HUGS 2014

29th Annual Hampton University Graduate Studies Program

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Spin Sum Rules and 3D Nucleon Structure (4/6)

Cédric Lorcé

IPN Orsay - IFPA Liège

June 2-20, 2014, JLab, Newport News, Virginia, USA

Outline

Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz

Lecture 2

- Photon point of view
- Galileo *vs* Lorentz : round 2
- Nucleon 1D picture

Lecture 3

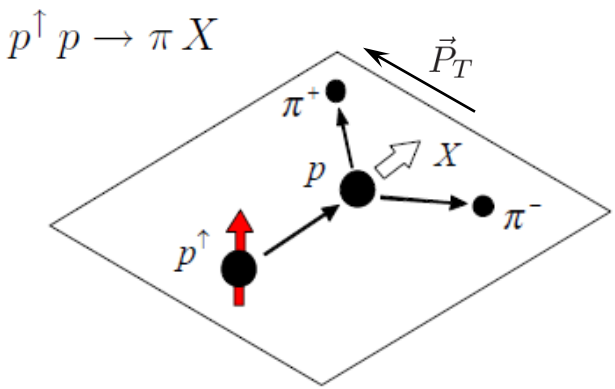
- Nucleon 2D picture
- 1D+2D=3D
- Galileo *vs* Lorentz : round 3

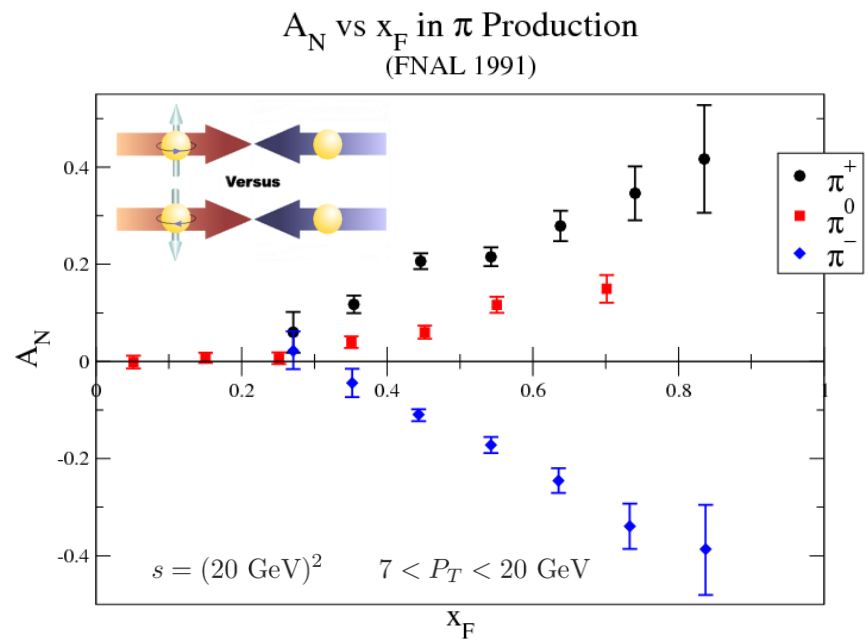
Lecture 4

- **Another nucleon 3D picture**
- **Tour in Fock space**
- **3D+3D=… 5D !**

What about k_T ?

Large single-spin asymmetries have been observed at high energy !





$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \propto \vec{S}_N \cdot (\vec{p}_N \times \vec{P}_T)$$

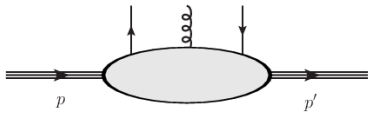
Partonic origin ?

Price for helicity flip

Collinear twist-2 ✗ $A_N \propto \frac{m_q}{E_q}\alpha$ → **Too small !** **[Kane, Pumplin, Repko (1978)]**

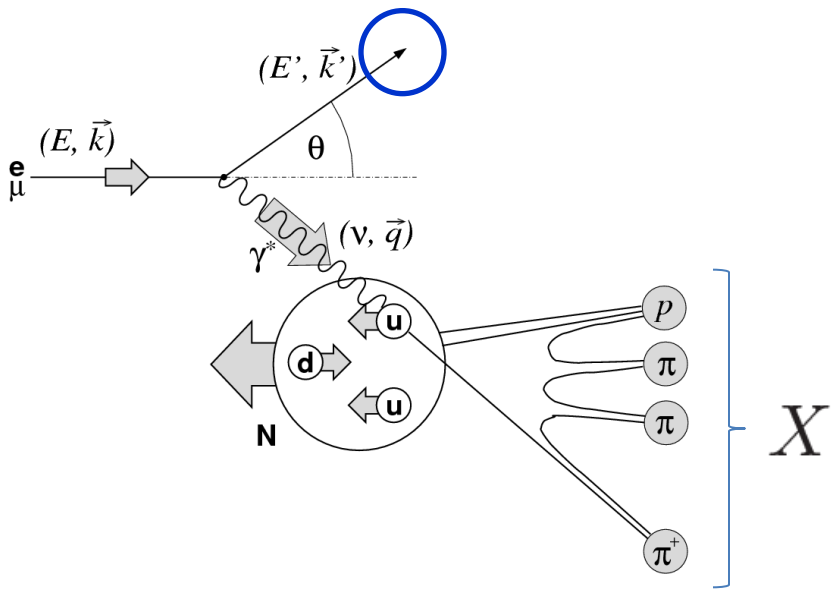
Intrinsic k_T ✓ $Q \gg P_T \sim \Lambda_{\mathrm{QCD}}$

Collinear twist-3 ✓ $Q \sim P_T \gg \Lambda_{\mathrm{QCD}}$

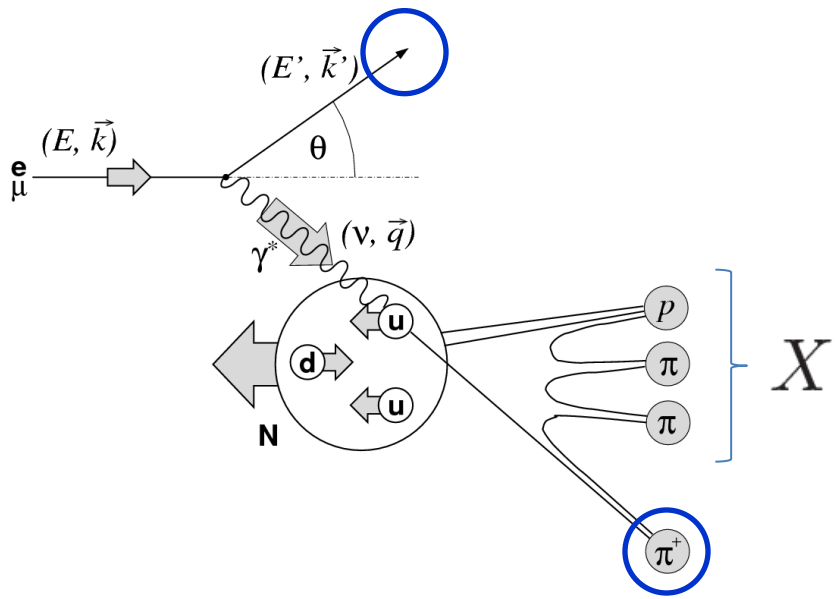


Semi-inclusive DIS

Inclusive DIS



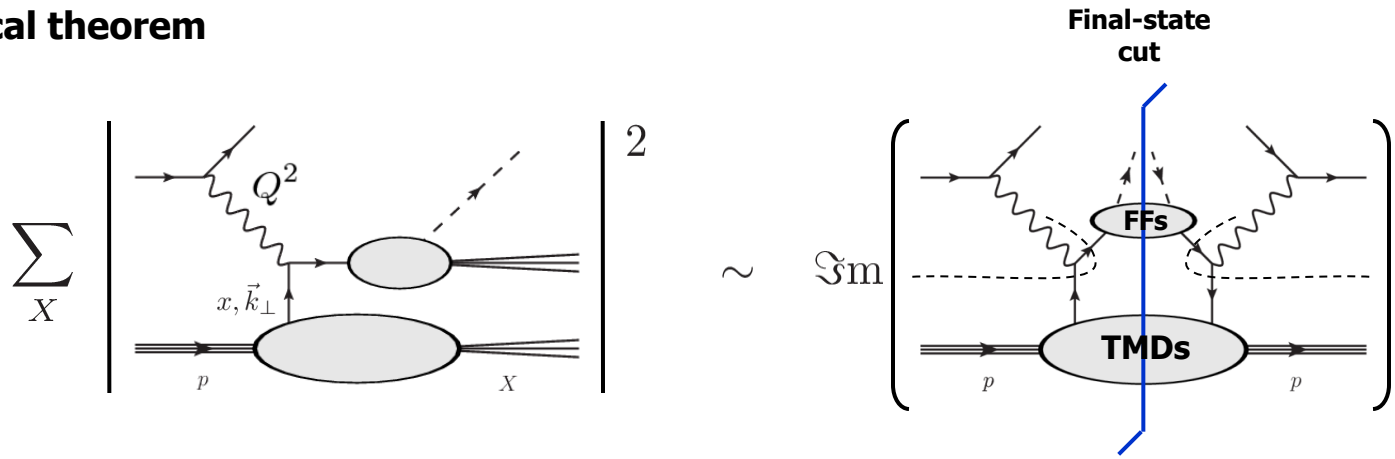
Semi-inclusive DIS (SIDIS)



Identified particles in final state

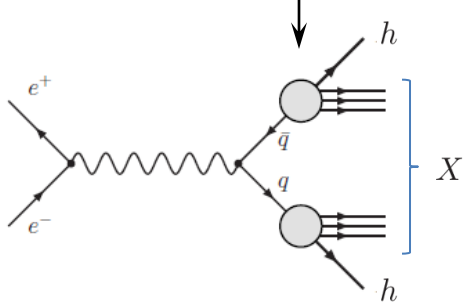
Transverse-momentum distributions (TMDs)

Optical theorem



$$\mathrm{d}\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes \mathrm{d}\hat{\sigma}_{hard} \otimes \underbrace{\mathrm{FF}(z, \vec{p}_\perp)}_{\text{Fragmentation Function}} + \mathcal{O}(\tfrac{P_T}{Q})$$

