Applications of Renormalization Group Methods in Nuclear Physics – 3

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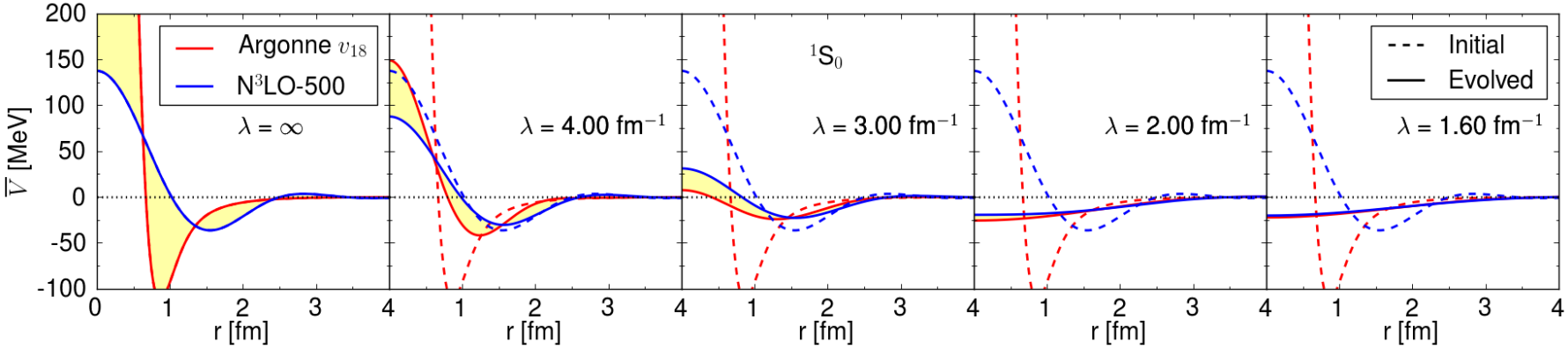
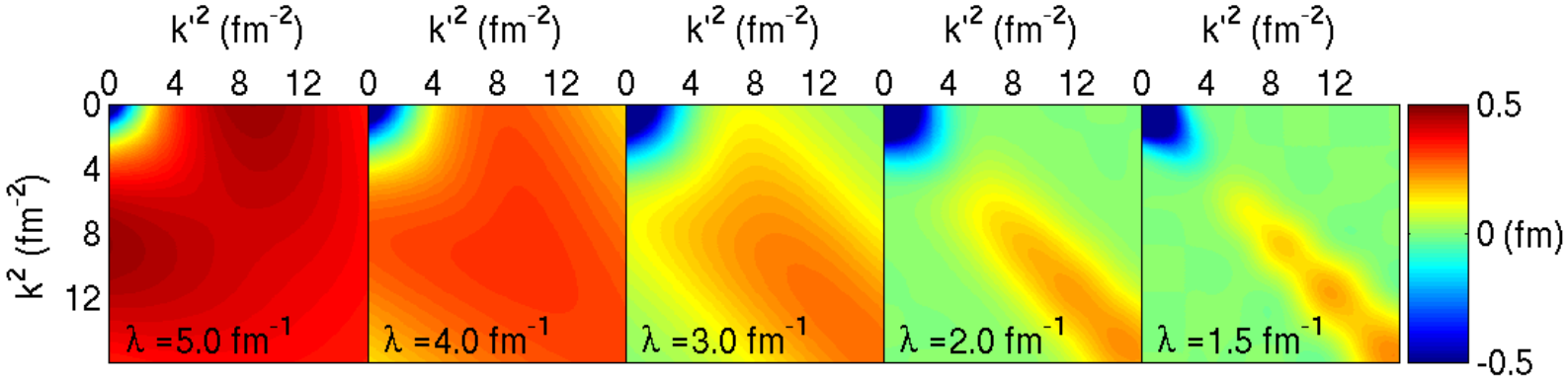
HUGS 2014





Outline: Lecture 3

Lecture 3: Effective field theory

- Recap from lecture 2: How SRG works
- Motivation for nuclear effective field theory
- Chiral effective field theory
- Universal potentials from RG evolution
- Extra: Quantitative measure of perturbativeness

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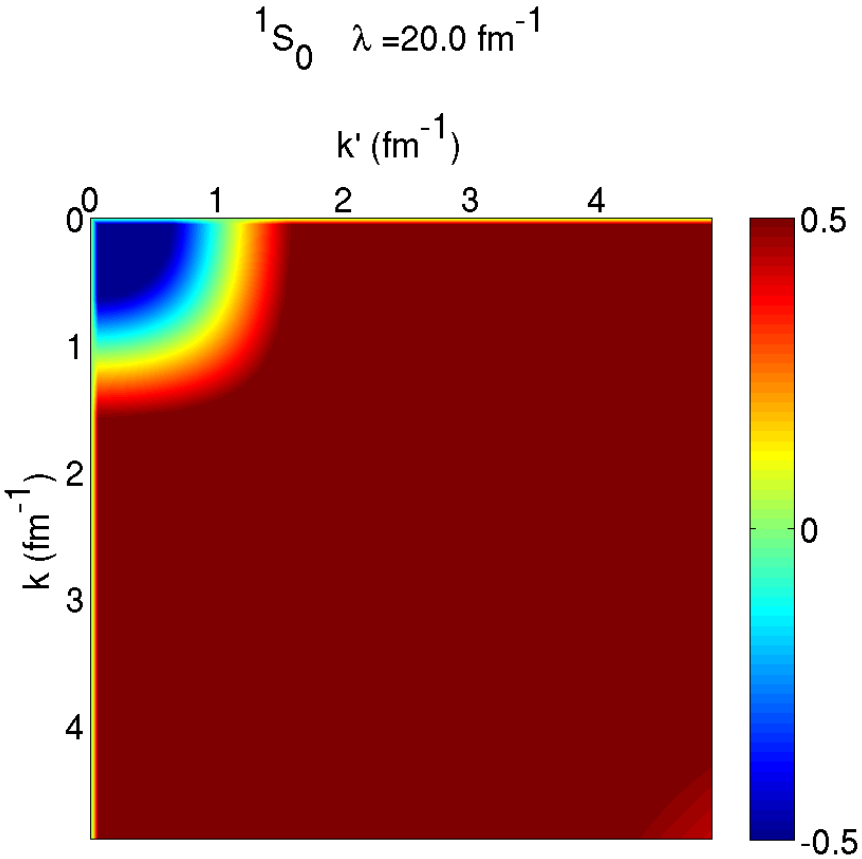
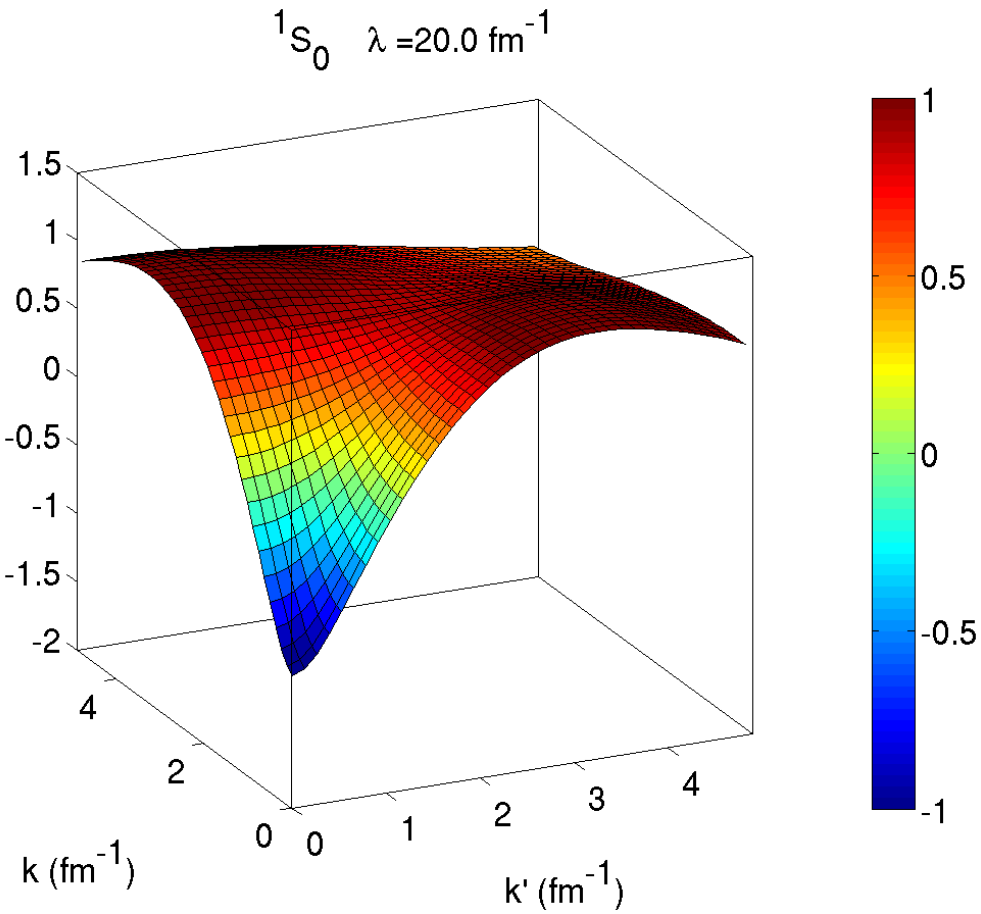
Flow equations in action: NN only

- In each partial wave with $\epsilon_k = \hbar^2 k^2/M$ and $\lambda^2 = 1/\sqrt{s}$

$$\frac{dV_\lambda}{d\lambda}(k,k') \propto -(\epsilon_k - \epsilon_{k'})^2 V_\lambda(k,k') + \sum_q (\epsilon_k + \epsilon_{k'} - 2\epsilon_q) V_\lambda(k,q) V_\lambda(q,k')$$





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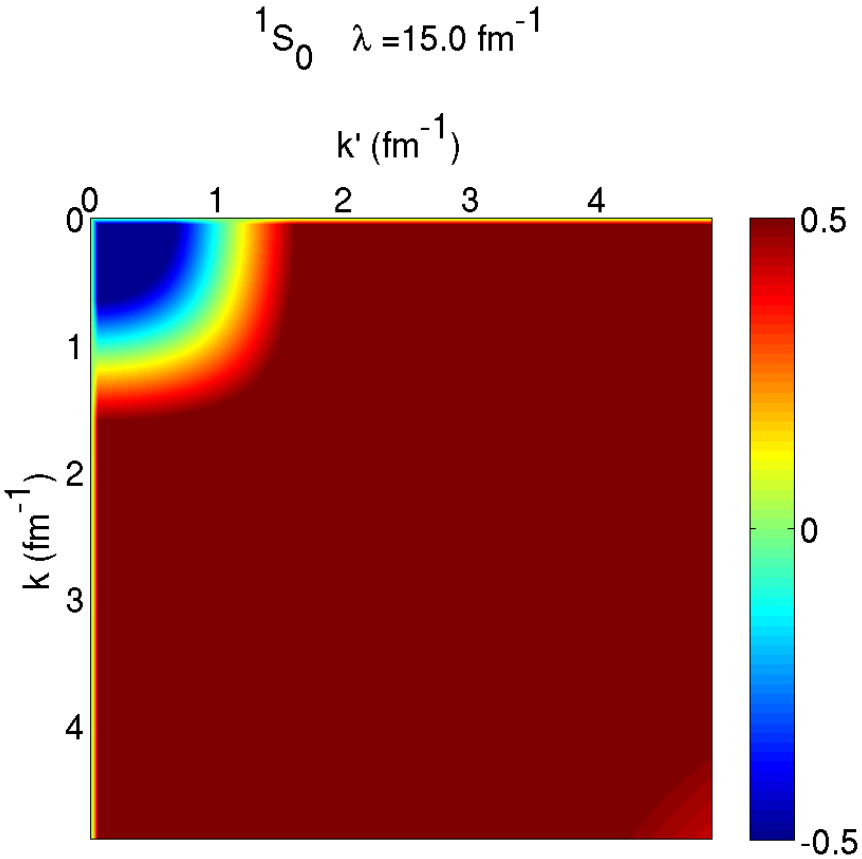

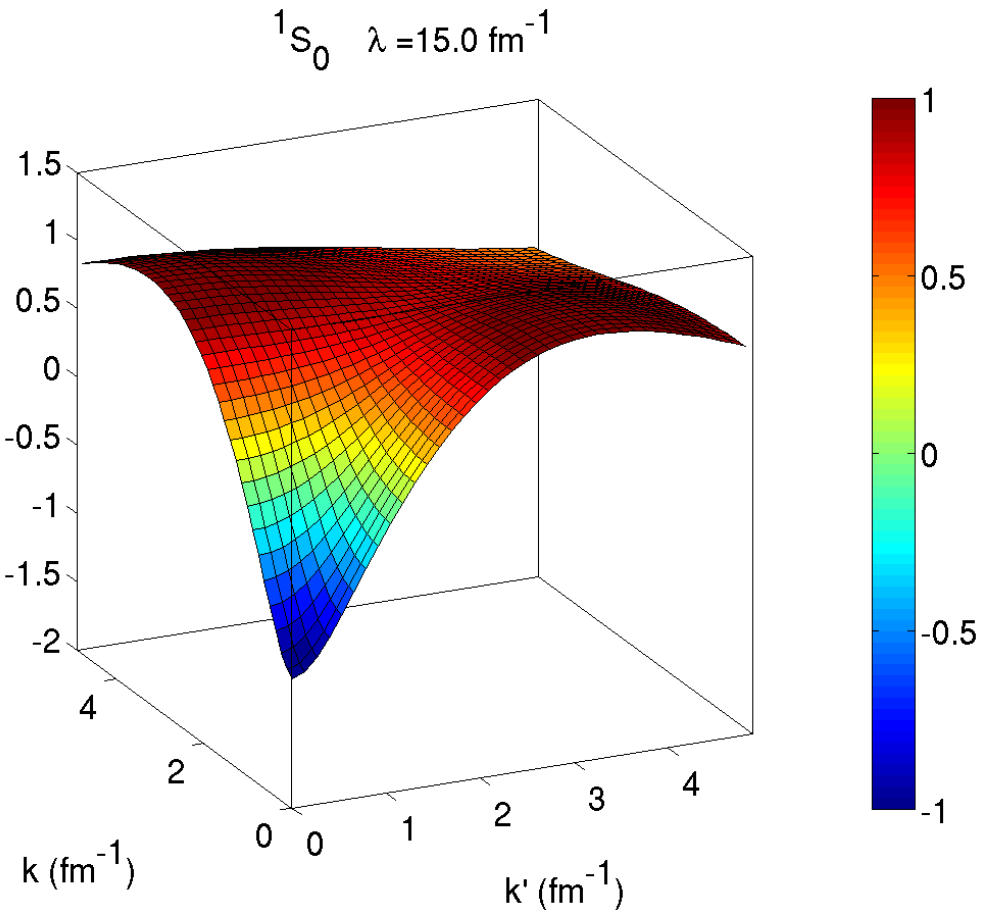

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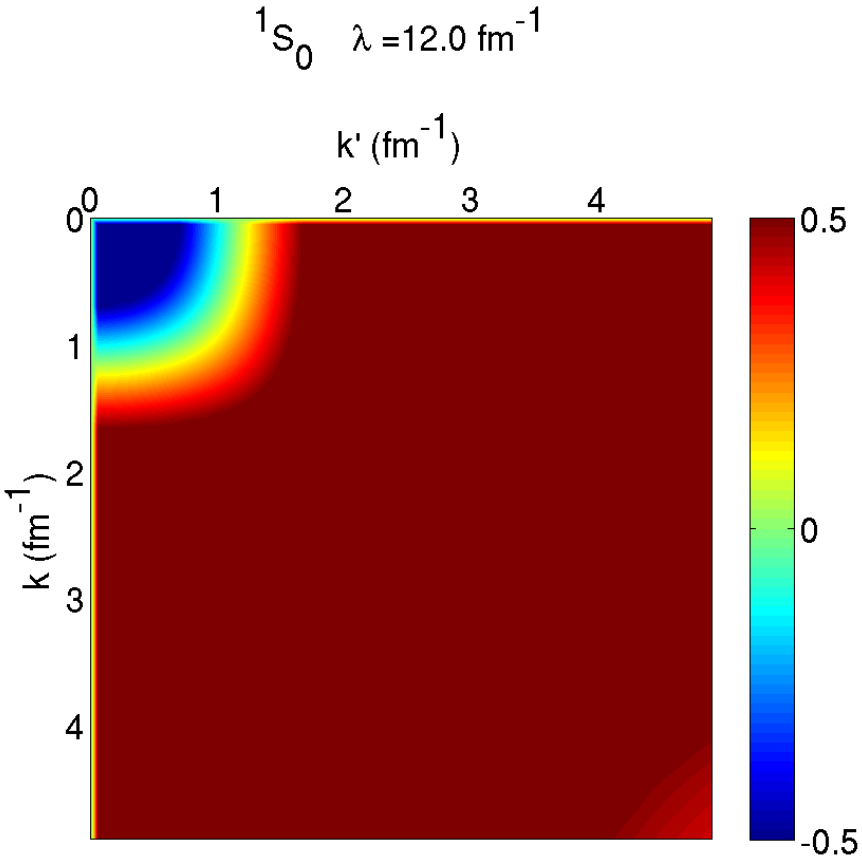

