



HUGS 2014

29th Annual Hampton University Graduate Studies Program

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Spin Sum Rules and 3D Nucleon Structure (5/6)

Cédric Lorcé

IPN Orsay - IFPA Liège



Outline

Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz

Lecture 2

- Photon point of view
- Galileo *vs* Lorentz : round 2
- Nucleon 1D picture

Lecture 3

- Nucleon 2D picture
- 1D+2D=3D
- Galileo *vs* Lorentz : round 3

Lecture 4

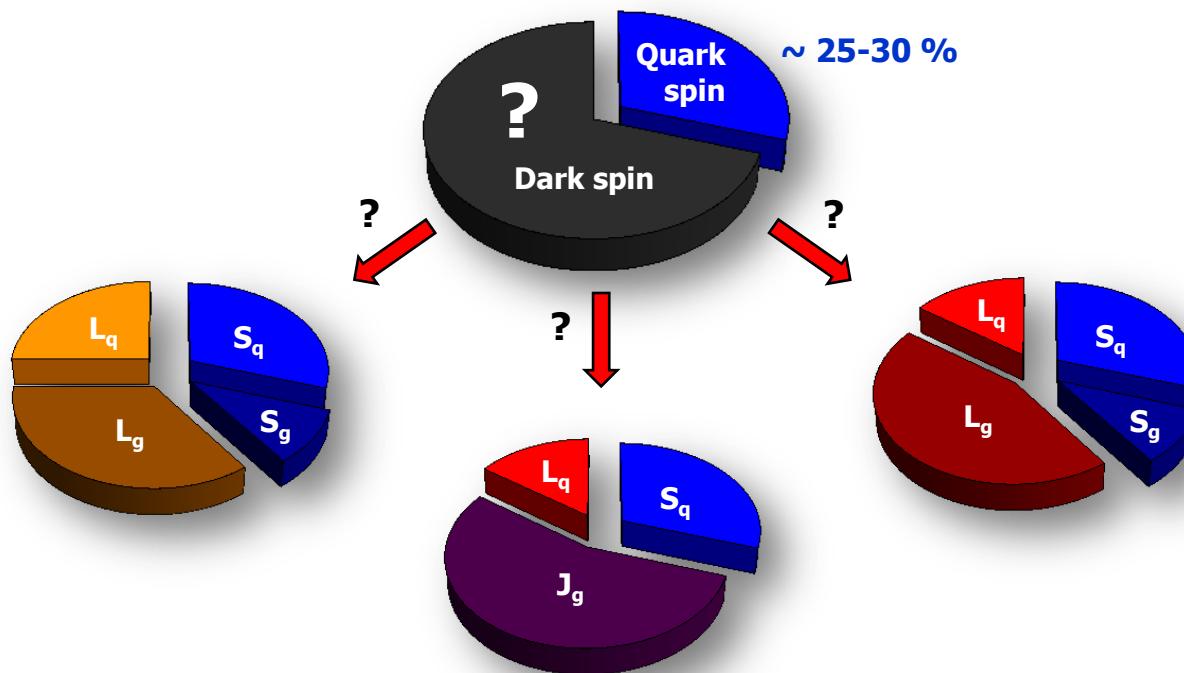
- Another nucleon 3D picture
- Tour in Fock space
- 3D+3D=... 5D !

Lecture 5

- Canonical *vs* kinetic
- Free fall in gauge space
- Physical interpretation

Nucleon spin puzzle

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Many questions/issues :

- Frame dependence ?
- Gauge invariance ?
- Uniqueness ?
- Measurability ?
- ...

Spin decompositions in a nutshell

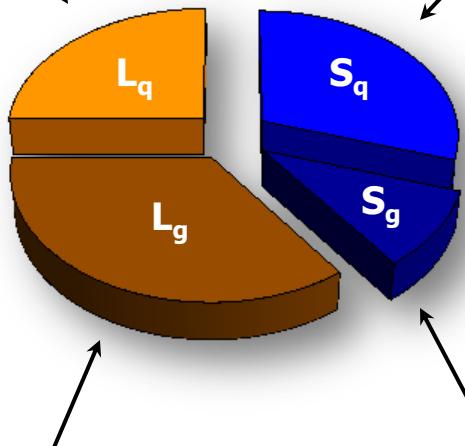
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$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

Canonical

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$



$$\vec{L}_g = \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}^a$$

Gauge non-invariant !

[Jaffe, Manohar (1990)]

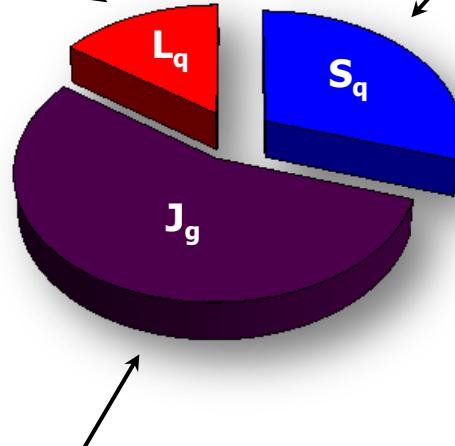
$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

Kinetic

$$\vec{D} = -\vec{\nabla} - ig\vec{A}$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$



$$\vec{J}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

« Incomplete »

[Ji (1997)]

Spin decompositions in a nutshell

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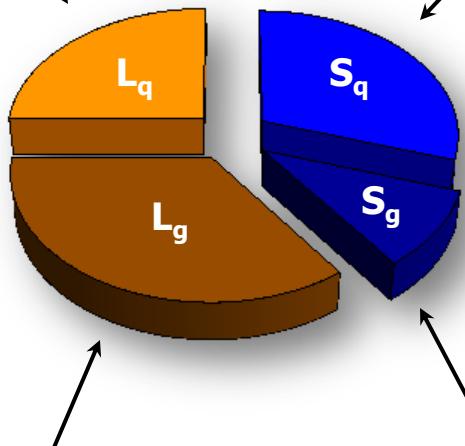
$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

Canonical

$$A = A_{\text{pure}} + A_{\text{phys}}$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D}_{\text{pure}}) \psi$$

$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$



$$\vec{L}_q = \int d^3r E^{ai} \vec{r} \times \vec{D}_{\text{pure}} A_{\text{phys}}^{ai}$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

Gauge-invariant extension (GIE)

[Chen *et al.* (2008)]