$e^+e^- \to hh'X$



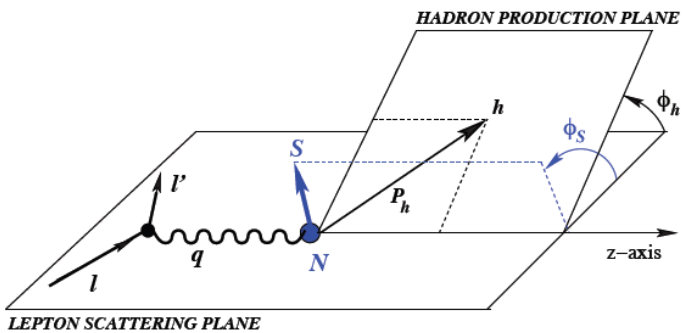
[Collins, Soper, Sterman (1985)]
[Ji, Ma, Yuan (2004)]
[Idilbi *et al.* (2004)]
[Cherednikov, Stefanis (2008)]
[Trentadue, Ceccopieri (2008)]
[Hautman (2008)]
[Echevarria, Idilbi, Scimemi (2011)]
[Collins (2011)]

Transverse-momentum distributions (TMDs)

TMD correlator

$$\Phi^{[\Gamma]}_{\Lambda'\Lambda}(x,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2 z_\perp}{(2\pi)^3}\, e^{ik\cdot z}\langle P,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|P,\Lambda\rangle\big|_{z^+=0}$$
$$= W^{[\Gamma]}_{\Lambda'\Lambda}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

SIDIS modulations



$$x = \frac{Q^2}{2\,P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot l}$$
$$z = \frac{P\cdot P_h}{P\cdot q}$$

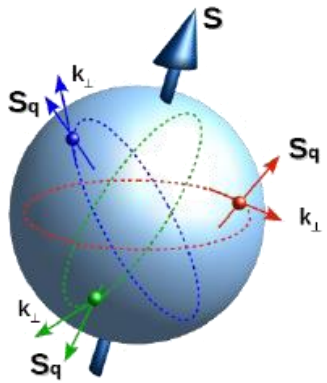
[Mulders, Tangermann (1996)]
[Boer, Mulders (1998)]
[Bacchetta *et al.* (2004)]
[Bacchetta *et al.* (2007)]
[Anselmino *et al.* (2011)]

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}\phi_S\,\mathrm{d}^2P_T} = F_{UU} + \cos(2\phi)\,F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\cos\phi\,F_{UU}^{\cos\phi} + \lambda_e\,\frac{1}{Q}\sin\phi\,F_{UU}^{\sin\phi}$$
$$+ S_L\left\{\sin(2\phi)\,F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\sin\phi\,F_{UL}^{\sin\phi} + \lambda_e\left[F_{LL} + \frac{1}{Q}\cos\phi\,F_{LL}^{\cos\phi}\right]\right\}$$
$$+ S_T\left\{\sin(\phi-\phi_S)\,F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S)\,F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)\,F_{UT}^{\sin(3\phi-\phi_S)}\right.$$
$$+\frac{1}{Q}\left[\sin(2\phi-\phi_S)\,F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\,F_{UT}^{\sin\phi_S}\right]$$
$$\left.+\lambda_e\left[\cos(\phi-\phi_S)\,F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q}\left(\cos\phi_S\,F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\,F_{LT}^{\cos(2\phi-\phi_S)}\right)\right]\right\}$$

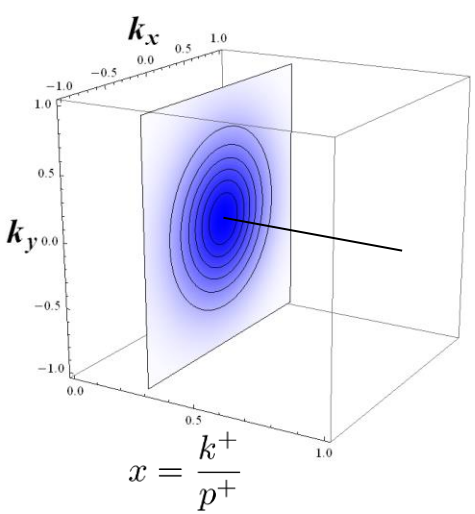
$$F_{S_qS} \propto \mathrm{TMD}\otimes\mathrm{FF}$$

Transverse-momentum distributions (TMDs)

Spin-orbit correlations



Momentum-space imaging



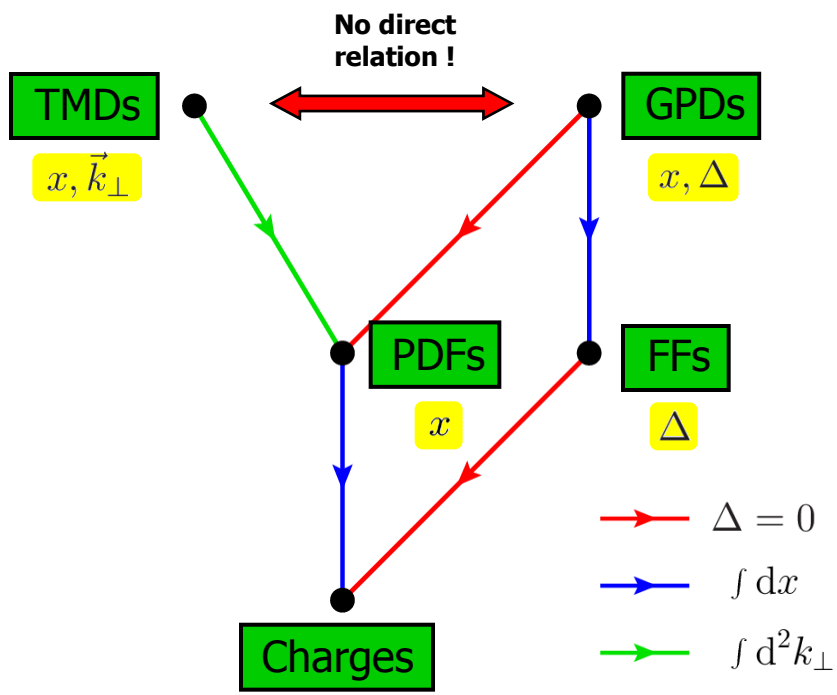
Average momentum — **Position shift**

$$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$$

$$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$$

Momentum space — **Position space**

Momentum transfer — **Average position**



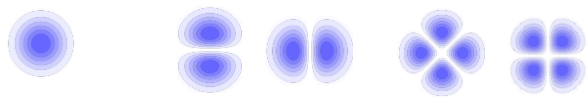
Complementary hadron 3D pictures !

Multipole structure

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M}\, h_1^\perp$	$-\frac{k_x}{M}\, h_1^\perp$	
T_x	$\frac{k_y}{M}\, f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$\frac{k_x}{M}\, g_{1T}$
T_y	$-\frac{k_x}{M}\, f_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_y}{M}\, g_{1T}$
L		$\frac{k_x}{M}\, h_{1L}^\perp$	$\frac{k_y}{M}\, h_{1L}^\perp$	g_{1L}



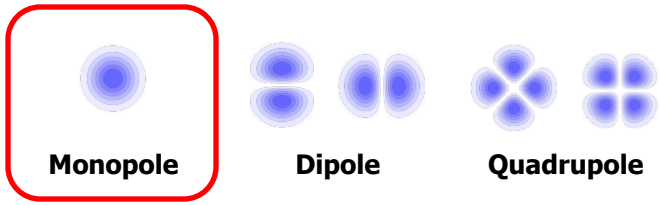
Monopole **Dipole** **Quadrupole**

Multipole structure

Quark polarization

Nucleon polarization

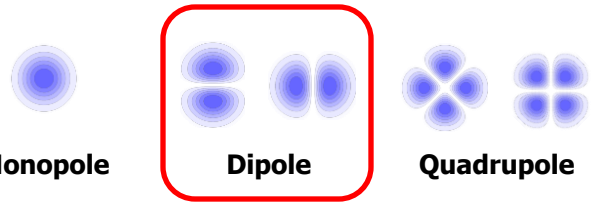
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}



Multipole structure

Quark polarization

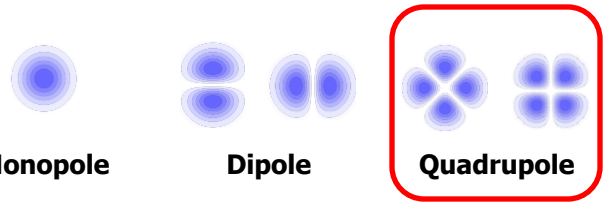
Nucleon polarization	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}



Multipole structure

Quark polarization

Nucleon polarization	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

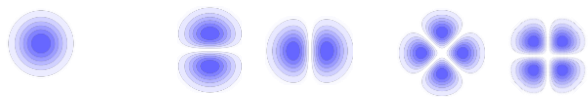


Multipole structure

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}



Monopole **Dipole** **Quadrupole**

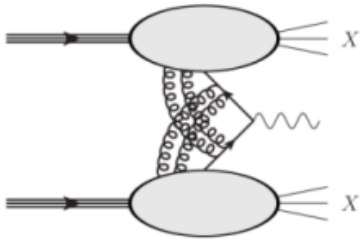
Naive T-odd !

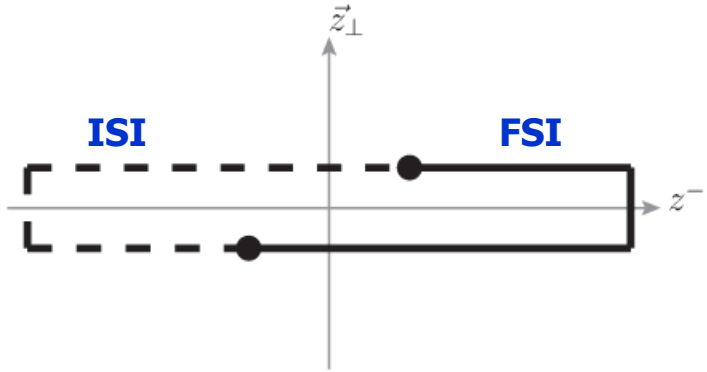
Transverse-momentum distributions (TMDs)

Non-trivial gauge link $\mathcal{W}$

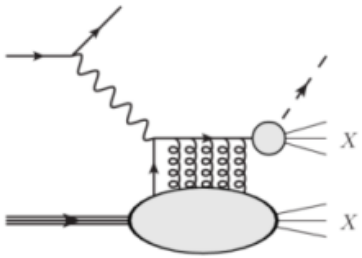
[Belitsky *et al.* (2003)]
[Boer *et al.* (2003)]

Drell-Yan





SIDIS



CPT invariance →

$$f_{1T}^{\perp}(x,\vec{k}_\perp)\big|_{\rm DY} = -f_{1T}^{\perp}(x,\vec{k}_\perp)\big|_{\rm SIDIS}$$
$$h_1^{\perp}(x,\vec{k}_\perp)\big|_{\rm DY} = -h_1^{\perp}(x,\vec{k}_\perp)\big|_{\rm SIDIS}$$

Naive T-odd

Fundamental test !