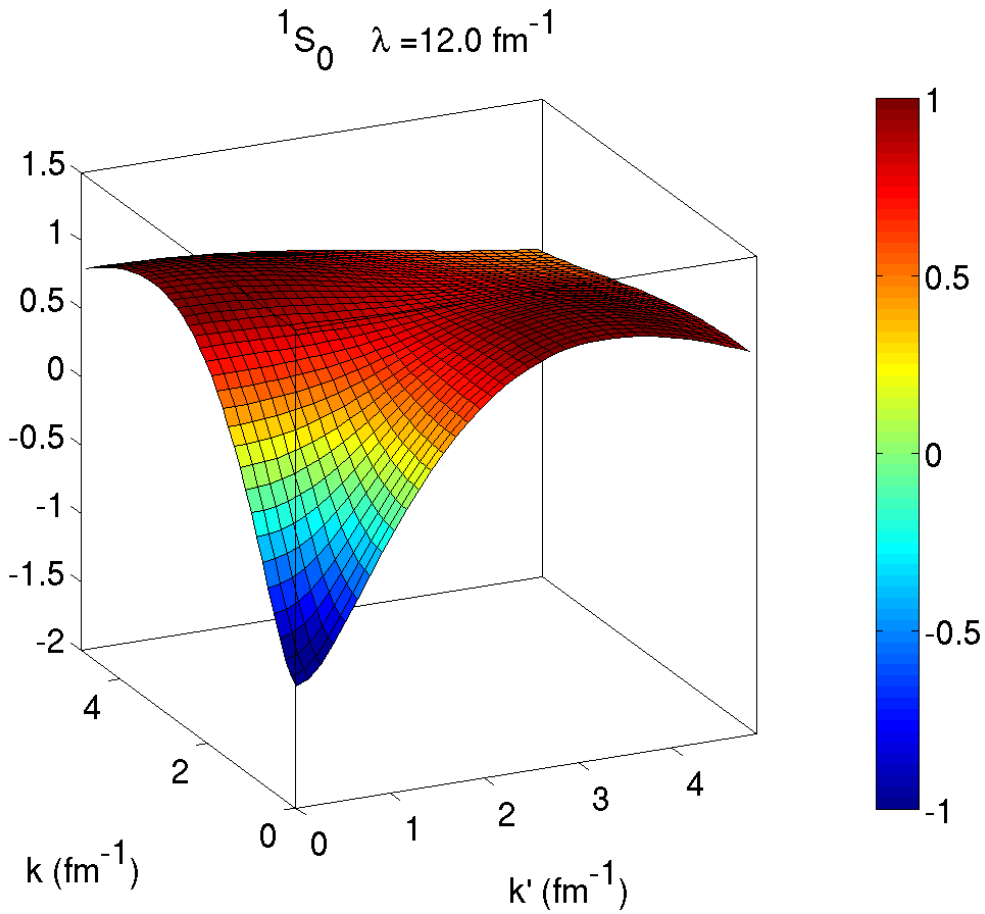

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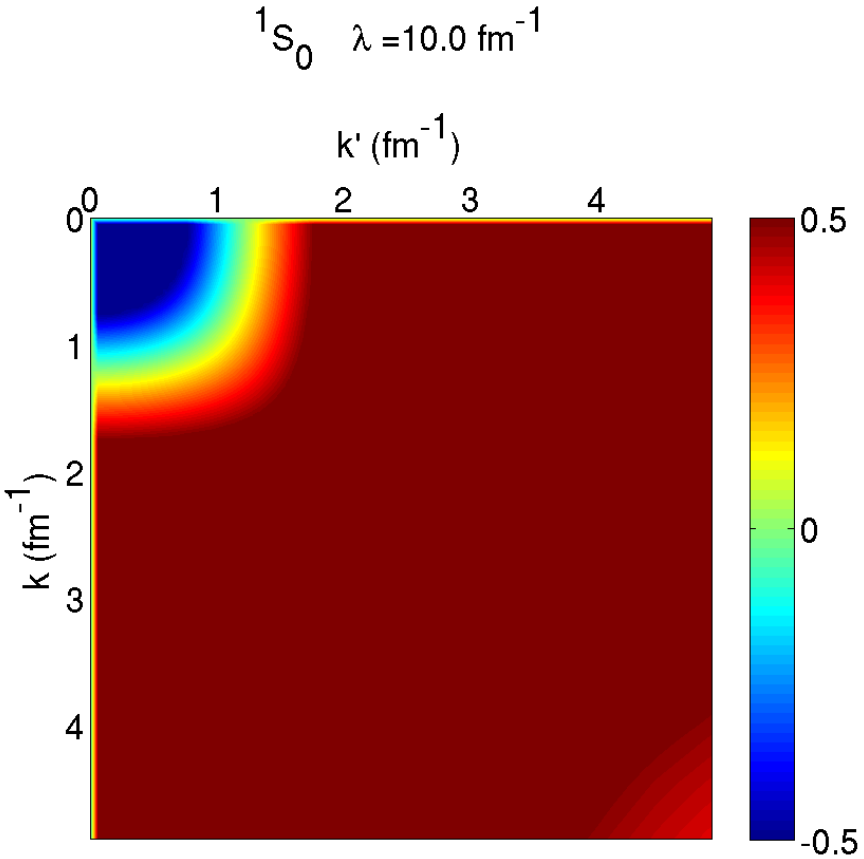

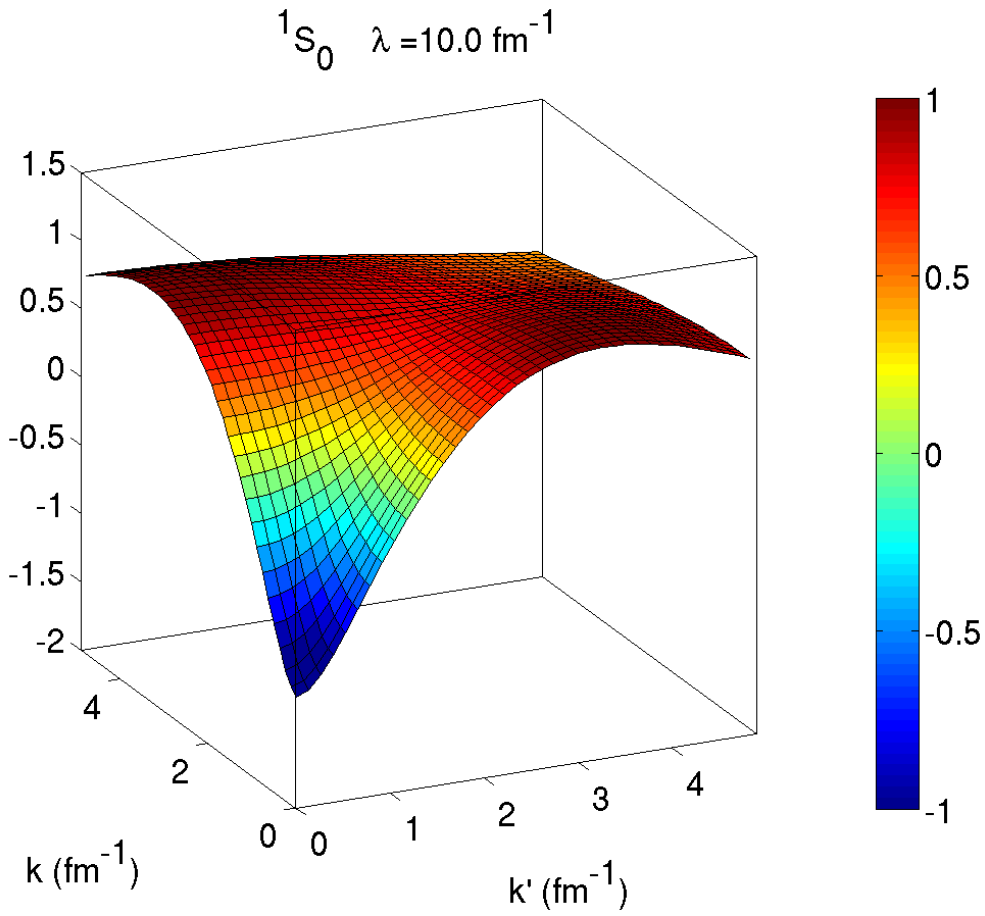

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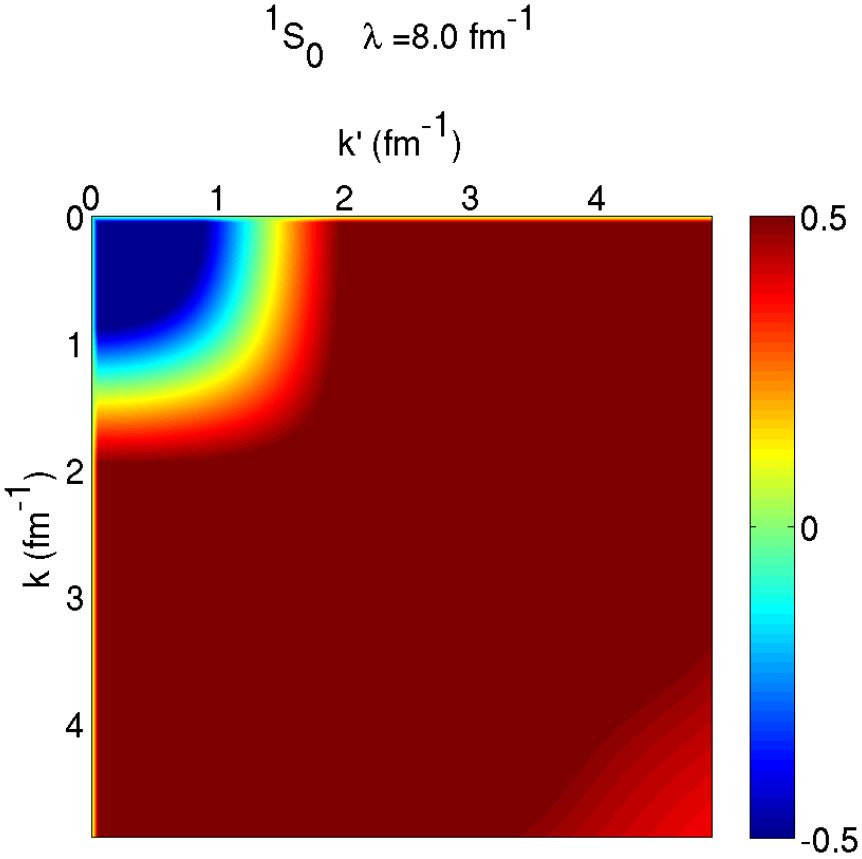

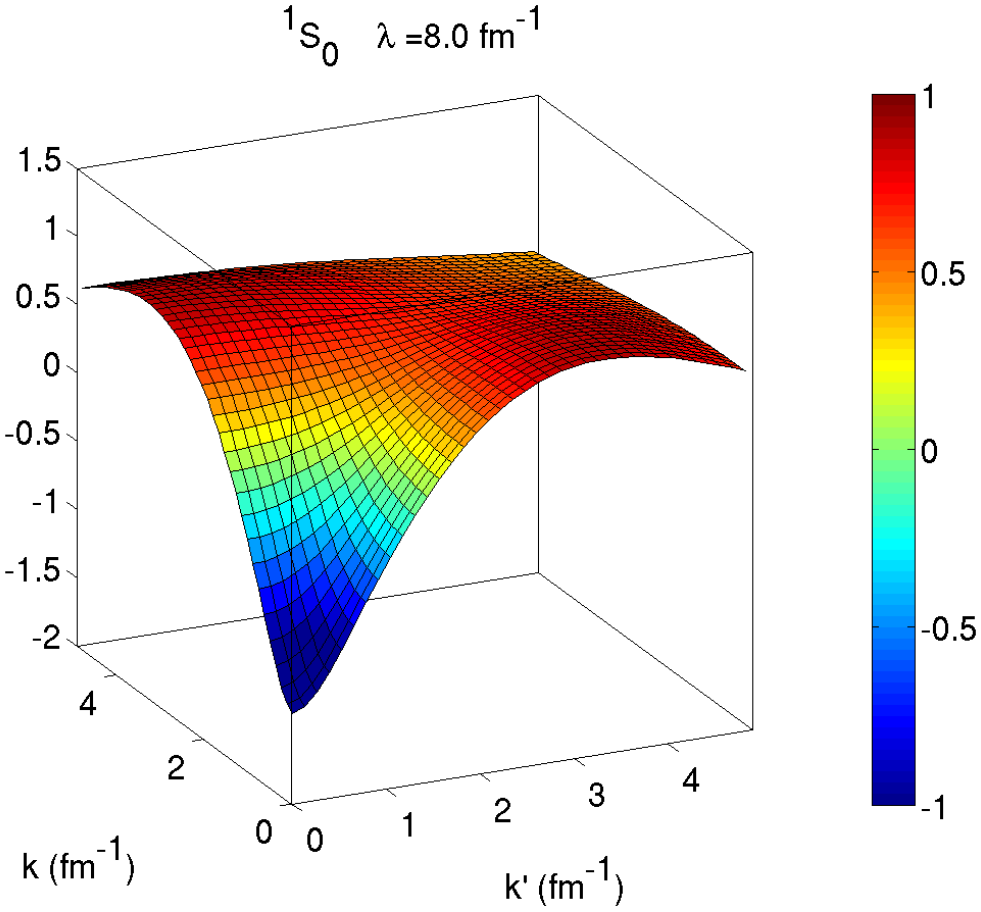

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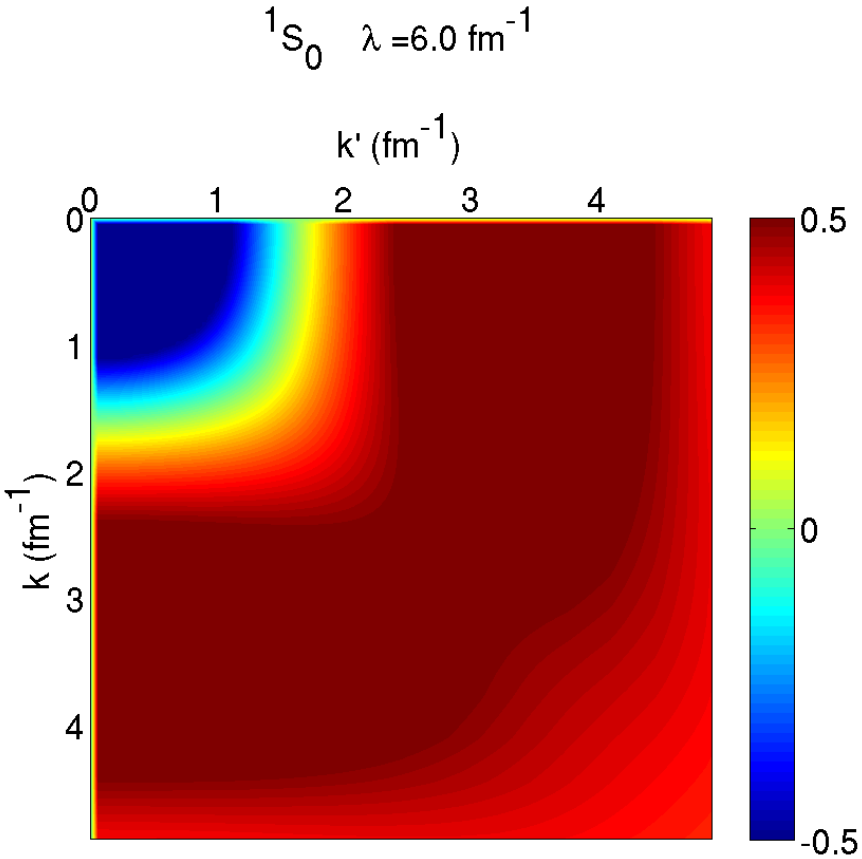

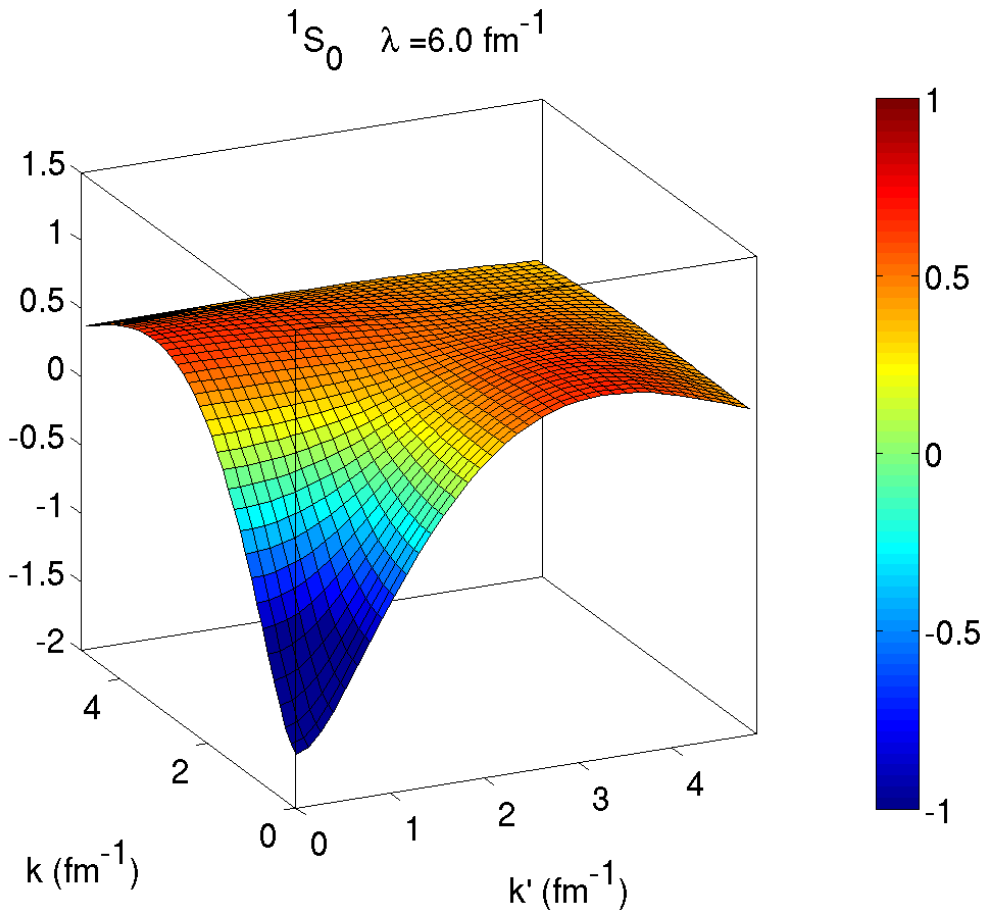

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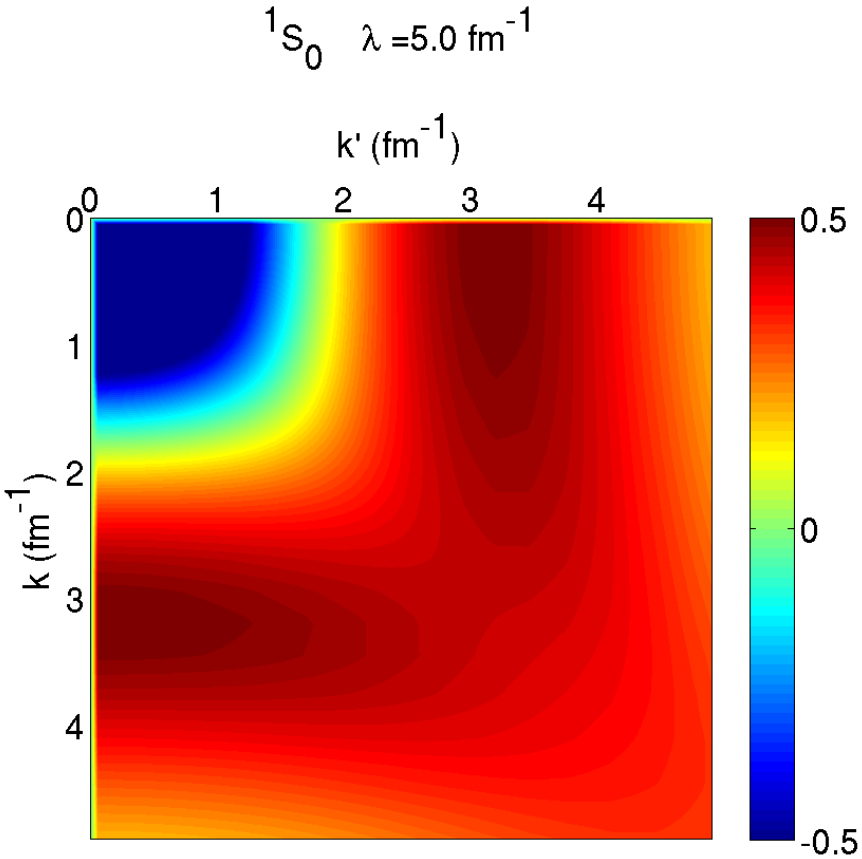