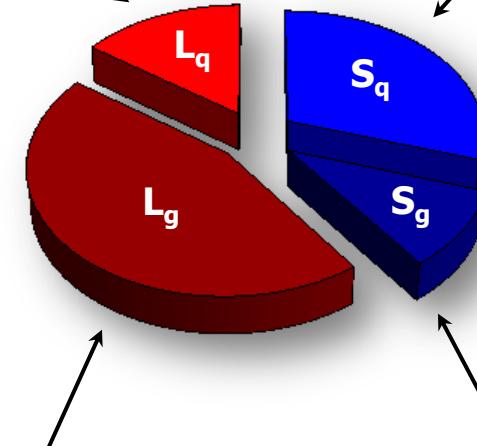
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Kinetic

$$A = A_{\text{pure}} + A_{\text{phys}}$$

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$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$



$$\begin{aligned} \vec{L}_g &= \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a) \\ &\quad - \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a \end{aligned}$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

[Wakamatsu (2010)]

Spin decompositions in a nutshell

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$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

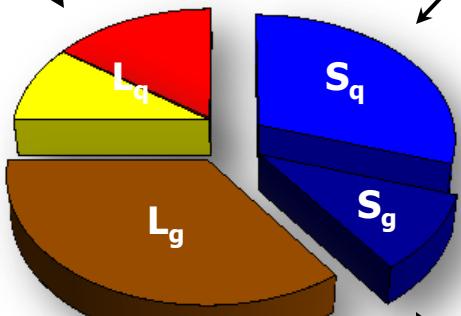
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Canonical

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Gauge-invariant extension (GIE)

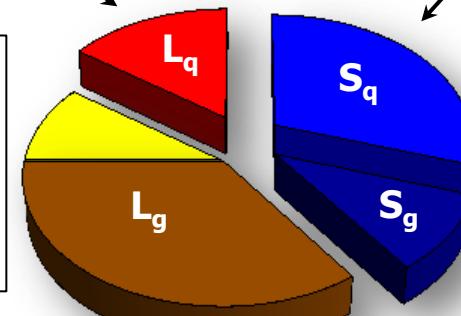
[Chen *et al.* (2008)]

Kinetic

$$A = A_{\text{pure}} + A_{\text{phys}}$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D}) \psi$$

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$$\begin{aligned} \vec{L}_g &= \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a) \\ &\quad - \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a \end{aligned}$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

[Wakamatsu (2010)]

$$A_\mu(x) = \textcolor{blue}{A}_\mu^{\text{pure}}(x) + \textcolor{red}{A}_\mu^{\text{phys}}(x)$$

[Chen *et al.* (2008,2009)]
 [Wakamatsu (2010,2011)]

Gauge transformation (assumed)

$$\begin{aligned} A_\mu^{\text{pure}}(x) &\mapsto U(x) \left[A_\mu^{\text{pure}}(x) + \frac{i}{g} \partial_\mu \right] U^{-1}(x) \\ A_\mu^{\text{phys}}(x) &\mapsto U(x) \textcolor{red}{A}_\mu^{\text{phys}}(x) U^{-1}(x) \end{aligned}$$

Pure-gauge covariant derivatives

$$\begin{aligned} D_\mu^{\text{pure}} &= \partial_\mu - ig A_\mu^{\text{pure}}(x) \\ \mathcal{D}_\mu^{\text{pure}} &= \partial_\mu - ig [A_\mu^{\text{pure}}(x), \quad] \end{aligned}$$

Field strength

$$F_{\mu\nu}^{\text{pure}}(x) = \frac{i}{g} [D_\mu^{\text{pure}}, D_\nu^{\text{pure}}] = 0$$

$$F_{\mu\nu}(x) = \textcolor{blue}{D}_\mu^{\text{pure}} \textcolor{red}{A}_\nu^{\text{phys}}(x) - \textcolor{blue}{D}_\nu^{\text{pure}} \textcolor{red}{A}_\mu^{\text{phys}}(x) - ig [A_\mu^{\text{phys}}(x), A_\nu^{\text{phys}}(x)]$$

Dual role

$$A_\mu(x) = \underbrace{\frac{i}{g} U_{\text{pure}}(x) \partial_\mu U_{\text{pure}}^{-1}(x)}_{A_\mu^{\text{pure}}(x)} + \underbrace{U_{\text{pure}}(x) \hat{A}_\mu^{\text{phys}}(x) U_{\text{pure}}^{-1}(x)}_{A_\mu^{\text{phys}}(x)}$$

Analogy with General Relativity

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Dual role

$$A_\mu(x) = \frac{i}{g} U_{\text{pure}}(x) \partial_\mu U_{\text{pure}}^{-1}(x) + U_{\text{pure}}(x) \hat{A}_\mu^{\text{phys}}(x) U_{\text{pure}}^{-1}(x)$$

Pure
gauge

Physical
polarizations

Degrees of freedom

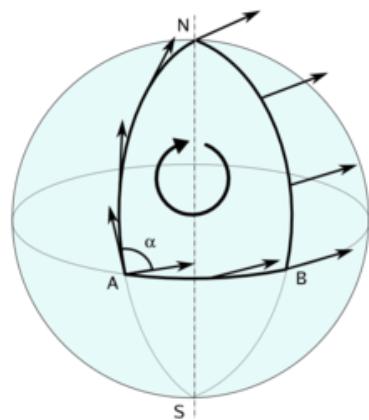
Analogy with General Relativity

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Dual role

$$A_\mu(x) = \frac{i}{g} U_{\text{pure}}(x) \partial_\mu U_{\text{pure}}^{-1}(x) + U_{\text{pure}}(x) \hat{A}_\mu^{\text{phys}}(x) U_{\text{pure}}^{-1}(x)$$