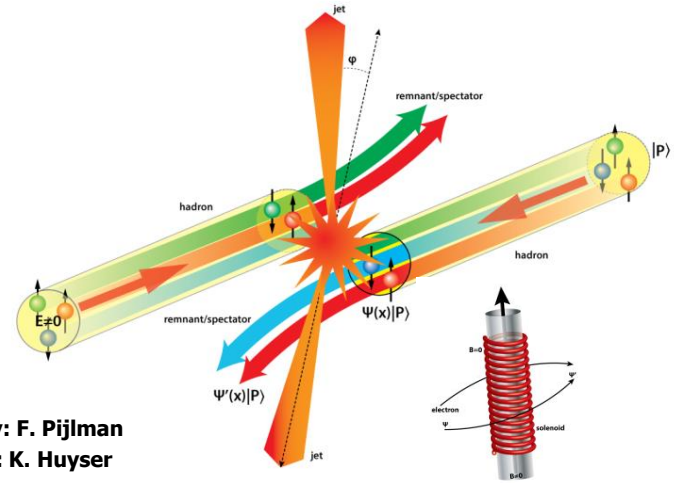
Generalized universality

$$\underbrace{\mathrm{TMD}(x,\vec{k}_\perp;\mathcal{W})}_{\text{Process dependent}} = \sum_i \underbrace{C_i(\mathcal{W})}_{\text{Calculable}}\,\underbrace{\mathrm{TMD}_i(x,\vec{k}_\perp)}_{\text{Universal}}$$

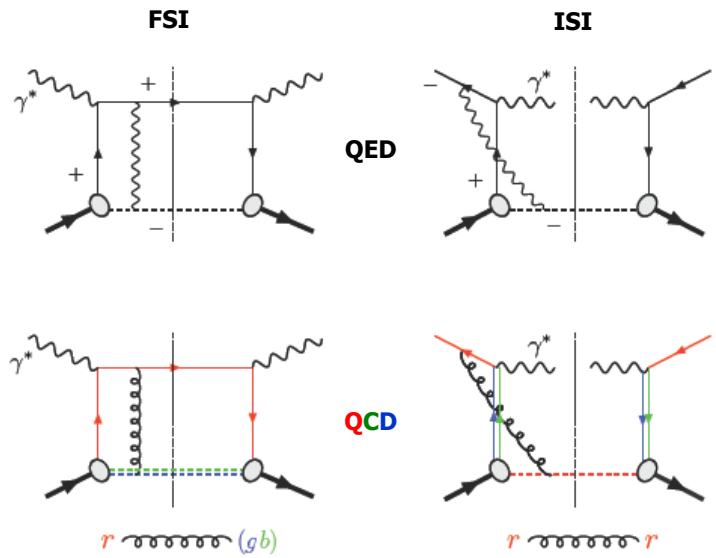
[Buffing *et al.* (2012)]
[Buffing *et al.* (2013)]

Transverse-momentum distributions (TMDs)

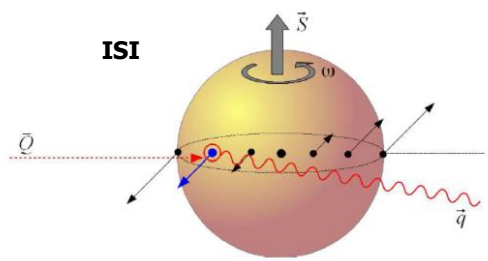
Color-induced phase !

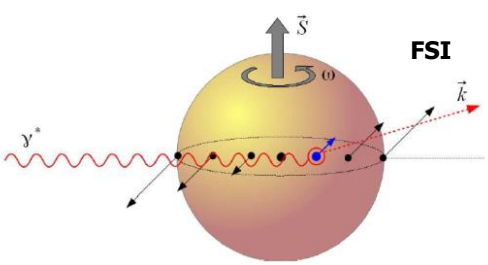


Analogy: F. Pijlman
Figure: K. Huyser



Quasi-classical interpretation

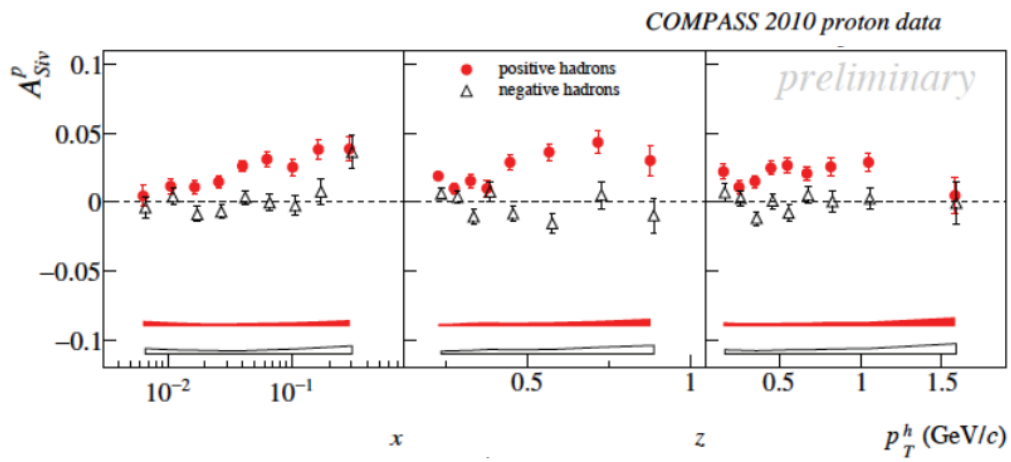




[Sivers (2006)]
[Sievert, Kovchegov (2014)]

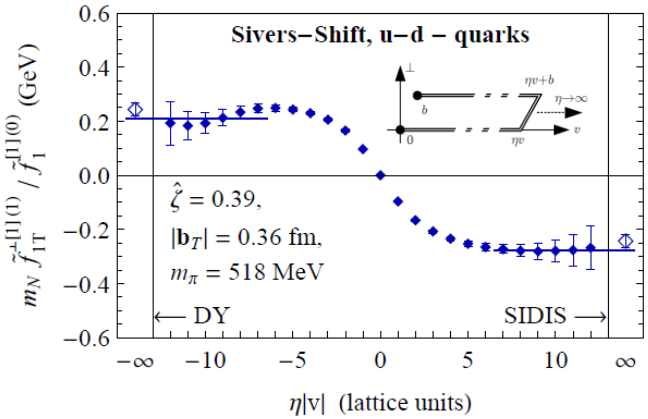
Transverse-momentum distributions (TMDs)

Clear experimental signal

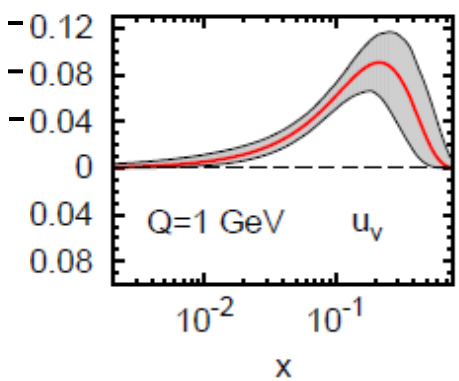


Lattice QCD

[Musch *et al.* (2012)]



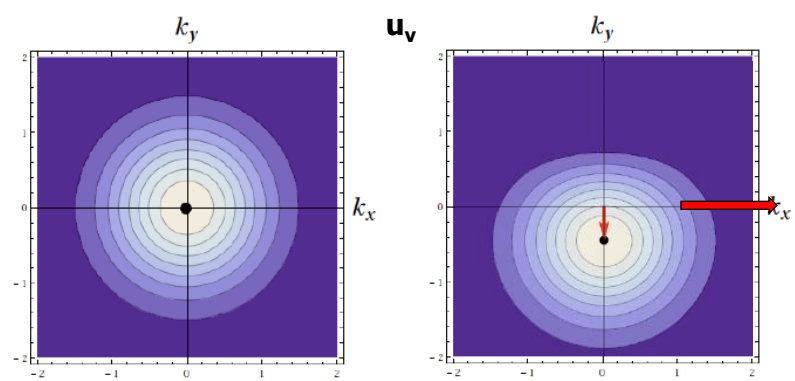
$$x f_{1T}^{\perp(1)}(x) = x \int d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} f_{1T}^\perp(x, \vec{k}_\perp)$$



[Anselmino *et al.* (2012)]

Dipole shift

$$\rho_X(x, \vec{k}_\perp) = f_1(x, \vec{k}_\perp) + \frac{k_y}{M} f_{1T}^\perp(x, \vec{k}_\perp)$$

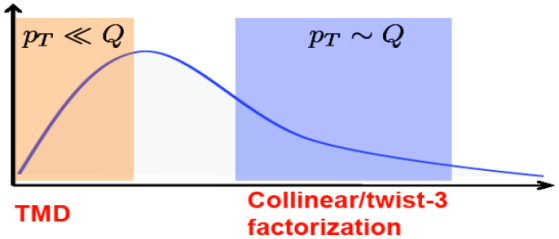


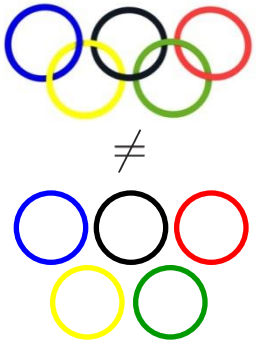
Courtesy of Alexei Prokudin

Transverse-momentum distributions (TMDs)

