Lecture 3

- FFs provide 2D pictures of the nucleon
- GPDs generalize both PDFs and FFs and provide 2+1D pictures
- Lorentz symmetry constrain some variables



$$\mathcal{F}[\mathrm{FF}](\vec{b}_\perp) = \int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$

$$\mathcal{F}[\mathrm{GPD}](x,\vec{b}_\perp) = \int \mathrm{d}^2k_\perp\,\rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$$