



HUGS 2014

29th Annual Hampton University Graduate Studies Program

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Spin Sum Rules and 3D Nucleon Structure (1/6)

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Outline

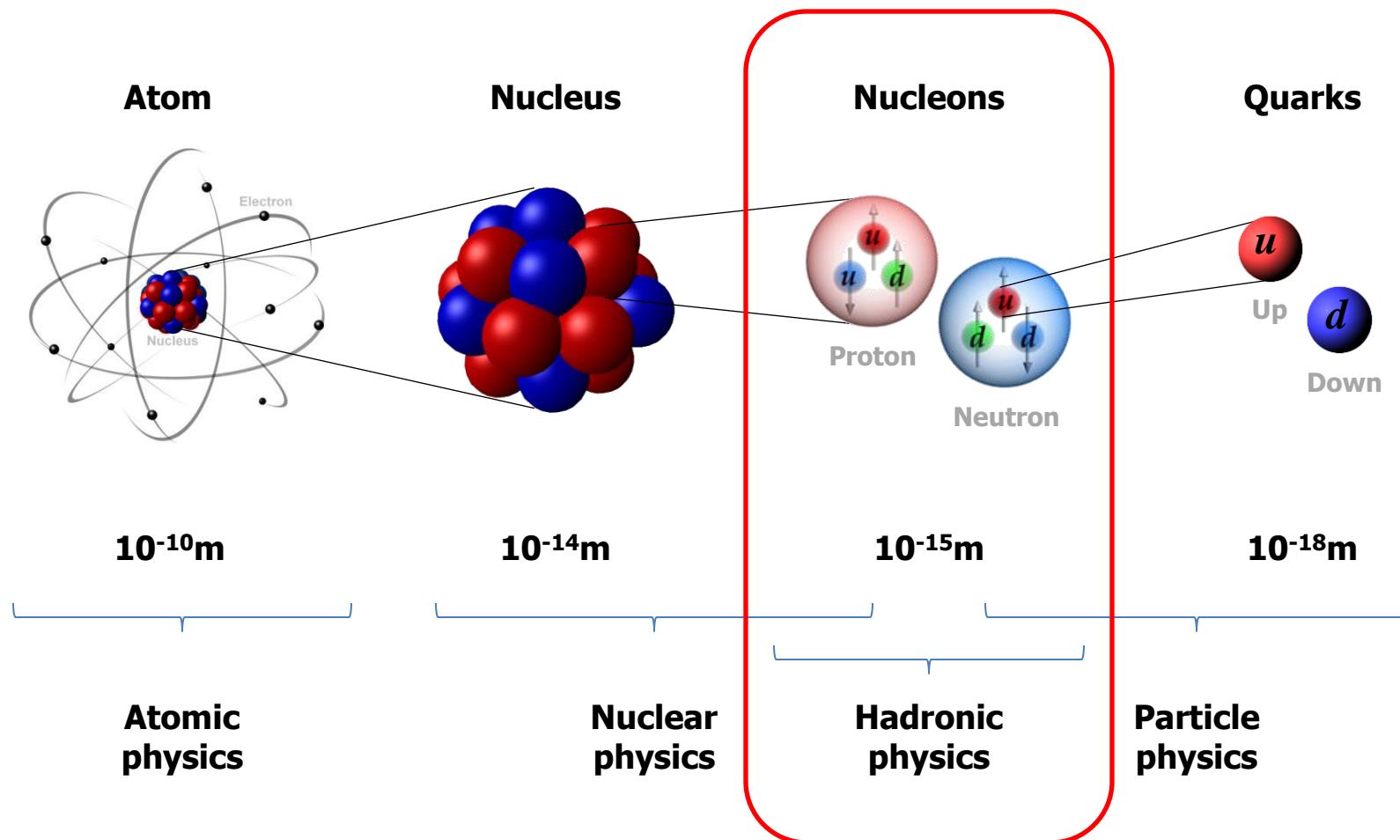
Lecture 1

- **Introduction**
- **Tour in phase space**
- **Galileo vs Lorentz**

Introduction

1/19

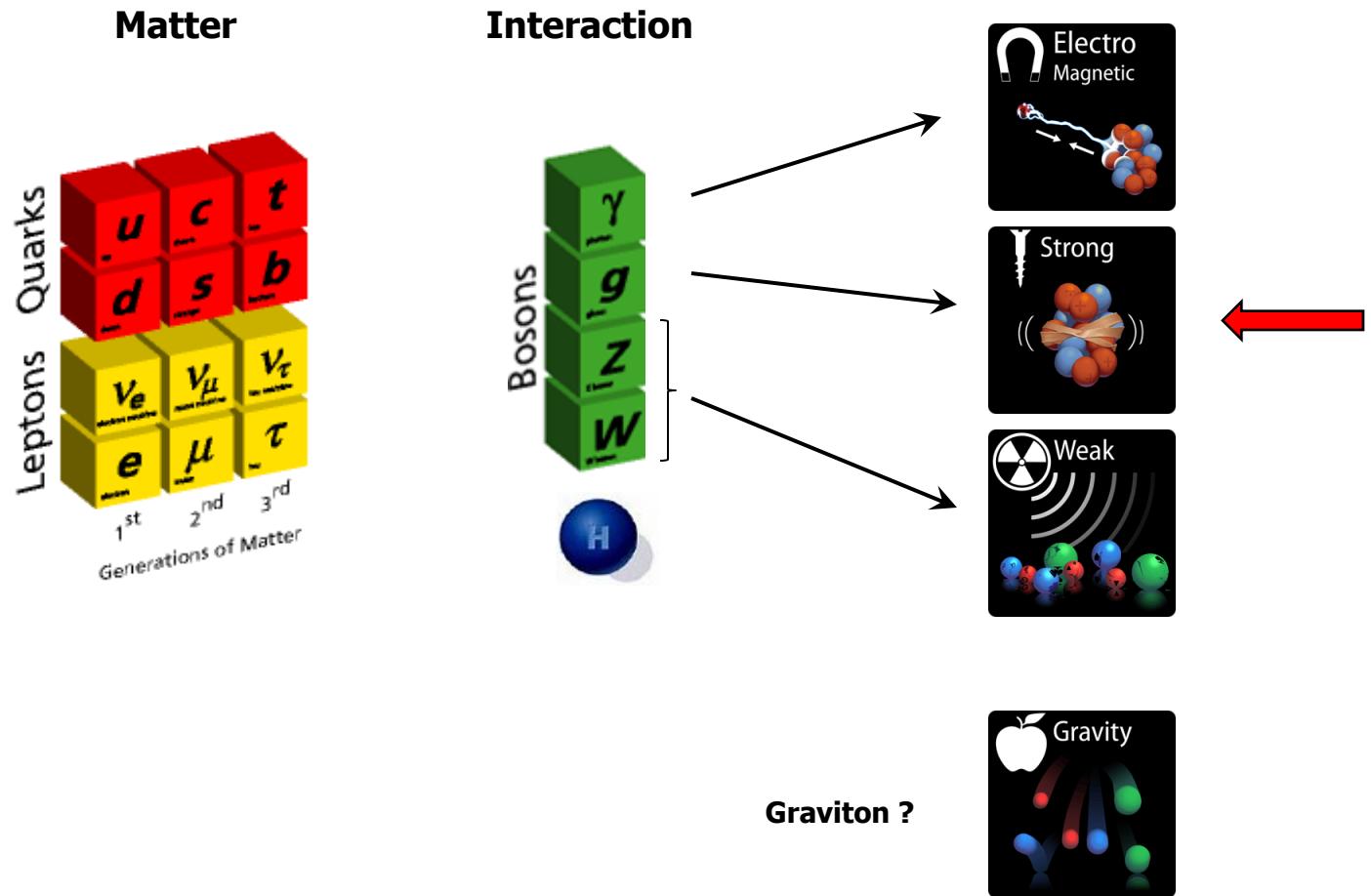
Structure of matter



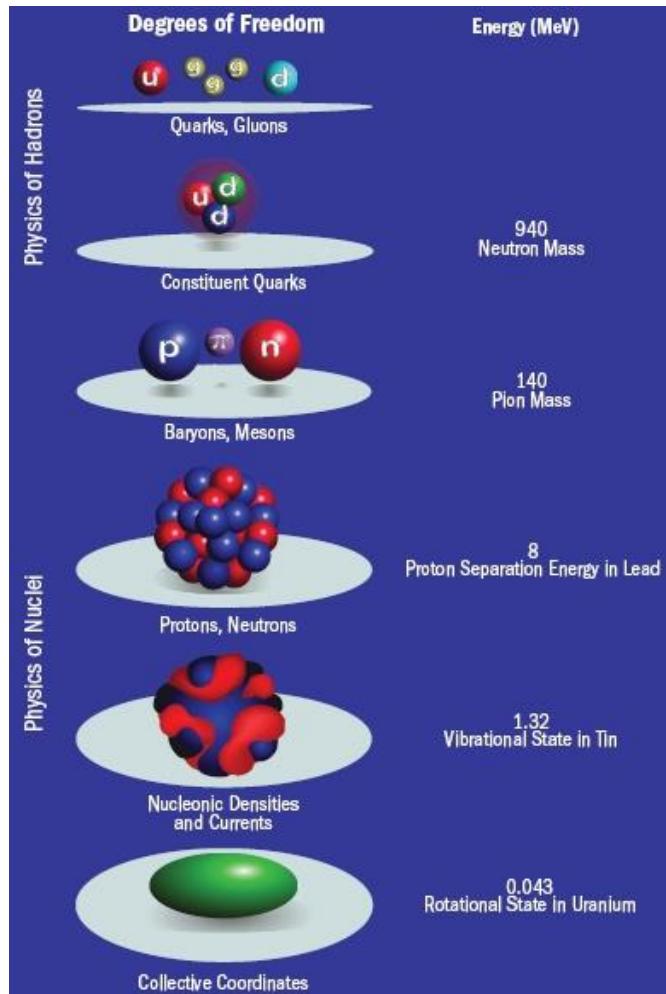