Open questions and problems

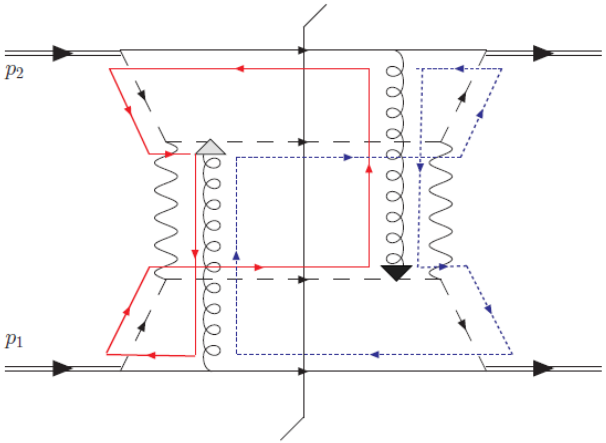
- **Tests of universality (*e.g.* with DY) and evolution**
- **Model dependence and extrapolations**
- **Precise determination of polarized TMDs**
- **Extraction of gluon TMDs**
- **Accessing higher-twist distributions**
- **Link with low x, k_T factorization**
- **Sign mismatch with collinear twist-3 approach**
- **Factorization breaks down in some pp scattering**
- **...**



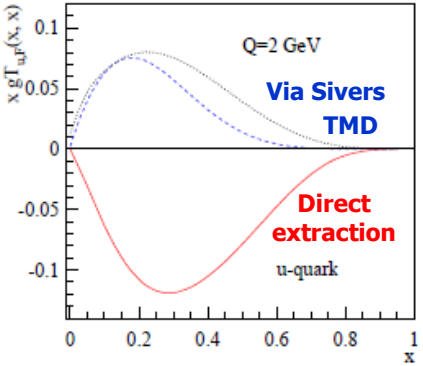


M. Buffing

Entangled color flow



[Rogers, Mulders (2011)]

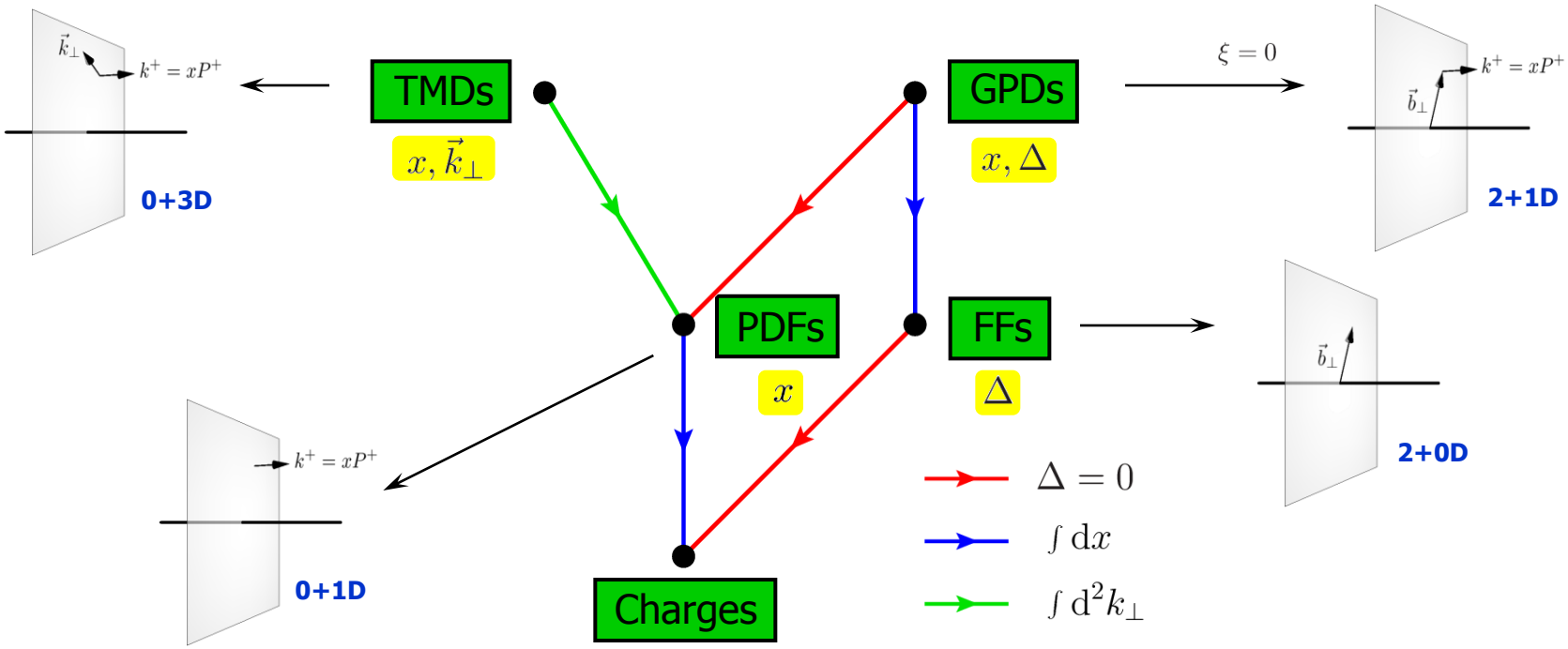


[Kang *et al.* (2011)]

