Applications of Renormalization Group Methods in Nuclear Physics – 1

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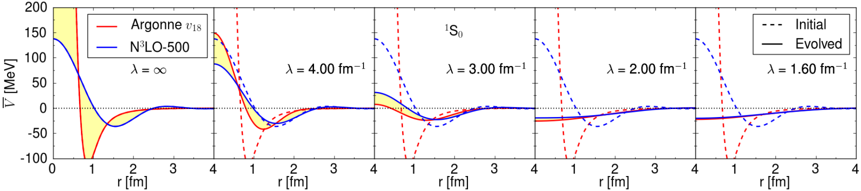
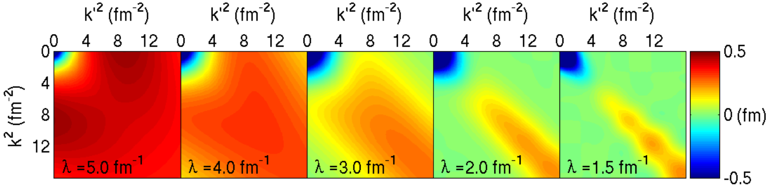
HUGS 2014





Outline: Lecture 1

Lecture 1: Overview

- Preview: Running couplings/potentials
- Goals of low-energy nuclear physics
- Breakthroughs in low-energy nuclear theory
- Nuclear scales and resolution

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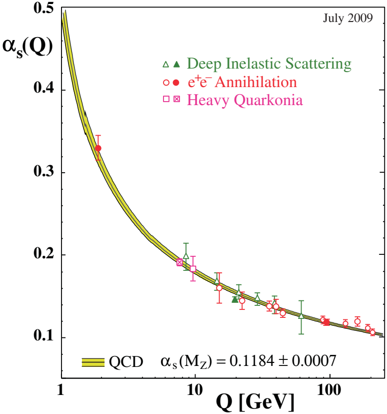
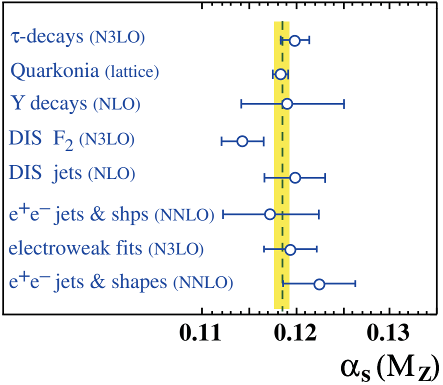
Renormalization group and QCD coupling: Running $\alpha_s(Q^2)$

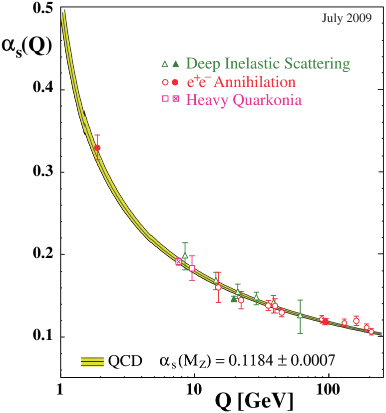


- Extractions from experiment can be compared (here at M_Z):



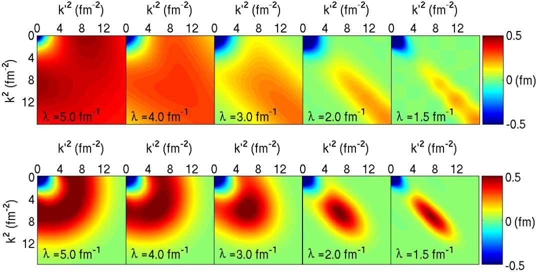
- The QCD coupling is *scale* dependent ("running"):
  $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\mathrm{QCD}})]^{-1}$
- From $\mu^2(d\alpha_s/d\mu^2) = \beta(\alpha_s)$ and
  $\beta(\alpha_s) = -\alpha_s^2(\beta_0 + \beta_1\alpha_s + \cdots)$
  (this is the "beta function")
- cf. QED, where $\alpha_{em}(Q^2)$ is effectively constant for soft Q^2:
  $$\alpha_{em}(Q^2 = 0) \approx 1/137$$
  $\therefore$ fixed H for quantum chemistry

Running QCD $\alpha_s(Q^2)$ vs. running nuclear $V_\lambda(k, k')$



- The QCD coupling is *scale* dependent (cf. low-E QED): $\alpha_s(Q^2) \approx [\beta_0 \ln(Q^2/\Lambda^2_{\rm QCD})]^{-1}$
- Different Hamiltonians? Do you get different answers from the same Feynman diagrams with $\alpha_s(\mu_1^2)$ and $\alpha_s(\mu_2^2)$?

- Vary scale ("resolution") with RG $\Longrightarrow$ diff. eq. for potential V
- Scale dependence: RG running of initial potential with scale λ (decoupling or separation scale)



- But all are (NN) phase equivalent! (predict the same NN cross sections)
- Shift contributions between interaction and sums over intermediate states

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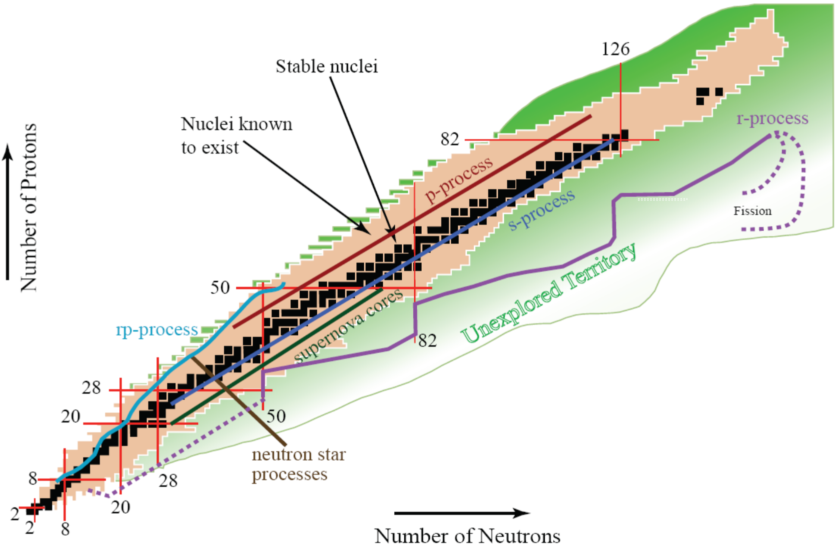
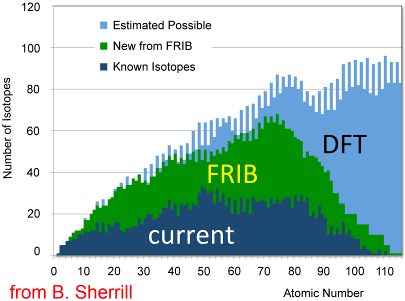
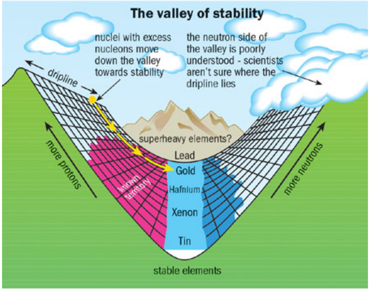
Low-energy playground: Table of the nuclides



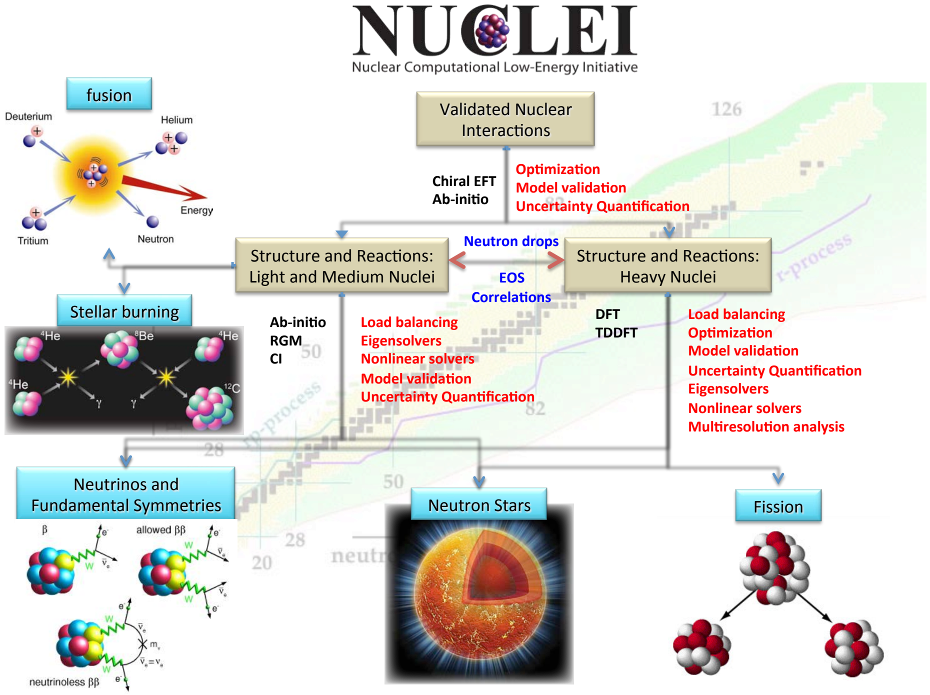
New frontiers from rare isotope facilities (worldwide)



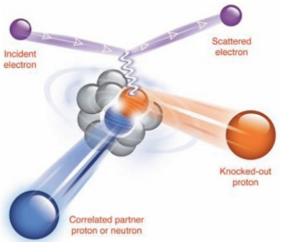
from B. Sherrill



Challenge of open quantum systems! (continuum channels, . . .)