$A_\mu^{\text{pure}}(x)$ $A_\mu^{\text{phys}}(x)$



Pure
gauge



Parallelism

Physical
polarizations



Curvature

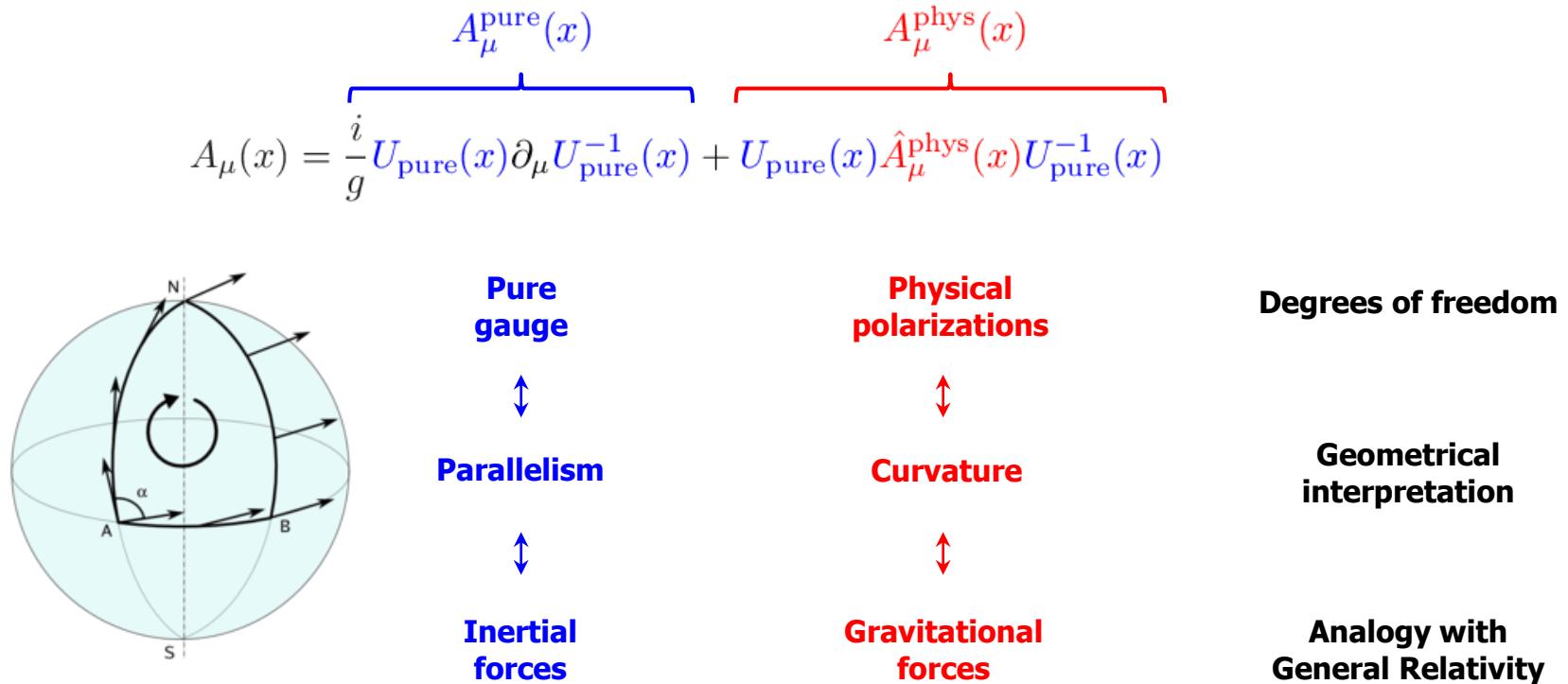
Degrees of freedom

Geometrical
interpretation

Analogy with General Relativity

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Dual role



$$\Gamma^\lambda_{\mu\nu}(x) = e_a^\lambda(x) \partial_\mu e_\nu^a(x) + e_a^\lambda(x) \omega_{\mu b}^a(x) e_\nu^b(x)$$

Stueckelberg symmetry

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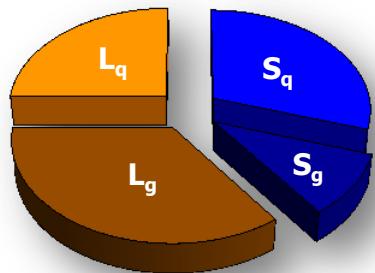
$$A = A_{\text{pure}} + A_{\text{phys}} = \underbrace{\bar{A}_{\text{pure}}}_{A_{\text{pure}}+C} + \underbrace{\bar{A}_{\text{phys}}}_{A_{\text{phys}}-C}$$

[Stoilov (2010)]
[C.L. (2013)]

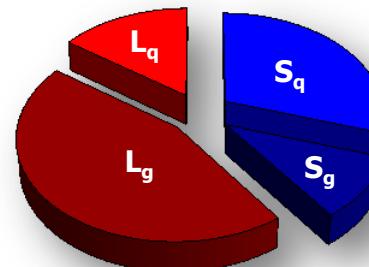
Ambiguous !
Infinitely many possibilities !

Coulomb GIE

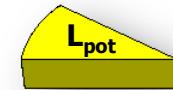
$$\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$$



[Chen *et al.* (2008)]

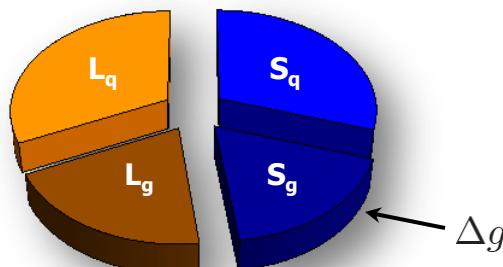


[Wakamatsu (2010)]

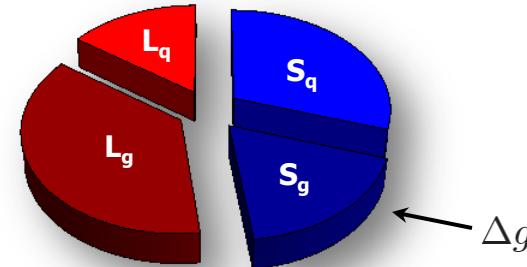


Light-front GIE

$$A_{\text{phys}}^+ = 0$$

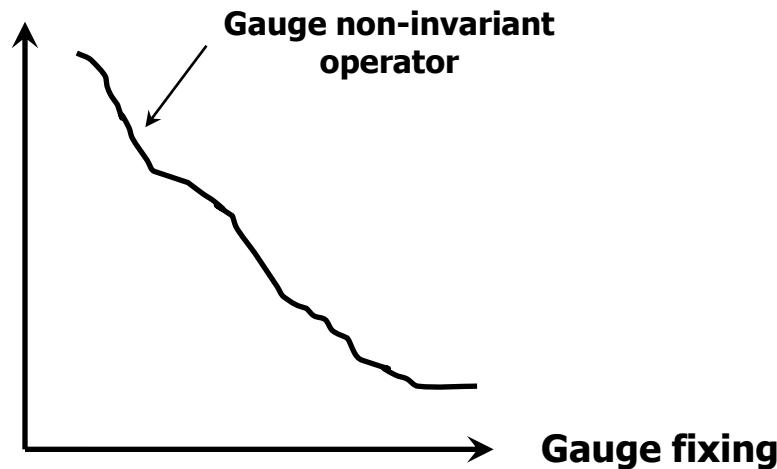


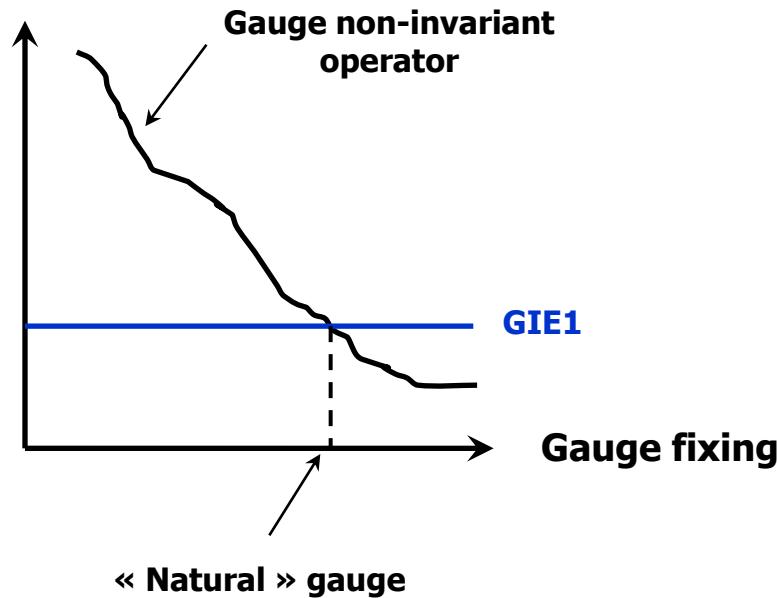
[Hatta (2011)]
[C.L. (2013)]