Introduction

2/19

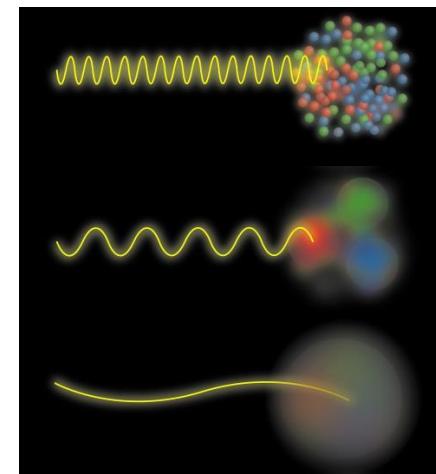
Elementary particles



Degrees of freedom

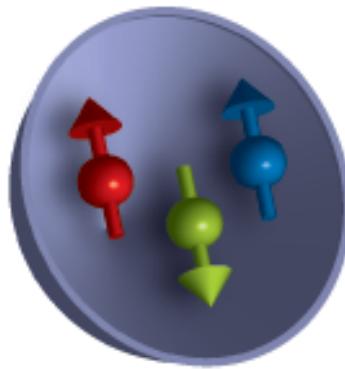


**Relevant degrees of freedom
depend on typical energy scale**



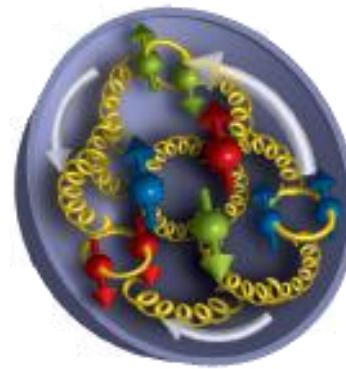
Nucleon pictures

« Naive »



3 non-relativistic
heavy quarks

Realistic



Indefinite # of relativistic
light quarks and gluons

$$M_N = m_{\text{const}} (\sim 2\%) + E_{\text{int}} (\sim 98\% !)$$