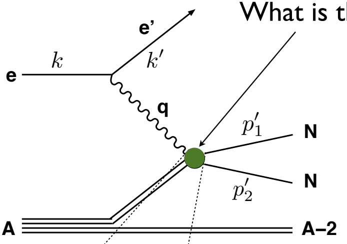

NUCLEI
Nuclear Computational Low-Energy Initiative
Validated Nuclear Interactions
Chiral EFT
Ab-initio
Optimization
Model validation
Uncertainty Quantification
Structure and Reactions: Light and Medium Nuclei
Neutron drops
EOS
Correlations
Structure and Reactions: Heavy Nuclei
Ab-initio
RGM
CI
Load balancing
Eigensolvers
Nonlinear solvers
Model validation
Uncertainty Quantification
DFT
TDDFT
Load balancing
Optimization
Model validation
Uncertainty Quantification
Eigensolvers
Nonlinear solvers
Multiresolution analysis
fusion
Deuterium
Helium
Energy
Tritium
Neutron
Stellar burning
4He
8Be
4He
4He
12C
γ
γ
Neutrinos and Fundamental Symmetries
β
allowed ββ
neutrinoless ββ
Neutron Stars
Fission
126
82
50
82
28
50
28
20
r-process
rp-process

JLab: Understanding "short-range correlations" in nuclei



Subedi et al., Science 320, 1476 (2008)

What is this vertex?



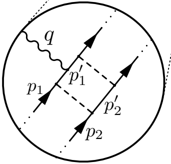
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

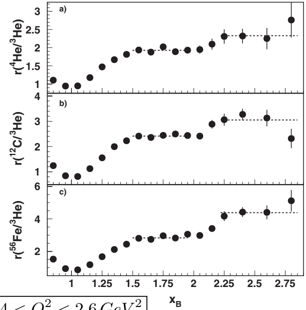
$$x_B = \frac{Q^2}{2m_N\nu}$$

Higinbotham, arXiv:1010.4433



SRC interpretation:

NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p'_1, p'_2 \gg k_F$ which are knocked out by the photon



$\boxed{1.4 < Q^2 < 2.6\, GeV^2}$

Egiyan et al. PRL 96, 1082501 (2006)

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Historical perspective: JLab experimentalist [L. Weinstein (2012)]

Comprehensive Theory Overview

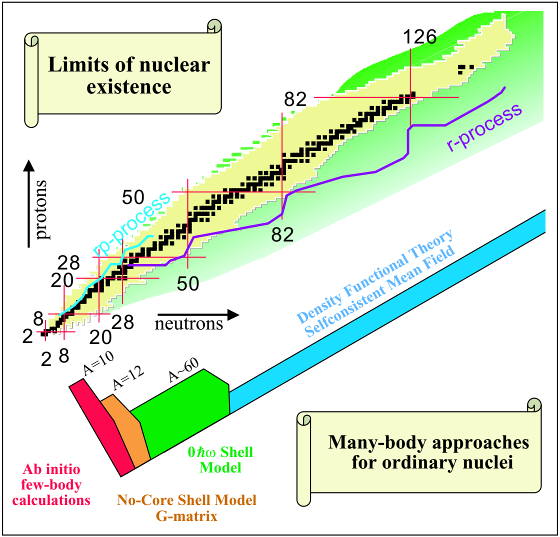
Nuclear Theory, circa 1980

Nuclear Theory - circa 2000

Nuclear Theory - today: 1, 2, 3, … 12, … many

Historical perspective: “Ab initio” structure 10–15 years ago

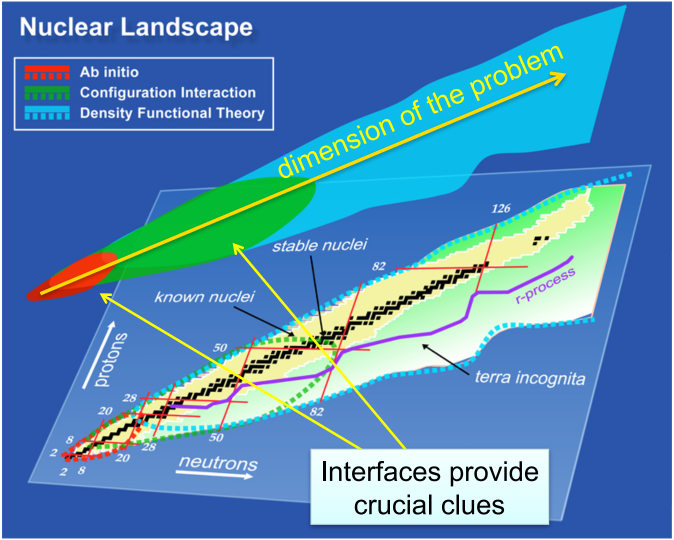
- Figure from the RIA (now FRIB) white paper (2000)



- Ab initio: Only up to ^{12}C (GFMC and NCSM with NN+3N)

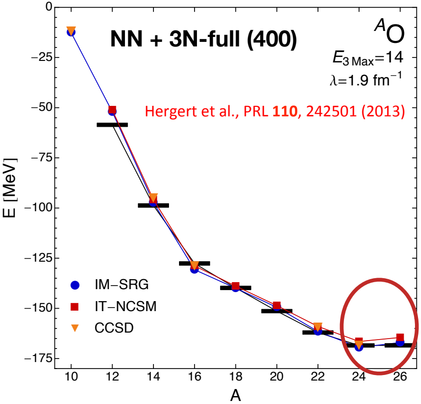
Historical perspective: "Ab initio" structure 10–15 years ago

- From the start of the SciDAC UNEDF project (2007)

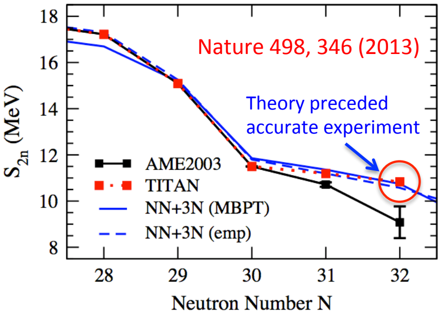


- Ab initio: Selected nuclei up to ^{40}Ca (with CC, but 3N?)

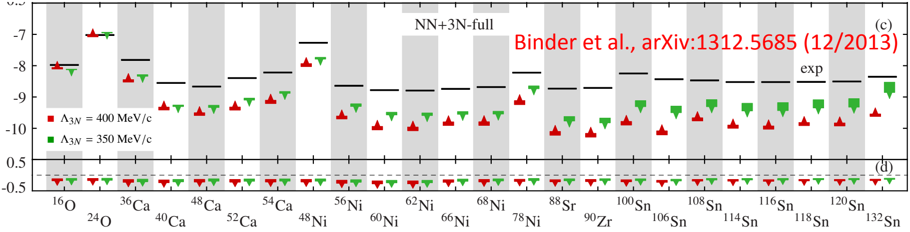
What is feasible for *ab initio* structure today? (examples)



Two-neutron separation energies for Calcium isotope chain



Energy/particle (in MeV) from oxygen to tin (A = 132!)



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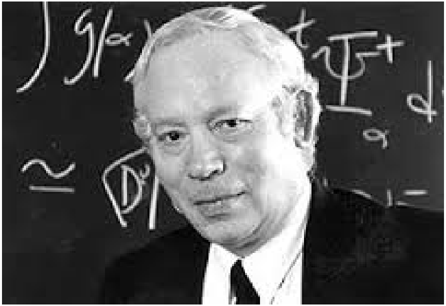
Standing on the shoulders of giants . . .



Steven Weinberg (1933–)

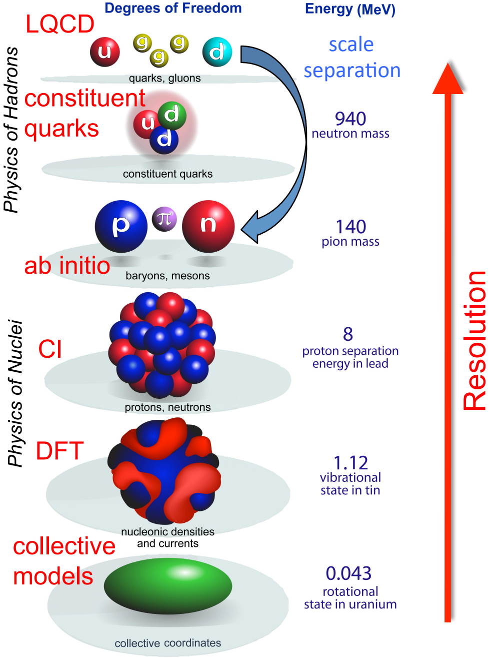
- Nobel Prize 1979
- electroweak theory (GWS), . . .
- effective field theory (EFT) applied to nuclear physics

Kenneth G. Wilson (1936–2013)

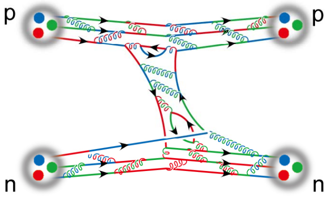
- Nobel Prize 1982
- renormalization group (RG) and critical phenomena, . . .
- similarity RG

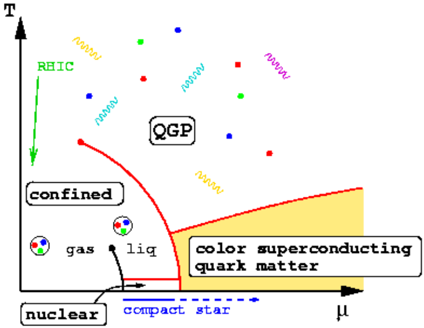
$\Longrightarrow$ Conceptual basis for changing “resolution” and tools to do it!