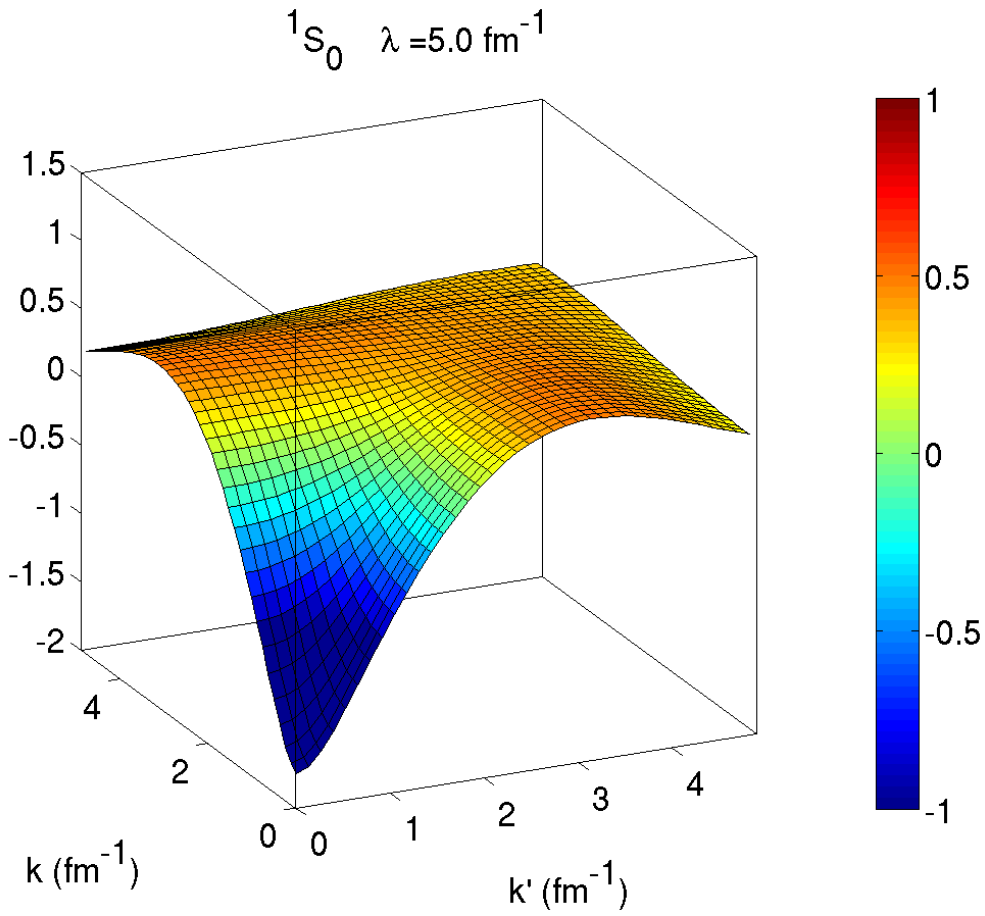

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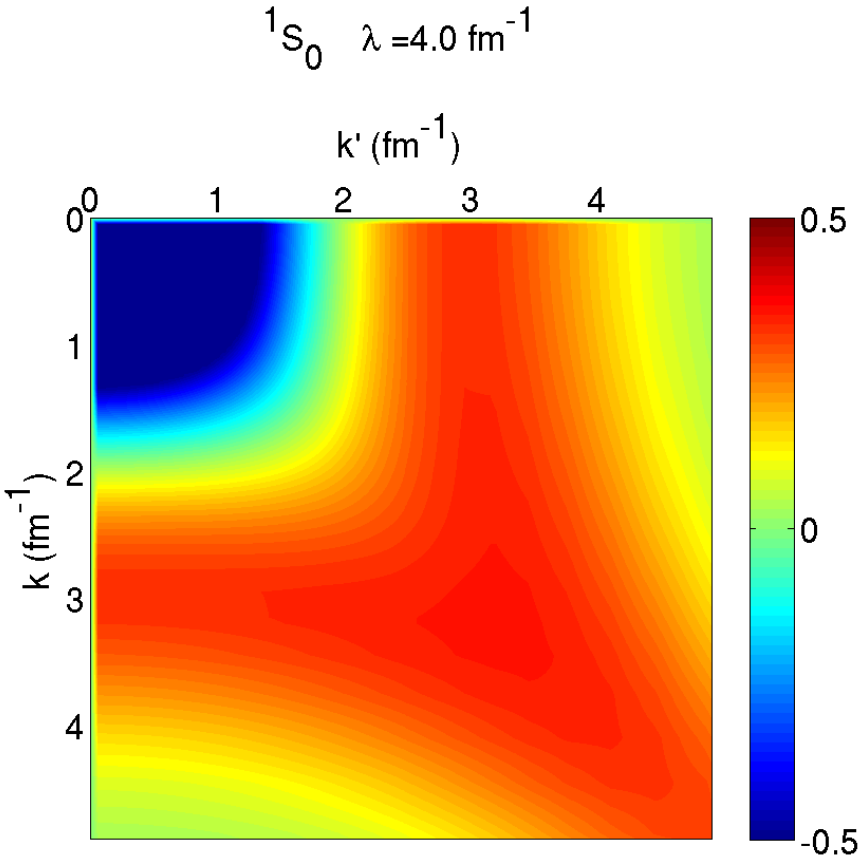

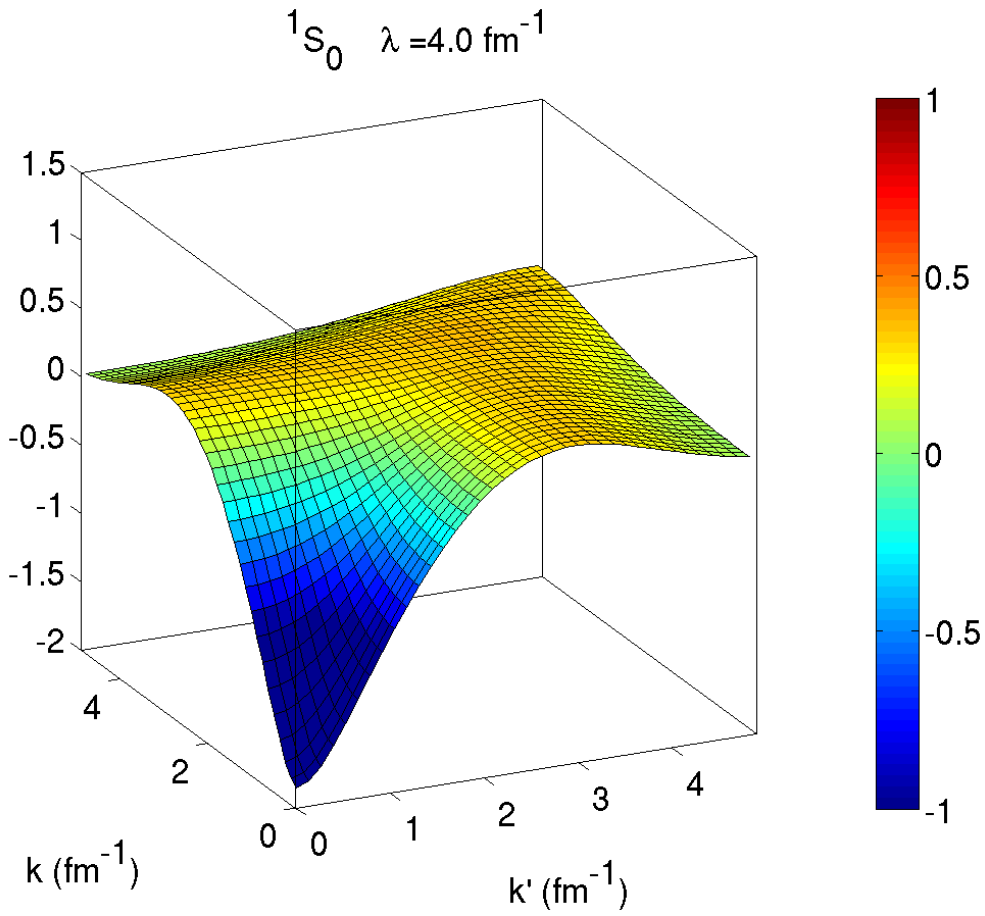

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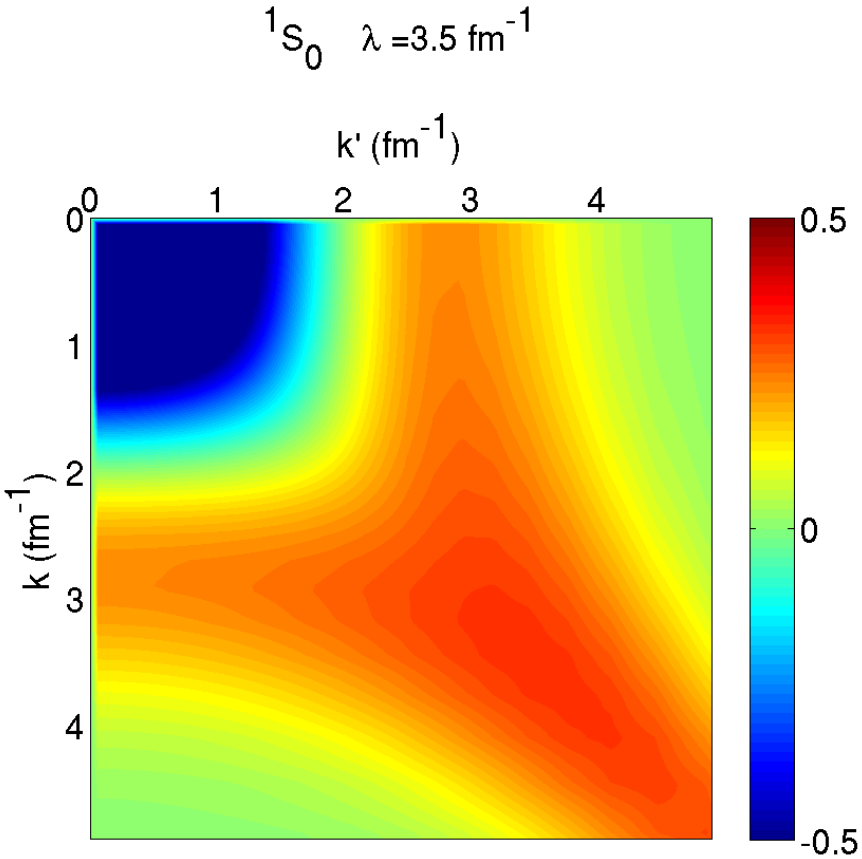

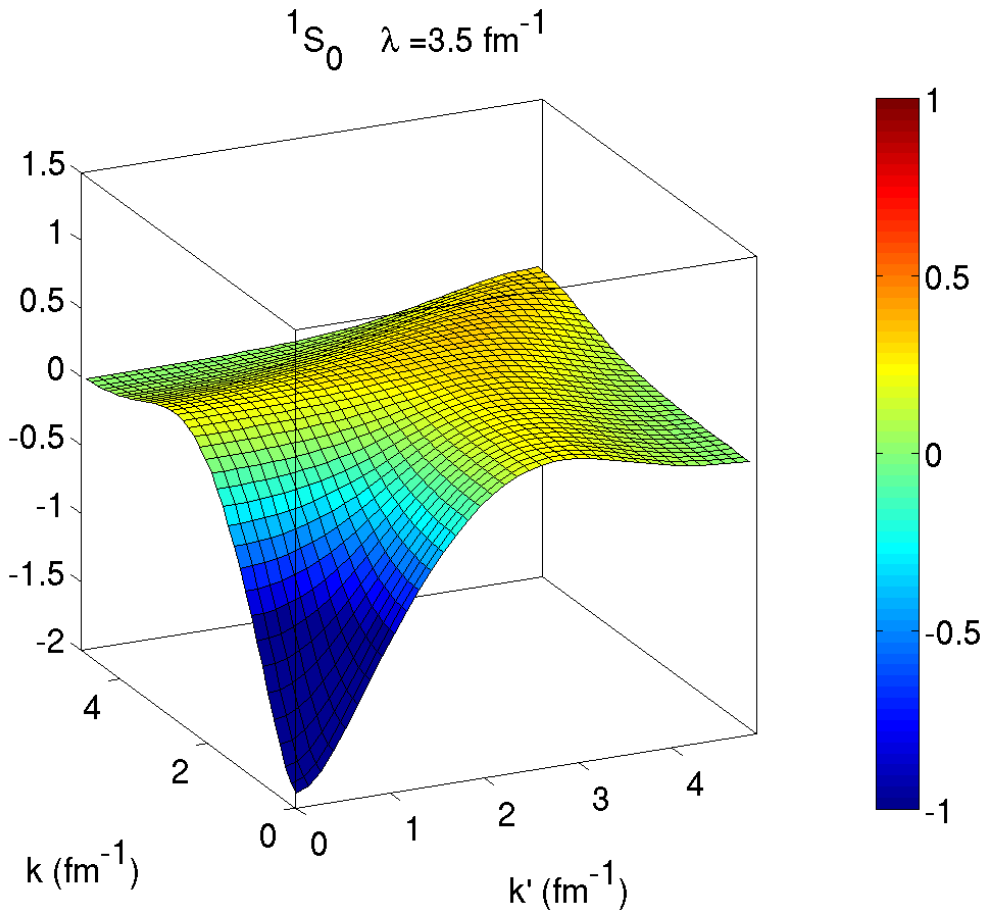

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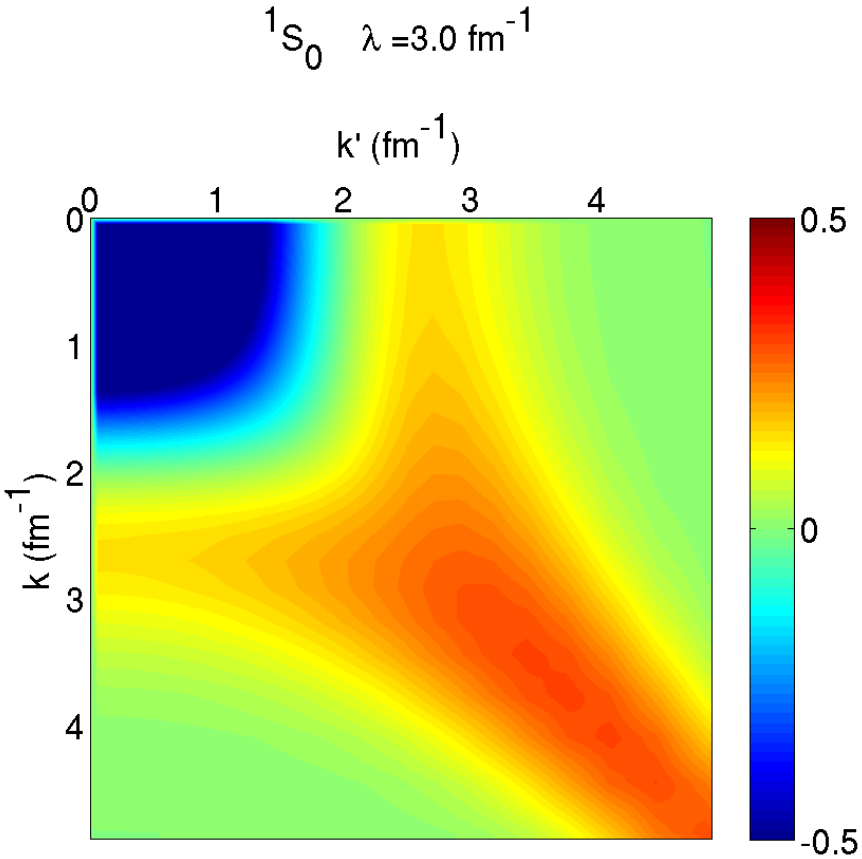

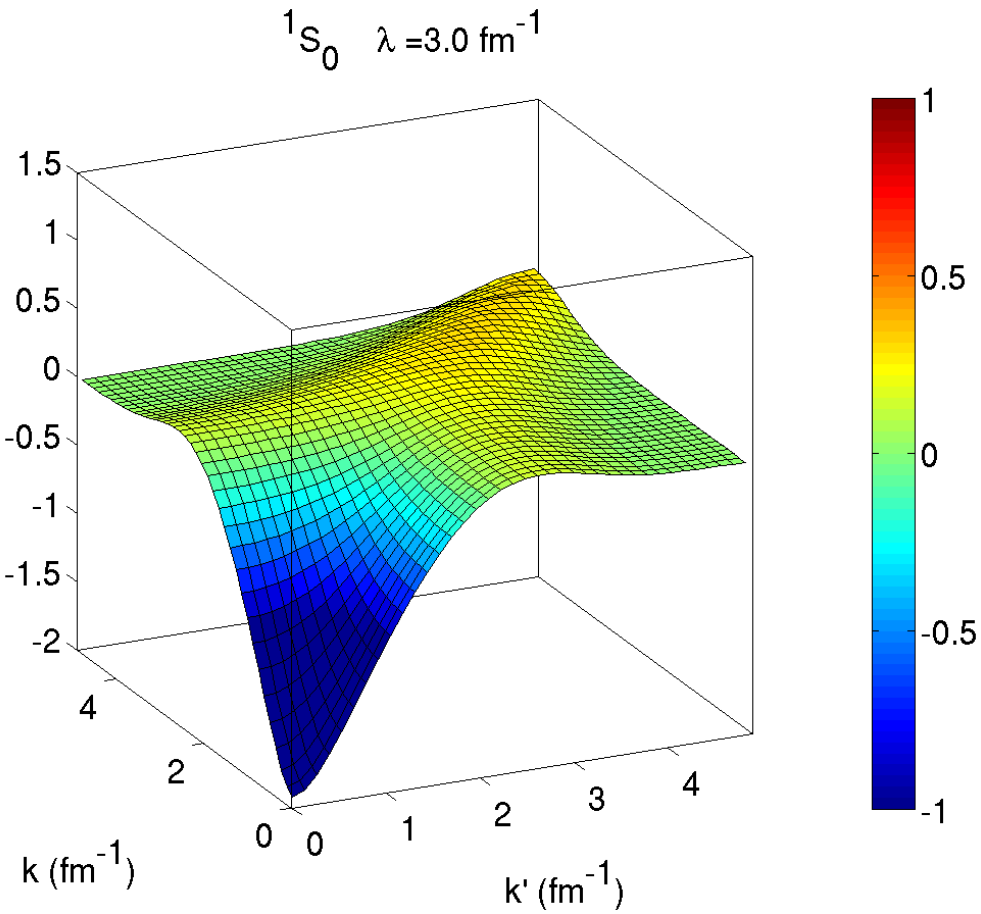

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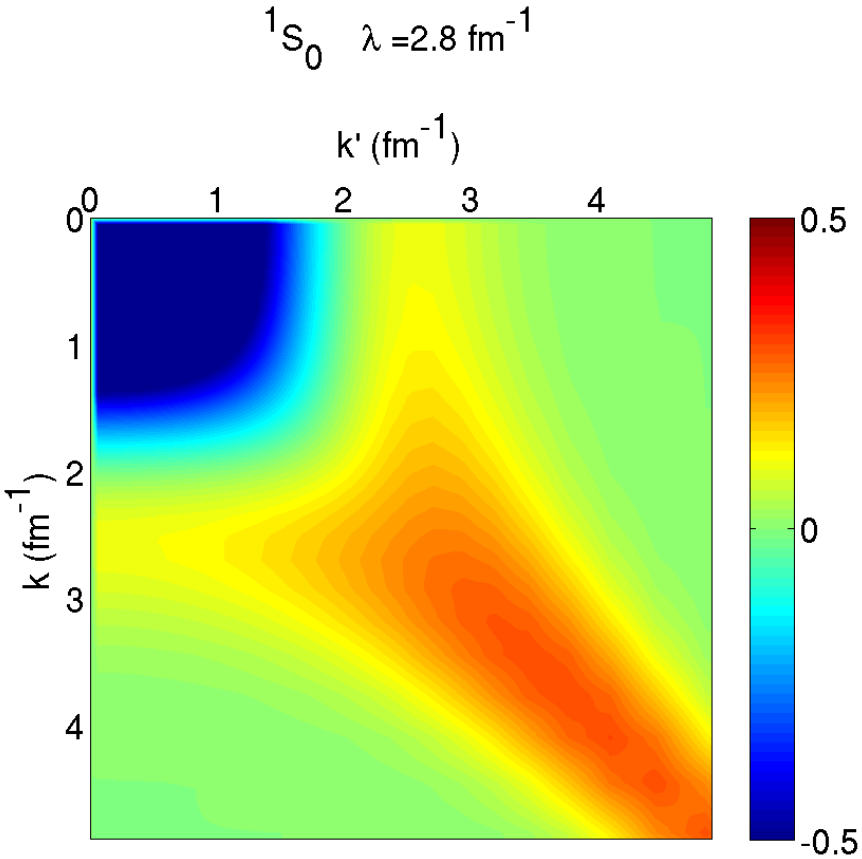

