HUGS 2014

29th Annual Hampton University Graduate Studies Program

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Spin Sum Rules and 3D Nucleon Structure (2/6)

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Université de Liège

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Outline

Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz

Lecture 2

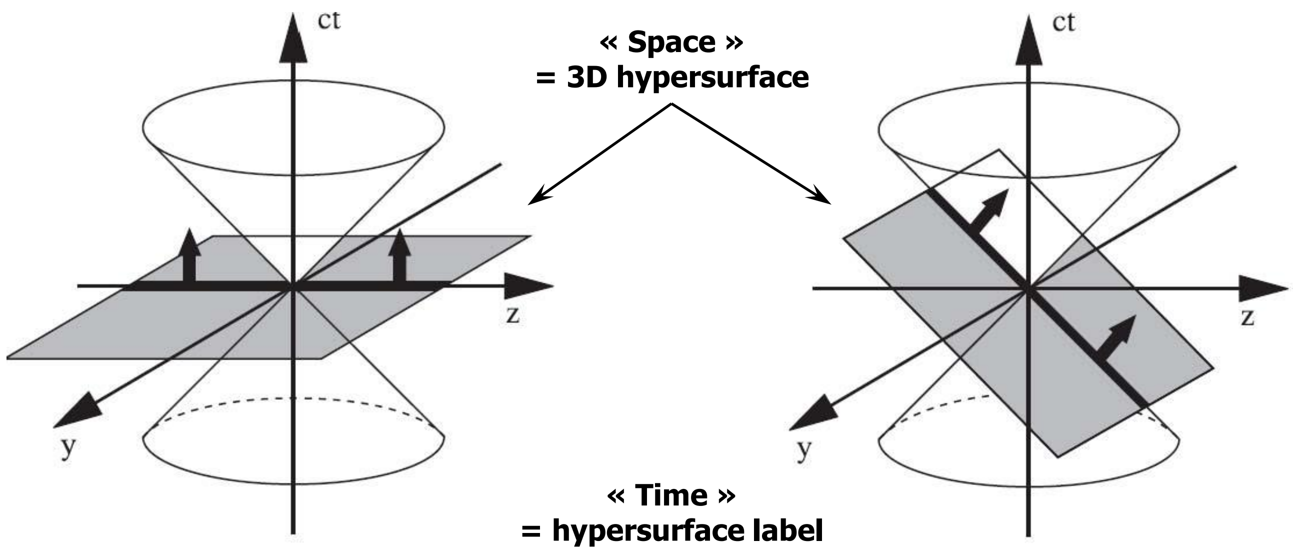
- **Photon point of view**
- **Galileo *vs* Lorentz : round 2**
- **Nucleon 1D picture**

Forms of dynamics

Space-time foliation

Light-front components

$$\boxed{a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^3)}$$

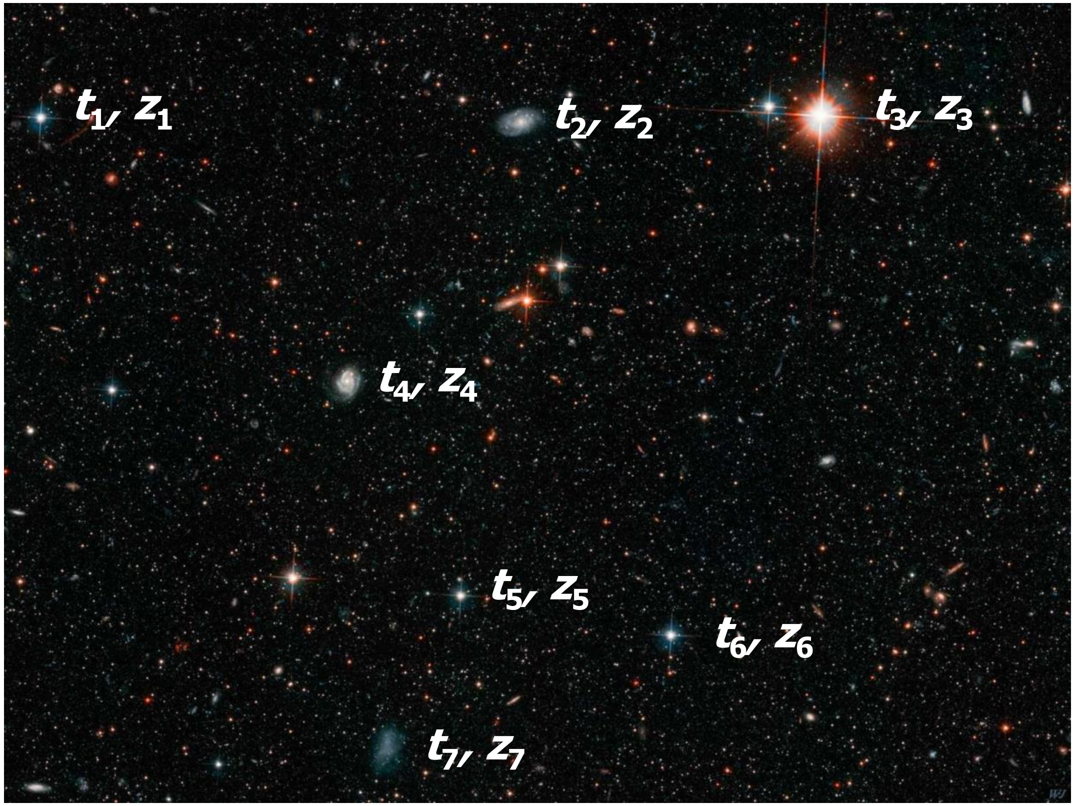


	Instant-form dynamics	**Light-front form dynamics**
Time	x^0	x^+
Space	$\vec{x}$	$\vec{x}_\perp, x^-$
Energy	p^0	p^-
Momentum	$\vec{p}$	$\vec{p}_\perp, p^+$

[Dirac (1949)]