↑
Brout-Englert-Higgs
mechanism

↑
QCD



Quantum Mechanics + Special Relativity = Quantum Field Theory

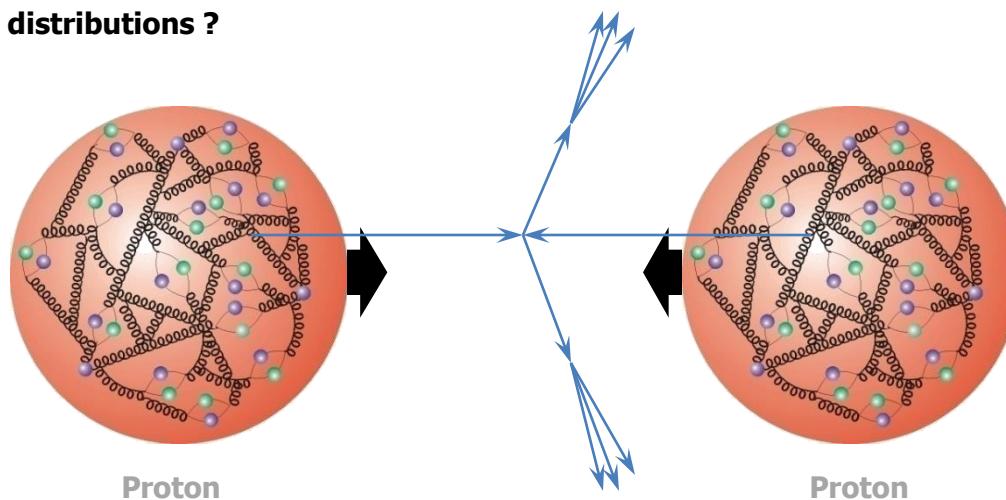
Goal : understanding the nucleon internal structure

Why ?

At the energy frontier



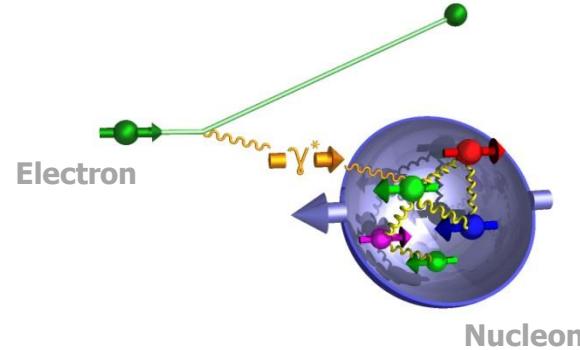
Quark & gluon distributions ?



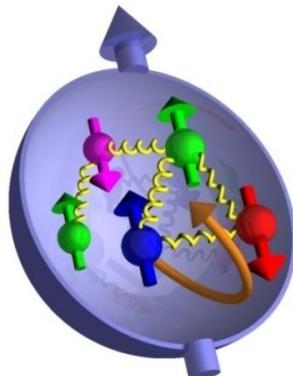
Goal : understanding the nucleon internal structure

Why ?

At the intensity frontier



Nucleon spin structure ?