Parton distribution zoo

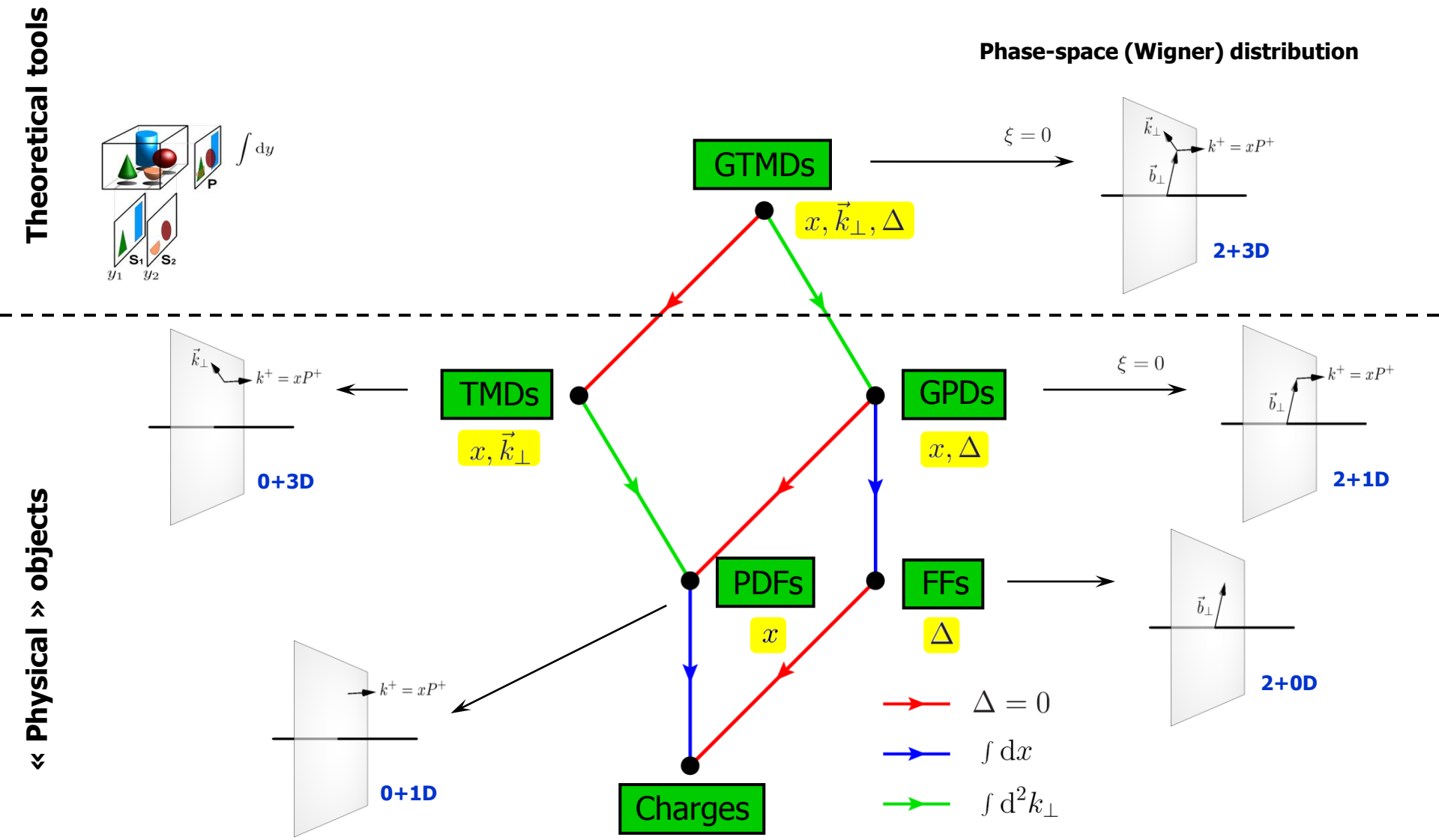
Theoretical tools

« Physical » objects



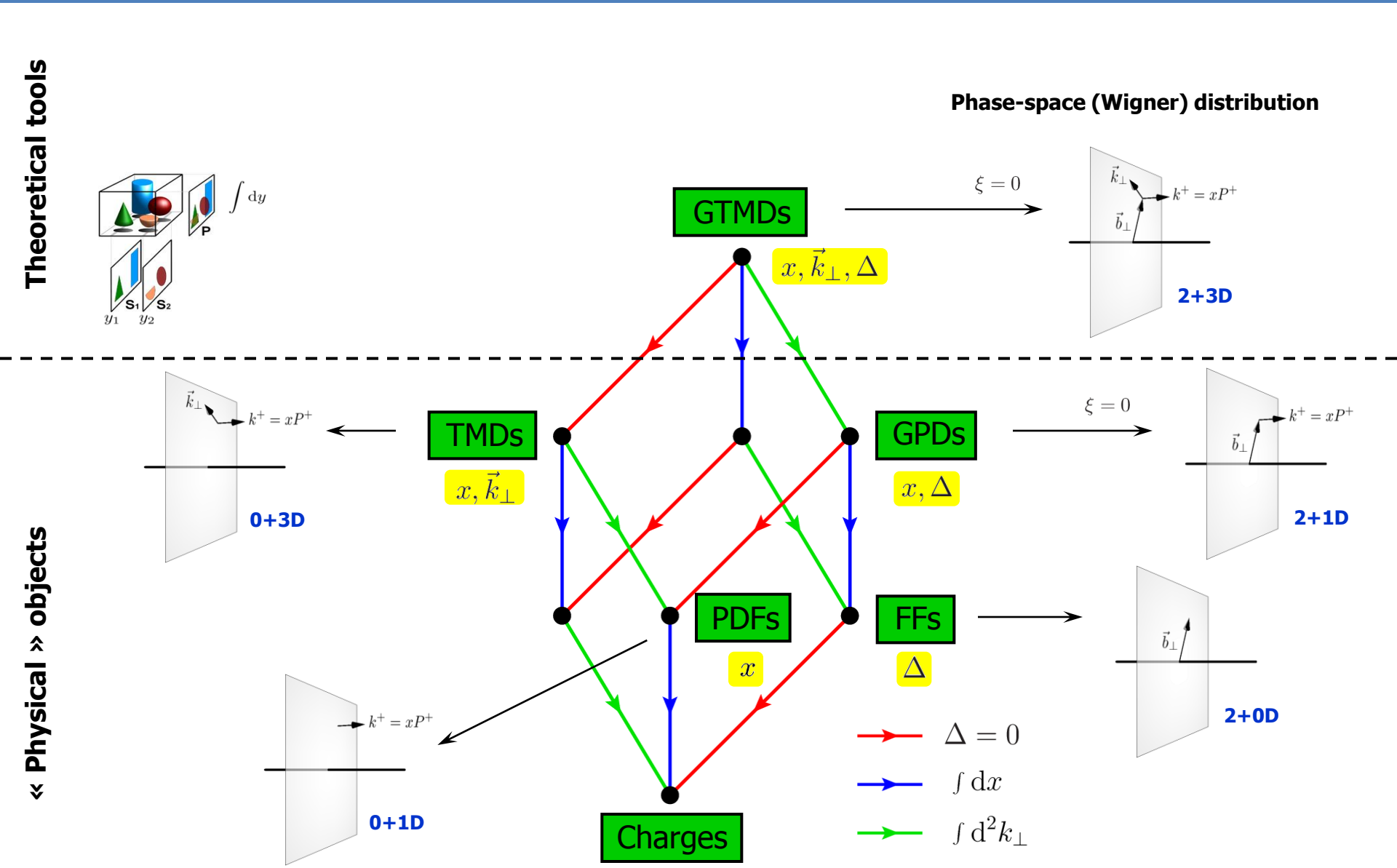
[C.L., Pasquini, Vanderhaeghen (2011)]

Parton distribution zoo



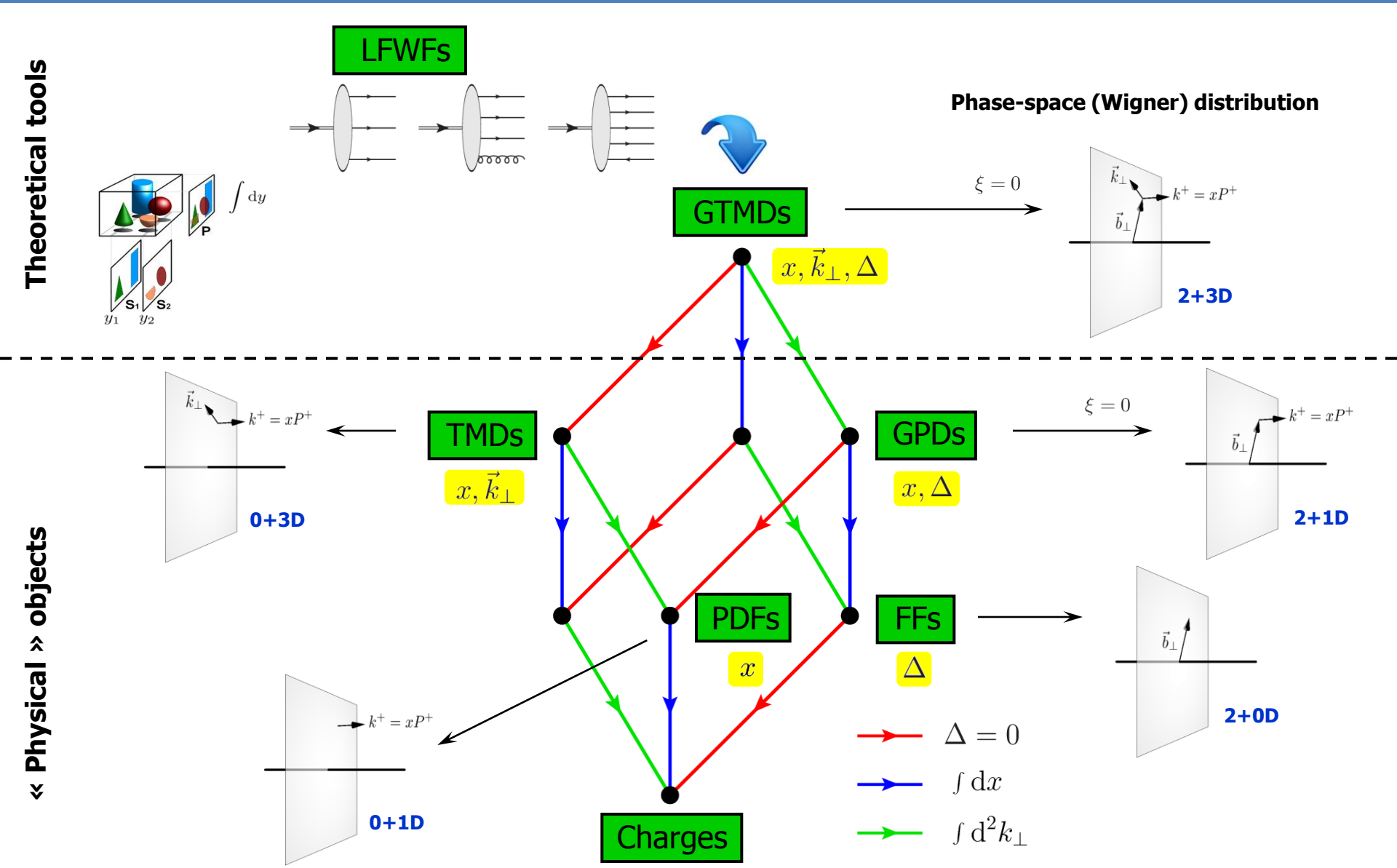
[C.L., Pasquini, Vanderhaeghen (2011)]

Parton distribution zoo



[C.L., Pasquini, Vanderhaeghen (2011)]

Parton distribution zoo

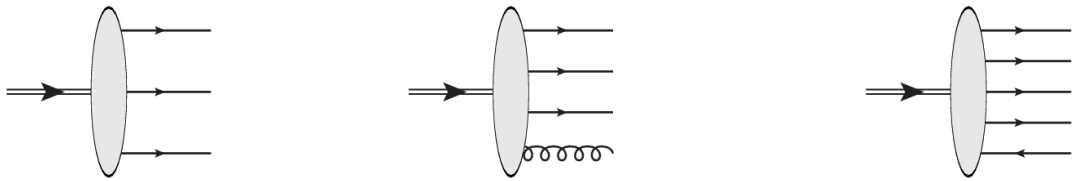


[C.L., Pasquini, Vanderhaeghen (2011)]

Light-front wave functions (LFWFs)

Fock expansion of the nucleon state

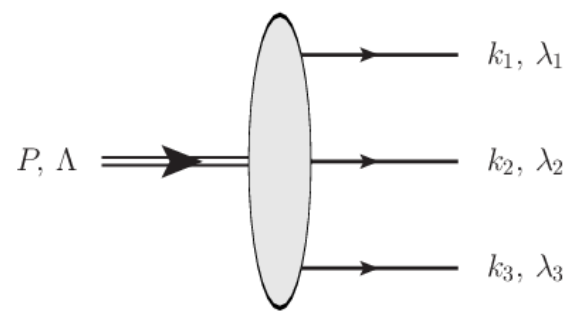
$$|p\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqqq\bar{q}}\,|qqqq\bar{q}\rangle + \cdots$$



Probability associated with the Fock states

$$\rho^{\Lambda}_{N,\beta} = \int [\mathrm{d}x]_N \left[\mathrm{d}^2 k_\perp\right]_N \left|\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N}\right|^2$$

$$\sum_{N,\beta} \rho^{\Lambda}_{N,\beta} = 1$$



Momentum and angular momentum conservation

$$P^+ = \sum_{i=1}^{N} k_i^+$$

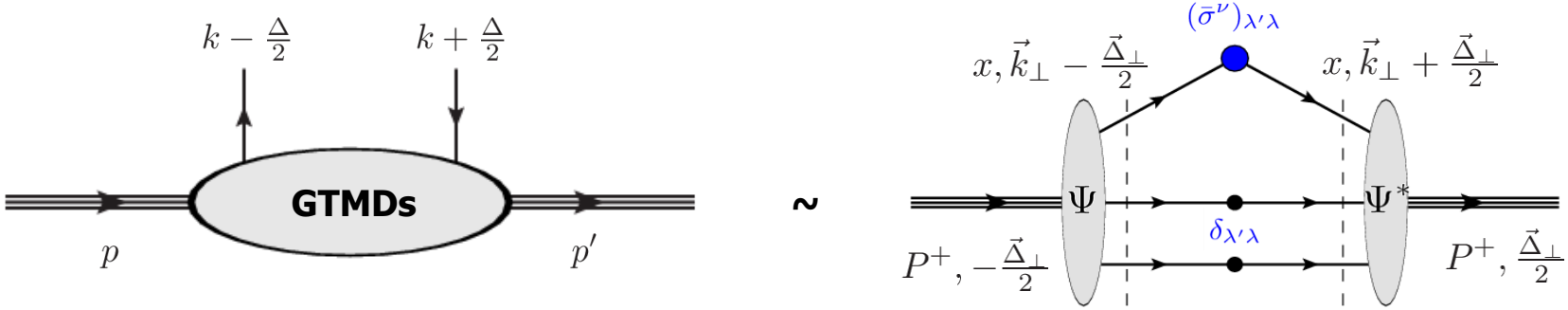
$$\vec{0}_\perp = \vec{P}_\perp = \sum_{i=1}^{N} \vec{k}_{i\perp}$$

$$\boxed{\Lambda = \sum_{i=1}^{N} \lambda_i + l_z}$$

$A^+ = 0$ **gauge**

Light-front wave functions (LFWFs)