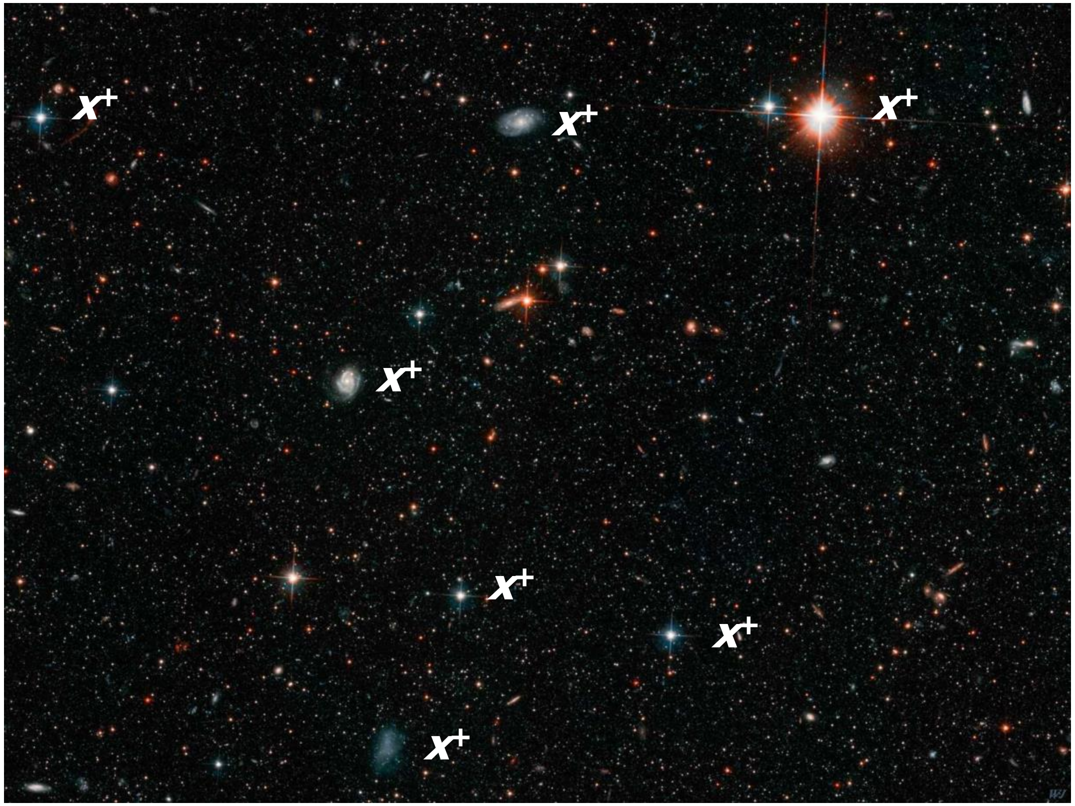
Instant form *vs* light-front form

Ordinary point of view

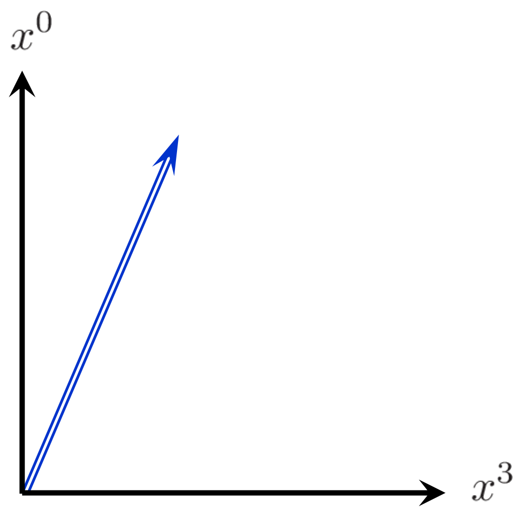


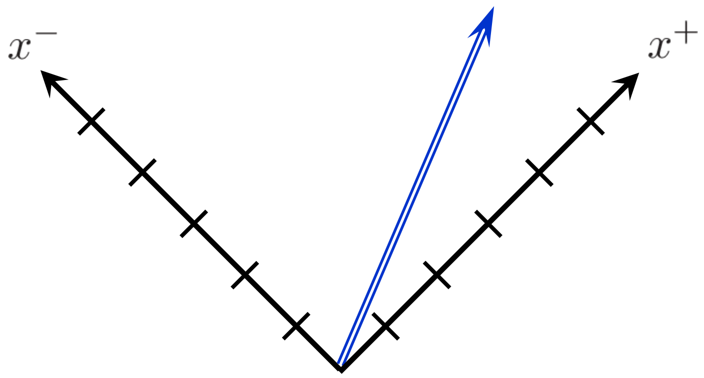
Photon point of view

$$\boxed{x^+ = \frac{1}{\sqrt{2}}(t+z)}$$

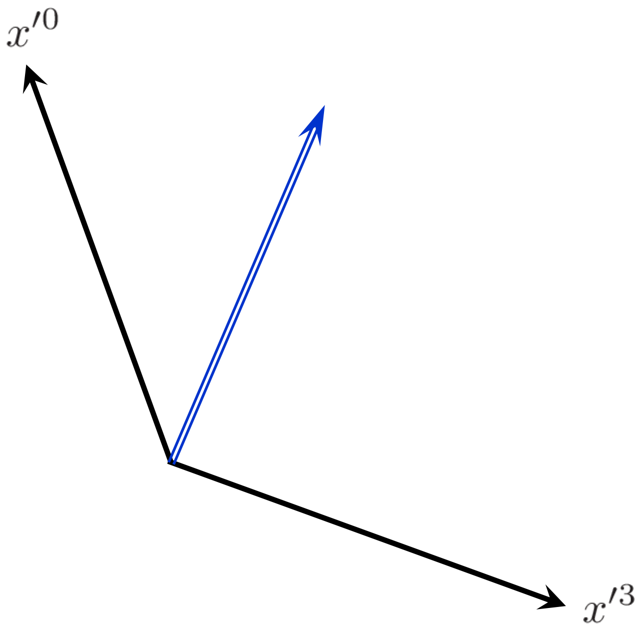


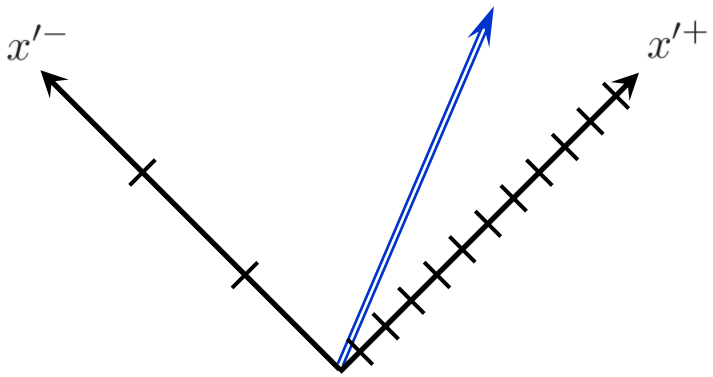
Initial frame





Boosted frame





$$x'^0 = \frac{x^0 + vx^3}{\sqrt{1-v^2}}$$

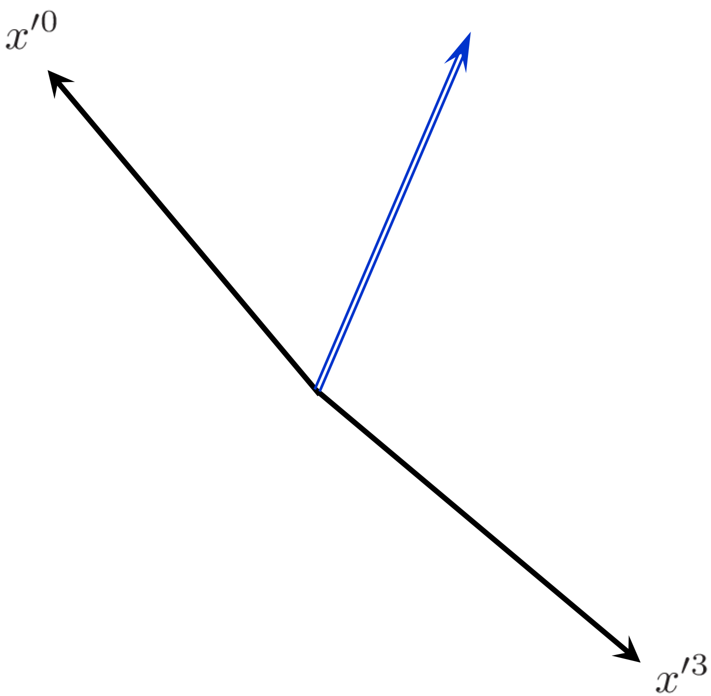
$$x'^3 = \frac{x^3 + vx^0}{\sqrt{1-v^2}}$$

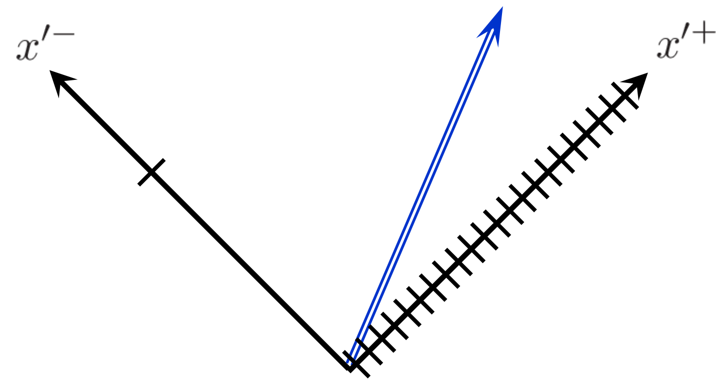
$$x'^+ = \sqrt{\frac{1+v}{1-v}}\, x^+$$

$$x'^- = \sqrt{\frac{1-v}{1+v}}\, x^-$$

Instant form *vs* light-front form

(Quasi) infinite-momentum frame





$$x'^0 \approx x'^3 \approx \frac{x'^+}{\sqrt{2}} \approx \frac{x^+}{\sqrt{1-v}} \qquad\qquad x'^- \approx 0$$

Light-front operators

$$K_\perp^1 = M^{+1} = \frac{1}{\sqrt{2}}(K^1 + J^2) \qquad J_\perp^1 = M^{-1} = \frac{1}{\sqrt{2}}(K^1 - J^2)$$

$$K_\perp^2 = M^{+2} = \frac{1}{\sqrt{2}}(K^2 - J^1) \qquad J_\perp^2 = M^{-2} = \frac{1}{\sqrt{2}}(K^2 + J^1)$$