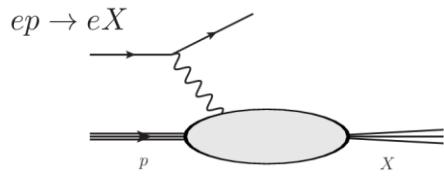
Nucleon shape & size ?



Goal : understanding the nucleon internal structure

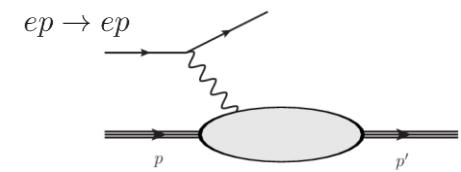
How ?

Deep Inelastic Scattering

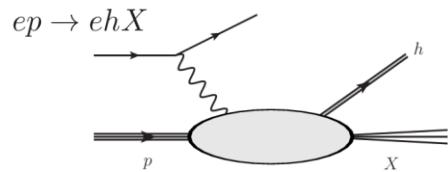


**Proton
« tomography »**

Elastic Scattering

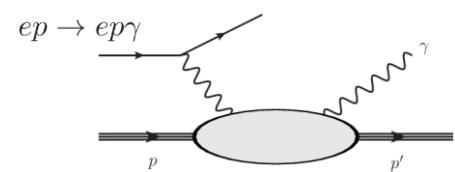


Semi-Inclusive DIS

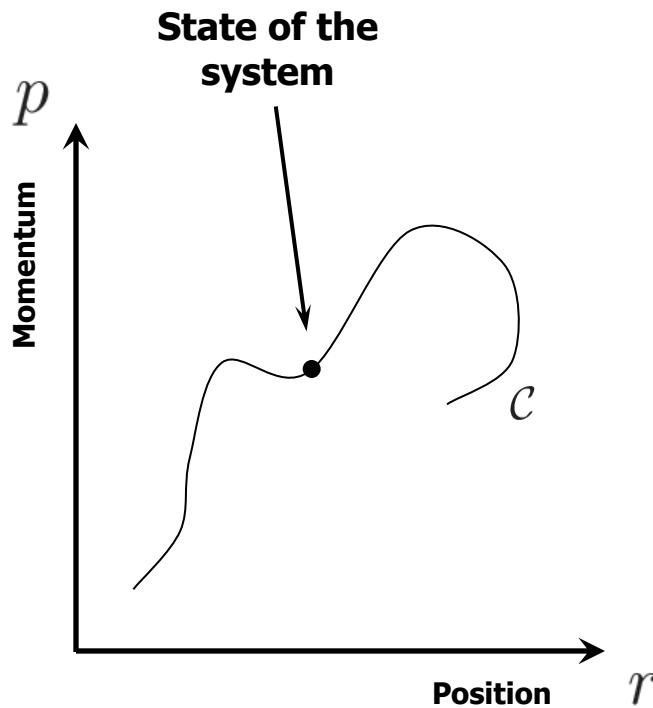


**Phase-space
distribution**

Deeply Virtual Compton Scattering



Classical Mechanics

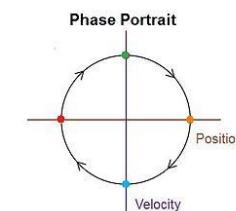
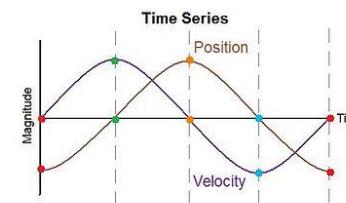