Connecting degrees of freedom with EFT and RG



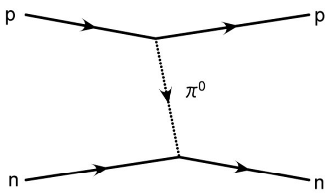
Quark (QCD) vs. hadronic NN· · · N interaction

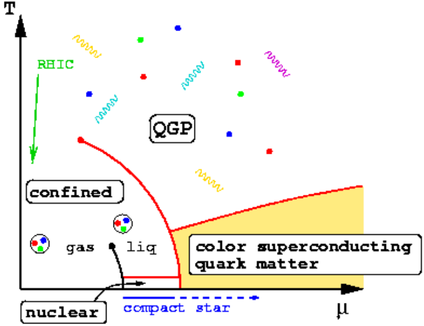




- Old goal: replace hadronic descriptions at ordinary nuclear densities with quark description (since QCD is *the* theory)

Quark (QCD) vs. hadronic NN$\cdots$N interaction





- Old goal: replace hadronic descriptions at ordinary nuclear densities with quark description (since QCD is *the* theory)
- New goal: use effective hadronic dof's *systematically*
  - Seek model independence and theory error estimates
  - Future: Use lattice QCD to match via "low-energy constants"
- Need quark dof's at higher densities (resolutions) where phase transitions happen or at high momentum transfers

Low resolution makes physics easier + efficient

- Weinberg's Third Law of Progress in Theoretical Physics:
  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

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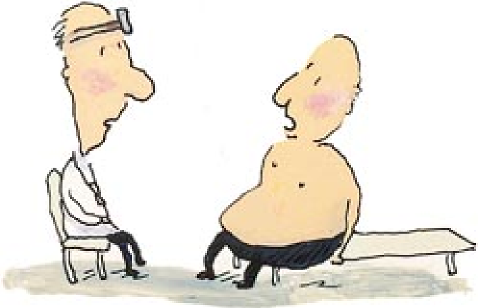
- There's an old joke about a doctor and patient . . .



Patient: Doctor, doctor, it hurts when I do this!

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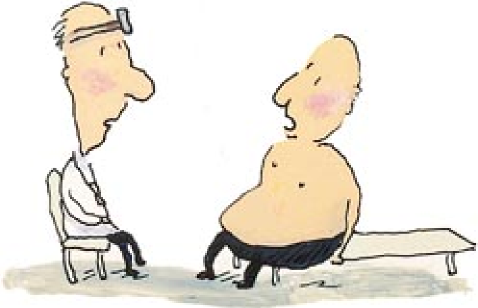
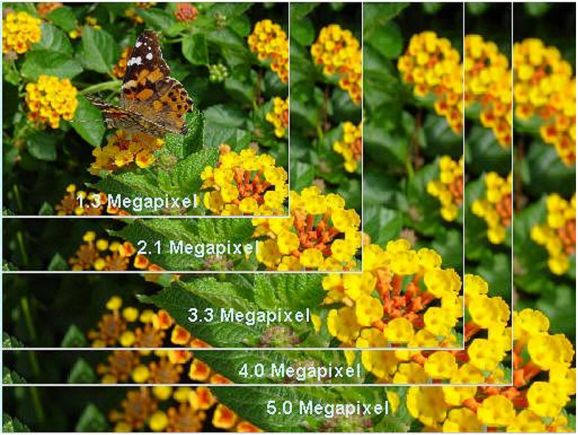


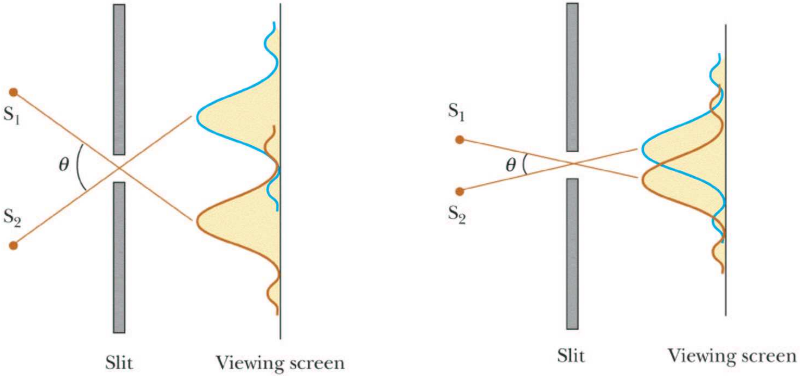
Patient: Doctor, doctor, it hurts when I do this!
Doctor: Then don't do that.

Digital resolution: Higher resolution is better (?)

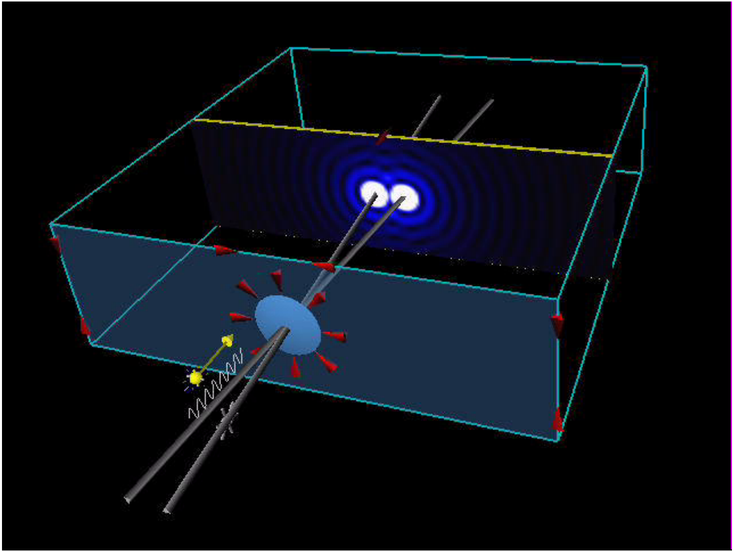
- Computer screens, printers, digital cameras, TV's . . .
- Higher resolution $\Longrightarrow$ more pixels
- Pixel size $\ll$ characteristic scale $\Longrightarrow$ greater detail



Diffraction and resolution



Wavelength and resolution



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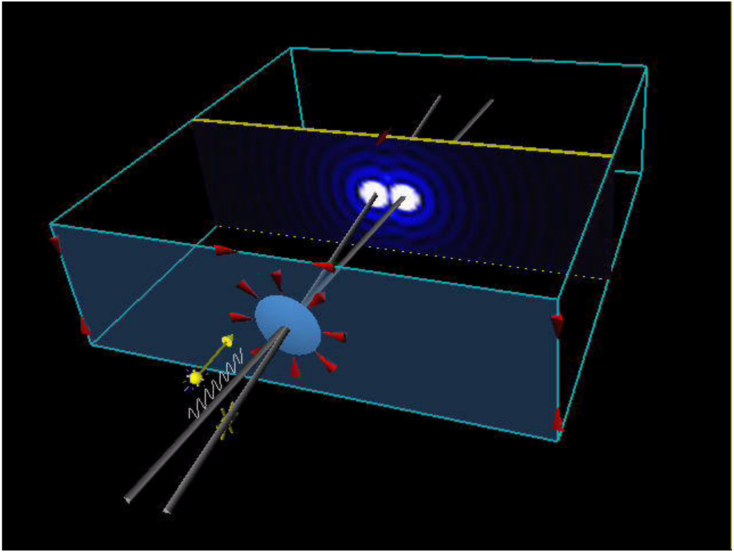

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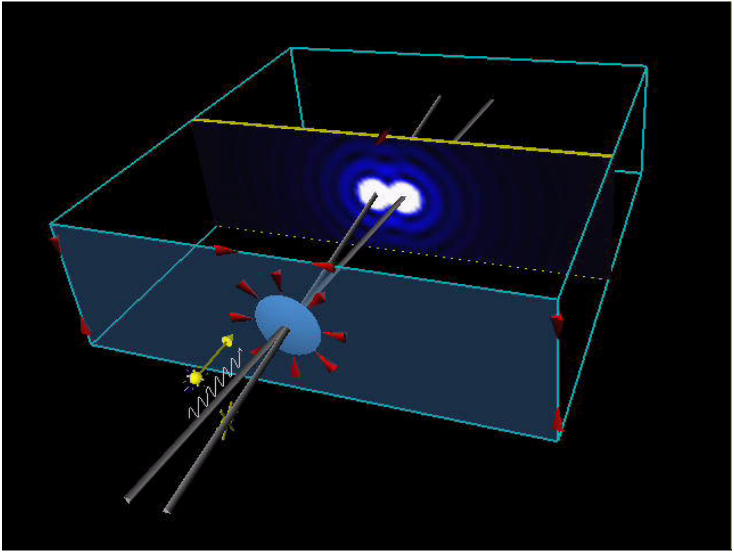