Transverse space-time symmetry is Galilean

$$[J^3, J_\perp^i] = i\epsilon^{3ij} J_\perp^j \qquad [K_\perp^i, P^-] = -iP_\perp^i$$

$$[J^3, K_\perp^i] = i\epsilon^{3ij} K_\perp^j \qquad [K_\perp^i, P_\perp^j] = -i\delta_\perp^{ij} P^+$$

$$[K_\perp^i, K_\perp^j] = 0 \qquad [K_\perp^i, P^+] = [J^3, P^+] = 0$$

Transverse position operator can be defined !

$$\boxed{K_\perp^i = -P^+ R_\perp^i} \quad \Longrightarrow \quad \begin{aligned} [R_\perp^i, P_\perp^j] &= i\delta_\perp^{ij} \mathbb{1} \\ [R_\perp^i, R_\perp^j] &= 0 \\ [J^3, R_\perp^i] &= i\epsilon^{3ij} R_\perp^j \end{aligned}$$

Longitudinal momentum plays the role of mass in the transverse plane $\quad P^+ \sim M$

Transverse boost $\quad p'^+ = p^+ \qquad \vec{p}'_\perp = \vec{p}_\perp + p^+ \vec{v}_\perp$

[Kogut, Soper (1970)]

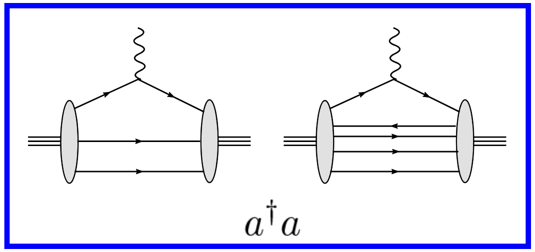
Quasi-probabilistic interpretation

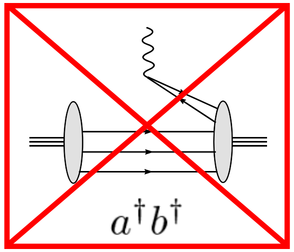
What about the further issues with Special Relativity ?