Particles follow well-defined trajectories

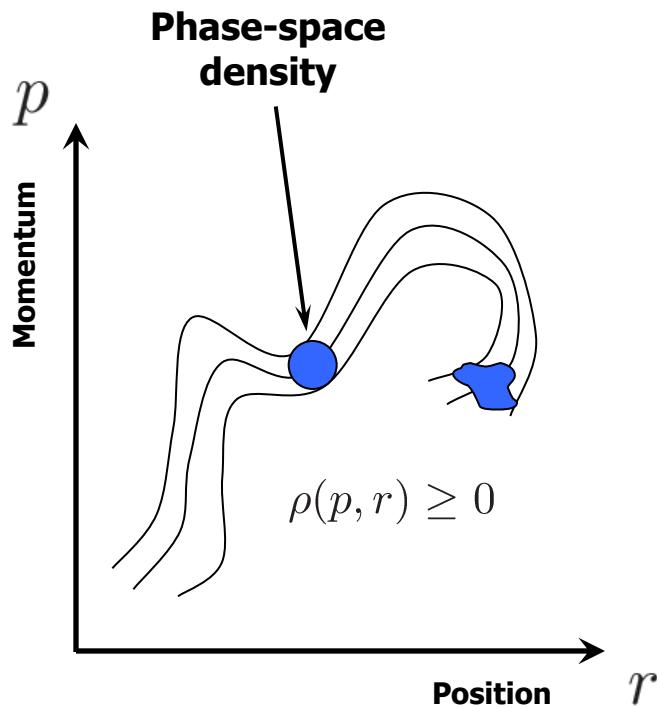


$$f[p(t), r(t), t] = 0$$

$$p = \frac{\partial L}{\partial \dot{r}}$$



Statistical Mechanics



Position-space density

$$\rho(r) = \int dp \rho(p, r)$$

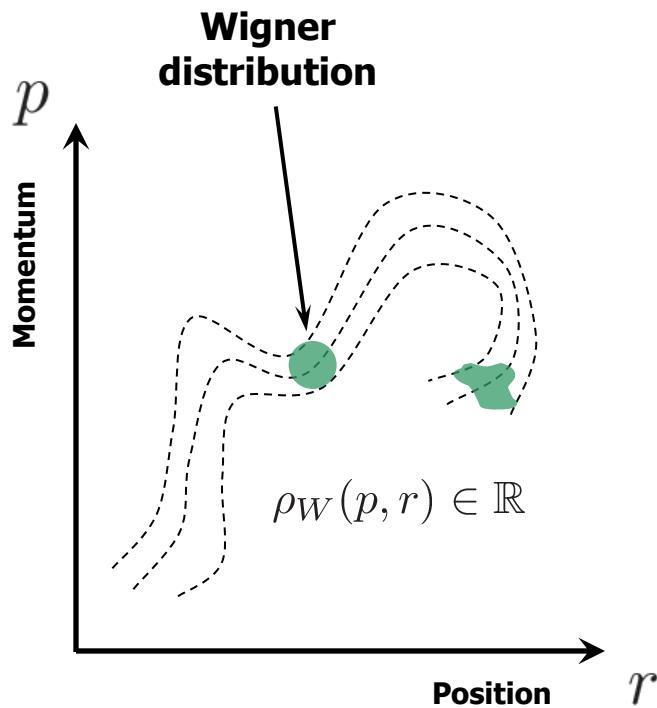
Momentum-space density

$$\rho(p) = \int dr \rho(p, r)$$

Phase-space average

$$\overline{O} = \int dp dr O(p, r) \rho(p, r)$$

Quantum Mechanics



Position-space density

$$|\psi(r)|^2 = \int dp \rho_W(p, r)$$

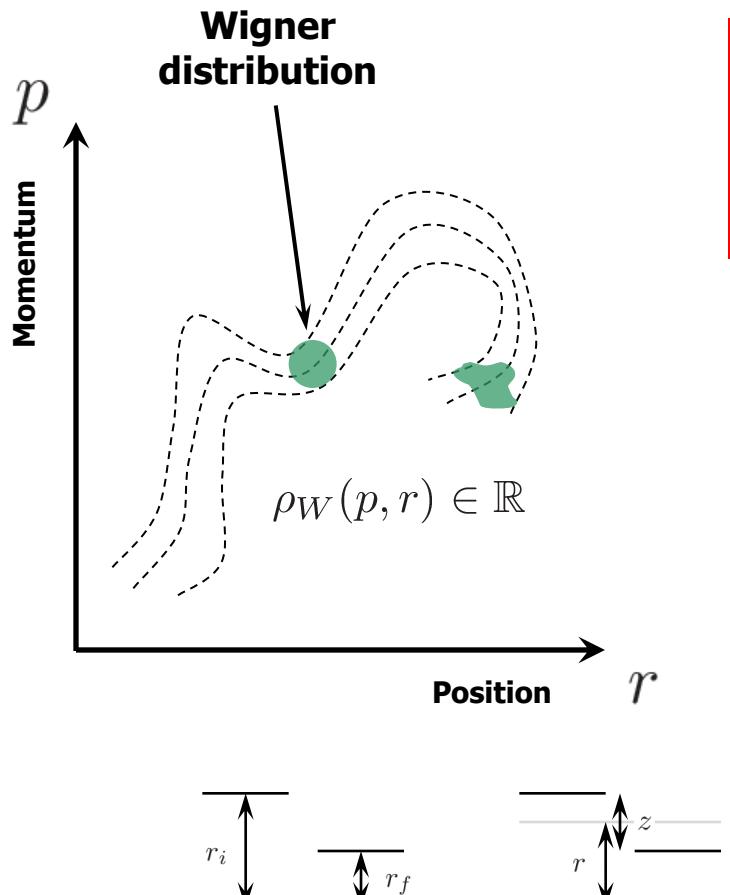
Momentum-space density

$$\frac{1}{2\pi} |\varphi(p)|^2 = \int dr \rho_W(p, r)$$

Phase-space average

$$\begin{aligned} \langle \hat{O} \rangle &= \int dr \psi^*(r) \hat{O}(-i \frac{\partial}{\partial r}, r) \psi(r) \\ &= \int \frac{dp}{2\pi} \varphi^*(p) \hat{O}(p, i \frac{\partial}{\partial p}) \varphi(p) \\ &= \int dp dr O(p, r) \rho_W(p, r) \end{aligned}$$