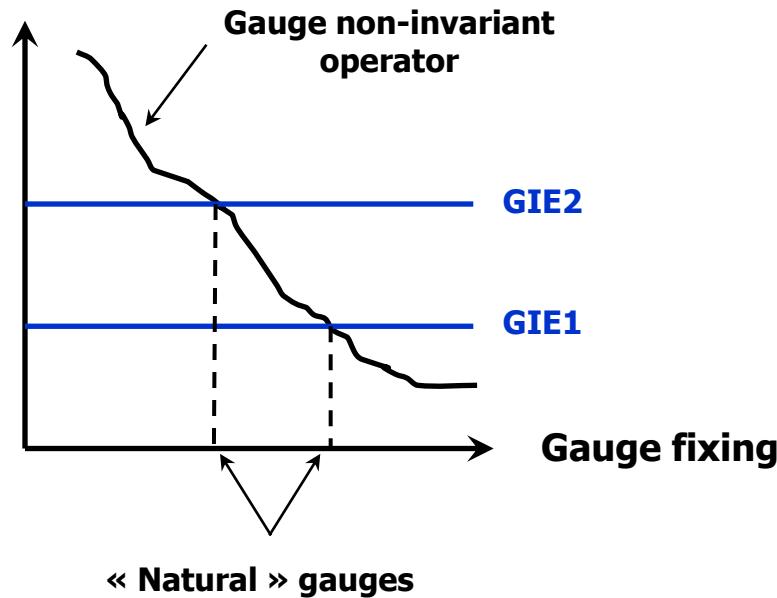


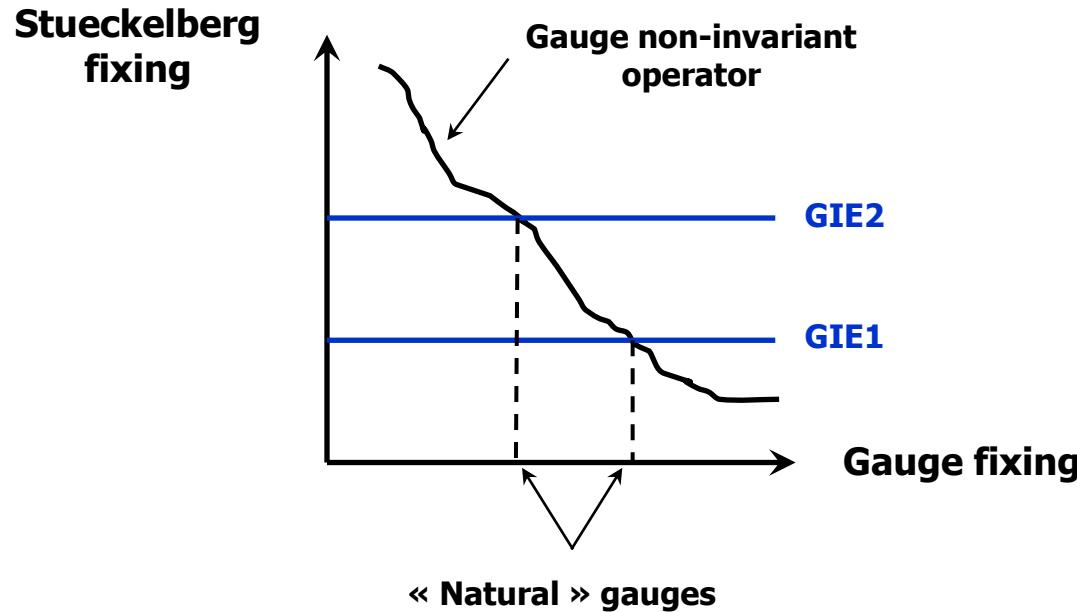
Stueckelberg symmetry

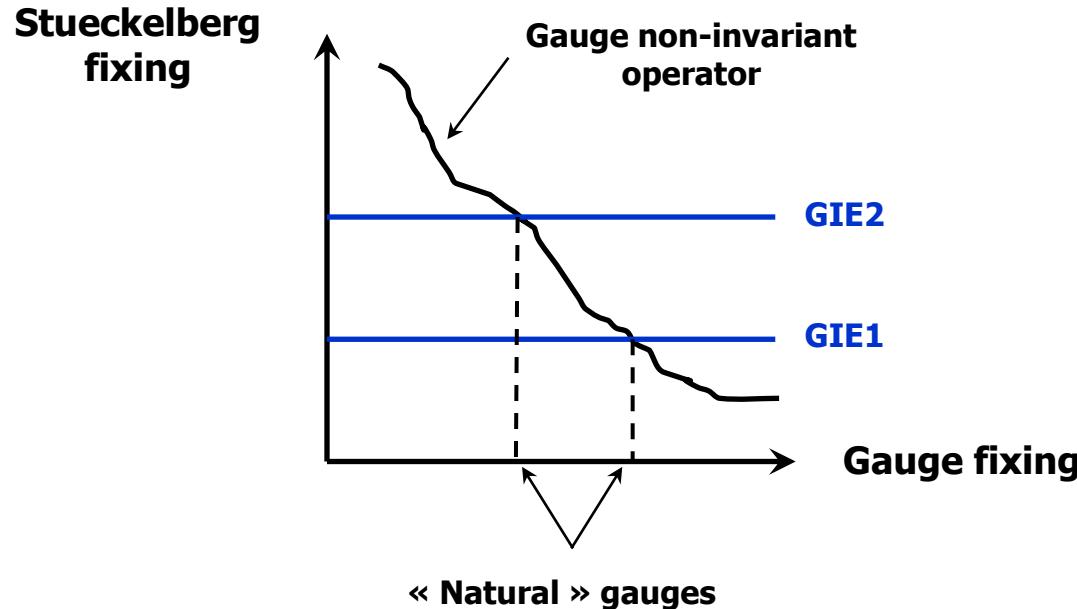
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Lorentz-invariant
extensions