- **Transverse boosts are Galilean** → **No transverse Lorentz contraction !**

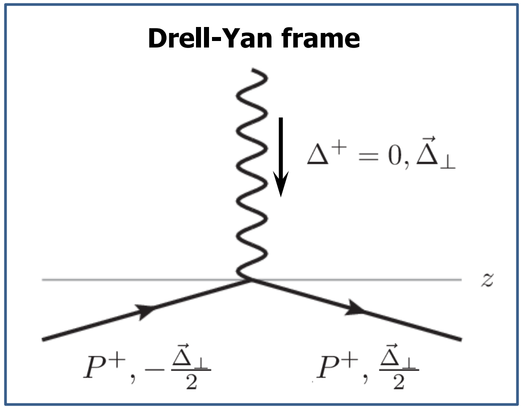
  $\int \mathrm{d}r^-$ → **No sensitivity to longitudinal Lorentz contraction !**

- **Particle number is conserved in Drell-Yan frame**





k^+ **is conserved and positive**



→ **Conclusion : quasi-probabilistic interpretation is possible !** ←

Non-relativistic phase space

Localized state in momentum space $|\Psi\rangle = |\vec{P}\rangle$

CoM momentum $\vec{P} = \sum_i \vec{k}_i$

in position space $|\Psi\rangle = |\vec{R}\rangle$

$$= \int \frac{\mathrm{d}^3 P}{(2\pi)^3}\, e^{-i\vec{P}\cdot\vec{R}}\, |\vec{P}\rangle$$

CoM position $\vec{R} = \dfrac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$

Equal-time Wigner operator

$$\widehat{W}(\vec{k},\vec{r},t) = \int \mathrm{d}k^0\, \widehat{W}(k,r)$$

Intrinsic phase-space/Wigner distribution

$$\rho_W(\vec{k},\vec{r},\cancel{t},\vec{0},\vec{0}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, \langle \tfrac{\vec{\Delta}}{2}|\widehat{W}(\vec{k},\vec{r},t)| -\tfrac{\vec{\Delta}}{2}\rangle$$

Identified with intrinsic variables

Time translation $\widehat{O}(t) = e^{i\widehat{H}t}\,\widehat{O}(0)\,e^{-i\widehat{H}t}$

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Relativistic phase space

Localized state in momentum space $|\Psi\rangle = |P^+, \vec{P}_\perp\rangle$

« CoM » momentum $\vec{P}_\perp = \sum_i \vec{k}_{i\perp}$

in position space $|\Psi\rangle = |P^+, \vec{R}_\perp\rangle$

$$= \int \frac{\mathrm{d}^2 P_\perp}{(2\pi)^2}\, e^{-i\vec{P}_\perp \cdot \vec{R}_\perp} |P^+, \vec{P}_\perp\rangle$$

« CoM » position $\vec{R}_\perp = \frac{\sum_i k_i^+ \vec{r}_{i\perp}}{\sum_i k_i^+}$

[Soper (1977)]
[Burkardt (2000)]
[Burkardt (2003)]

Equal light-front time Wigner operator

$$\widehat{W}(k^+, \vec{k}_\perp, r^-, \vec{r}_\perp, r^+) = \int \mathrm{d}k^- \, \widehat{W}(k, r)$$

[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

Intrinsic relativistic phase-space/Wigner distribution

$$\rho_W(x, \vec{k}_\perp, \vec{b}_\perp) = \frac{P^+}{\langle P^+|P^+\rangle} \int \mathrm{d}r^- \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \langle P^+, \tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+, \vec{k}_\perp, r^-, \vec{b}_\perp, r^+)|P^+, -\tfrac{\vec{\Delta}_\perp}{2}\rangle$$

$$= \frac{1}{2} \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \langle P^+, \tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+, \vec{k}_\perp, 0, \vec{b}_\perp, 0)|P^+, -\tfrac{\vec{\Delta}_\perp}{2}\rangle$$

$x = \frac{k^+}{P^+}$

Boost invariant !

$$\int \mathrm{d}k^+ \, f(k^+) = \int \mathrm{d}x \, P^+ f(xP^+)$$

Space-time translation $\widehat{O}(r) = e^{i\widehat{P}\cdot r}\, \widehat{O}(0)\, e^{-i\widehat{P}\cdot r}$

Normalization $\langle P^+|P^+\rangle = 2P^+\, 2\pi\, \delta(0)$

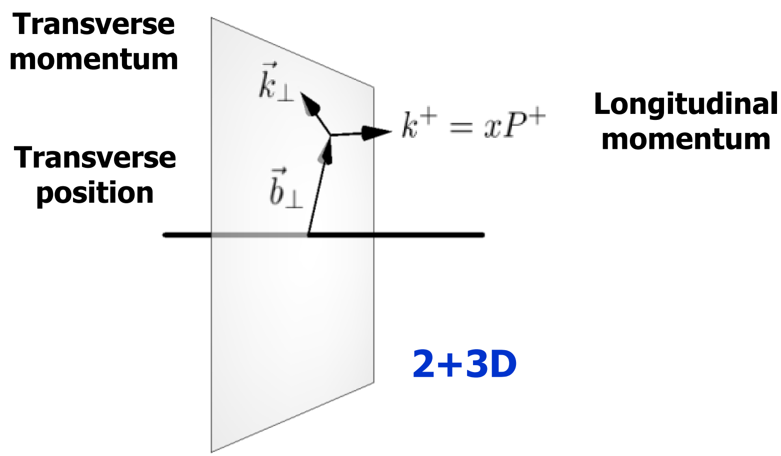
[C.L., Pasquini (2011)]