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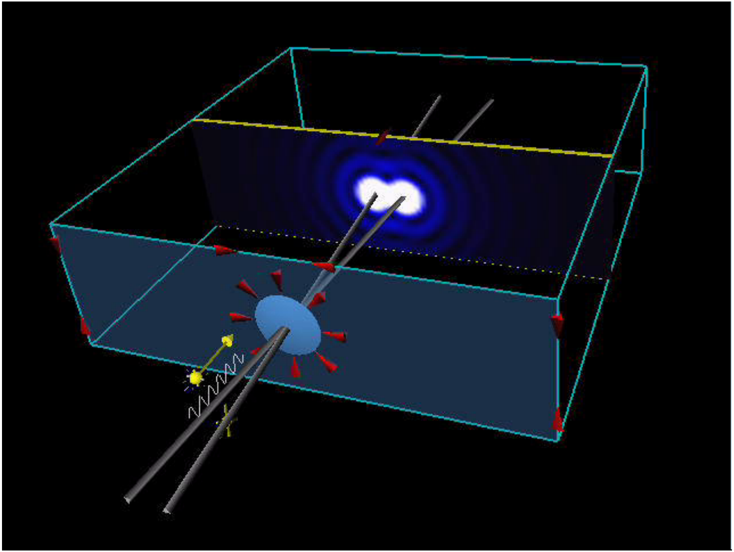

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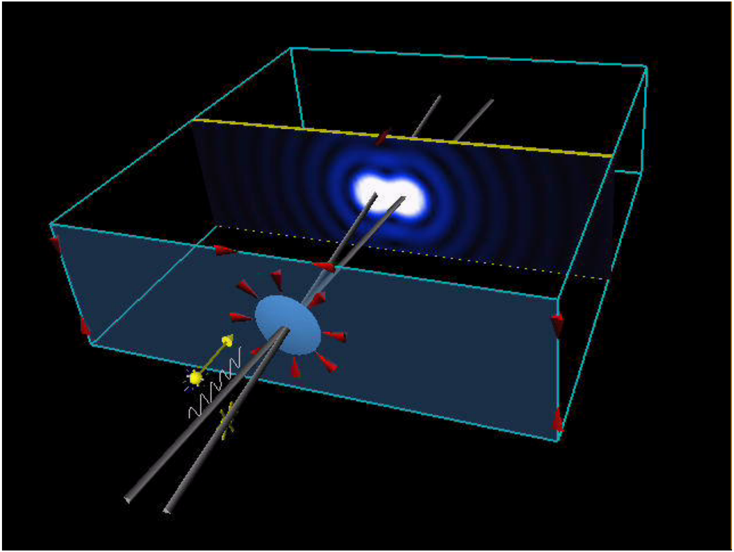

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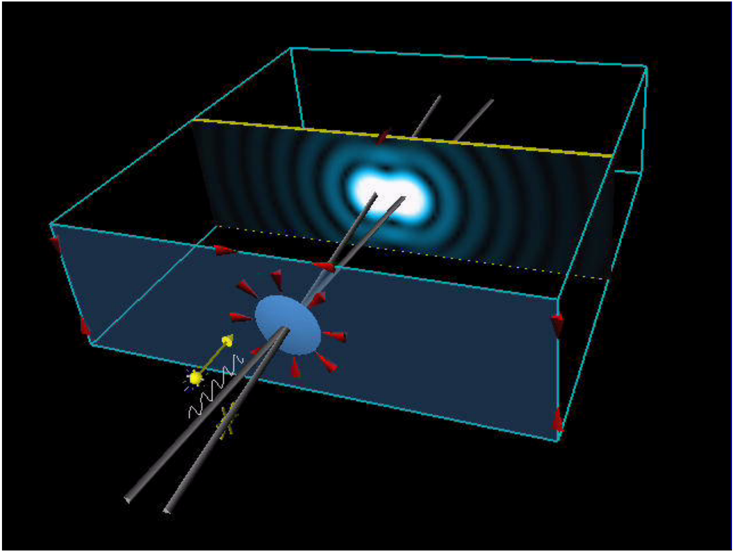

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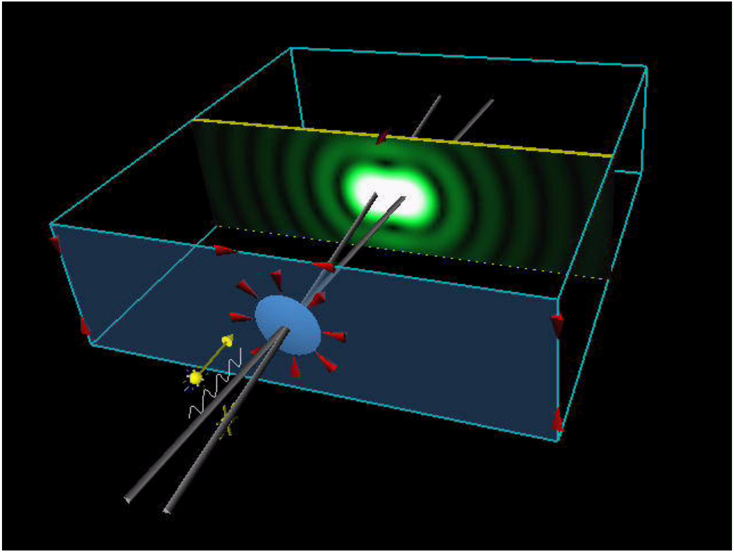

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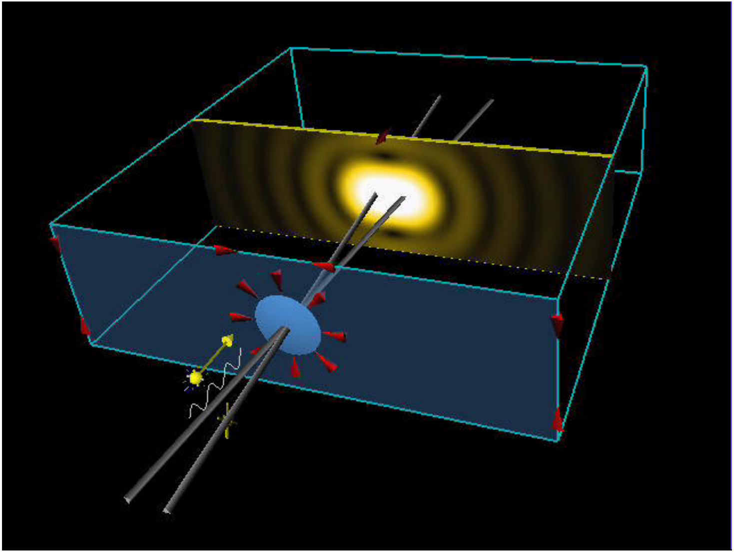

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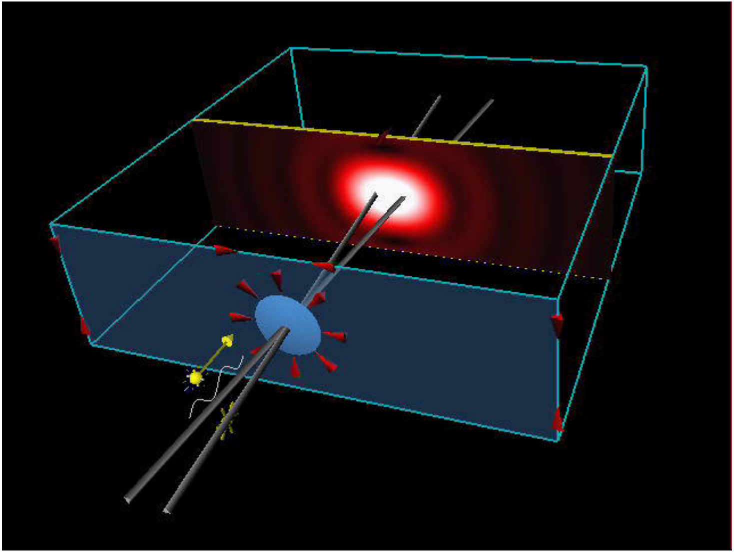
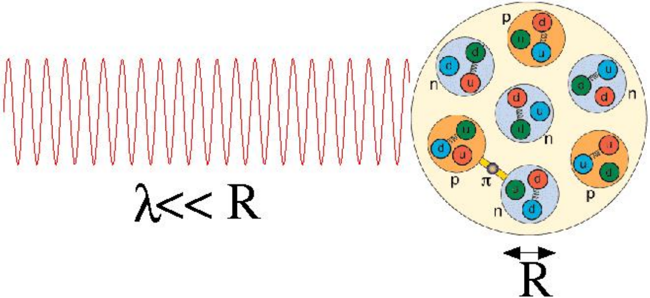