$$p^2 = m_0^2$$

Rest

$$s = E_{\text{CM}}^2$$

Center-of-mass

$$x = k_{\text{IMF}}^z / p_{\text{IMF}}^z$$

Infinite momentum

<< Natural >> frames

Geometrical interpretation

$$\mathcal{W}(x + dx, x) = 1 + igA_\mu(x)dx^\mu$$

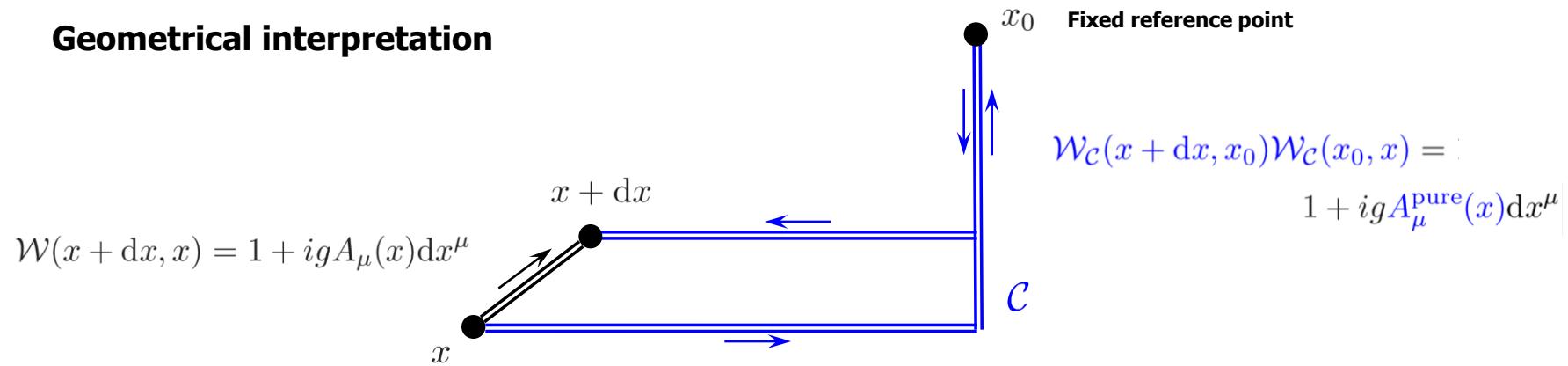
Finite Wilson line

$$\begin{aligned} \mathcal{W}(y, x) &= \mathcal{P} \left[e^{ig \int_x^y A_\mu(s) ds^\mu} \right] \\ &= 1 + ig \int_x^y A_\mu(s) ds^\mu + (ig)^2 \int_x^y \int_x^{s_1} A_\mu(s_1) A_\nu(s_2) ds_1^\mu ds_2^\nu + \dots \end{aligned}$$

Gauge transformation

$$\mathcal{W}(y, x) \mapsto U(y) \mathcal{W}(y, x) U^{-1}(x) \quad \longleftrightarrow \quad A_\mu(x) \mapsto U(x) \left[A_\mu(x) + \frac{i}{g} \partial_\mu \right] U^{-1}(x)$$

Geometrical interpretation



$$\mathcal{W}(x + dx, x) = 1 + igA_\mu(x)dx^\mu$$

$$\mathcal{W}_C(x + dx, x_0)\mathcal{W}_C(x_0, x) =$$

$$1 + igA_\mu^{\text{pure}}(x)dx^\mu$$

Explicit expressions

$$A_\mu^{\text{pure}}(x) = \frac{i}{g} \mathcal{W}_C(x, x_0) \frac{\partial}{\partial x^\mu} \mathcal{W}_C(x_0, x)$$

Non-local !

$$A_\mu^{\text{phys}}(x) = A_\mu(x) - A_\mu^{\text{pure}}(x)$$

Derivative of parametric integral

$$\frac{\partial}{\partial x} \int_a^b f(y) dy = \frac{\partial b}{\partial x} f(b) - \frac{\partial a}{\partial x} f(a) + \int_a^b \frac{\partial f}{\partial x} dy$$

Generalization to path-ordered exponentials

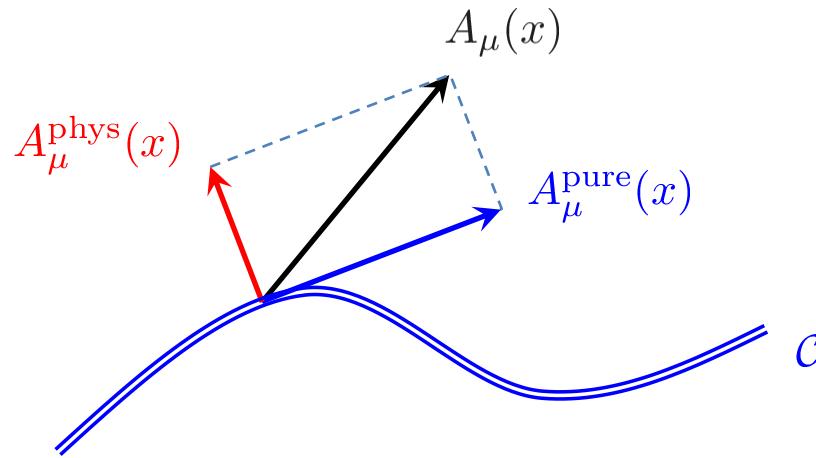
$$y^\mu = s^\mu(\lambda_f) \quad x^\mu = s^\mu(\lambda_i)$$

$$\begin{aligned} \frac{\partial}{\partial z^\mu} \mathcal{W}_C(y, x) &= ig \left[A_\alpha(y) \frac{\partial y^\alpha}{\partial z^\mu} \mathcal{W}_C(y, x) - \mathcal{W}_C(y, x) A_\alpha(x) \frac{\partial x^\alpha}{\partial z^\mu} \right] \\ &\quad + ig \int_x^y \mathcal{W}_C(y, s) F_{\alpha\beta}(s) \mathcal{W}_C(s, x) \frac{\partial s^\alpha}{\partial z^\mu} ds^\beta \end{aligned}$$

Derivative along the path

$$\begin{aligned} \frac{\partial}{\partial \lambda} \mathcal{W}_C(y, x) &= ig \left[A_\mu(y) \frac{\partial s^\mu}{\partial \lambda} \Big|_{\lambda_f} \mathcal{W}_C(y, x) - \mathcal{W}_C(y, x) A_\mu(x) \frac{\partial s^\mu}{\partial \lambda} \Big|_{\lambda_i} \right] \\ &\quad + ig \int_x^y \mathcal{W}_C(y, s) F_{\alpha\beta}(s) \mathcal{W}_C(s, x) \frac{\partial s^\alpha}{\partial \lambda} \frac{\partial s^\beta}{\partial \lambda} d\lambda \end{aligned}$$

Decomposition is path-dependent !



$$\frac{\partial s^\mu}{\partial \lambda} D_\mu \mathcal{W}_C(x, x_0) = 0$$

$$D_\mu^{\text{pure}} \mathcal{W}_C(x, x_0) = 0$$

$$A_\mu^{\text{pure}}(x) = \frac{i}{g} \mathcal{W}_C(x, x_0) \frac{\partial}{\partial x^\mu} \mathcal{W}_C(x_0, x)$$

$$A_\mu^{\text{phys}}(x) = - \int_{x_0}^x \mathcal{W}_C(x, s) F_{\alpha\beta}(s) \mathcal{W}_C(s, x) \frac{\partial s^\alpha}{\partial x^\mu} ds^\beta$$

Non-local !