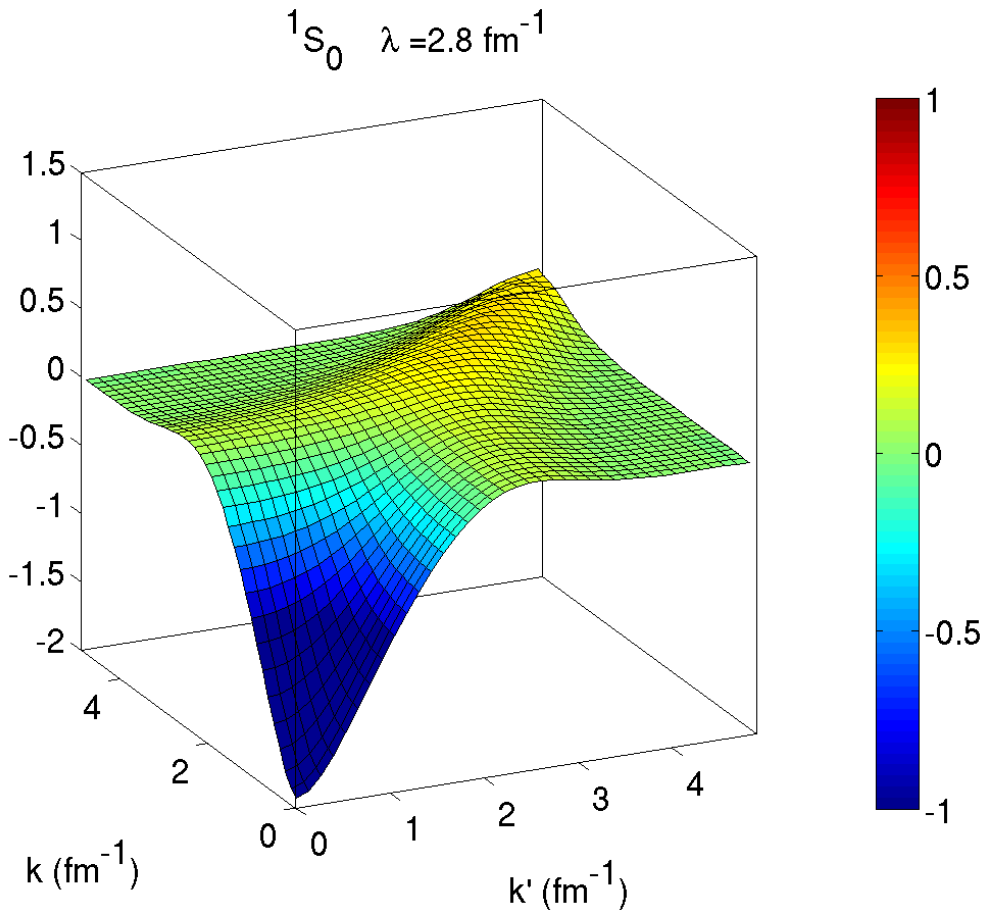

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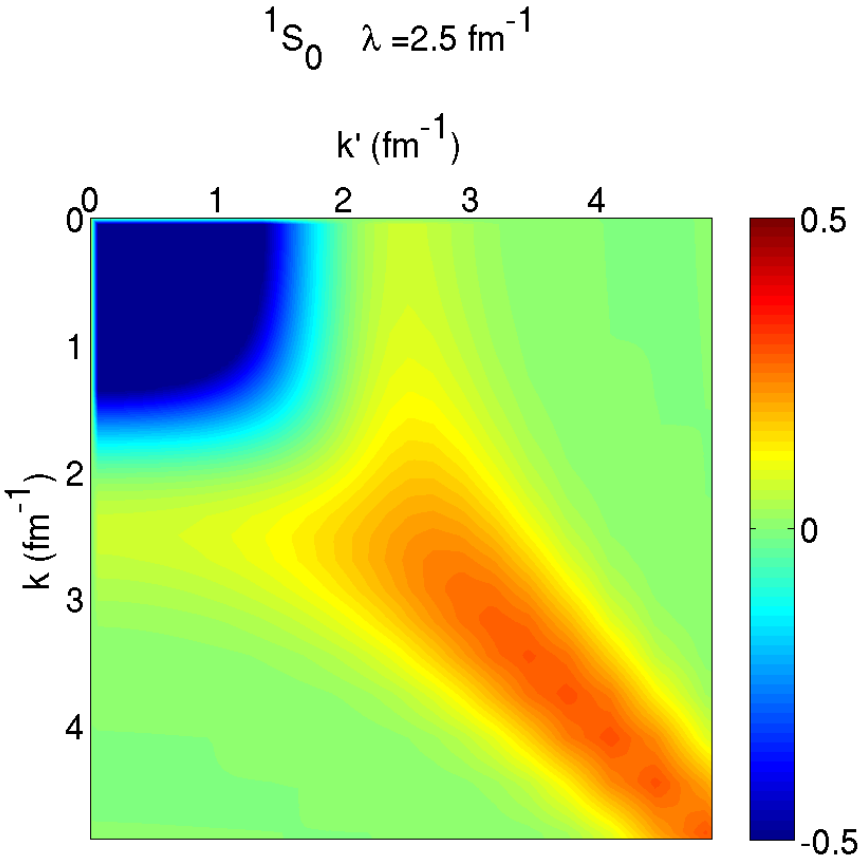

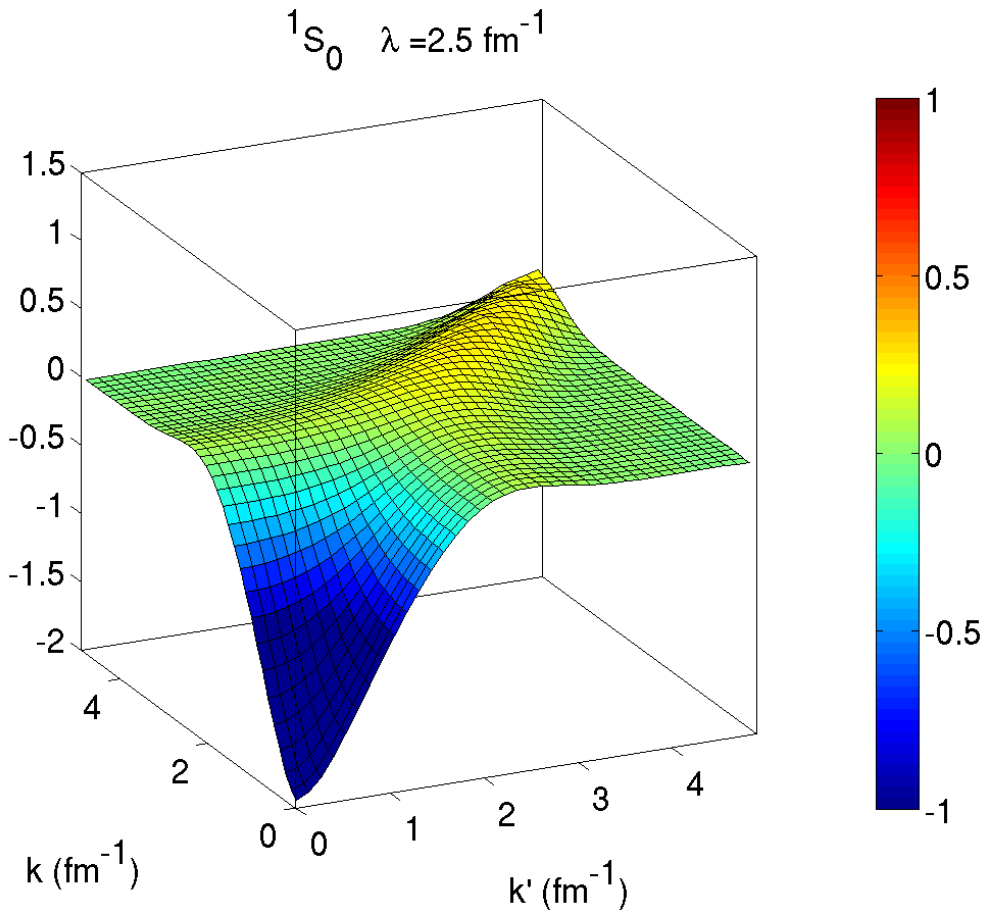

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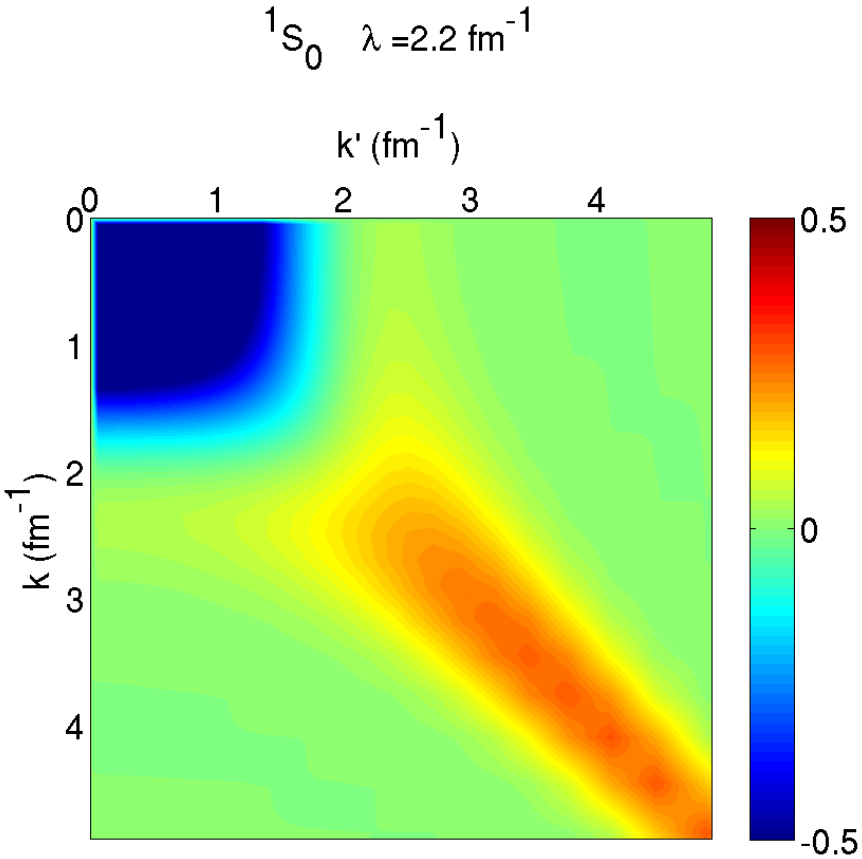

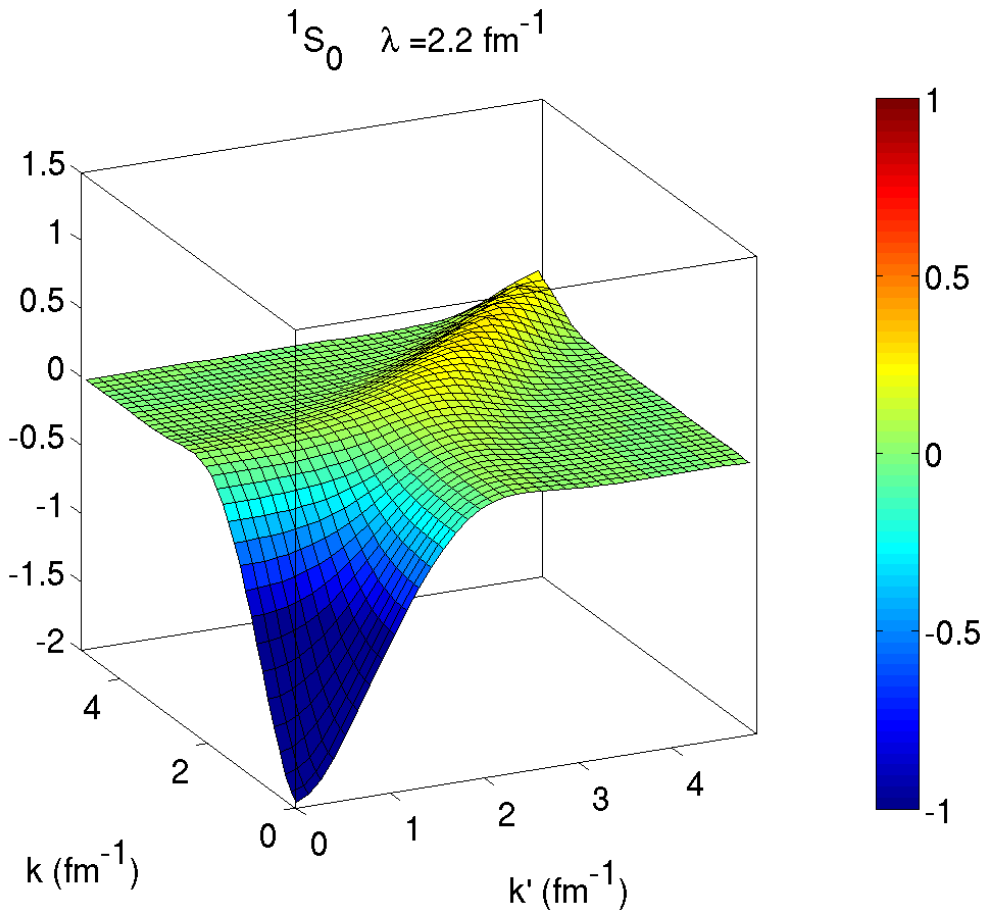

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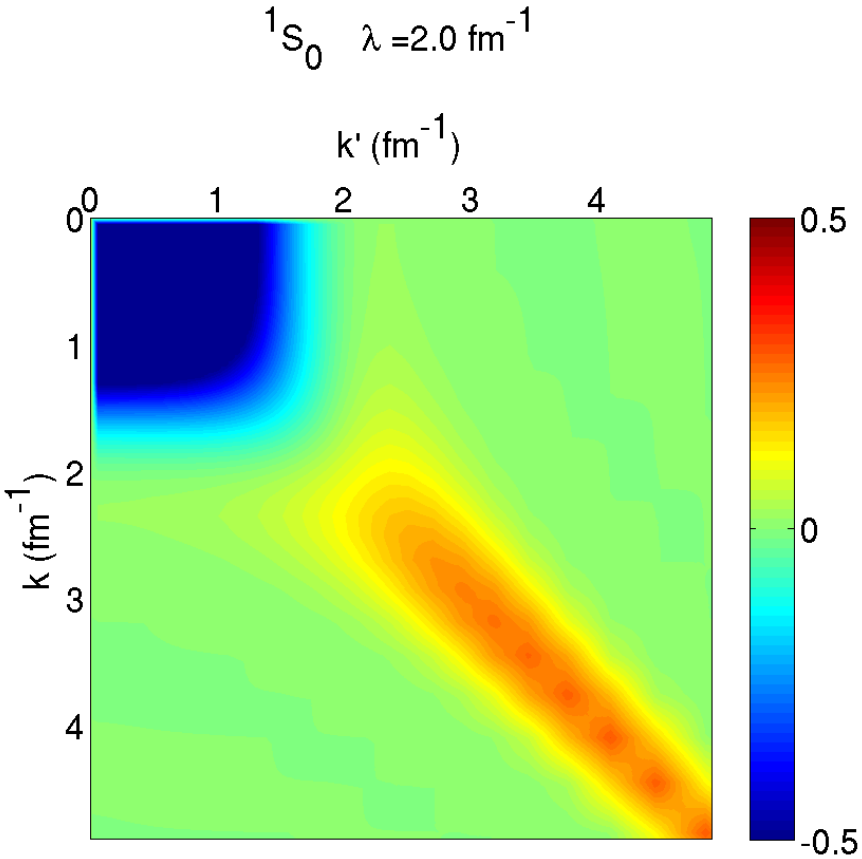

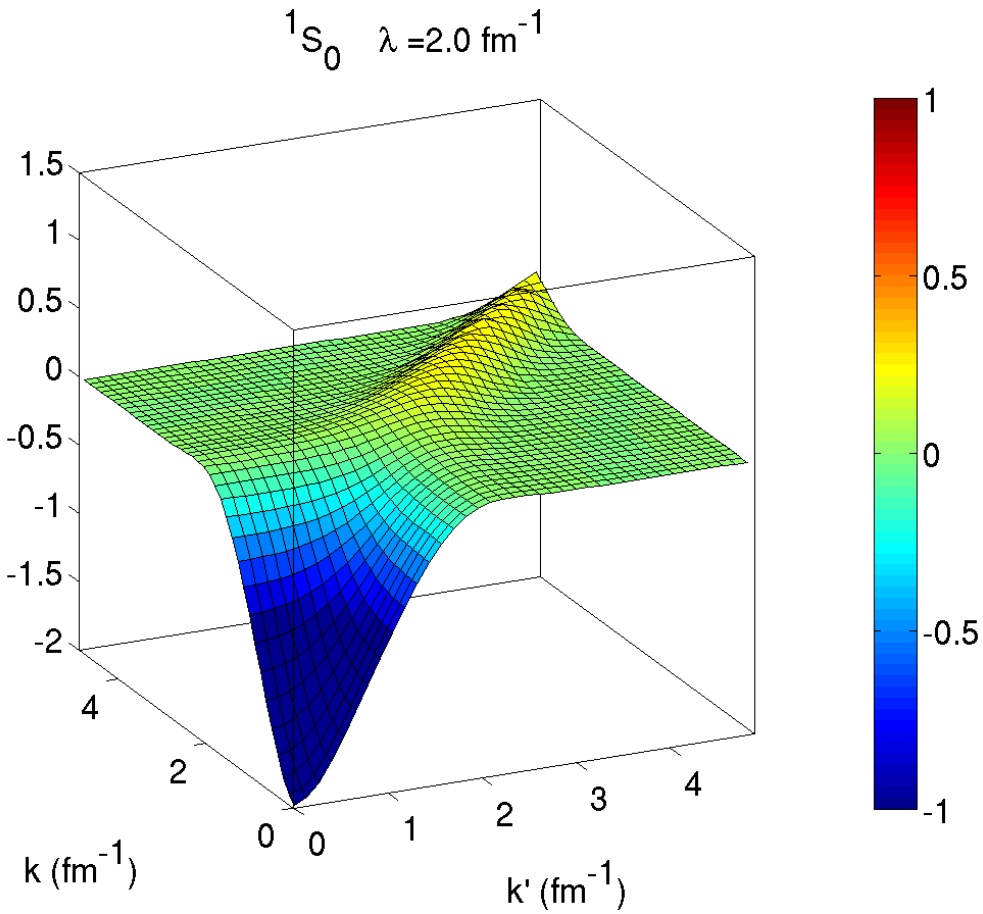

Basics: SRG flow equations [e.g., see arXiv:1203.1779]

- Transform an initial hamiltonian, $H = T + V$, with U_s:

$$H_s = U_s H U_s^\dagger \equiv T + V_s \ ,$$

where s is the *flow parameter*. Differentiating wrt s:

$$\frac{dH_s}{ds} = [\eta_s, H_s] \qquad \text{with} \qquad \eta_s \equiv \frac{dU_s}{ds} U_s^\dagger = -\eta_s^\dagger \ .$$

- η_s is specified by the commutator with Hermitian G_s:

$$\eta_s = [G_s, H_s] \ ,$$

which yields the unitary flow equation (T held fixed),

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[G_s, H_s], H_s] \ .$$

  - Very simple to implement as matrix equation (e.g., MATLAB)
- G_s determines flow $\Longrightarrow$ many choices (T, H_D, H_{BD}, ...)