Principle of *any* effective low-energy description



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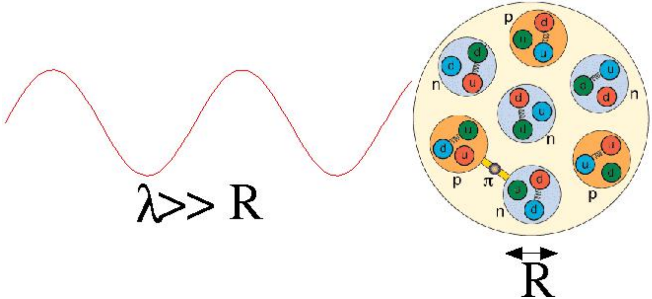
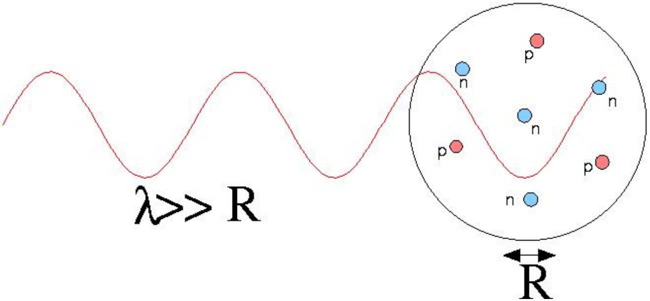


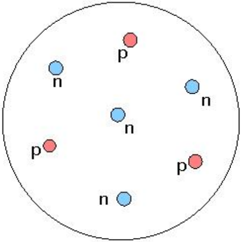
- If system is probed at low energies, fine details not resolved

Principle of *any* effective low-energy description



- If system is probed at low energies, fine details not resolved
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables (cf. multipoles in classical electrodynamics)
  - Could be a model or systematic (e.g., effective field theory)

Principle of *any* effective low-energy description



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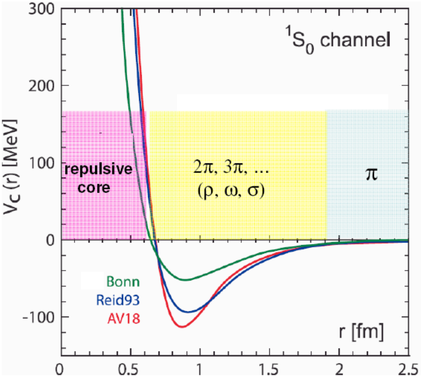
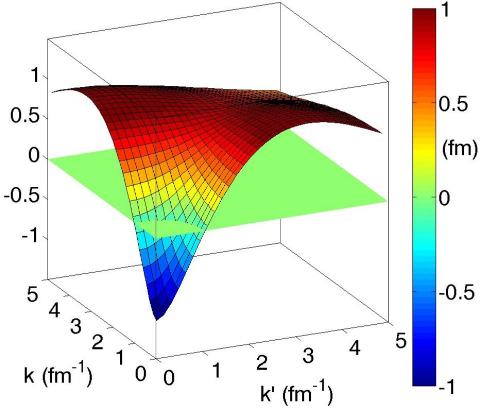
  - Use low-energy variables for low-energy processes
  - Short-distance structure can be replaced by something simpler without distorting low-energy observables (cf. multipoles in classical electrodynamics)
  - Could be a model or systematic (e.g., effective field theory)
- Density in Pb $\Leftrightarrow$ low momentum $\Leftrightarrow$ low resolution ($\lambda = h/p$) but not so fast: the interaction can affect the resolution

Why is textbook nuclear physics so hard?





$$V_{L=0}(k,k') = \int d^3r\, j_0(kr)\, V(r)\, j_0(k'r) = \langle k|V_{L=0}|k'\rangle \implies V_{kk'} \text{ matrix}$$

- Momentum units ($\hbar = c = 1$): typical relative momentum in large nucleus $\approx 1\,\mathrm{fm}^{-1} \approx 200\,\mathrm{MeV}$ but ...
- Repulsive core $\Longrightarrow$ large high-k ($\gtrsim 2\,\mathrm{fm}^{-1}$) components

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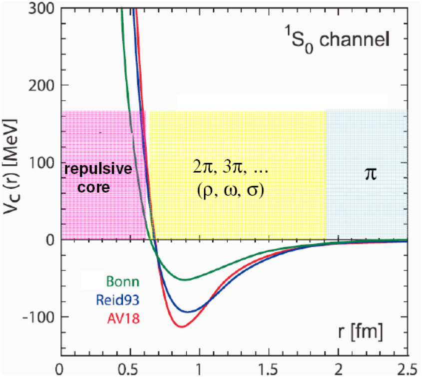

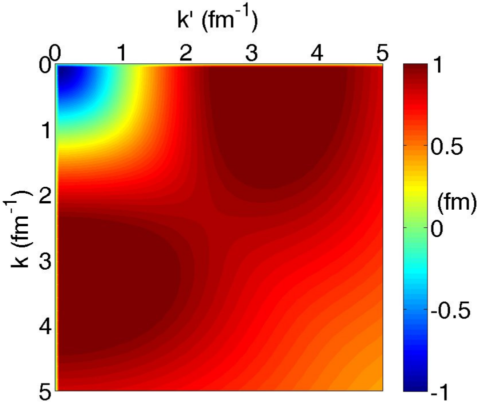

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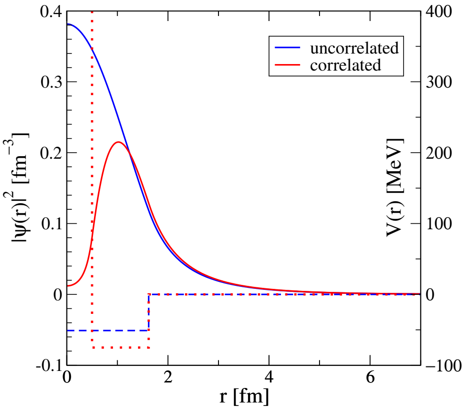
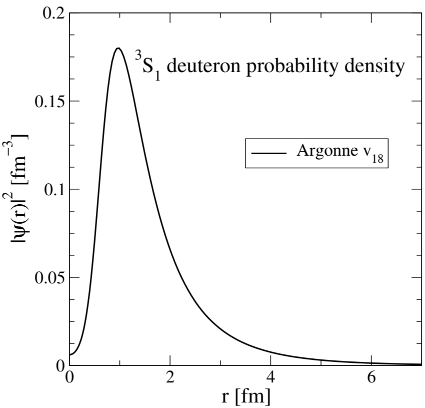
Consequences of a repulsive core





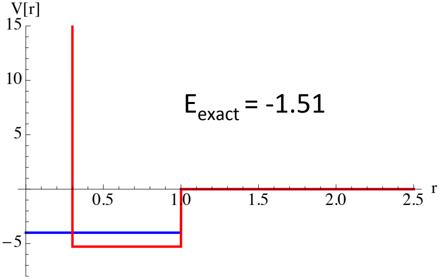
- Probability at short separations suppressed $\Longrightarrow$ "correlations"
- Short-distance structure $\Leftrightarrow$ high-momentum components
- Greatly complicates expansion of many-body wave functions

Expanding wave functions in an HO basis

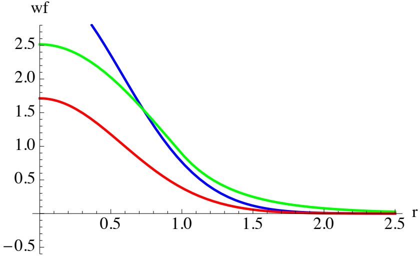
- Single-particle radial wf $\psi(r)$
- Expand in harmonic oscillator wfs:

$$\psi_{N_{\max}}(r) = \sum_{\alpha=0}^{N_{\max}} c_\alpha \phi_\alpha(r)$$

- Find c_αs by diagonalizing $\widehat{H}\Psi = E\Psi$

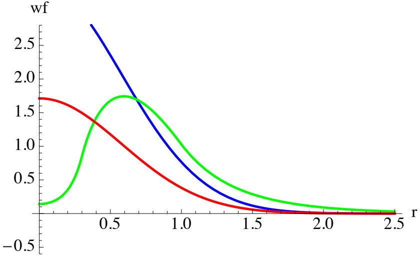


$\psi_{\text{exact}}(r)$, $\psi_0(r)$, $0.5 * \phi_0$



$N_{\max} = 0$, $E_0 = -1.30$

$\psi_{\text{exact}}(r)$, $\psi_0(r)$, $0.5 * \phi_0$



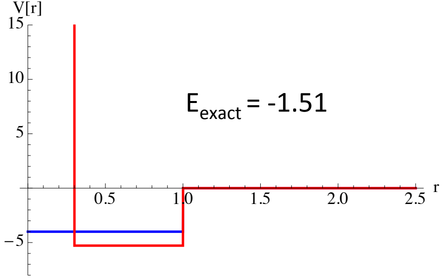
$N_{\max} = 0$, $E_0 = +5.23$

Expanding wave functions in an HO basis

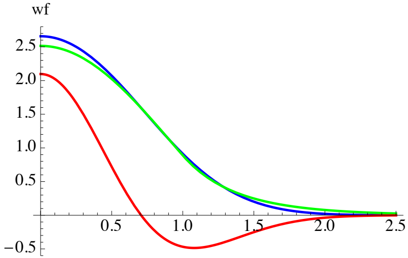
- Single-particle radial wf $\psi(r)$
- Expand in harmonic oscillator wfs:

$$\psi_{N_{\max}}(r) = \sum_{\alpha=0}^{N_{\max}} c_\alpha \phi_\alpha(r)$$

- Find c_αs by diagonalizing $\widehat{H}\Psi = E\Psi$

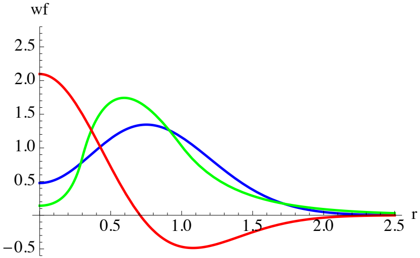


$\psi_{\text{exact}}(r)$, $\psi_2(r)$, $0.5 * \phi_2$



$N_{\max} = 2$, $E_2 = -1.46$

$\psi_{\text{exact}}(r)$, $\psi_2(r)$, $0.5 * \phi_2$



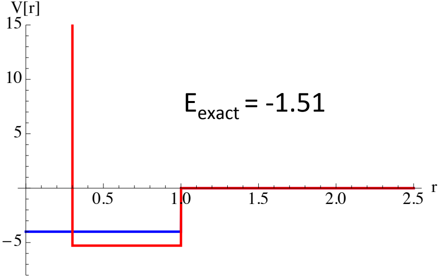
$N_{\max} = 2$, $E_2 = -0.87$

Expanding wave functions in an HO basis

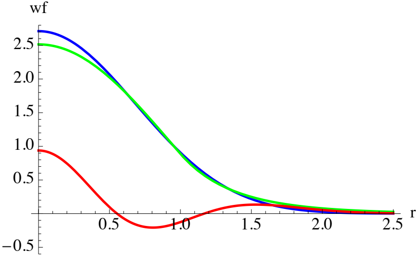
- Single-particle radial wf $\psi(r)$
- Expand in harmonic oscillator wfs:

$$\psi_{N_{\max}}(r) = \sum_{\alpha=0}^{N_{\max}} c_\alpha \phi_\alpha(r)$$

- Find c_αs by diagonalizing $\widehat{H}\Psi = E\Psi$

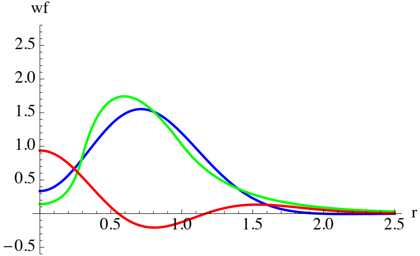


$\psi_{\text{exact}}(r)$, $\psi_4(r)$, $0.5 * \phi_4$



$N_{\max} = 4$, $E_4 = -1.46$

$\psi_{\text{exact}}(r)$, $\psi_4(r)$, $0.5 * \phi_4$



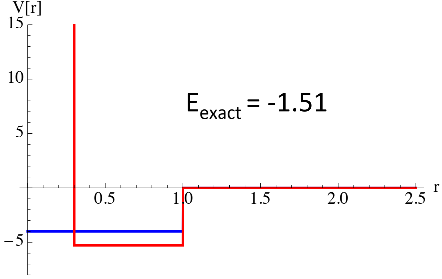
$N_{\max} = 4$, $E_4 = -1.04$

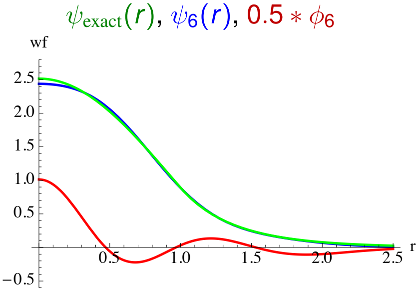
Expanding wave functions in an HO basis

- Single-particle radial wf $\psi(r)$
- Expand in harmonic oscillator wfs:

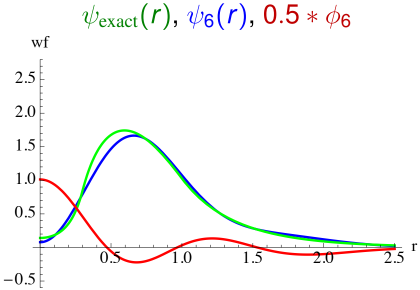
$$\psi_{N_{\max}}(r) = \sum_{\alpha=0}^{N_{\max}} c_\alpha \phi_\alpha(r)$$

- Find c_αs by diagonalizing $\widehat{H}\Psi = E\Psi$





$N_{\max} = 6$, $E_6 = -1.50$



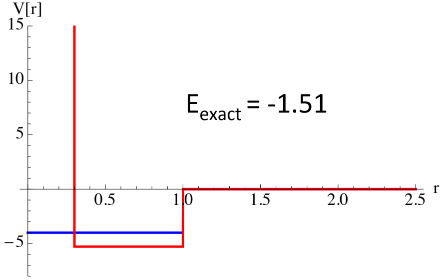
$N_{\max} = 6$, $E_6 = -1.40$

Expanding wave functions in an HO basis

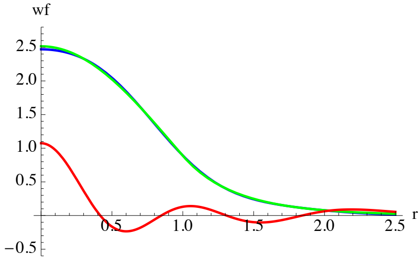
- Single-particle radial wf $\psi(r)$
- Expand in harmonic oscillator wfs:

$$\psi_{N_{\max}}(r) = \sum_{\alpha=0}^{N_{\max}} c_\alpha \phi_\alpha(r)$$

- Find c_αs by diagonalizing $\widehat{H}\Psi = E\Psi$
- Extend to many-body system

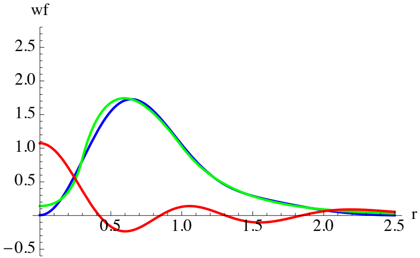


$\psi_{\text{exact}}(r)$, $\psi_8(r)$, $0.5 * \phi_8$



$N_{\max} = 8$, $E_8 = -1.50$

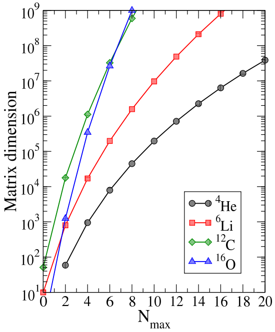
$\psi_{\text{exact}}(r)$, $\psi_8(r)$, $0.5 * \phi_8$

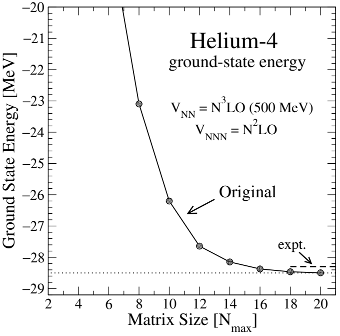


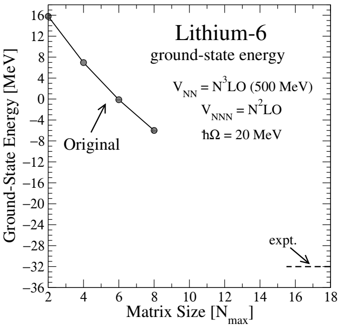
$N_{\max} = 8$, $E_8 = -1.43$

Many-body physics by matrix diagonalization

- Harmonic oscillator basis with $N_{\max}$ shells for excitations
- Graphs show convergence for *soft* chiral EFT potential (although not at optimal $\hbar\Omega$ for ^{6}Li)







- Factorial growth of basis with $A \Longrightarrow$ limits calculations
- Too much resolution from potential $\Longrightarrow$ mismatch of scales

What if your theory has too much resolution?

What if your theory has too much resolution?

What if your theory has too much resolution?

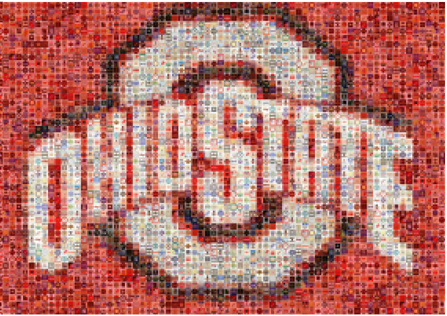
What if your theory has too much resolution?

Claim: Nuclear physics with textbook $V(\mathbf{r})$ is like using beer coasters!

Less painful to use a low-resolution version!