Quantum Mechanics



$$\begin{aligned}\rho_W(p, r) &= \int \frac{dz}{2\pi} e^{-ipz} \psi^*(r - \frac{z}{2}) \psi(r + \frac{z}{2}) \\ &= \int \frac{d\Delta}{(2\pi)^2} e^{-i\Delta r} \varphi^*(p + \frac{\Delta}{2}) \varphi(p - \frac{\Delta}{2})\end{aligned}$$

$$\begin{aligned}p \rho_W(p, r) &= \int \frac{dz}{2\pi} p e^{-ipz} \psi^*(r - \frac{z}{2}) \psi(r + \frac{z}{2}) \\ &= \int \frac{dz}{2\pi} \left[(i \frac{\partial}{\partial z}) e^{-ipz} \right] \psi^*(r - \frac{z}{2}) \psi(r + \frac{z}{2}) \\ &= \int \frac{dz}{2\pi} e^{-ipz} \left(-i \frac{\partial}{\partial z} \right) [\psi^*(r - \frac{z}{2}) \psi(r + \frac{z}{2})] \\ &= \int \frac{dz}{2\pi} e^{-ipz} \psi^*(r - \frac{z}{2}) \underbrace{\left(-\frac{i}{2} \frac{\partial}{\partial r} \right)}_{\text{Hermitian (symmetric) derivative}} \psi(r + \frac{z}{2})\end{aligned}$$

$$\text{Hermitian (symmetric) derivative} \equiv -\frac{i}{2} \left(\frac{\partial}{\partial r} - \frac{\partial}{\partial r} \right)$$

Fourier conjugate variables

$$p_f r_f - p_i r_i$$

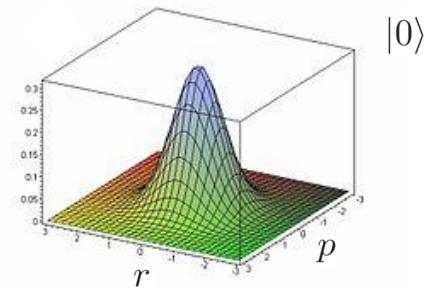
$$\Delta r - pz$$

[Wigner (1932)]
[Moyal (1949)]

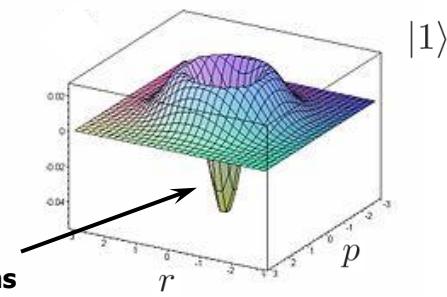
Wigner distributions have applications in:

- Nuclear physics
- Quantum chemistry
- Quantum molecular dynamics
- Quantum information
- Quantum optics
- Classical optics
- Signal analysis
- Image processing
- Quark-gluon plasma
- ...

Harmonic oscillator



$|0\rangle$



$|1\rangle$

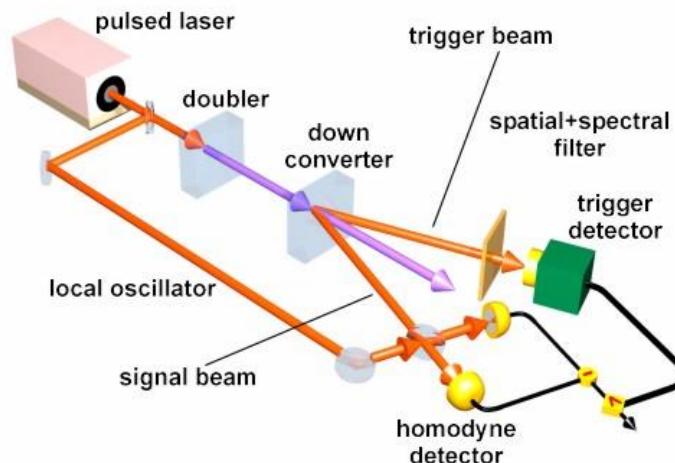


Quasi-probabilistic

**Heisenberg's
uncertainty relations**

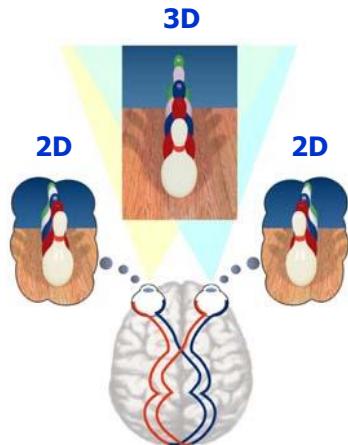
$$\Delta p \Delta r \geq \frac{\hbar}{2}$$

In quantum optics, Wigner distributions are « measured » using homodyne tomography