Path dependence



Stueckelberg non-invariance



Non-local color phase factor

$$U_{\text{pure}}(x) = \mathcal{W}_{\mathcal{C}}(x, x_0) U_{\text{pure}}(x_0) \quad \text{Path-dependent}$$

$$U_{\text{pure}}(x) = e^{-ie \frac{\vec{\nabla} \cdot \vec{A}(x)}{\vec{\nabla}^2}}$$

↑

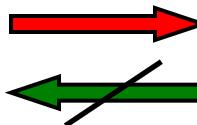
$$\frac{1}{\vec{\nabla}^2} f(\vec{x}) \equiv -\frac{1}{4\pi} \int d^3 x' \frac{f(\vec{x}')}{|\vec{x} - \vec{x}'|}$$

Path-independent

$$A_\mu^{\text{pure}}(x) = \frac{i}{g} U_{\text{pure}}(x) \frac{\partial}{\partial x^\mu} U_{\text{pure}}^{-1}(x)$$

$$A_\mu^{\text{phys}}(x) = - \int_{x_0}^x U_{\text{pure}}(x) [U_{\text{pure}}^{-1}(s) F_{\alpha\beta}(s) U_{\text{pure}}(s)] U_{\text{pure}}^{-1}(x) \frac{\partial s^\alpha}{\partial x^\mu} ds^\beta$$

Path dependence



Stueckelberg non-invariance

Degrees of freedom

$$A_\mu(x) = A_\mu^{\text{pure}}(x) + A_\mu^{\text{phys}}(x)$$

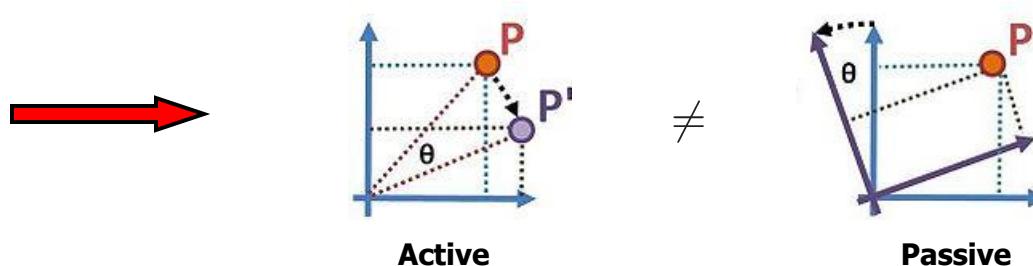
↗ ↘

**Classical
Non-dynamical** **Quantum
Dynamical**

$$F_{\mu\nu}^{\text{pure}}(x) = \frac{i}{g} [D_\mu^{\text{pure}}, D_\nu^{\text{pure}}] = 0$$

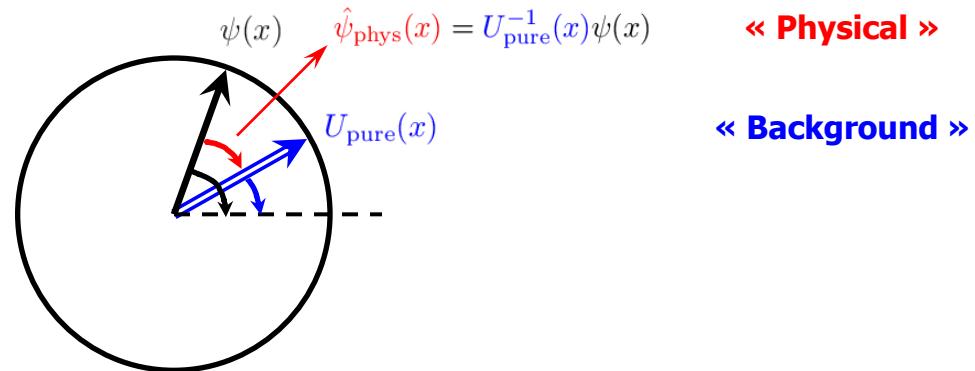
$$F_{\mu\nu}(x) = \mathcal{D}_\mu^{\text{pure}} A_\nu^{\text{phys}}(x) - \mathcal{D}_\nu^{\text{pure}} A_\mu^{\text{phys}}(x) - ig [A_\mu^{\text{phys}}(x), A_\nu^{\text{phys}}(x)]$$

$A_\mu^{\text{pure}}(x)$ plays the role of a background field !



Quantum Electrodynamics

Phase in
internal space

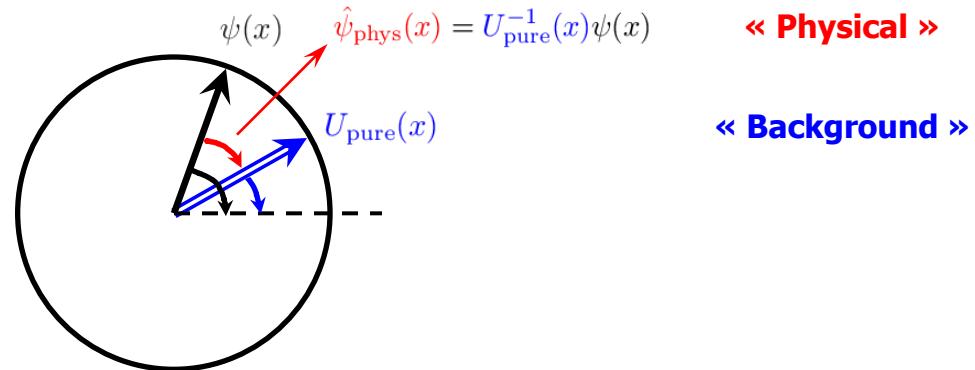


« Physical »

« Background »