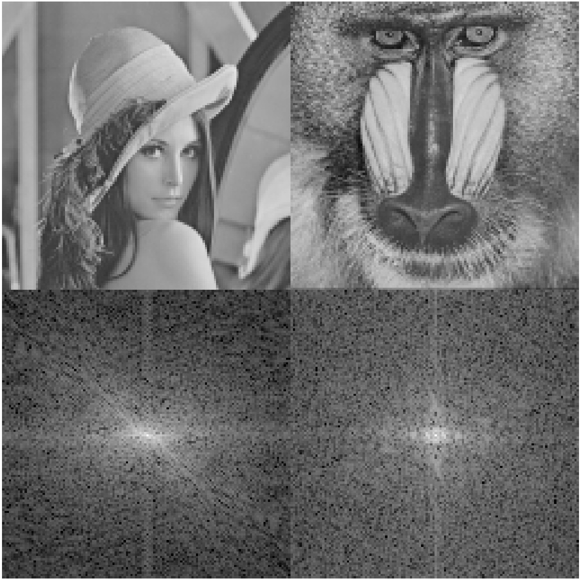
High resolution

Low resolution



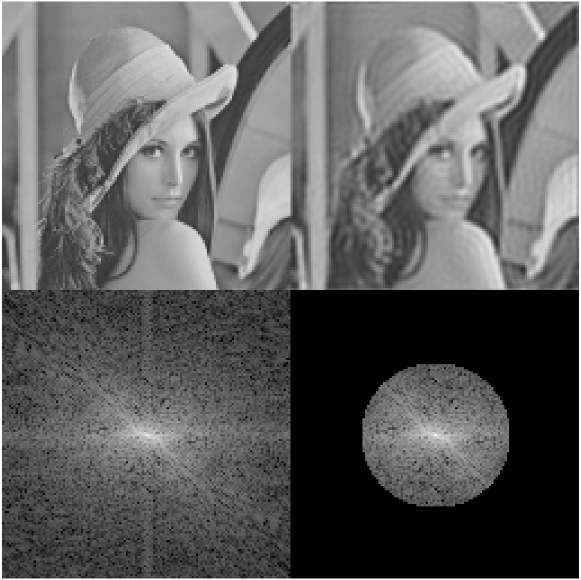
- Can greatly reduce storage without distorting message
- Resolution was lowered here by “block spinning”
- Alternative: apply a low-pass filter

Low-pass filter on an image



- Use 2D Fourier transform
- Long and short wavelengths separated

Low-pass filter on an image

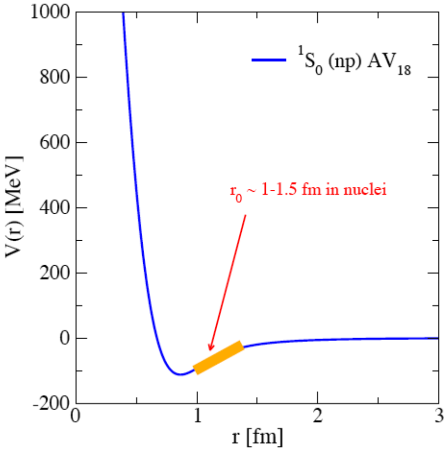


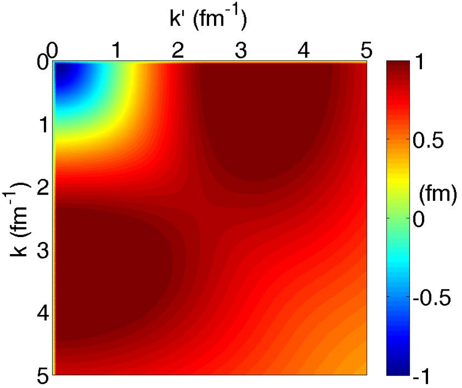
- Use 2D Fourier transform
- Long and short wavelengths separated

After low-pass filter:

- Much less information needed
- Long-wavelength info is preserved

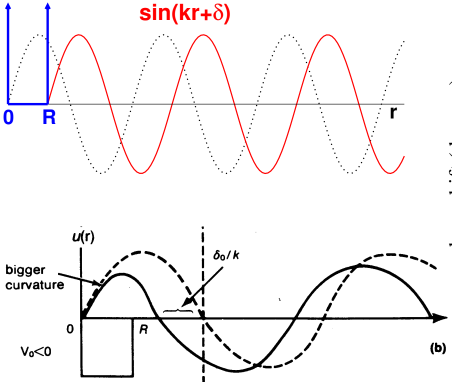
Try a low-pass filter on nuclear $V(\mathrm{r})$

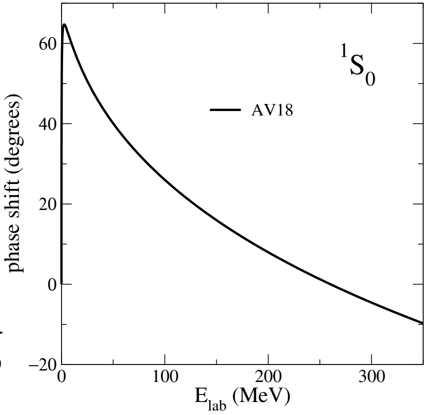




$\Longrightarrow$ Set to zero high momentum ($k \geqslant 2\,\mathrm{fm}^{-1}$) matrix elements and see the effect on low-energy observables

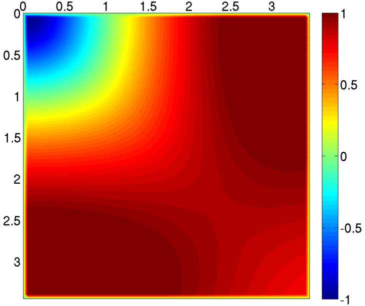
Use Phase Shifts to Test

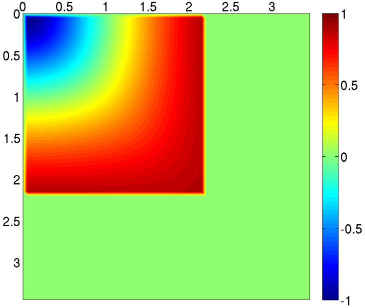


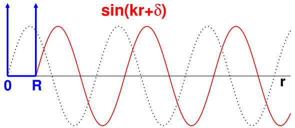


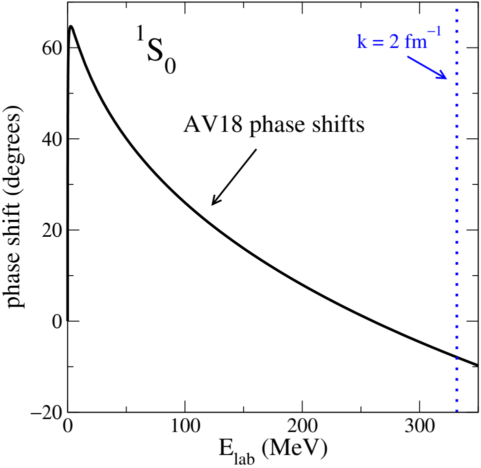
- Here: ${}^1S_0$ (spin-singlet, $L = 0$, $J = 0$) neutron-proton scattering
- Different phase shifts in each partial wave channel

Effect of low-pass filter on observables

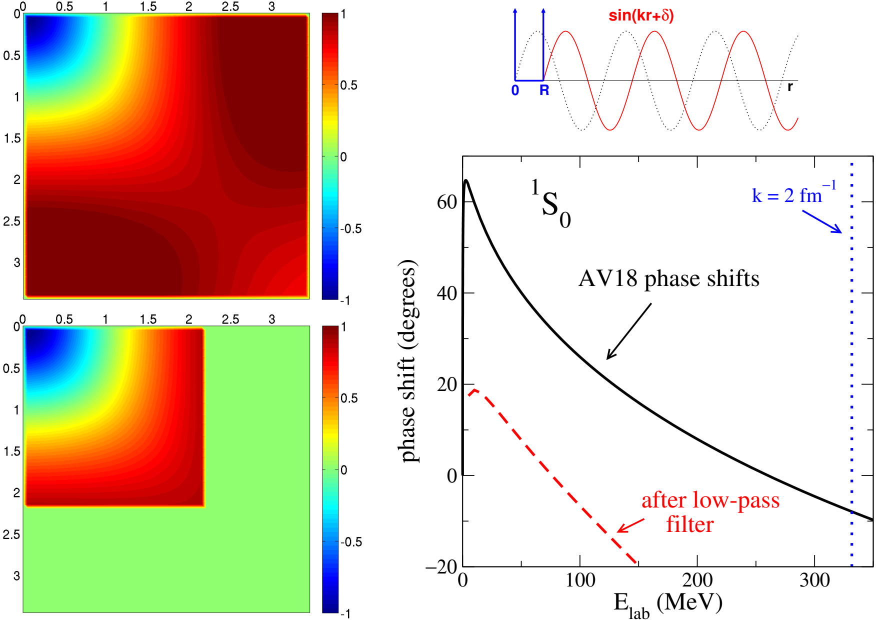








Effect of low-pass filter on observables



Why did our low-pass filter fail?