SRG flow of $H = T + V$ in momentum basis

- Takes $H \longrightarrow H_s = U_s H U_s^\dagger$ in small steps labeled by s or λ

$$\frac{dH_s}{ds} = \frac{dV_s}{ds} = [[T_{\rm rel}, V_s], H_s] \quad \text{with} \quad T_{\rm rel}|k\rangle = \epsilon_k |k\rangle \quad \text{and} \quad \lambda^2 = 1/\sqrt{s}$$

- For NN, project on relative momentum states $|k\rangle$, but generic

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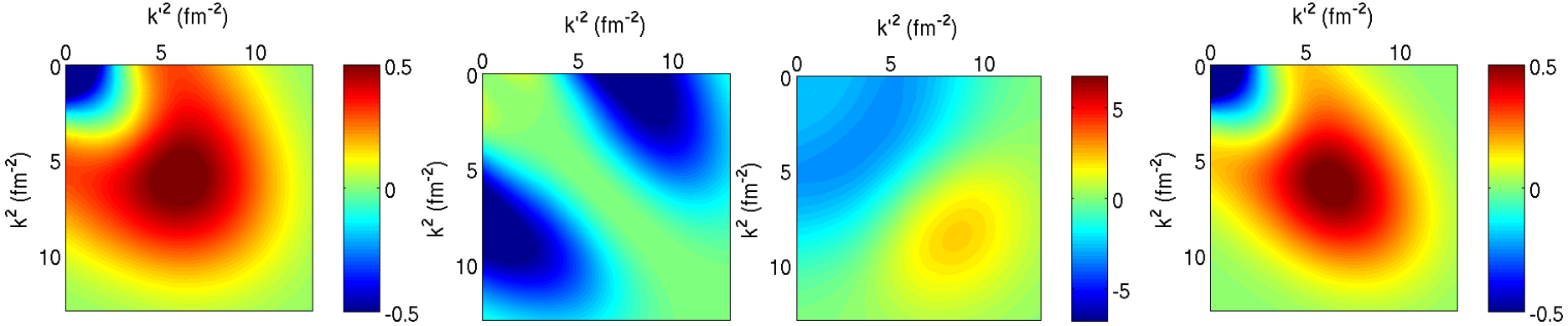

$V_{\lambda=3.0}(k,k')$ 1st term 2nd term $V_{\lambda=2.5}(k,k')$

- First term drives $^1S_0$ V_λ toward diagonal:

$$V_\lambda(k,k') = V_{\lambda=\infty}(k,k')\, e^{-[(\epsilon_k - \epsilon_{k'})/\lambda^2]^2} + \ldots$$

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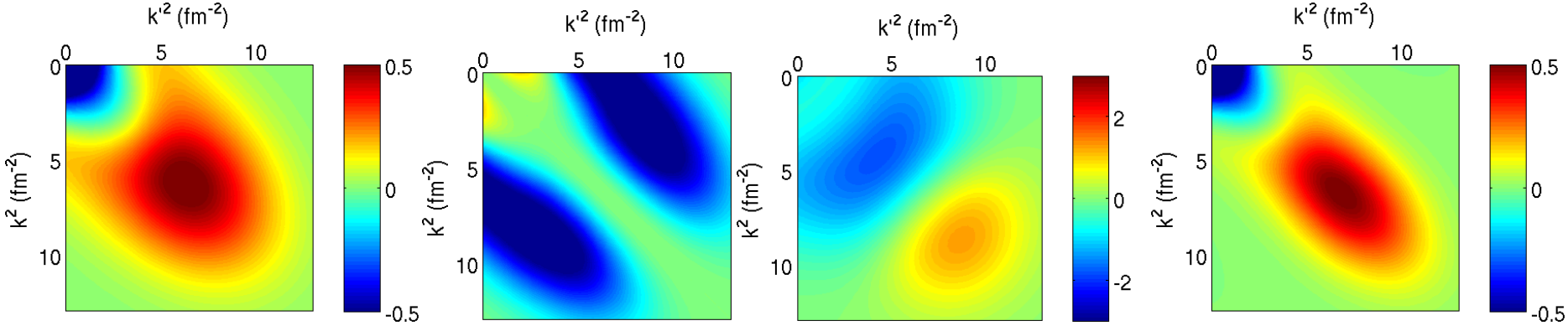

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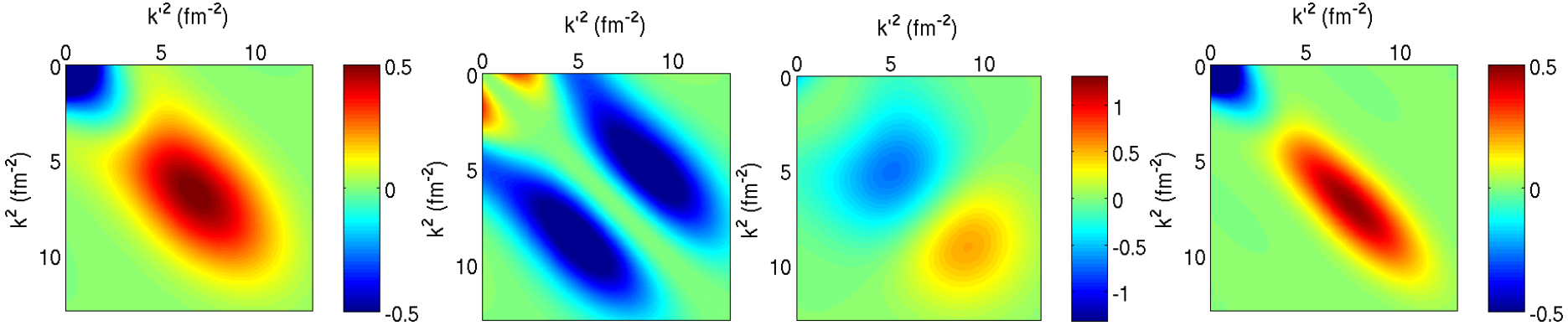

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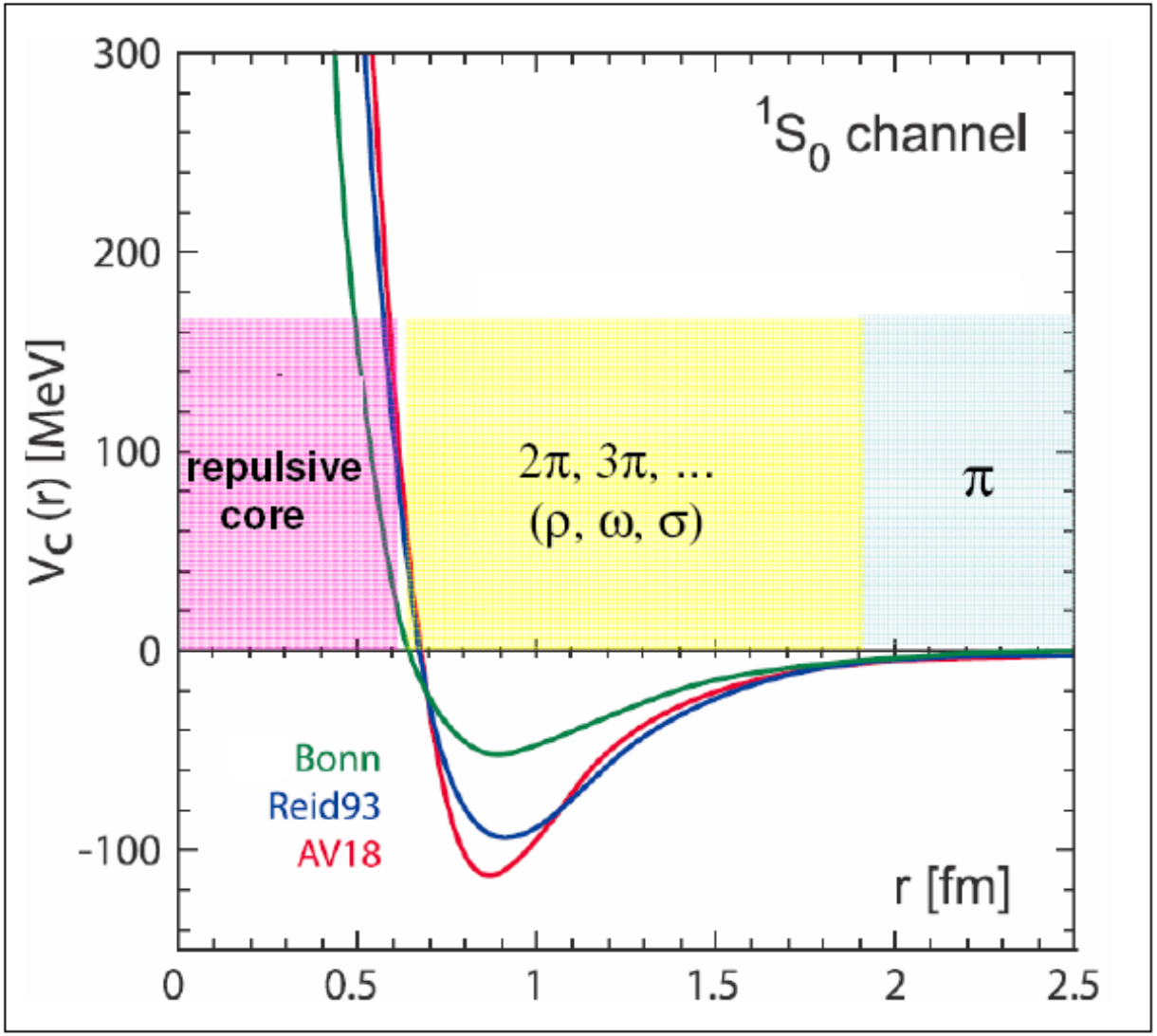
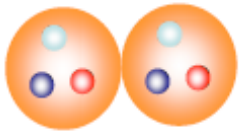
“Traditional” nucleon-nucleon interaction (from T. Papenbrock)



From T. Hatsuda (Oslo 2008)

One-pion exchange
by Yukawa (1935)

Multi-pions
by Taketani (1951)



Repulsive core
by Jastrow (1951)

Local nucleon-nucleon interaction for non-rel S-eqn

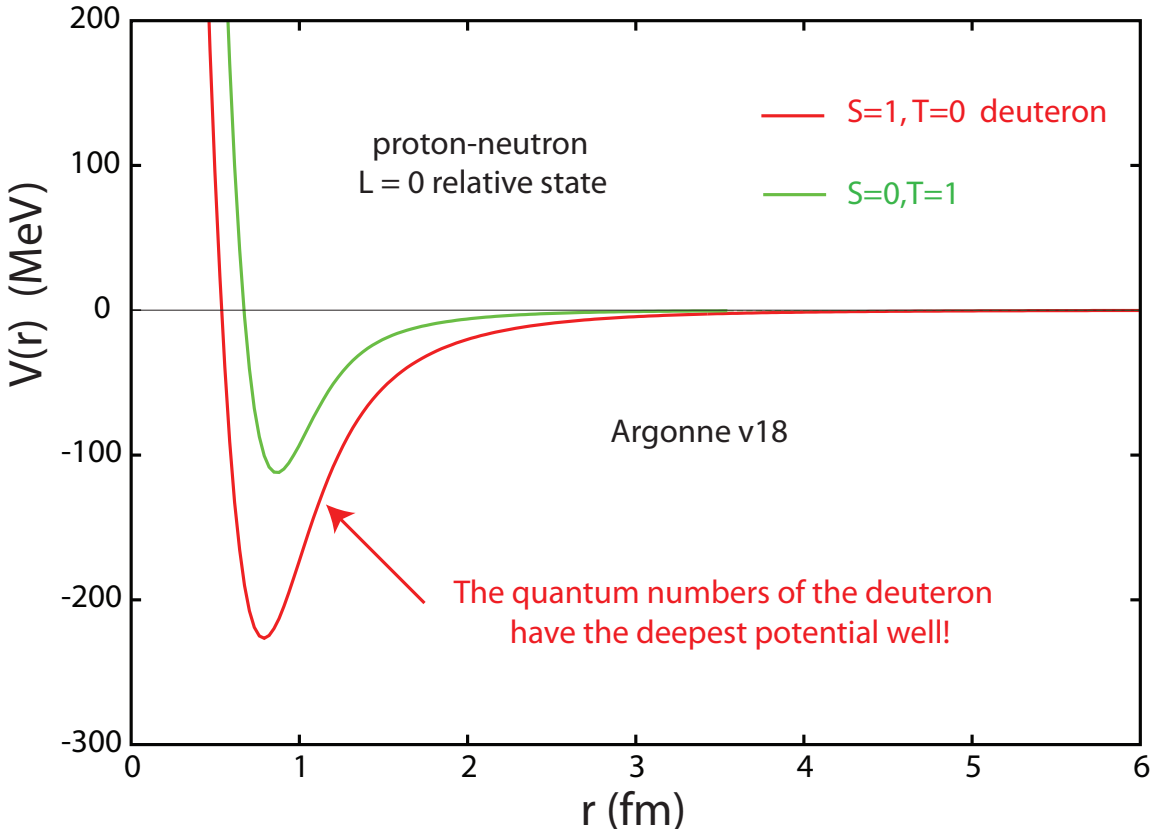
- Depends on spins *and* isospins of nucleons; non-central
  - longest-range part is one-pion-exchange potential

$$V_\pi(\mathbf{r}) \propto (\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_2})\left[(3\boldsymbol{\sigma_1}\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma_2}\cdot\hat{\mathbf{r}} - \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2})(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}) + \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}\right]\frac{e^{-m_\pi r}}{r}$$

- Characterize operator structure of shorter-range potential
  - central, spin-spin, non-central tensor and spin-orbit

$$\{1, \boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}, S_{12}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2\boldsymbol{\sigma_1}\cdot\boldsymbol{\sigma_2}, (\mathbf{L}\cdot\mathbf{S})^2\} \otimes \{1, \boldsymbol{\tau_1}\cdot\boldsymbol{\tau_2}\}$$

- Tensor $\Longrightarrow$ deuteron wf is mixed S ($L=0$) and D ($L=2$)
- Non-zero quadrupole moment



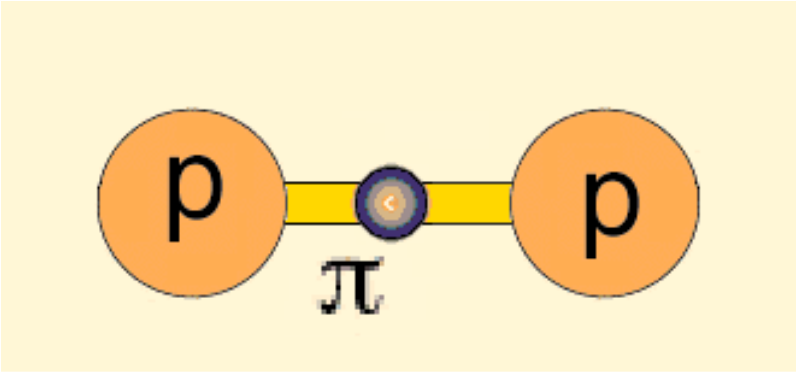
Problems with Phenomenological Potentials

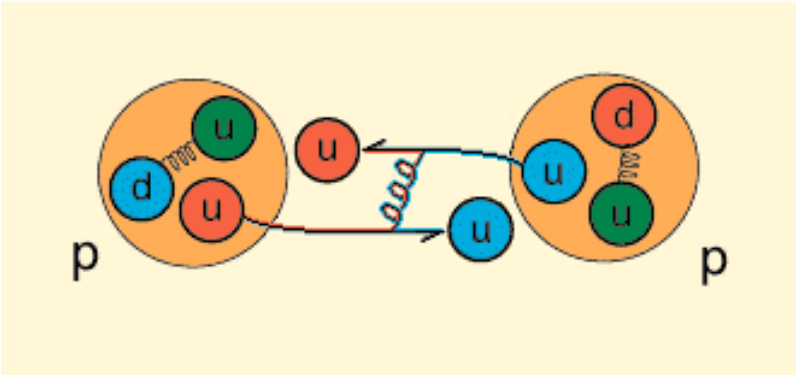
- The best potential models can describe with $\chi^2/\text{dof} \approx 1$ all of the NN data (about 6000 points) below the pion production threshold. So what more do we need?
- Some drawbacks:
  - Usually have very strong repulsive short-range part $\Longrightarrow$ requires special (non-systematic) treatment in many-body calculations (e.g. nuclear structure).
  - Difficult to estimate theoretical errors and range of applicability.
  - Three-nucleon forces (3NF) are largely unconstrained and unsystematic models. How to define *consistent* 3NF's and operators (e.g., meson exchange currents)?
  - Models are largely unconnected to QCD (e.g., chiral symmetry). Don't connect NN and other strongly interacting processes (e.g., $\pi\pi$ and πN). Lattice QCD will be able to predict NN, 3N observables for high pion masses. How to extrapolate to physical pion masses?