Quantum Electrodynamics

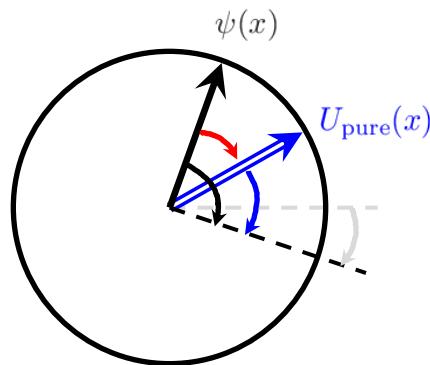
Phase in
internal space



« Physical »

« Background »

Passive



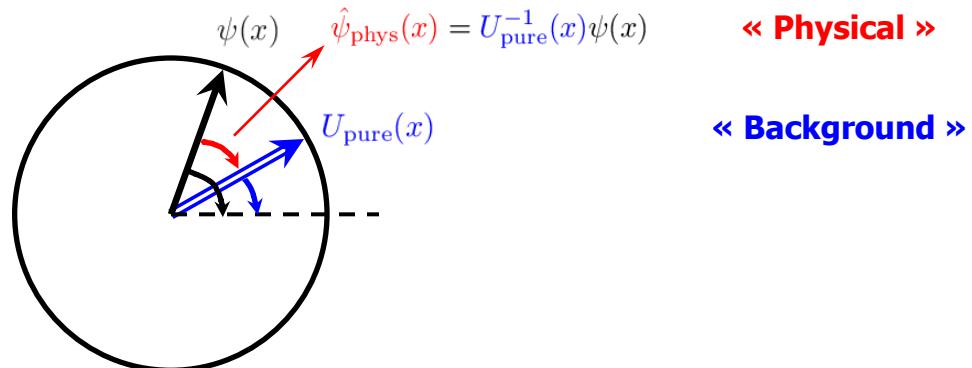
$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$

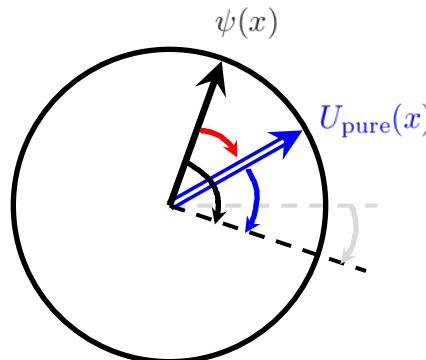
$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Quantum Electrodynamics

Phase in
internal space



Passive

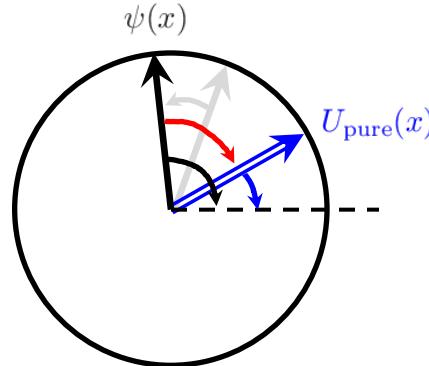


$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Active



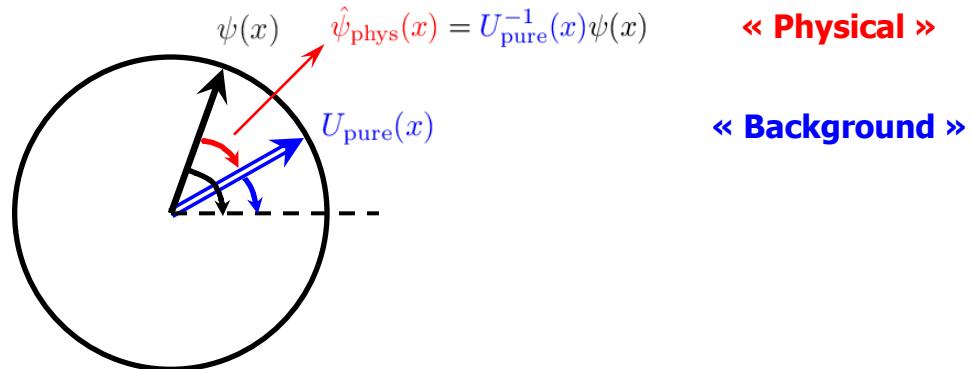
$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)$$

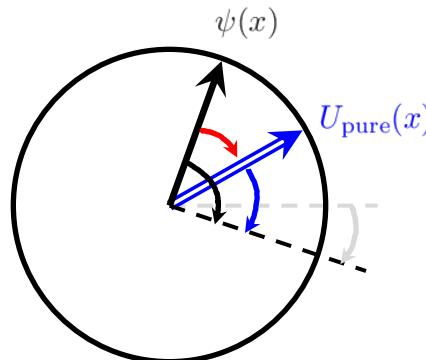
$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Quantum Electrodynamics

Phase in internal space



Passive

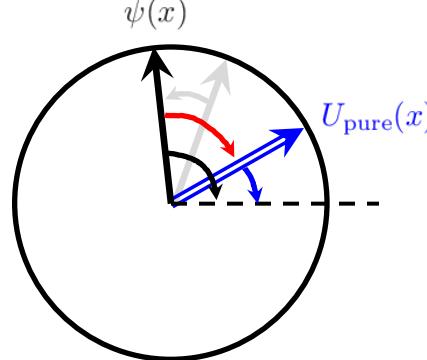


$$\psi(x) \mapsto U(x)\psi(x)$$

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$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Active

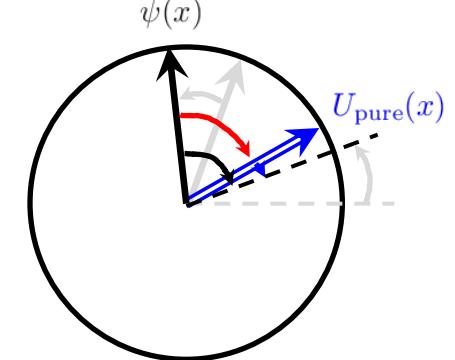


$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Active x (Passive)⁻¹



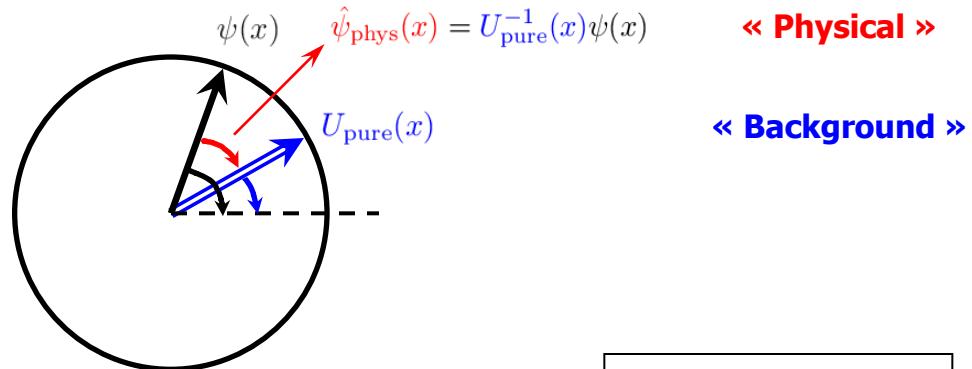
$$\psi(x) \mapsto \psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)U^{-1}(x)$$