Relativistic phase space

Our intuition is instant form and not light-front form

NB : $\rho_W(x,\vec{k}_\perp,\vec{b}_\perp)$ **is invariant under light-front boosts**

→ **It can be thought as instant form phase-space/Wigner distribution in IMF !**



In IMF, the nucleon looks like a pancake 

[C.L., Pasquini (2011)]

Link with parton correlators

Phase-space/Wigner distributions are Fourier transforms

$$\rho_W(x,\vec{k}_\perp,\vec{b}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,\langle P^+,\tfrac{\vec{\Delta}_\perp}{2}|\widehat{W}(xP^+,\vec{k}_\perp,0,\vec{b}_\perp,0)|P^+,-\tfrac{\vec{\Delta}_\perp}{2}\rangle$$

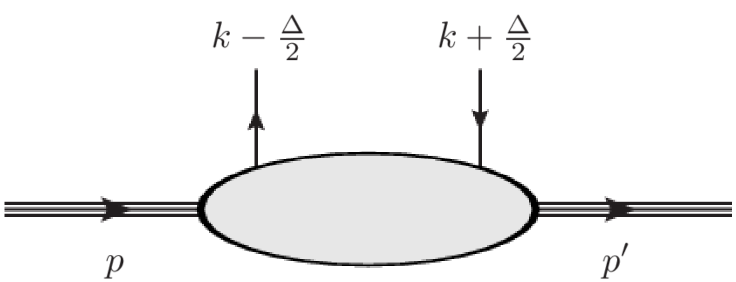
$$= \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,W(x,0,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Space-time translation

$$\widehat{O}(r) = e^{i\widehat{P}\cdot r}\,\widehat{O}(0)\,e^{-i\widehat{P}\cdot r}$$

General parton correlator

$$W(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p'|\phi^*(-\tfrac{z}{2})\,\phi(\tfrac{z}{2})|p\rangle\big|_{z^+=0}$$



$$P = \tfrac{1}{2}(p'+p) \qquad k^+ = xP^+$$

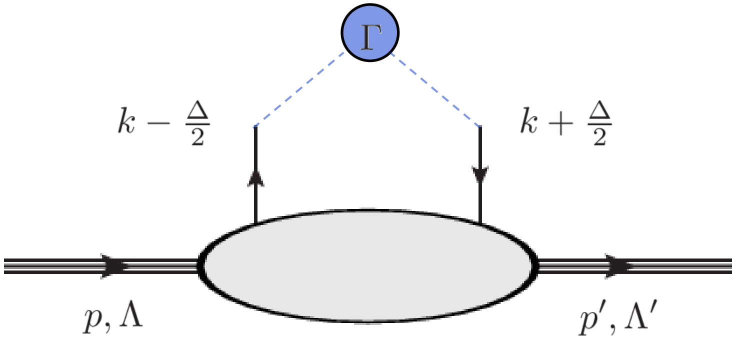
$$\Delta = p'-p \qquad \Delta^+ = -2\xi P^+$$

Link with parton correlators

Adding spin to the picture

$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\, e^{ik\cdot z}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\big|_{z^+=0}$$

Dirac matrix
~ quark polarization



Leading twist

$$\gamma^+ \sim \delta_{\lambda'\lambda}$$
$$\gamma^+\gamma_5 \sim (\tau_3)_{\lambda'\lambda}$$
$$i\sigma^{j+}\gamma_5 \sim (\tau_j)_{\lambda'\lambda}$$

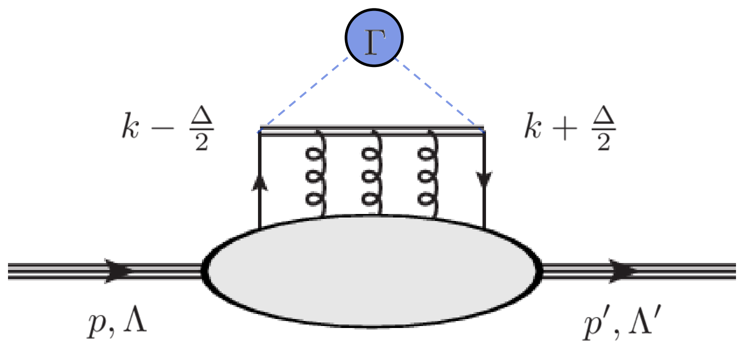
Link with parton correlators

Adding spin and color to the picture

$$W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp;\mathcal{W})=\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{-\frac{z}{2}\,\frac{z}{2}}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\big|_{z^+=0}$$

Wilson line

$$\mathcal{W}_{ba}=\mathcal{P}\left[e^{ig\int_a^b \mathrm{d}x^-A^+}\right]$$



Gauge transformation

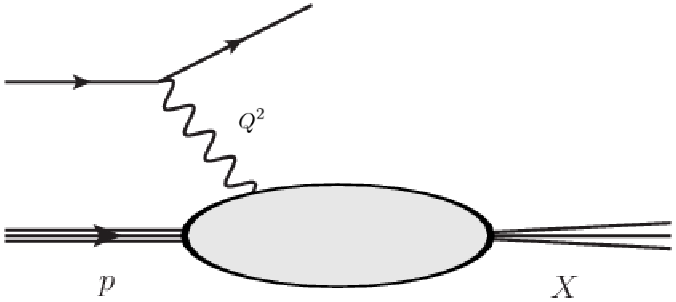
$$\psi(x)\mapsto U(x)\psi(x)$$
$$\mathcal{W}_{yx}\mapsto U(y)\mathcal{W}_{yx}U^{-1}(x)$$

Very very very complicated object not directly measurable

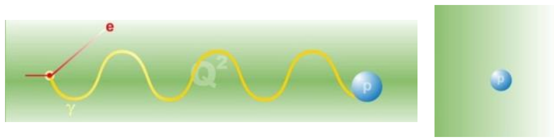
➡ **Let's look for simpler measurable correlators**