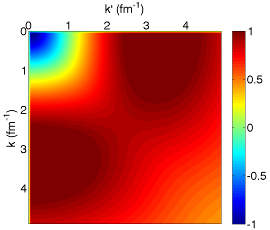
- Basic problem: low k and high k are coupled (mismatched dof's!)
- E.g., perturbation theory for (tangent of) phase shift:

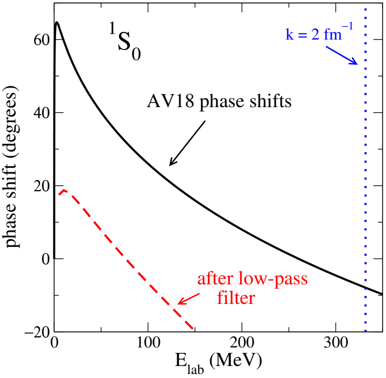
$$\langle k|V|k\rangle + \sum_{k'} \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2 - k'^2)/m} + \cdots$$

- Solution: Unitary transformation of the H matrix $\Longrightarrow$ decouple!

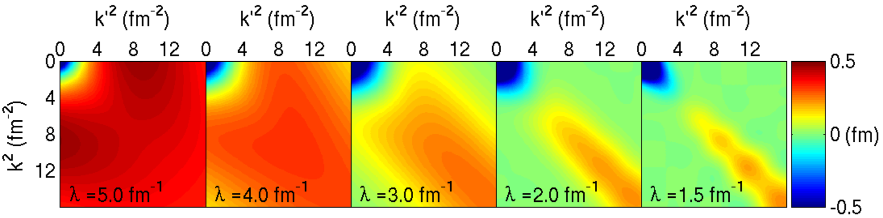
$$\begin{aligned} E_n &= \langle \Psi_n|H|\Psi_n\rangle \qquad U^\dagger U = 1 \\ &= (\langle \Psi_n|U^\dagger)UHU^\dagger(U|\Psi_n\rangle) \\ &= \langle \widetilde{\Psi}_n|\widetilde{H}|\widetilde{\Psi}_n\rangle \end{aligned}$$

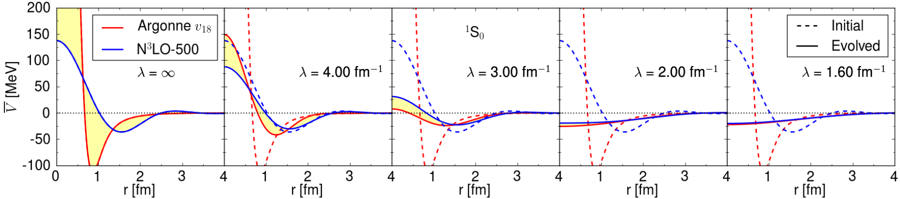
- Here: Decouple using RG



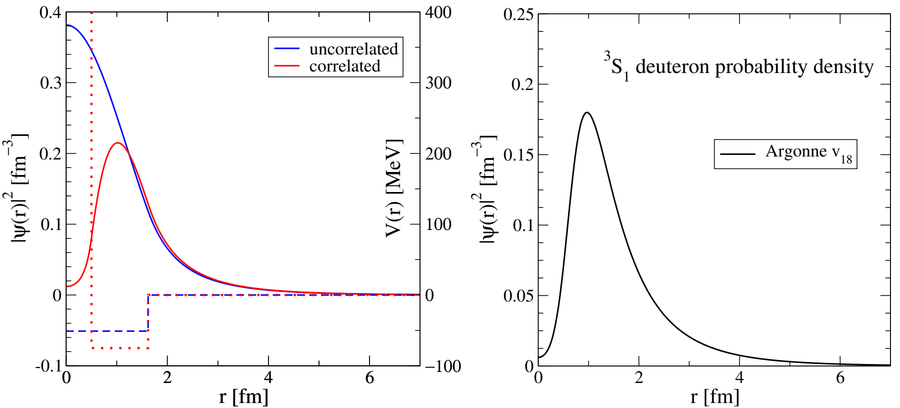


Preview: Decoupling with the similarity RG



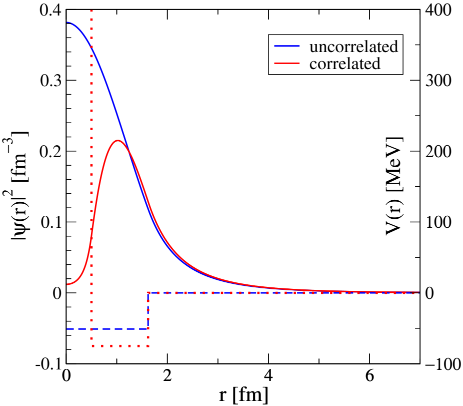


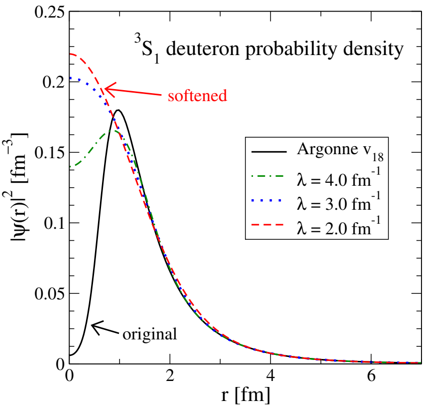
Preview: Consequences of a repulsive core revisited



- Probability at short separations suppressed $\Longrightarrow$ “correlations”
- Short-distance structure $\Leftrightarrow$ high-momentum components
- Greatly complicates expansion of many-body wave functions

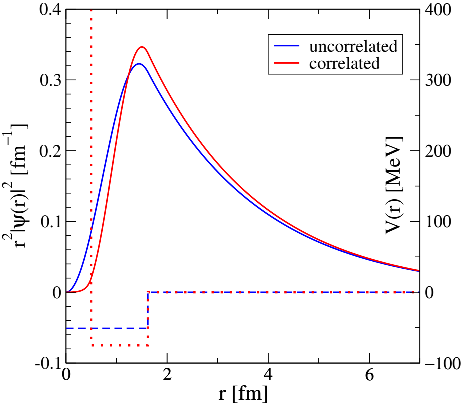
Preview: Consequences of a repulsive core revisited

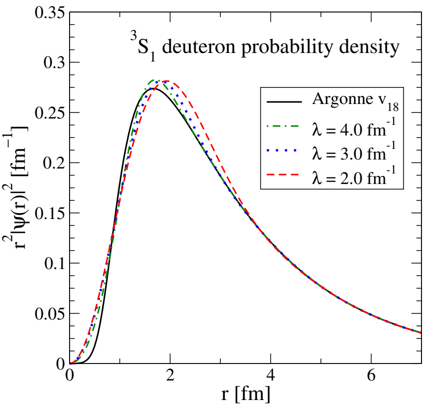




- Transformed potential $\Longrightarrow$ no short-range correlations in wf!
- Can it really be so different in the interior?

Preview: Consequences of a repulsive core revisited

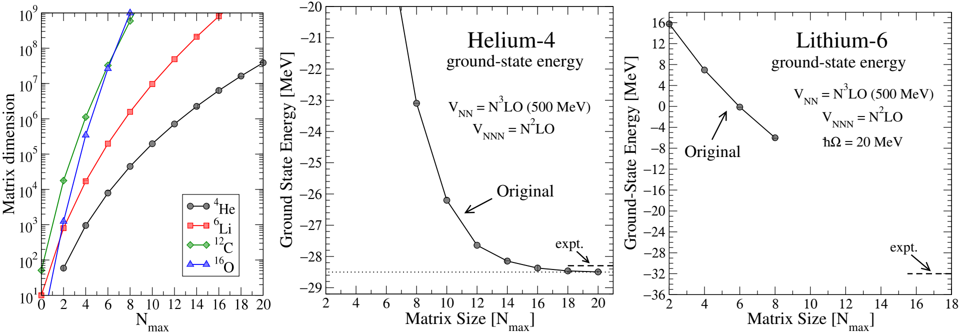




- Transformed potential $\Longrightarrow$ no short-range correlations in wf!
- What part of the coordinate-space wave function is measurable?
- What about the high-momentum tail in momentum space?

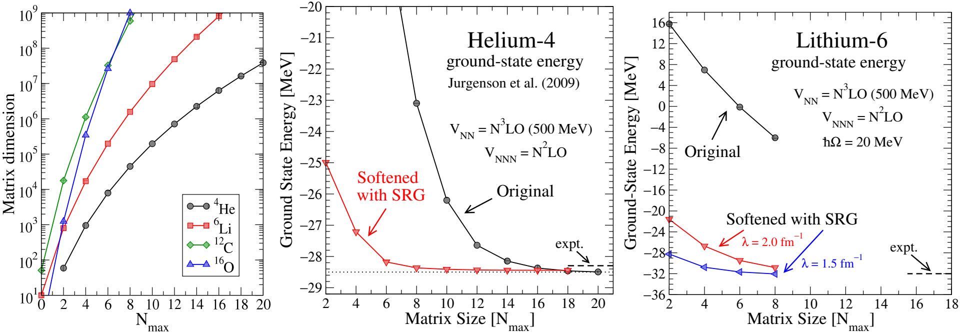
Preview: Revisit the convergence with matrix size ($N_{\max}$)

- Harmonic oscillator basis with $N_{\max}$ shells for excitations



Preview: Revisit the convergence with matrix size ($N_{\max}$)

- Harmonic oscillator basis with $N_{\max}$ shells for excitations



- Graphs show that convergence for *soft* chiral EFT potential is accelerated for evolved SRG potentials
- Nuclear structure/reaction calculations more “perturbative”