Overlap representation



$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}} \sum_{\beta',\beta} \int [\mathrm{d}x]_3 [\mathrm{d}^2k_\perp]_3 \, \bar{\delta}(\tilde{k}) \, \psi^*_{\Lambda'\beta'}(r') \, \psi_{\Lambda\beta}(r) \, M^{[\Gamma]\beta'\beta}$$

Momentum **Polarization**

$$[\mathrm{d}x]_3 \equiv \left[\prod_{i=1}^{3} \mathrm{d}x_i\right] \delta\left(1 - \sum_{i=1}^{3} x_i\right)$$

$$[\mathrm{d}^2k_\perp]_3 \equiv \left[\prod_{i=1}^{3} \frac{\mathrm{d}^2k_{i\perp}}{2(2\pi)^3}\right] 2(2\pi)^3 \, \delta^{(2)}\left(\sum_{i=1}^{3} \vec{k}_{i\perp}\right)$$

$$\bar{\delta}(\tilde{k}) \equiv \sum_{i=1}^{3} \Theta(x) \, \delta(x - x_i) \, \delta^{(2)}(\vec{k}_\perp - \vec{k}_{i\perp})$$

$$M^{[\Gamma]\beta'\beta} = M^{[\Gamma]\lambda'_1\lambda_1} \, \delta^{\lambda'_2\lambda_2} \, \delta^{\lambda'_3\lambda_3}$$

$$M^{[\Gamma]\lambda'\lambda} \equiv \frac{\overline{u}(p',\lambda')\Gamma u(p,\lambda)}{2P^+\sqrt{1-\xi^2}}$$

[C.L., Pasquini, Vanderhaeghen (2011)]

Light-front wave functions (LFWFs)

Light-front quark models

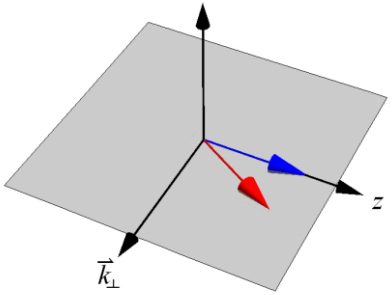
$$\psi_{\Lambda\beta}(r) = \mathcal{N}\,\Psi(r) \sum_{\sigma_i} \Phi_{\Lambda}^{\sigma_1\sigma_2\sigma_3} \prod_{i=1}^{3} D_{\lambda_i\sigma_i}(\tilde{k}_i)$$

SU(6) spin-flavor wave function

Wigner rotation

$$q_{\lambda}^{LC}(k) = \sum_{s} D_{\lambda s}^{(1/2)*}(k)\, q_{s}^{C}(k)$$

Light-front helicity **Canonical spin**



$$D(\tilde{k}) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

$$K_{R,L} = K_x \pm iK_y$$

Model	$\Psi(r)$	K_z	$\vec{K}_\perp$	κ_z
LFCQM	$\tilde{\psi}(r)$	$m + y\mathcal{M}_0$	$\vec{\kappa}_\perp$	$y\mathcal{M}_0 - \omega$
LFχQSM	$\prod_{i=1}^{3} \lvert\vec{K}_i\rvert$	$f_{/\!/}(y,\kappa_\perp)$	$\vec{\kappa}_\perp\, f_\perp(y,\kappa_\perp)$	$y\mathcal{M}_N - E_{\rm lev}$

[C.L., Pasquini, Vanderhaeghen (2011)]

Twist-2

GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs $\xi = 0$

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$	$-i\,\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$	
T_x	$i\,\frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				$\tilde{H}$

Complete parametrizations : Quarks [Meissner, Metz, Schlegel (2009)]

Quarks & gluons [C.L., Pasquini (2013)]

Parametrization

Twist-2

GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs $\xi = 0$

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$	$-i\,\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$	
T_x	$i\,\frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				$\tilde{H}$

Complete parametrizations : **Quarks** **[Meissner, Metz, Schlegel (2009)]**

Quarks & gluons **[C.L., Pasquini (2013)]**

Parametrization

Twist-2

GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs $\xi = 0$

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$	$-i\,\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$	
T_x	$i\,\frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				$\tilde{H}$

Complete parametrizations : **Quarks** **[Meissner, Metz, Schlegel (2009)]**

Quarks & gluons **[C.L., Pasquini (2013)]**

Parametrization

Twist-2

GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L	✗	$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs $\xi = 0$

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$	$-i\,\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$	
T_x	$i\,\frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L	✗			$\tilde{H}$

Complete parametrizations : **Quarks** **[Meissner, Metz, Schlegel (2009)]**

Quarks & gluons **[C.L., Pasquini (2013)]**

Parametrization

Twist-2

GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2}F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2}F_{11})\right)$	$\cdots$	$\cdots$	$\cdots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\vec{\Delta}_\perp = \vec{0}_\perp$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	✗
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs $\xi = 0$

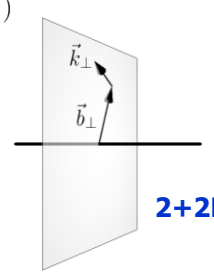
	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\left(2\tilde{H}_T + E_T\right)$	$-i\,\frac{\Delta_x}{2M}\left(2\tilde{H}_T + E_T\right)$	✗
T_x	$i\,\frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2}\tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2}\tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2}\tilde{H}_T$	
L				$\tilde{H}$

Complete parametrizations : **Quarks** **[Meissner, Metz, Schlegel (2009)]**

Quarks & gluons **[C.L., Pasquini (2013)]**

Model results

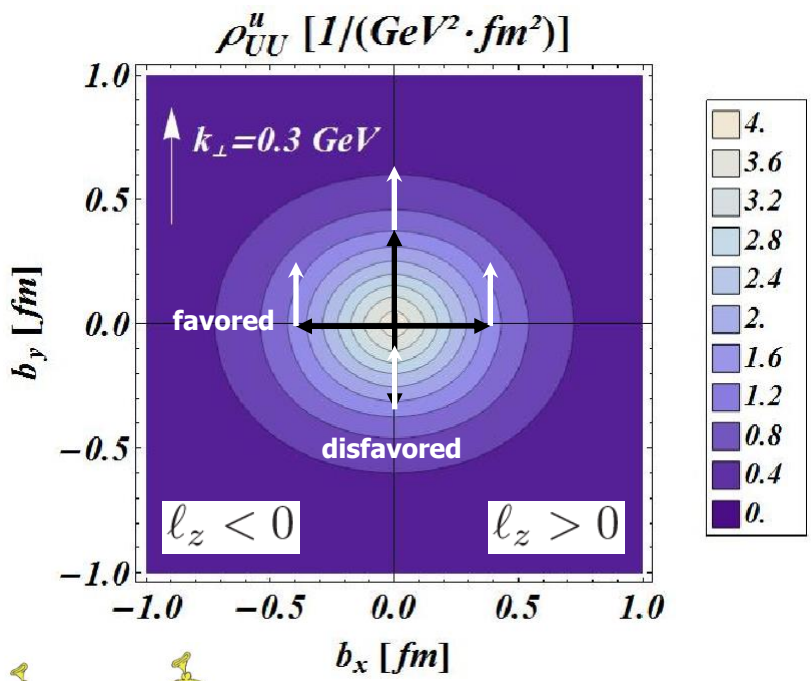
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

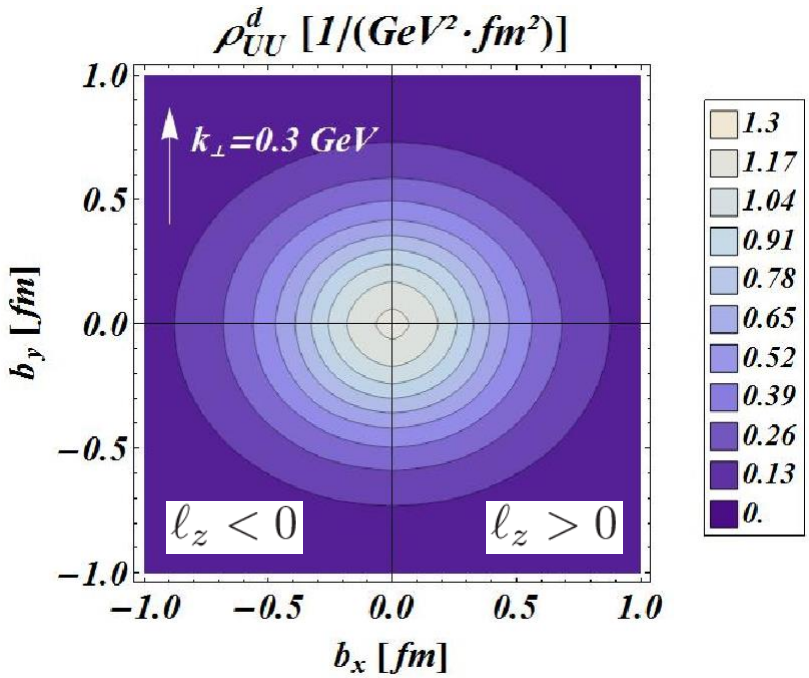


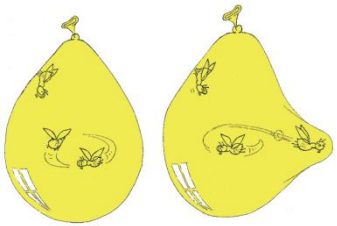
2+2D

Wigner distribution of unpolarized quark in unpolarized nucleon

$$\rho_{UU} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+]} + \rho_{--}^{[\gamma^+]}\right) \propto F_{11}$$







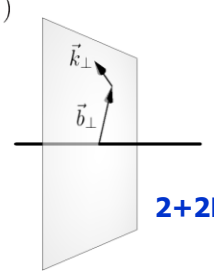
favored disfavored

Left-right symmetry → $\ell_z^{\mathrm{tot}} = 0$

[C.L., Pasquini (2011)]