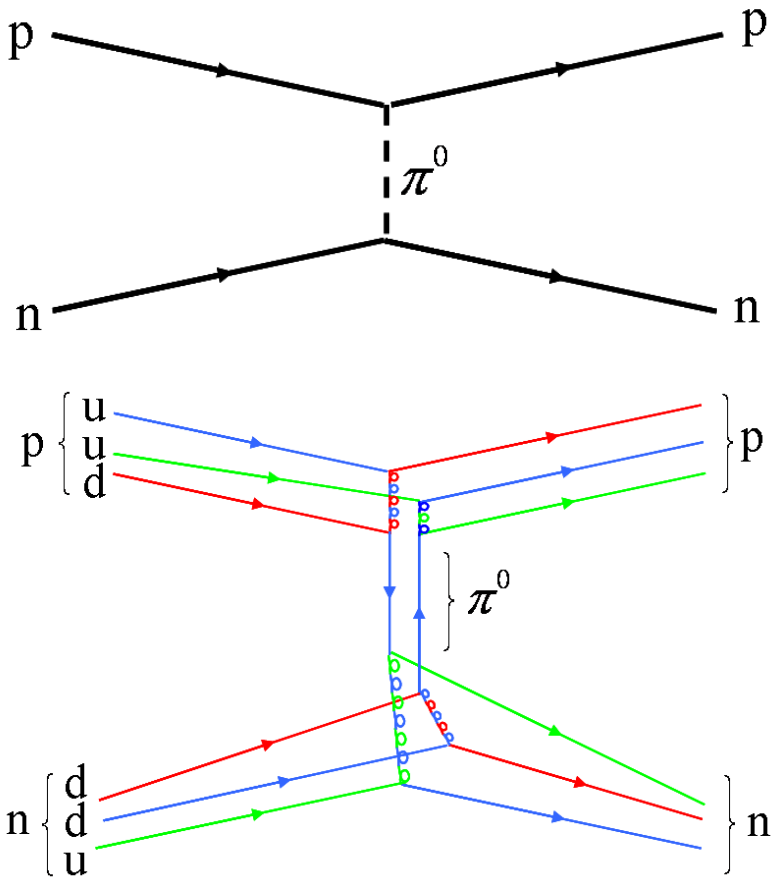
Alternative: Use Chiral Effective Field Theory (EFT)

QCD and Nuclear Forces

- Quarks and gluons are the fundamental QCD dof's, but . . .

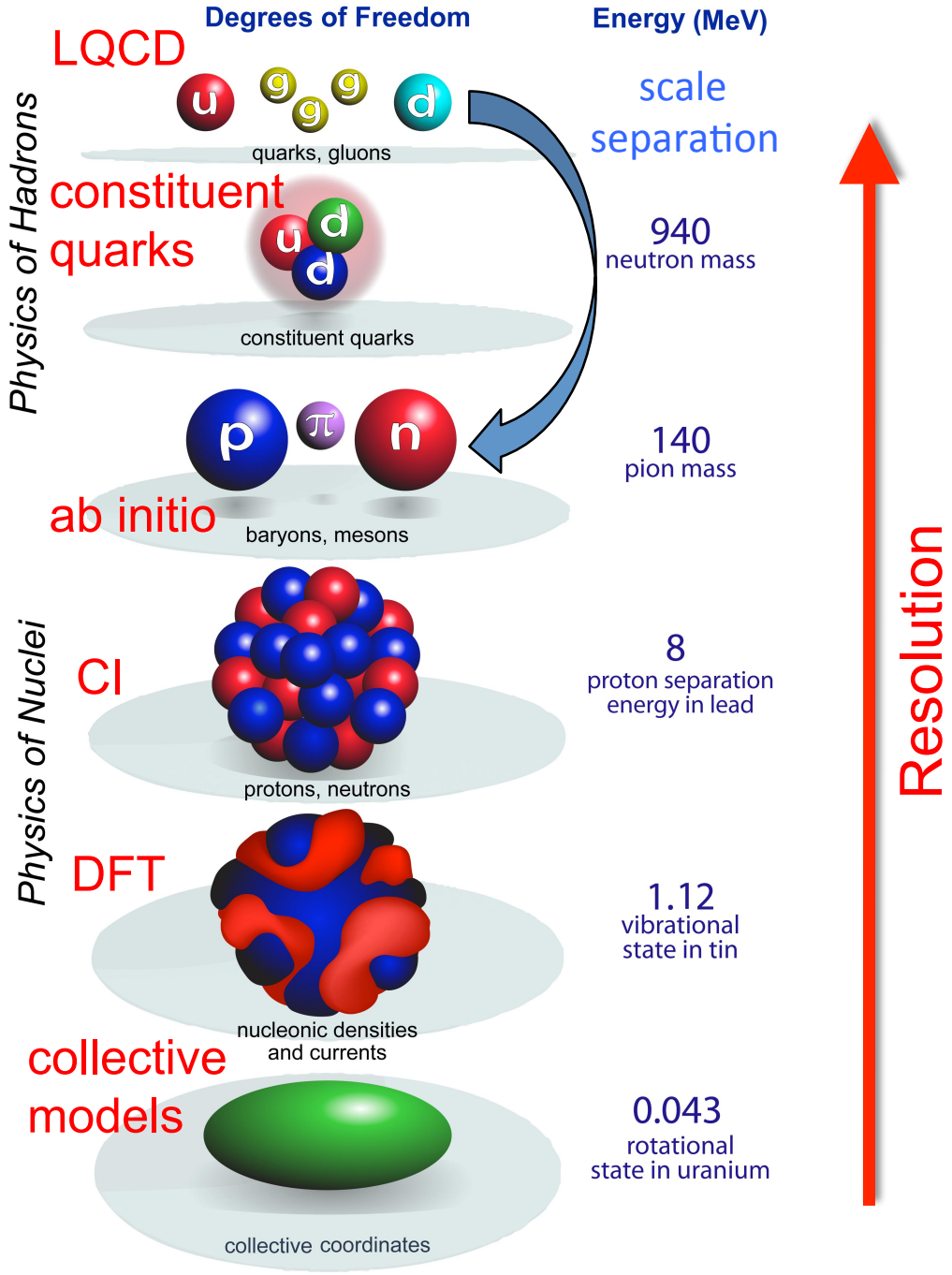






- At low energies (low resolution), use "collective" degrees of freedom $\Longrightarrow$ (colorless) hadrons. Which ones?

Different EFTs depending on scale of interest

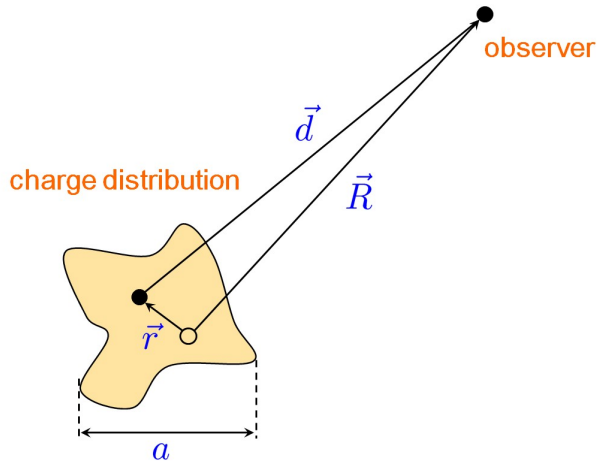


Effective theories [H. Georgi, Ann. Rev. Nucl. Part. Sci. 43, 209 (1993)]

- *One of the most astonishing things about the world in which we live is that there seems to be interesting physics at all scales.*
- *To do physics amid this remarkable richness, it is convenient to be able to isolate a set of phenomena from all the rest, so that we can describe it without having to understand everything. . . . We can divide up the parameter space of the world into different regions, in each of which there is a different appropriate description of the important physics. Such an appropriate description of the important physics is an "effective theory."*
- *The common idea is that if there are parameters that are very large or very small compared to the physical quantities (with the same dimension) that we are interested in, we may get a simpler approximate description of the physics by setting the small parameters to zero and the large parameters to infinity. Then the finite effects of the parameters can be included as small perturbations about this simple approximate starting point.*
- E.g., non-relativistic QM: $c \to \infty$
- E.g., chiral effective *field* theory (EFT): $m_\pi \to 0$, $M_N \to \infty$
- E.g., pionless effective *field* theory (EFT): $m_\pi, M_N \to \infty$
- Goals: model independence (completeness) and error estimates

Classical analogy to EFT: Multipole expansion

If we have a localized charge distribution $\rho(\mathbf{r})$ within a volume characterized by distance a, the electrostatic potential is



$$\phi(\mathbf{R}) \propto \int d^3r \, \frac{\rho}{|\mathbf{R}-\mathbf{r}|}$$

If we expand $1/|\mathbf{R}-\mathbf{r}|$ for $r \ll R$, we get the multipole expansion

$$\int d^3r \, \frac{\rho}{|\mathbf{R}-\mathbf{r}|} = \frac{q}{R} + \frac{1}{R^3}\sum_i R_i P_i + \frac{1}{6R^5}\sum_{ij}(3R_iR_j - \delta_{ij}R^2)Q_{ij} + \cdots$$

$\Longrightarrow$ *pointlike* total charge q, dipole moment P_i, quadrupole Q_{ij}:

$$q = \int d^3r \, \rho(\mathbf{r}) \qquad P_i = \int d^3r \, \rho(\mathbf{r}) \, r_i \qquad Q_{ij} = \int d^3r \, \rho(\mathbf{r})(3r_ir_j - \delta_{ij}r^2)$$

- Hierarchy of terms from separation of scales $\Longrightarrow$ a/R expansion
- Can determine coefficients (LECs) by matching to actual distribution (if known) or comparing to experimental measurements
- Completeness $\Longrightarrow$ model independent (cf. model of distribution)

Effective Field Theory Ingredients

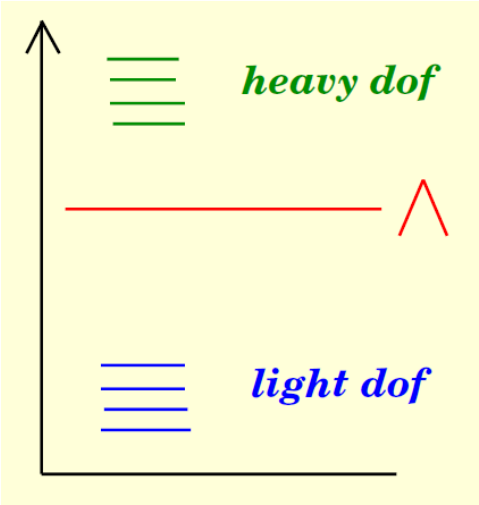
General procedure for building an EFT . . .

1. Use the most general $\mathcal{L}$ with low-energy dof's consistent with global and local symmetries of underlying theory

2. Declaration of regularization and renormalization scheme

3. Well-defined power counting $\Longrightarrow$ small expansion parameter

- General procedures:
  - QFT: trees + loops $\rightarrow$ renormalization
  - Include long-range physics explicitly
  - Short-distance physics captured in a few LEC's (calculated from underlying or fit to data). Check naturalness.

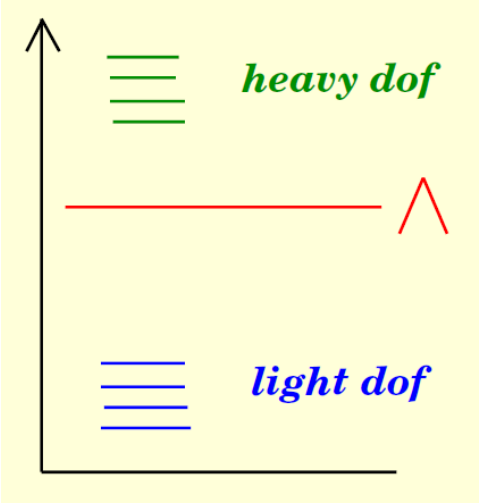


Effective Field Theory Ingredients

General procedure for building an EFT . . .

1. Use the most general $\mathcal{L}$ with low-energy dof's consistent with global and local symmetries of underlying theory
   - What are the low-energy dof's for QCD?
   - What are the relevant symmetries?
2. Declaration of regularization and renormalization scheme
   - What choices are there?
   - Will we be able to use dimensional regularization?
3. Well-defined power counting $\Longrightarrow$ small expansion parameter
   - Usually Q/Λ. What are the QCD scales?

- General procedures:
  - QFT: trees + loops $\rightarrow$ renormalization
  - Include long-range physics explicitly
  - Short-distance physics captured in a few LEC's (calculated from underlying or fit to data). Check naturalness.



Outline: Lecture 3

Lecture 3: Effective field theory

- Recap from lecture 2: How SRG works
- Motivation for nuclear effective field theory
- Chiral effective field theory
- Universal potentials from RG evolution
- Extra: Quantitative measure of perturbativeness

Symmetries of the QCD Lagrangian

- Besides space-time symmetries and parity, what else?
- Is $SU(3)$ color gauge "symmetry" in the EFT?
- Consider chiral symmetry ...

$$\mathcal{L}_{\rm QCD} = \overline{q}_L i\not{D} q_L + \overline{q}_R i\not{D} q_R - \frac{1}{2}{\rm Tr}\, G_{\mu\nu} G^{\mu\nu} - \overline{q}_R \mathcal{M} q_L - \overline{q}_L \mathcal{M} q_R$$

$$\not{D} \equiv \not{\partial} - i g_s \not{G}^a T^a \; ; \qquad T^a = SU(3) \text{ Gell-Mann matrices}$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \qquad SU(2) \text{ quark mass matrix}$$

$$q_{L,R} = \frac{1}{2}(1 \pm \gamma_5) q \; , \qquad \text{projection on left,right-handed quarks}$$

- m_u and m_d are small compared to typical hadron masses (5 and 9 MeV at 1 GeV renormalization scale vs. about 1 GeV)

$$\mathcal{M} \approx 0 \Longrightarrow \text{approximate } SU(2)_L \otimes SU(2)_R \text{ chiral symmetry}$$

Chiral Symmetry of QCD

- What happens if we have a symmetry of the Hamiltonian?
  - Could have a multiplet of equal mass particles
  - Could be a spontaneously broken (hidden) symmetry
- Experimentally we notice
  - Isospin multiplets like p,n or Σ^+,Σ^-,Σ^0 (that is, they have close to the same mass). So isospin symmetry is manifest.
  - But we don't find opposite parity partners for these states with close to the same mass. The "axial" part of chiral symmetry is spontaneously broken down!
- Isospin symmetry is "vectorial subgroup" with $L = R$
- The pions are pseudo-Goldstone bosons. The symmetry is *explicitly* broken by the quark masses, which means the pion is light ($m_\pi^2 \ll M_{\rm QCD}^2$) but not massless.
- Chiral symmetry relates states with different numbers of pions and dictates that pion interactions get weak at low energy $\Longrightarrow$ pion as calculable long-distance dof in χEFT!

Effective Field Theory Ingredients

Specific answers for chiral EFT:

1. Use the most general $\mathcal{L}$ with low-energy dof's consistent with the global and local symmetries of the underlying theory

2. Declaration of regularization and renormalization scheme

3. Well-defined power counting $\Longrightarrow$ expansion parameters

Effective Field Theory Ingredients: Chiral NN

Specific answers for chiral EFT:

1. Use the most general $\mathcal{L}$ with low-energy dof's consistent with the global and local symmetries of the underlying theory
   - $\mathcal{L}_{\rm eft} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN}$
   - chiral symmetry $\Longrightarrow$ systematic long-distance pion physics
2. Declaration of regularization and renormalization scheme
   - momentum cutoff and "Weinberg counting" (still unsettled!) $\Longrightarrow$ define irreducible potential and sum with LS eqn
   - use cutoff sensitivity as measure of uncertainties
3. Well-defined power counting $\Longrightarrow$ expansion parameters
   - use the separation of scales $\Longrightarrow \dfrac{\{\mathbf{p}, m_\pi\}}{\Lambda_\chi}$ with $\Lambda_\chi \sim 1$ GeV
   - chiral symmetry $\Longrightarrow V_{NN} = \sum_{\nu=\nu_{\rm min}}^{\infty} c_\nu Q^\nu$ with $\nu \geq 0$
   - naturalness: LEC's are $\mathcal{O}(1)$ in appropriate units

Chiral Lagrangian

- Unified description of $\pi\pi$, πN, and $NN\cdots N$
- Lowest orders [Can you identify the vertices?]:

$$
\begin{aligned}
\mathcal{L}^{(0)} &= \frac{1}{2}\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + N^\dagger\left[i\partial_0 + \frac{g_A}{2f_\pi}\boldsymbol{\tau}\sigma\cdot\nabla\boldsymbol{\pi} - \frac{1}{4f_\pi^2}\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}})\right]N \\
&\quad - \frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\sigma N)(N^\dagger\sigma N) + \ldots, \\
\mathcal{L}^{(1)} &= N^\dagger\left[4c_1 m_\pi^2 - \frac{2c_1}{f_\pi^2}m_\pi^2\boldsymbol{\pi}^2 + \frac{c_2}{f_\pi^2}\dot{\boldsymbol{\pi}}^2 + \frac{c_3}{f_\pi^2}(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})\right. \\
&\qquad \left. - \frac{c_4}{2f_\pi^2}\epsilon_{ijk}\,\epsilon_{abc}\,\sigma_i\tau_a(\nabla_j\,\pi_b)(\nabla_k\,\pi_c)\right]N \\
&\quad - \frac{D}{4f_\pi}(N^\dagger N)(N^\dagger\sigma\boldsymbol{\tau}N)\cdot\nabla\boldsymbol{\pi} - \frac{1}{2}E\,(N^\dagger N)(N^\dagger\boldsymbol{\tau}N)\cdot(N^\dagger\boldsymbol{\tau}N) + \ldots
\end{aligned}
$$

- Infinite # of unknown parameters (LEC's), but leads to hierarchy of diagrams: $\nu = -4 + 2N + 2L + \sum_i(d_i + n_i/2 - 2) \geq 0$

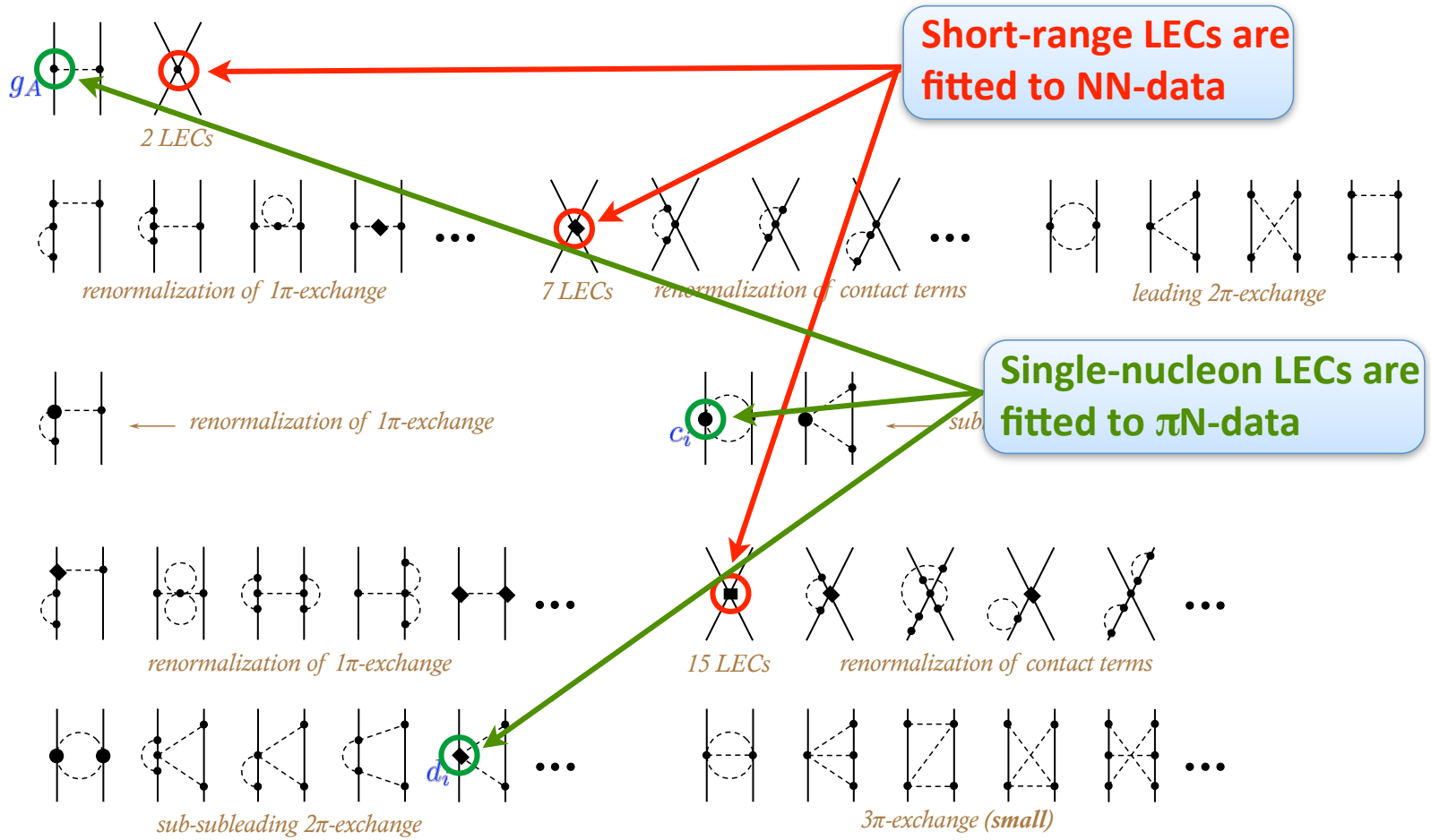
Nucleon-nucleon force up to N^{3}LO

Ordonez et al. '94; Friar & Coon '94; Kaiser et al. '97; Epelbaum et al. '98,'03; Kaiser '99-'01; Higa et al. '03; ...