Deep inelastic scattering

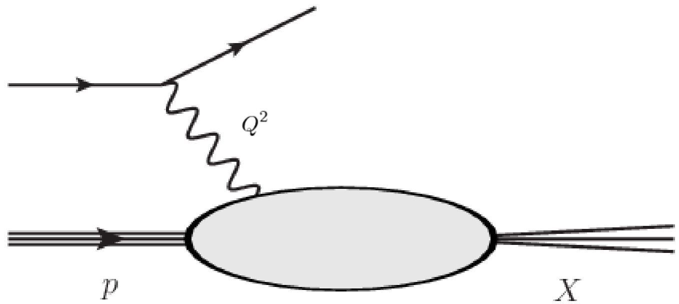
Coherent scattering



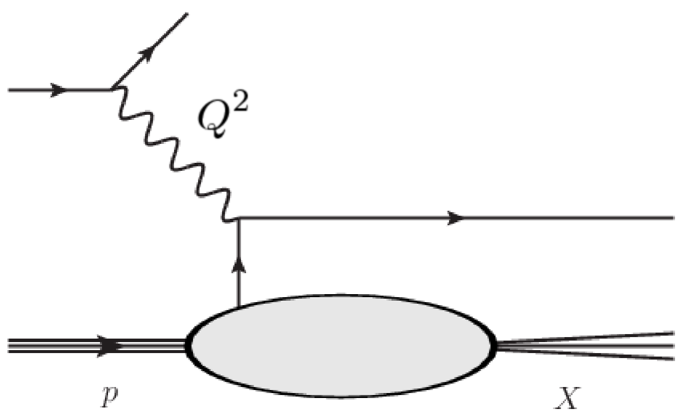
Deep inelastic scattering



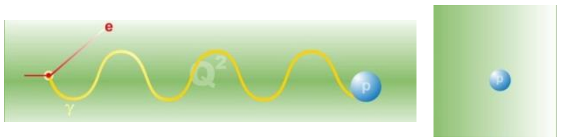
Coherent scattering



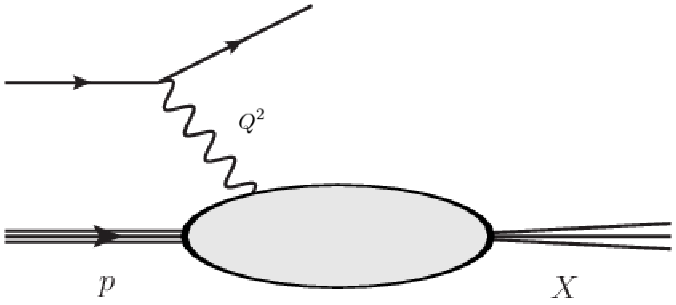
Incoherent scattering



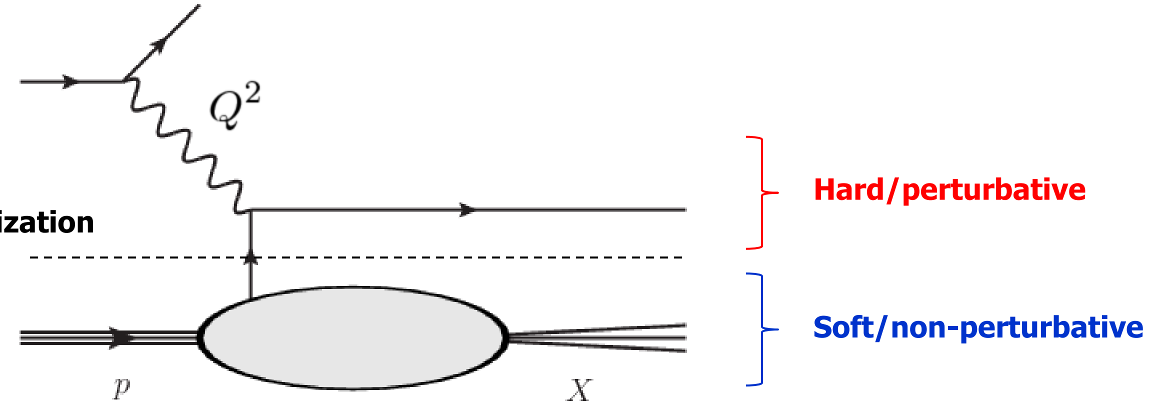
Deep inelastic scattering



Coherent scattering

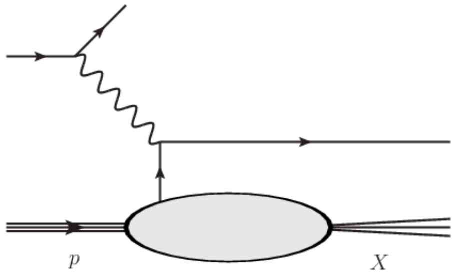


Incoherent scattering



Deep inelastic scattering

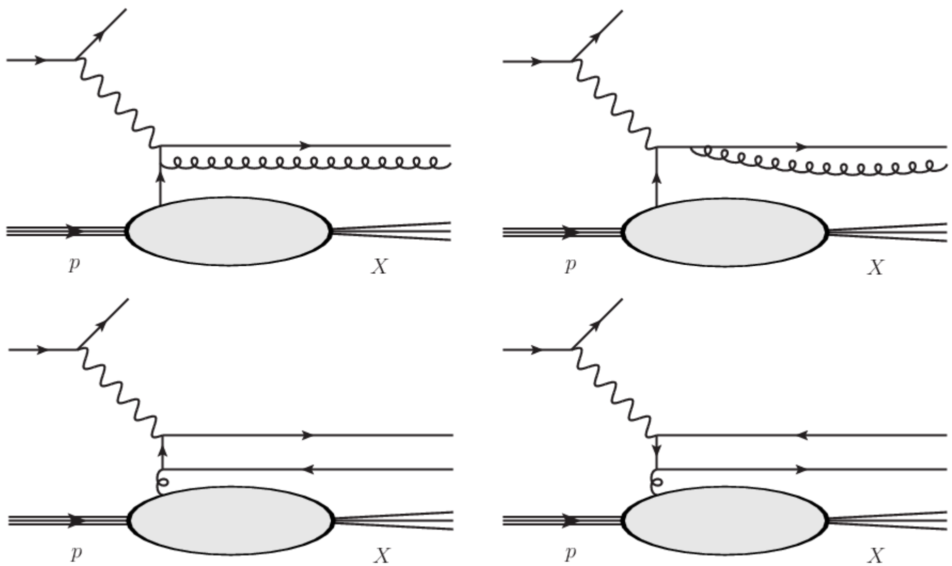
Parton model



$$\mathrm{d}\sigma^{lp\to lX} \sim \sum_i f_i(x)$$

Probability to find parton i with momentum fraction x

QCD corrections



+ virtual diagrams

$$\boxed{\mathrm{d}\sigma^{lp\to lX} \sim \sum_i \int_x^1 \frac{\mathrm{d}y}{y}\, C_i\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R)\right) f_i(y, \mu_F^2)}$$