Model results

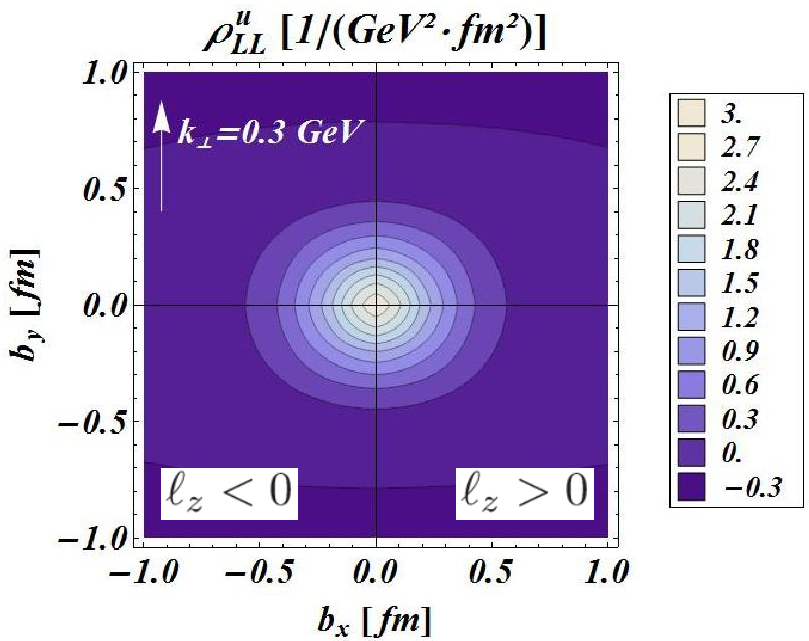
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

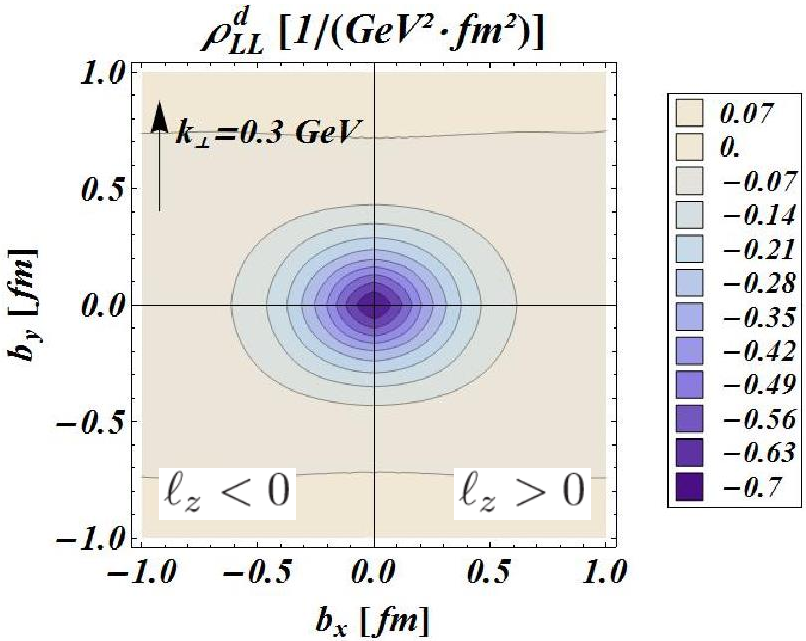


2+2D

Quark spin-nucleon spin correlation

$$\rho_{LL} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+\gamma_5]} - \rho_{--}^{[\gamma^+\gamma_5]}\right) \propto G_{14}$$





Proton spin

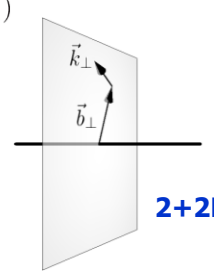
***u*-quark spin**

***d*-quark spin**

[C.L., Pasquini (2011)]

Model results

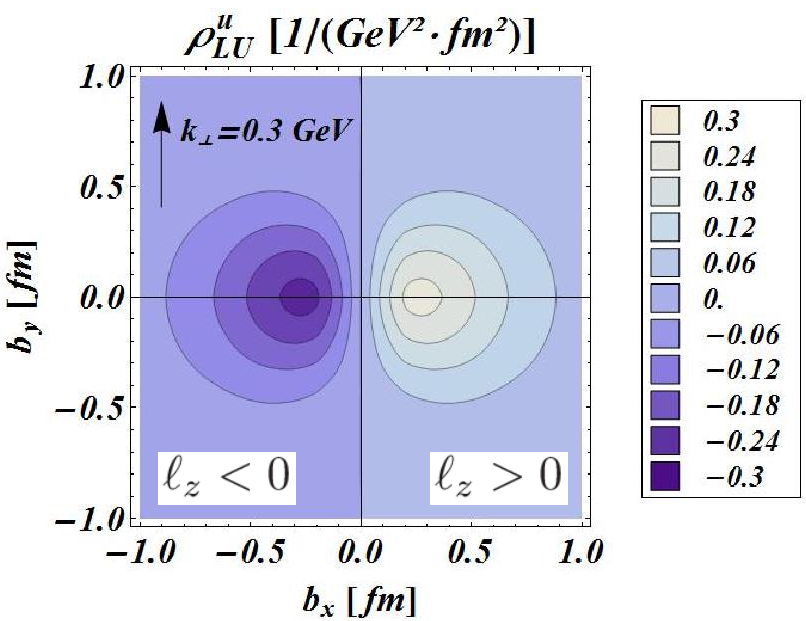
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

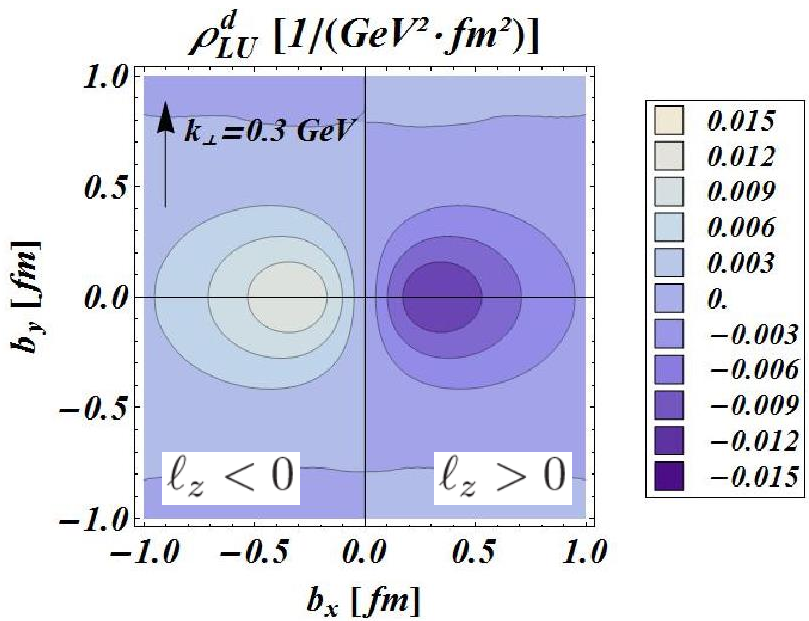


2+2D

Distortion correlated to nucleon spin

$$\rho_{LU} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+]} - \rho_{--}^{[\gamma^+]}\right) \propto F_{14}$$





Proton spin

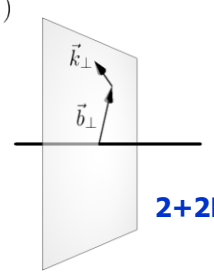
***u*-quark OAM**

***d*-quark OAM**

[C.L., Pasquini (2011)]

Model results

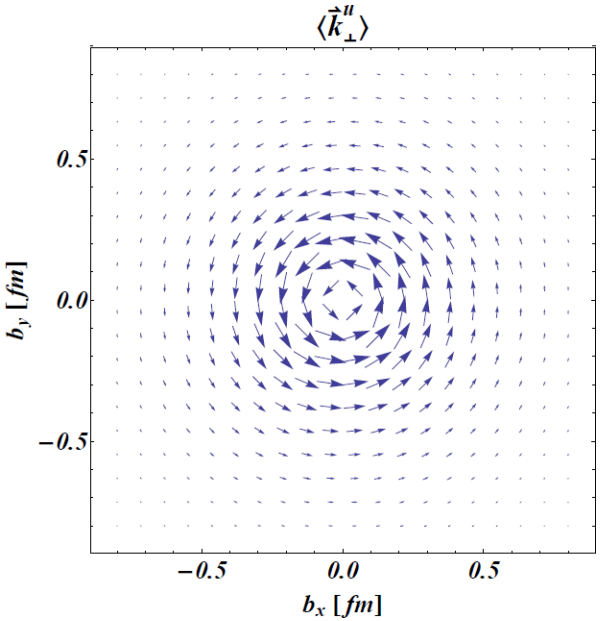
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

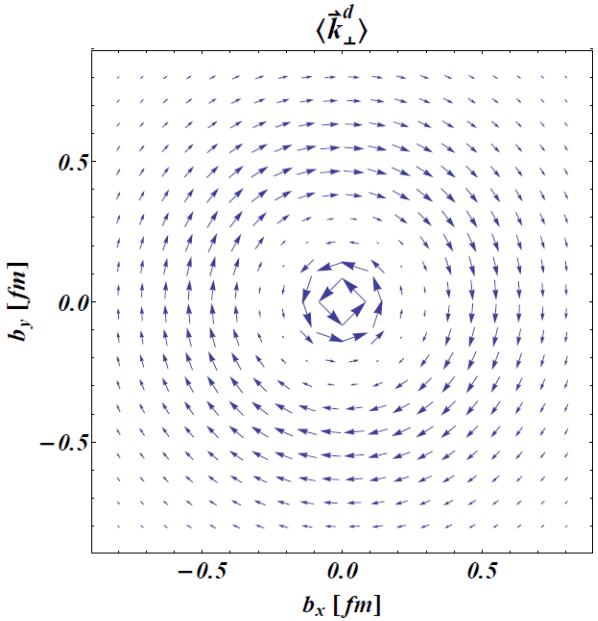


2+2D

Average transverse quark momentum correlated to nucleon spin

$$\langle \vec{k}_\perp \rangle (\vec{b}_\perp) = \int \mathrm{d}x\, \mathrm{d}^2 k_\perp\, \vec{k}_\perp\, \rho_{LU}(x, \vec{k}_\perp, \vec{b}_\perp)$$





$$\ell_z = \int \mathrm{d}^2 b_\perp\, \vec{b}_\perp \times \langle \vec{k}_\perp \rangle (\vec{b}_\perp)$$