Chiral expansion for the 2N force:

$$V_{2N} = V_{2N}^{(0)} + V_{2N}^{(2)} + V_{2N}^{(3)} + V_{2N}^{(4)} + \ldots$$

- **LO:**
- **NLO:**
- **N^{2}LO:**
- **N^{3}LO:**



Short-range LECs are fitted to NN-data

Single-nucleon LECs are fitted to πN-data

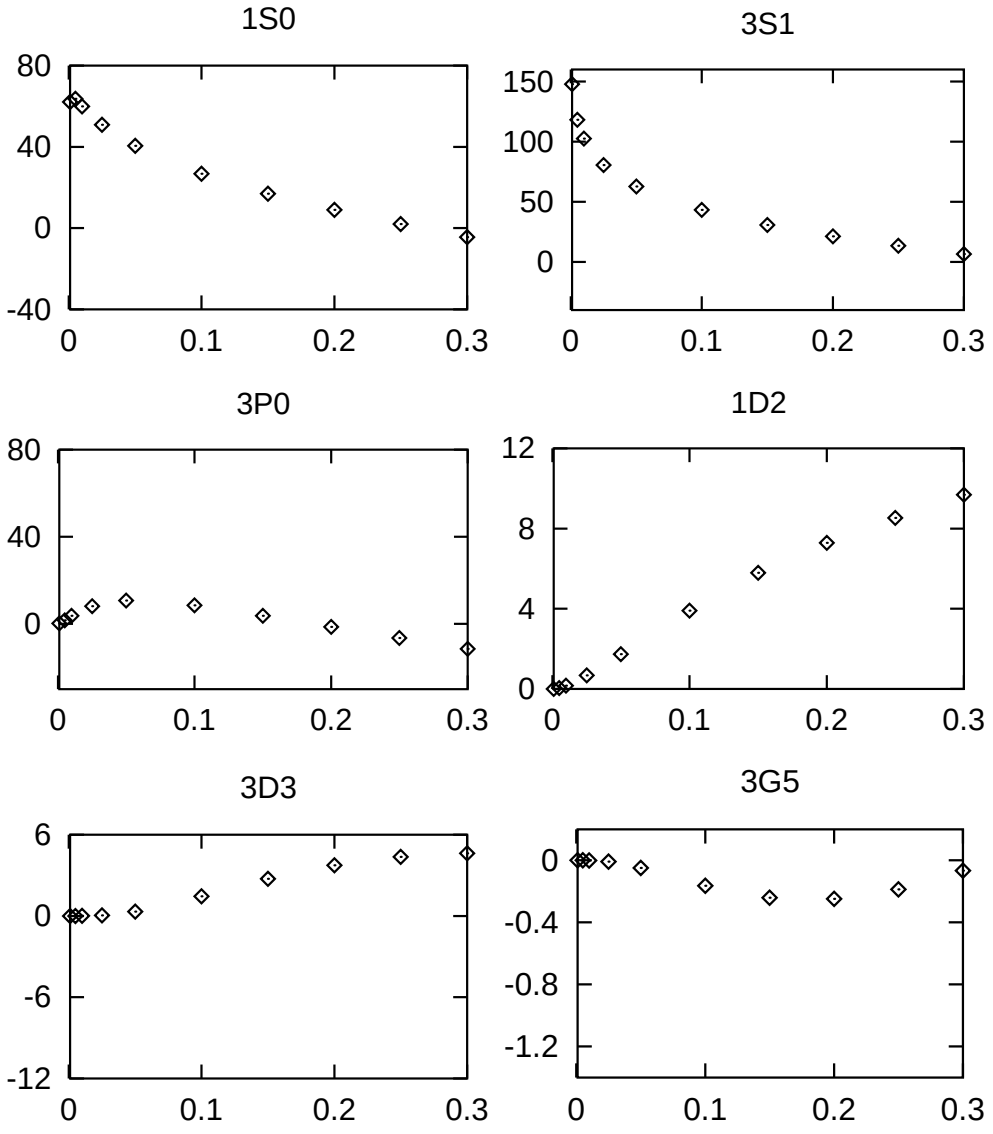
+ 1/m and isospin-breaking corrections…

figure from H. Krebs

Chiral effective field theory for two nucleons

- Epelbaum, Meißner, et al.
- Also Entem, Machleidt
- Organize by $(Q/\Lambda)^\nu$ where $Q = \{\mathbf{p}, m_\pi\}$, $\Lambda \sim 0.5$–1 GeV
- $\mathcal{L}_{\pi N}$ + match at low energy

Q^ν	1π	2π	$4N$



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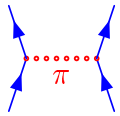
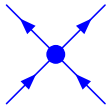
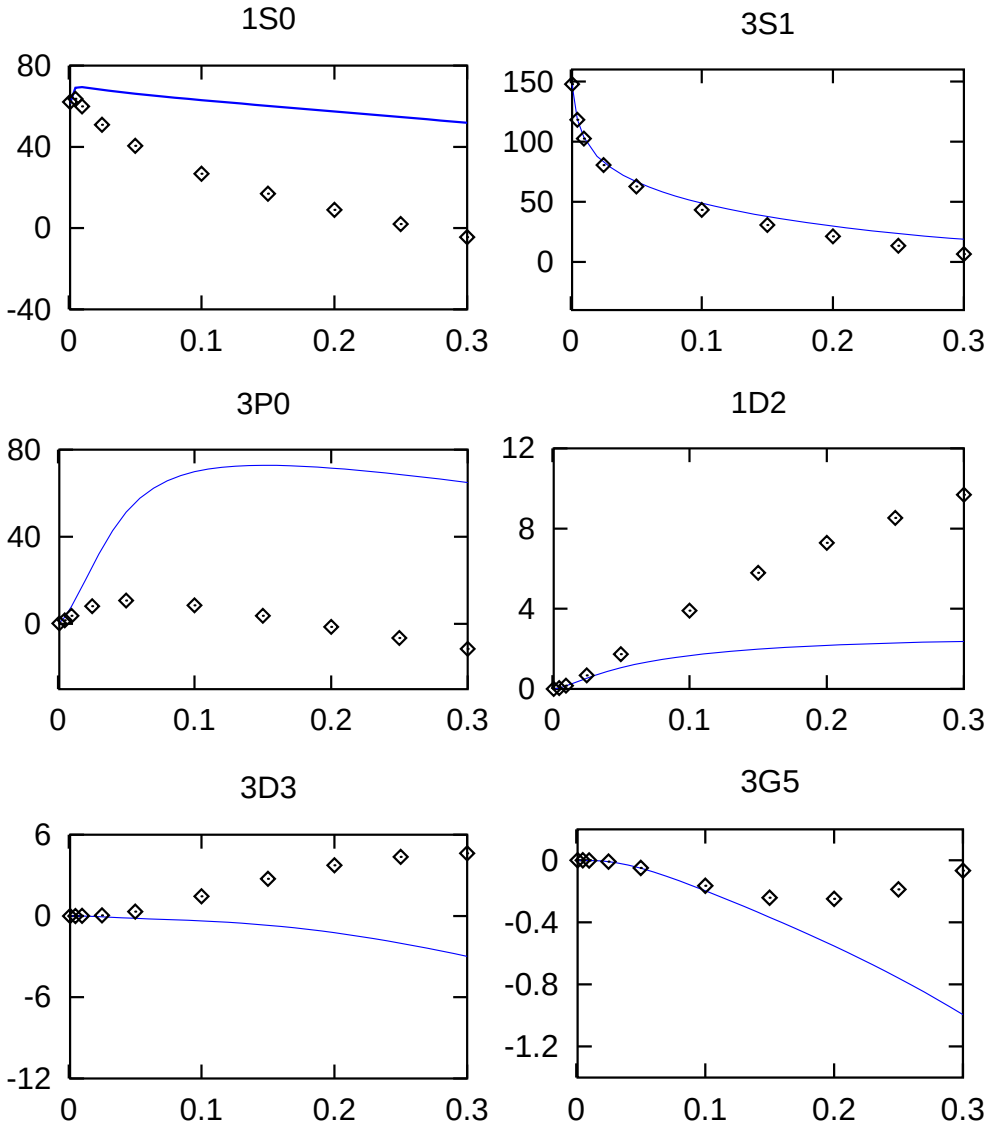
Q^0		—	 (2)



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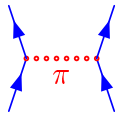
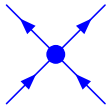
Q^0		—	 (2)
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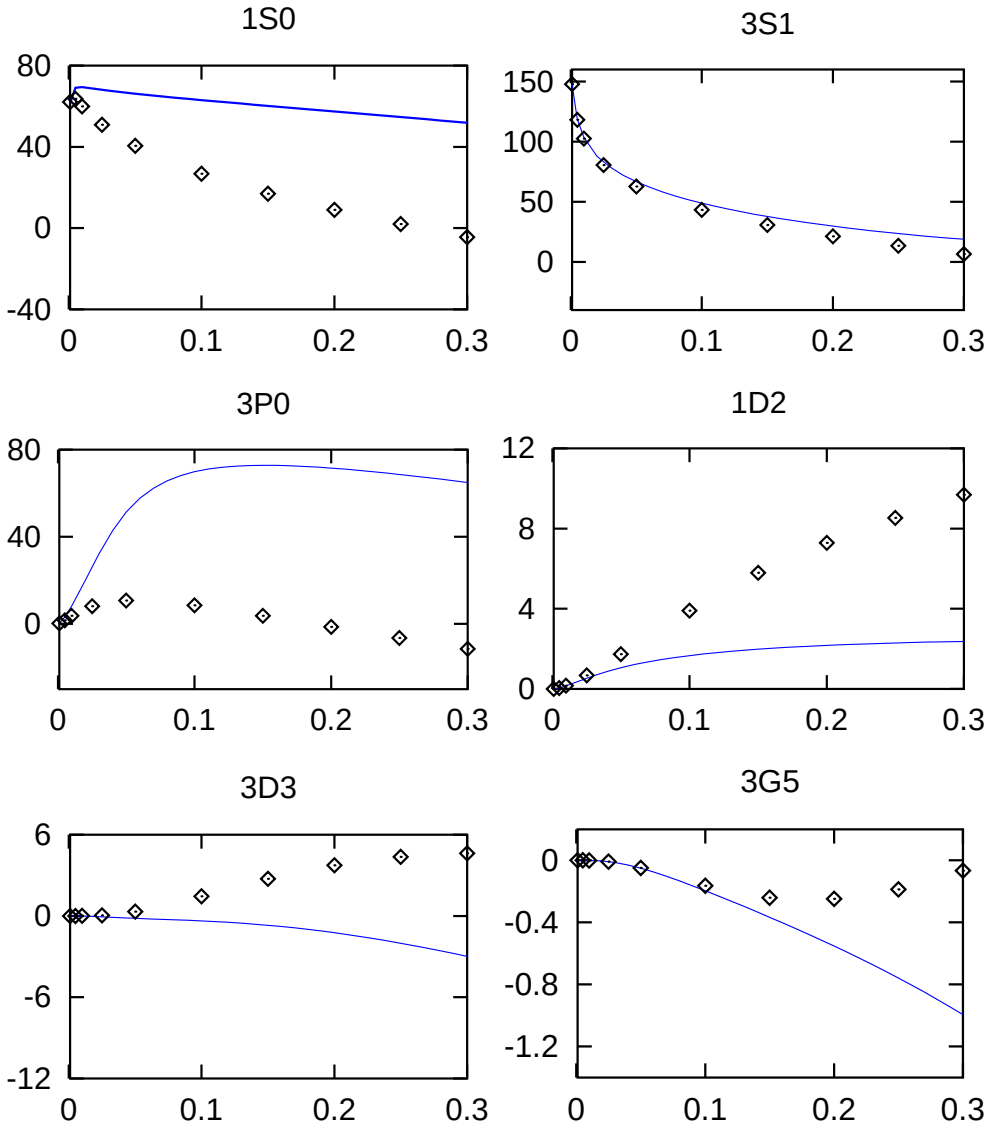

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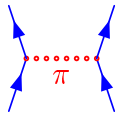
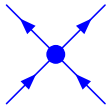
Q^0		—	 (2)
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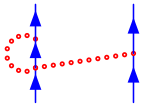
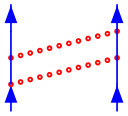
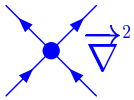
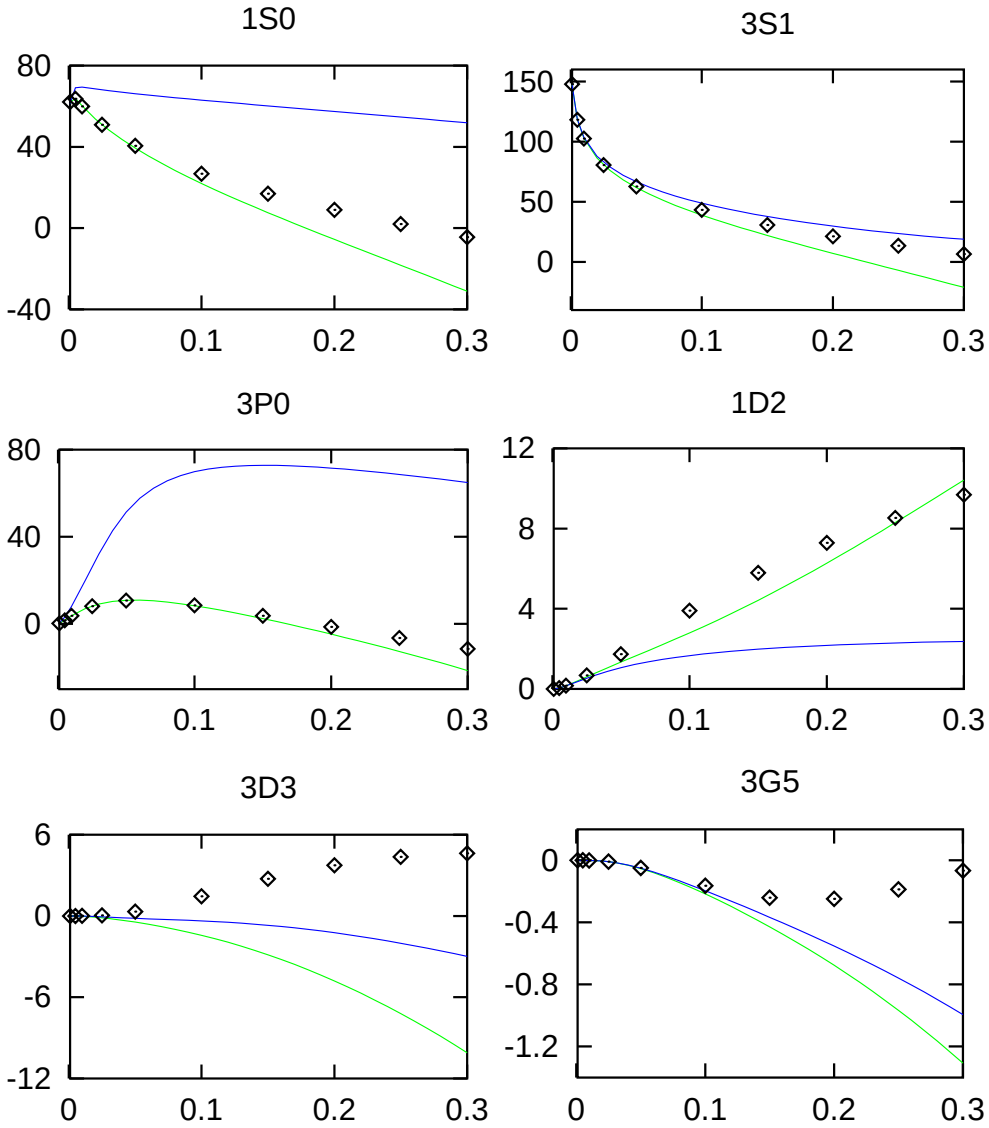
Q^2			 (7)