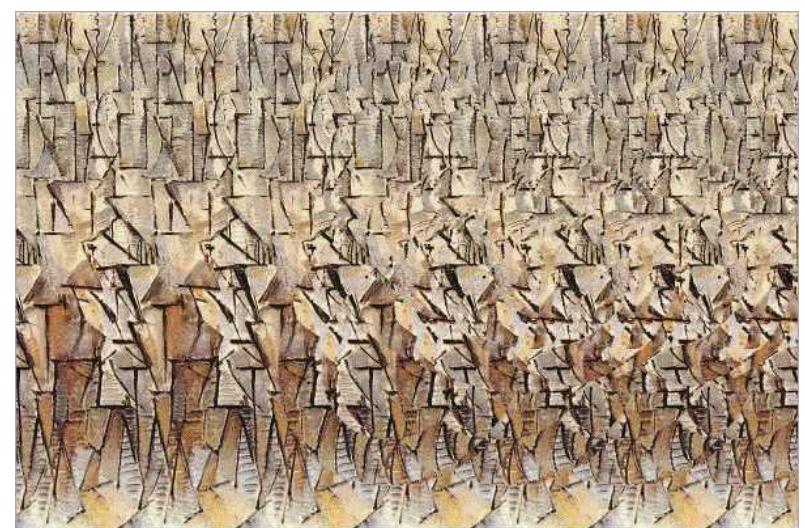
[Lvovski *et al.* (2001)]
 [Bimbard *et al.* (2014)]

Idea : measuring projections of Wigner distributions from different directions



Binocular vision
in phase space !

Find the hidden 3D picture



Quantum Field Theory

Covariant Wigner operator

$$\widehat{W}(k, r) = \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \phi^*(r - \frac{z}{2}) \phi(r + \frac{z}{2})$$

Scalar fields

Time ordering ?

Equal-time Wigner operator

$$\begin{aligned} \widehat{W}(\vec{k}, \vec{r}, t) &= \int dk^0 \widehat{W}(k, r) \\ &= \int \frac{d^3 z}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{z}} \phi^*(\vec{r} - \frac{\vec{z}}{2}, t) \phi(\vec{r} + \frac{\vec{z}}{2}, t) \end{aligned}$$

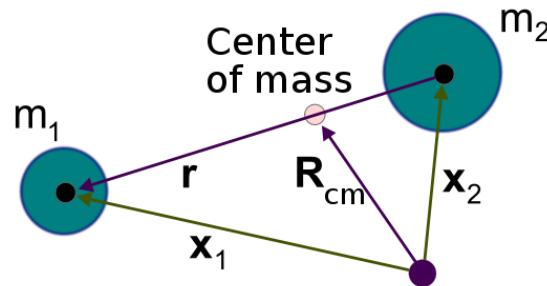
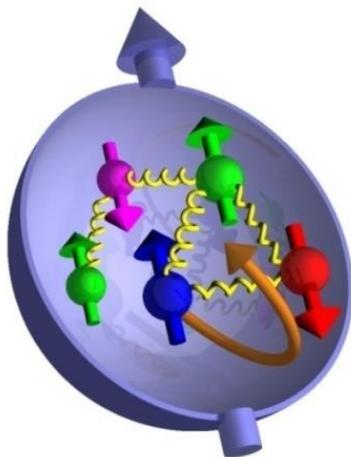
$$\boxed{\int \frac{dk^0}{2\pi} e^{ik^0 z^0} = \delta(z^0)}$$

Phase-space/Wigner distribution

$$\rho_W(\vec{k}, \vec{r}, t; \Psi) = \langle \Psi | \widehat{W}(\vec{k}, \vec{r}, t) | \Psi \rangle$$

Nucleons are composite systems

~ internal motion \otimes center-of-mass motion



Localized state in momentum space

$$|\Psi\rangle = |\vec{P}\rangle$$

CoM momentum $\vec{P} = \sum_i \vec{k}_i$

in position space

$$|\Psi\rangle = |\vec{R}\rangle$$

CoM position $\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

$$= \int \frac{d^3 P}{(2\pi)^3} e^{-i\vec{P}\cdot\vec{R}} |\vec{P}\rangle$$

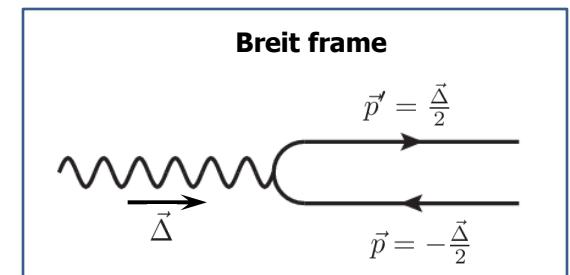
Phase-space compromise

$$\begin{aligned} \rho_W(\vec{k}, \vec{r}, t; \vec{P}, \vec{R}) &= \int \frac{d^3 \Delta}{(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{R}} \langle \vec{P} + \frac{\vec{\Delta}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | \vec{P} - \frac{\vec{\Delta}}{2} \rangle \\ &= \int d^3 Z e^{i\vec{P}\cdot\vec{Z}} \langle \vec{R} - \frac{\vec{Z}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | \vec{R} + \frac{\vec{Z}}{2} \rangle \end{aligned}$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k}, \vec{r}, t; \vec{0}, \vec{0}) = \int \frac{d^3 \Delta}{(2\pi)^3} \langle \frac{\vec{\Delta}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | -\frac{\vec{\Delta}}{2} \rangle$$