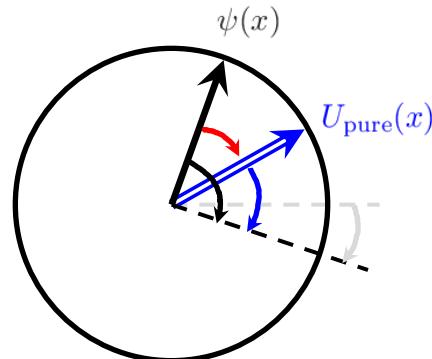
$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Quantum Electrodynamics

Phase in internal space



Passive

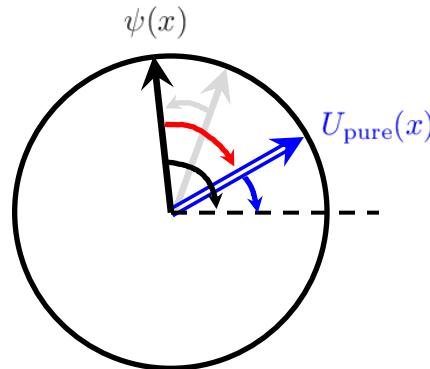


$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Active



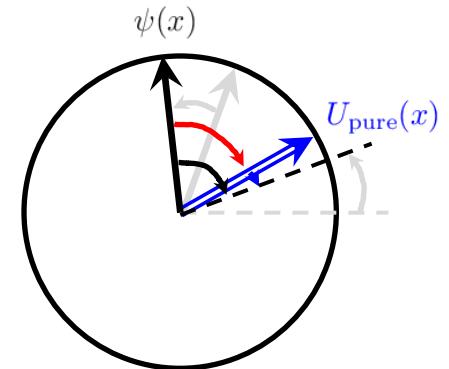
$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Stueckelberg

Active \times **(Passive)**⁻¹

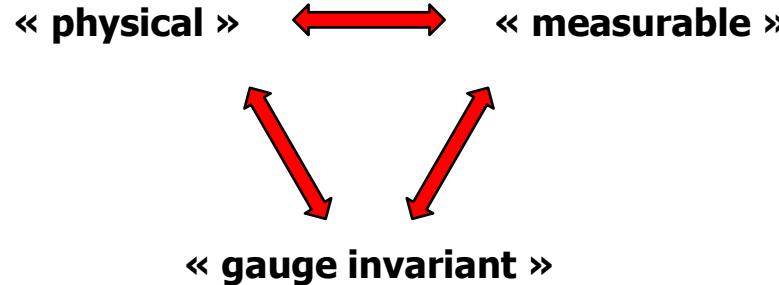


$$\psi(x) \mapsto \psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)U^{-1}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

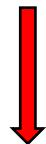
Quid ?



Observables

E.g. cross-sections

Measurable, physical, gauge invariant and local
(**active** and **passive**)



Expansion scheme

E.g. collinear factorization

$\left[\begin{array}{c} \text{Path} \\ \text{Stueckelberg} \\ \text{Background} \end{array} \right]$ -dependent

Quasi-observables

E.g. parton distributions

« Measurable », « physical », gauge invariant and non-local
(only **passive**)

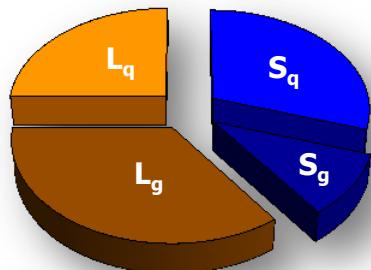
Observable

Quasi-observable

Not observable

Canonical

[Jaffe-Manohar (1990)]



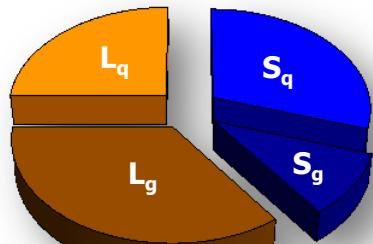
$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}^a$$

$$\vec{L}_g = \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}$$

[Chen *et al.* (2008)]



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

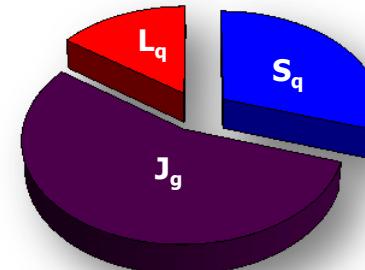
$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D}_{\text{pure}})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

$$\vec{L}_g = \int d^3r E^{ai} \vec{r} \times \vec{\mathcal{D}}_{\text{pure}} A_{\text{phys}}^{ai}$$

Kinetic

[Ji (1997)]

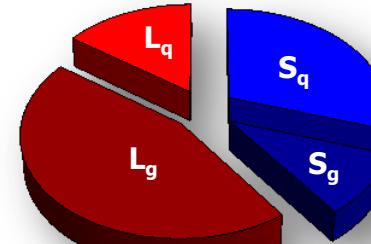


$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{J}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

[Wakamatsu (2010)]



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (i\vec{D})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

$$\vec{L}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a) - \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

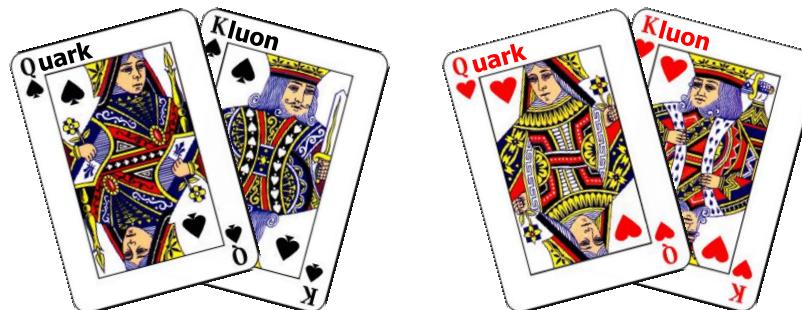
Summary

Lecture 5

- Essentially 2 types of spin decomposition : canonical and kinetic
- Measurability requires gauge invariance but not local expressions
- Physical interpretation is usually Lorentz and gauge non-invariant !



Canonical



Kinetic