Perturbative Process-dependent

Non-perturbative Process-independent

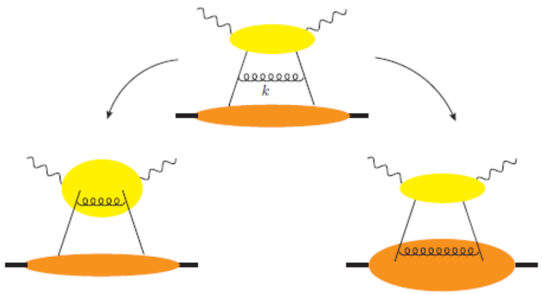
$i = q_f, \bar{q}_f, g$

$\mu_F \approx \mu_R$

Deep inelastic scattering

Cross-section must not depend on μ

$$\frac{d\sigma^{lp\to lX}}{d\ln\mu} = 0$$

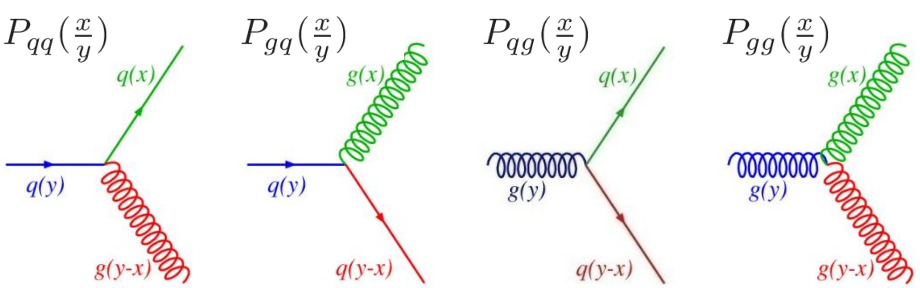


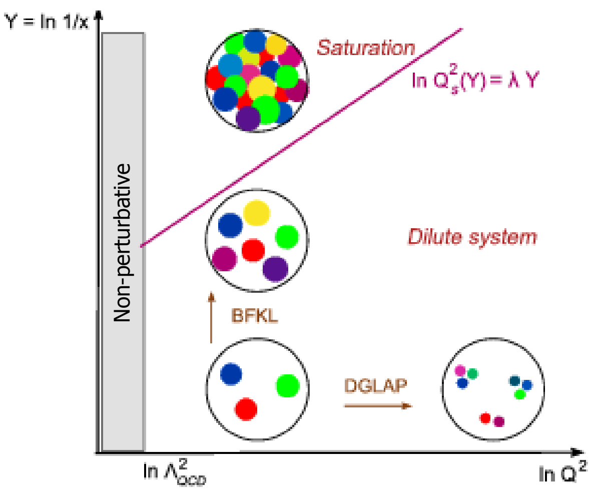
→ **DGLAP evolution equations**

$$\frac{d}{d\ln\mu} f_i(x,\mu^2) = \sum_j \int_x^1 \frac{dy}{y} P_{ij}(\tfrac{x}{y})\, f_j(y,\mu^2)$$

$$\frac{d}{d\ln\mu} C_i(x,\tfrac{Q^2}{\mu^2}) = -\sum_j \int_x^1 \frac{dy}{y} C_j(y,\tfrac{Q^2}{\mu^2})\, P_{ji}(\tfrac{x}{y})$$

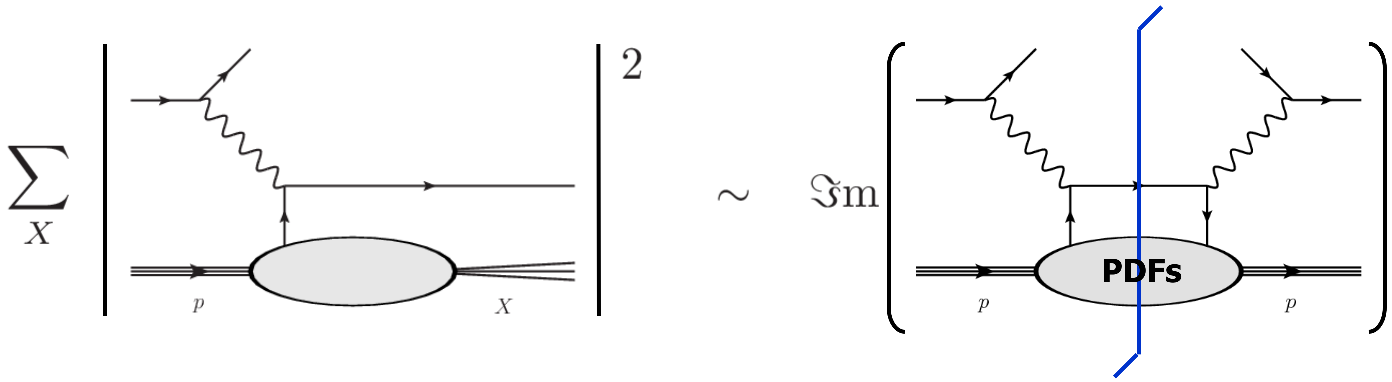
Splitting functions





Parton distribution functions (PDFs)

Optical theorem



PDF correlator

$$\mathcal{F}^{[\Gamma]}_{\Lambda'\Lambda}(x) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\,\langle P,\Lambda'|\overline{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|P,\Lambda\rangle\big|_{z^+=z_\perp=0}$$