Identified with
intrinsic variables



Localized state in momentum space

$$|\Psi\rangle = |\vec{P}\rangle$$

CoM momentum $\vec{P} = \sum_i \vec{k}_i$

in position space

$$|\Psi\rangle = |\vec{R}\rangle$$

CoM position $\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

$$= \int \frac{d^3 P}{(2\pi)^3} e^{-i\vec{P}\cdot\vec{R}} |\vec{P}\rangle$$

Phase-space compromise

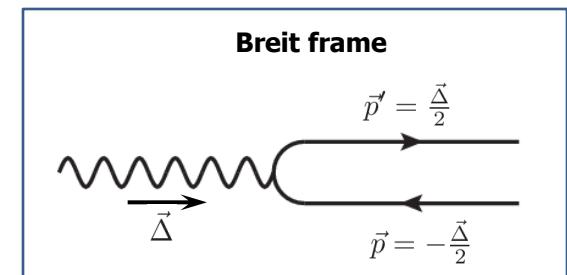
$$\begin{aligned} \rho_W(\vec{k}, \vec{r}, t; \vec{P}, \vec{R}) &= \int \frac{d^3 \Delta}{(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{R}} \langle \vec{P} + \frac{\vec{\Delta}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | \vec{P} - \frac{\vec{\Delta}}{2} \rangle \\ &= \int d^3 Z e^{i\vec{P}\cdot\vec{Z}} \langle \vec{R} - \frac{\vec{Z}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | \vec{R} + \frac{\vec{Z}}{2} \rangle \end{aligned}$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k}, \vec{r}, \cancel{0}; \vec{0}, \vec{0}) = \int \frac{d^3 \Delta}{(2\pi)^3} \langle \frac{\vec{\Delta}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | -\frac{\vec{\Delta}}{2} \rangle$$

↑ ↑
Same energy !

Identified with
intrinsic variables



Time translation $\hat{O}(t) = e^{i\hat{H}t} \hat{O}(0) e^{-i\hat{H}t}$

All is fine as long as space-time symmetry is **Galilean**

$$[J^i, J^j] = i\epsilon^{ijk} J^k$$

$$[J^i, B^j] = i\epsilon^{ijk} B^k$$

$$[B^i, B^j] = 0$$

$$[B^i, H] = -iP^i$$

$$[B^i, P^j] = -i\delta^{ij} M$$

$$[B^i, M] = [J^i, M] = 0$$

Position operator can be defined

$$B^i = -MR^i$$



$$[R^i, P^j] = i\delta^{ij} \mathbb{1}$$

$$[R^i, R^j] = 0$$

$$[J^i, R^j] = i\epsilon^{ijk} R^k$$

But in relativity, space-time symmetry is Lorentzian

$$J^i = \frac{1}{2} \epsilon^{ijk} M^{jk}$$

$$K^i = M^{0i}$$

$$[J^i, J^j] = i\epsilon^{ijk} J^k$$

$$[K^i, P^0] = -iP^i$$

$$[J^i, K^j] = i\epsilon^{ijk} K^k$$

$$[K^i, P^j] = -i\delta^{ij} P^0$$

$$[K^i, K^j] = -i\epsilon^{ijk} J^k$$

Position operator is ill-defined !

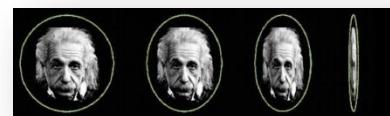


No separation of CoM
and internal coordinates

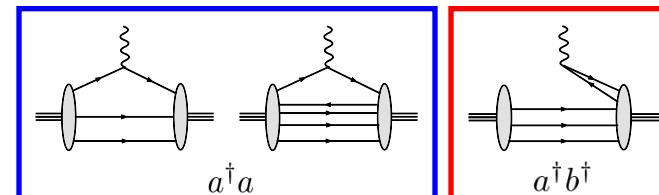


Further issues :

Lorentz contraction



Creation/annihilation of pairs

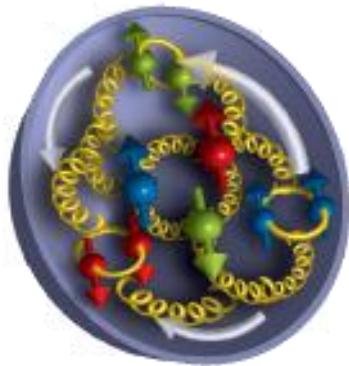


Spoils (quasi-)
probabilistic
interpretation

Summary

Lecture 1

- Understanding nucleon internal structure is essential
- Concept of phase space can be generalized to QM and QFT
- Special Relativity spoils probabilistic interpretation



$$\widehat{W}(k, r) = \int \frac{d^4 z}{(2\pi)^4} e^{ik \cdot z} \phi^*(r - \frac{z}{2}) \phi(r + \frac{z}{2})$$

$$\rho_W(\vec{k}, \vec{r}, \cancel{\vec{0}}, \vec{0}) = \int \frac{d^3 \Delta}{(2\pi)^3} \langle \frac{\vec{\Delta}}{2} | \widehat{W}(\vec{k}, \vec{r}, t) | - \frac{\vec{\Delta}}{2} \rangle$$