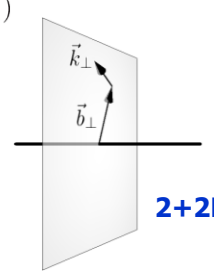
F_{14}

« Vorticity »

[C.L., Pasquini, Xiong, Yuan (2012)]

Model results

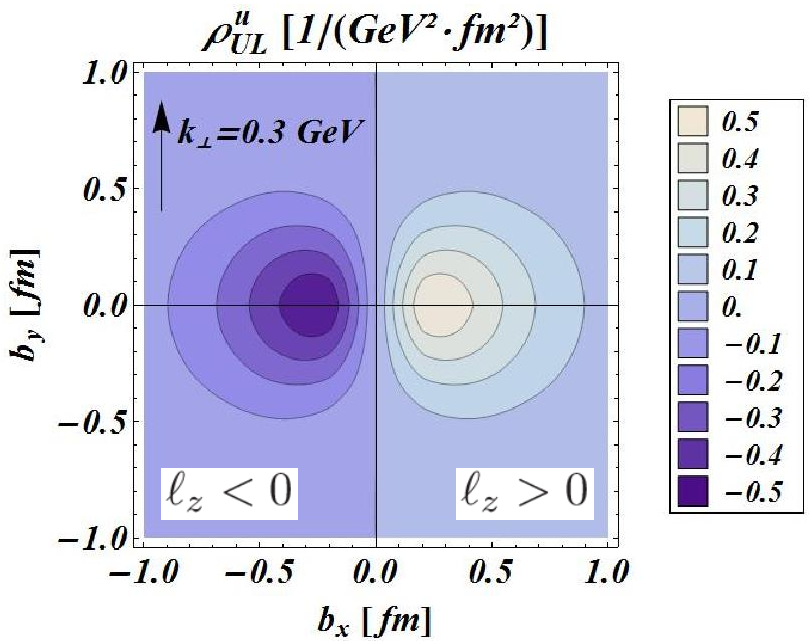
$$\int \mathrm{d}x\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$

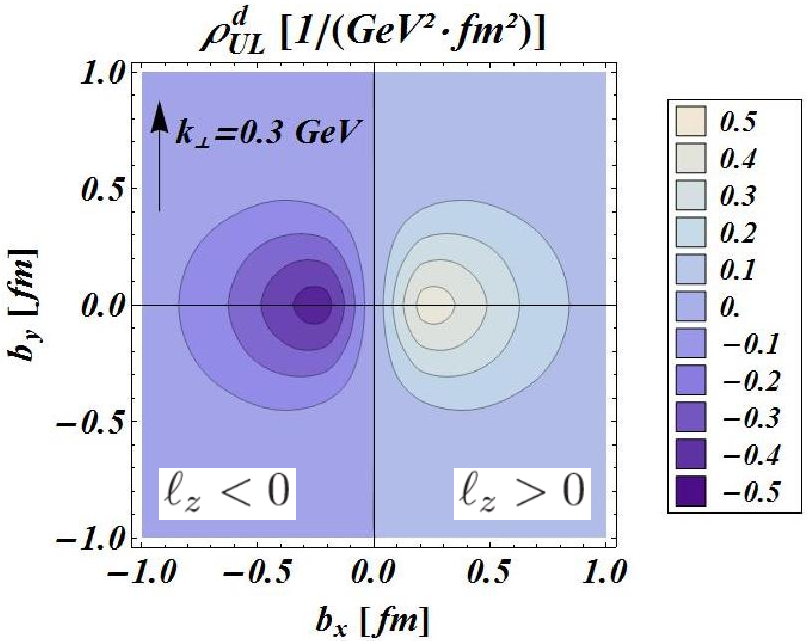


2+2D

Distortion correlated to quark spin

$$\rho_{UL} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+\gamma_5]} + \rho_{--}^{[\gamma^+\gamma_5]}\right) \propto G_{11}$$





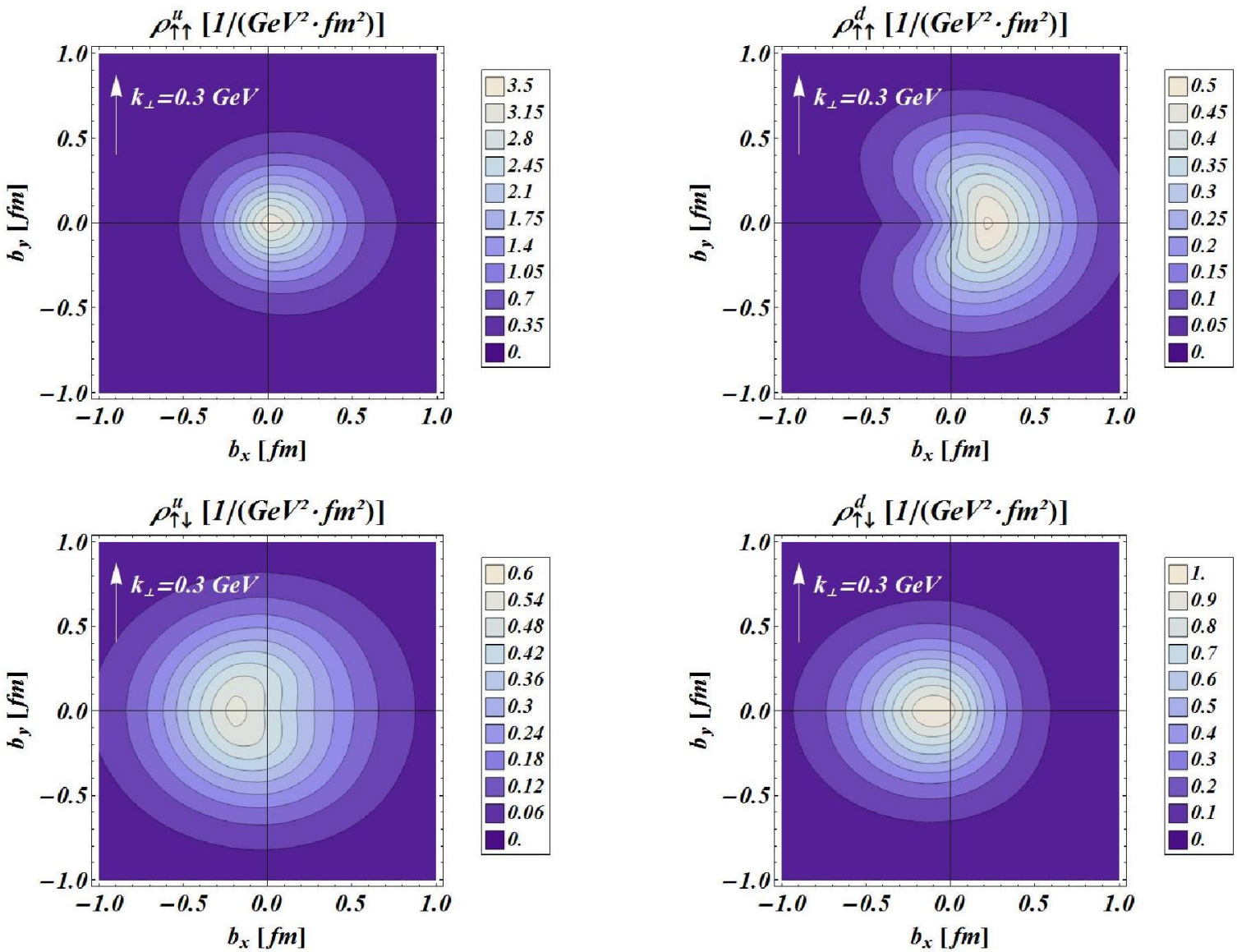
Quark spin

***u*-quark OAM**

***d*-quark OAM**

[C.L., Pasquini (2011)]

Model results

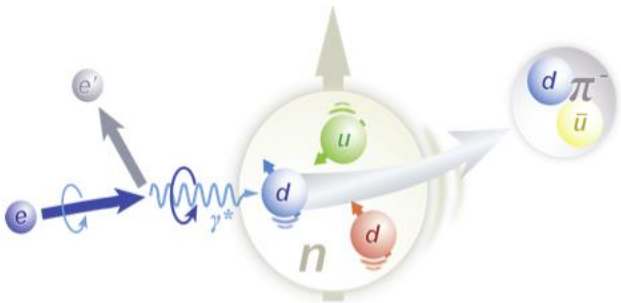


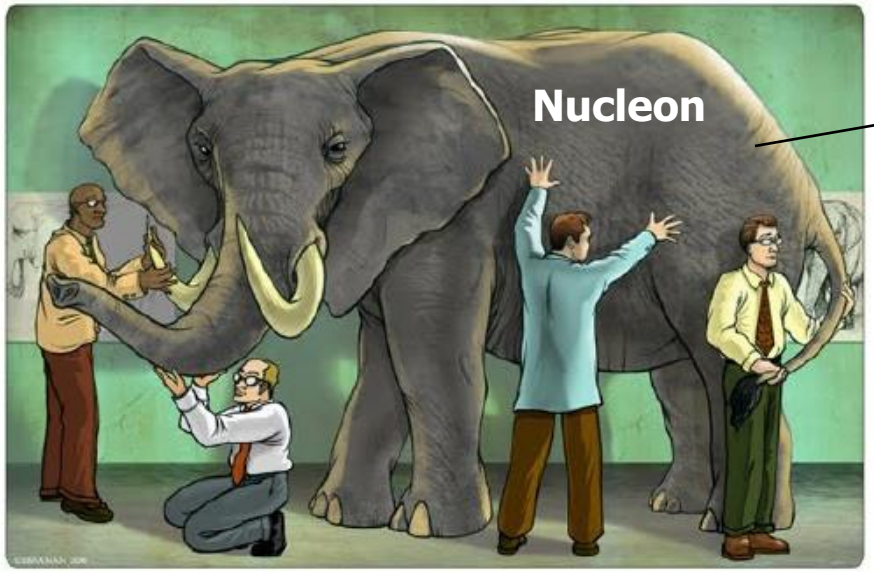
[C.L., Pasquini (2011)]

Summary

Lecture 4

- **TMDs provide complementary 3D pictures of the nucleon**
- **All distributions can be seen as overlaps of light-front wave functions**
- **Models constrained by data give access to Wigner distributions**





LFWFs

GPDs **TMDs** **FFs** **PDFs**