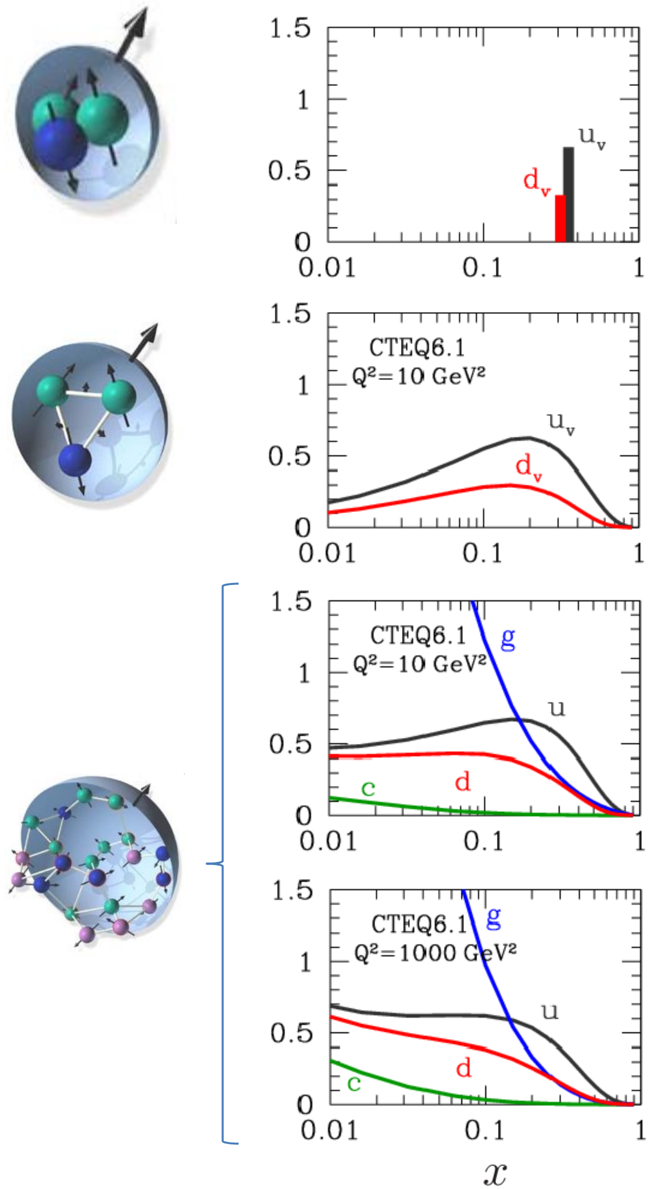
$$= \int \mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

Parametrization

$$\mathcal{F}^{[\Gamma]}_{\Lambda'\Lambda}(x) = \sum_i \left[\overline{u}(P,\Lambda')\,\Gamma_i\,u(P,\Lambda)\right]\,\mathrm{PDF}_i(x)$$

Constrained by Lorentz and discrete space-time symmetries

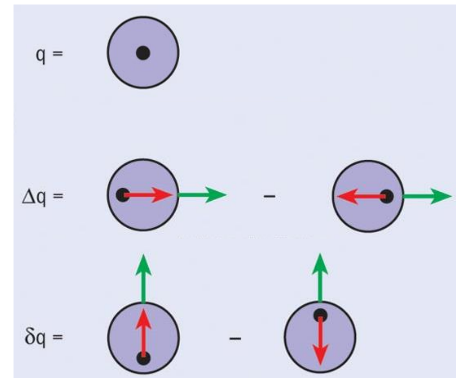
Parton distribution functions (PDFs)

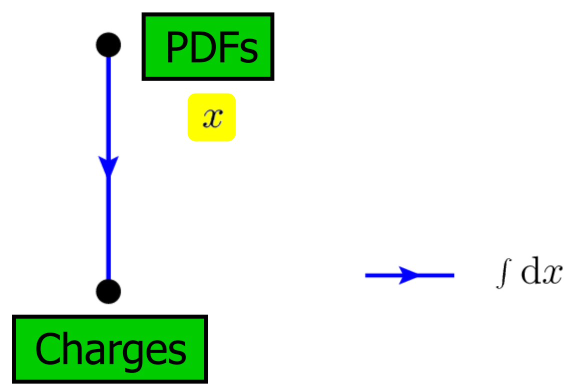


Vector
Quark number

Axial
Quark helicity

Tensor
Quark transversity

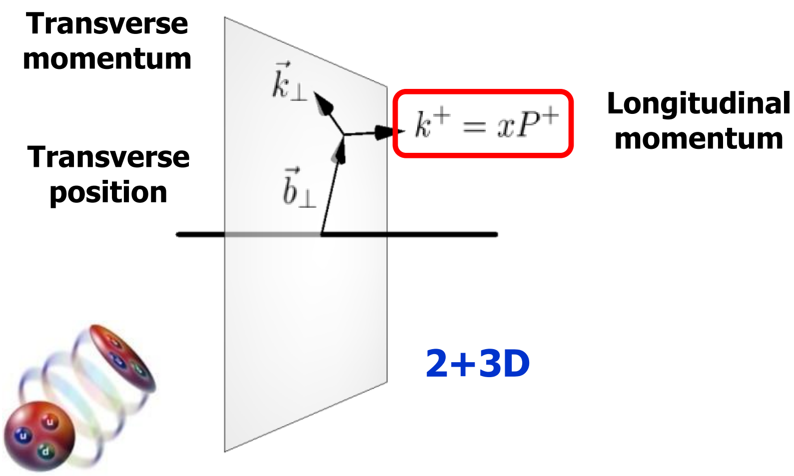




Summary

Lecture 2

- **Light-front coordinates are very convenient**
- **Relativistic phase space has 2+3 dimensions**
- **PDFs provide 1D pictures of the nucleon**



$$\mathrm{PDF}(x) = \int \mathrm{d}^2k_\perp\, \mathrm{d}^2b_\perp\, \rho_W(x, \vec{k}_\perp, \vec{b}_\perp)$$