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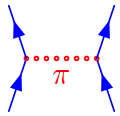
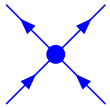
Q^0		—	 (2)
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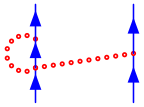
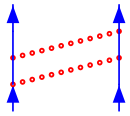
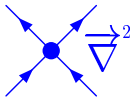
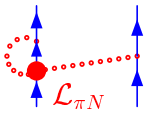
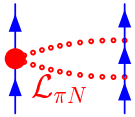
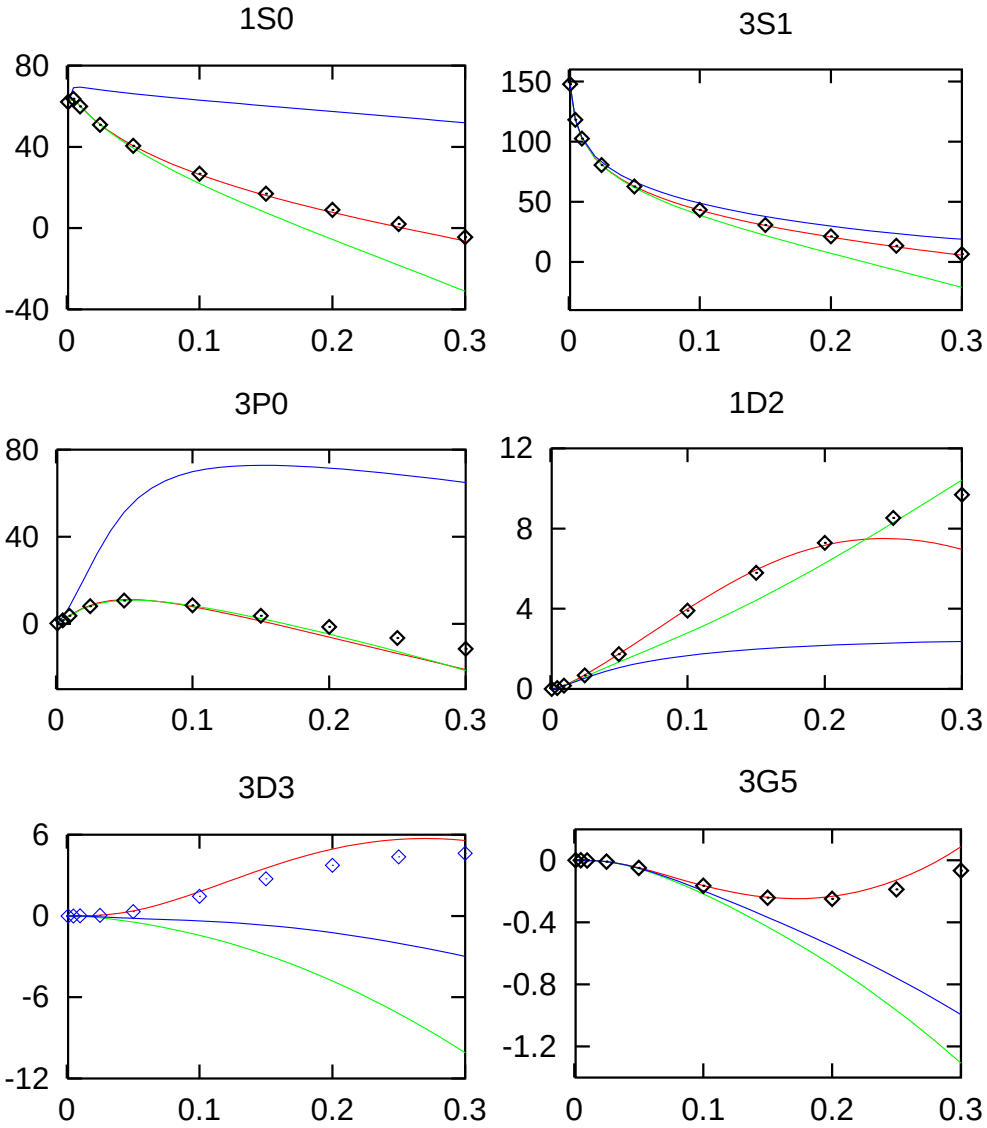
Q^2			 (7)
Q^3			



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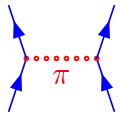
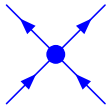
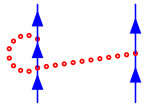
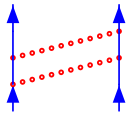
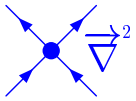
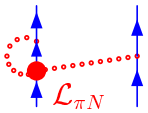
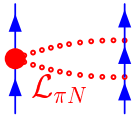
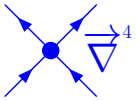
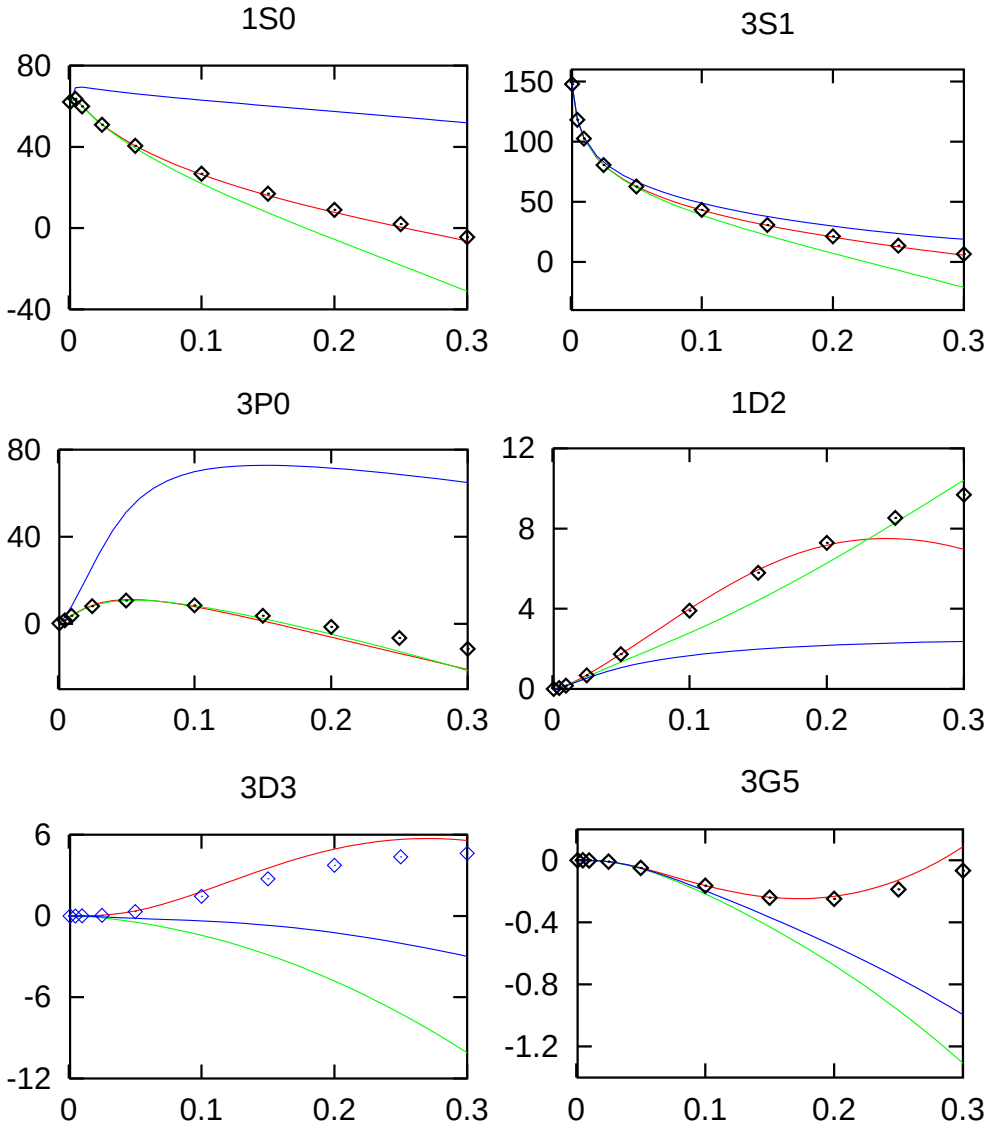
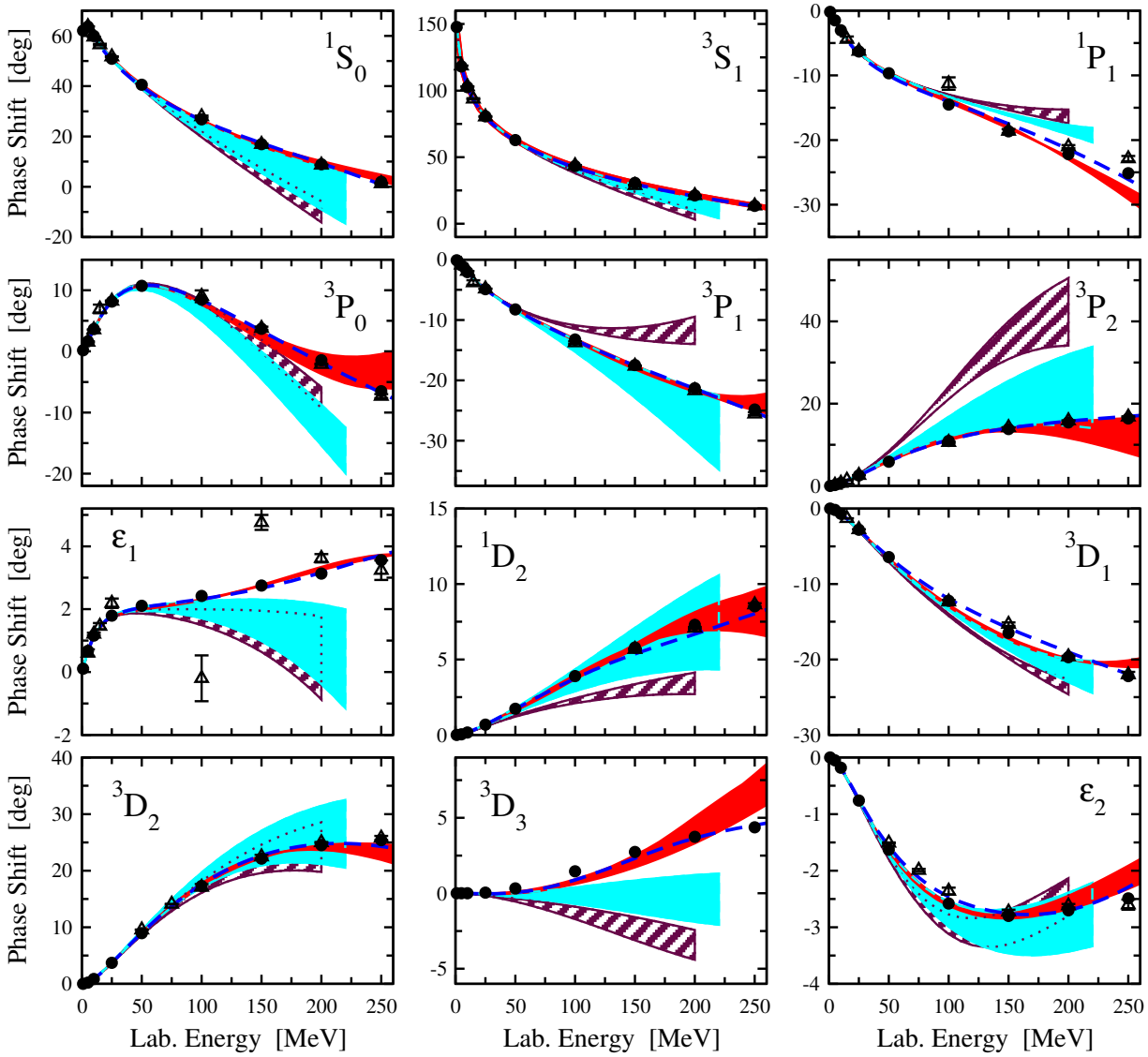
Q^0		—	 (2)
Q^1			
Q^2			 (7)
Q^3			
Q^4	many	many	 (15)



NN scattering up to N^{3}LO (Epelbaum, nucl-th/0509032)



- Theory error bands from varying cutoff over “natural” range

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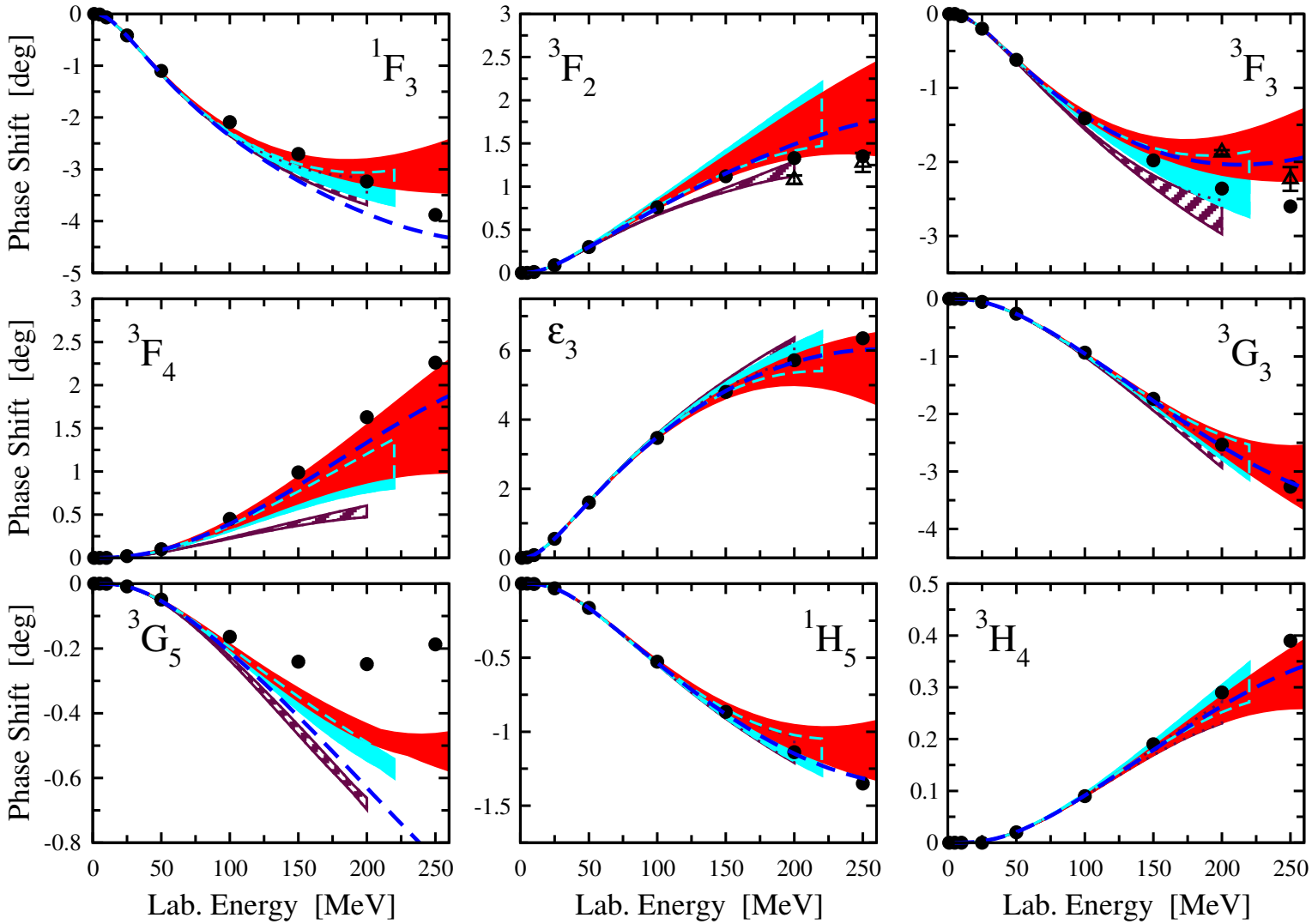
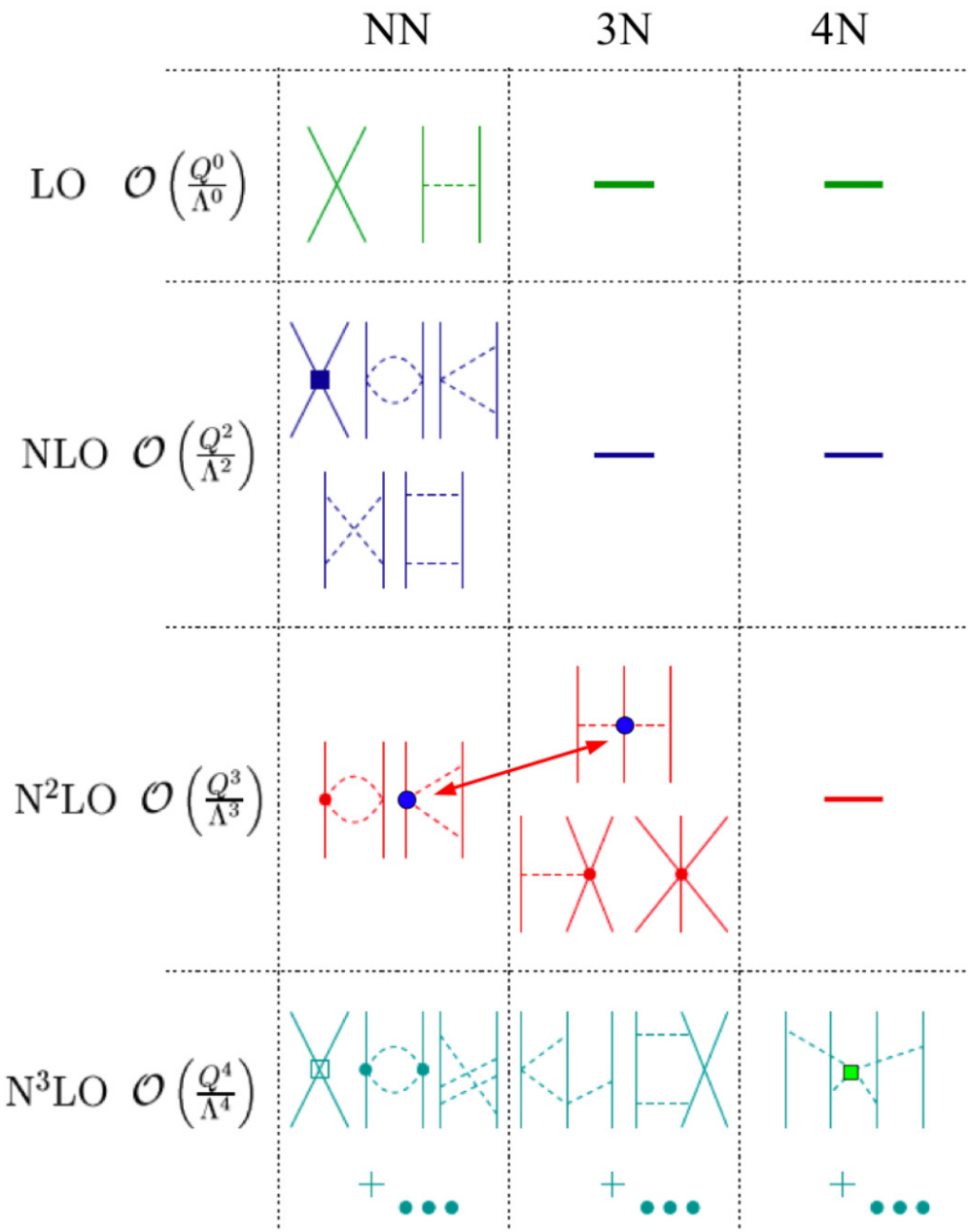


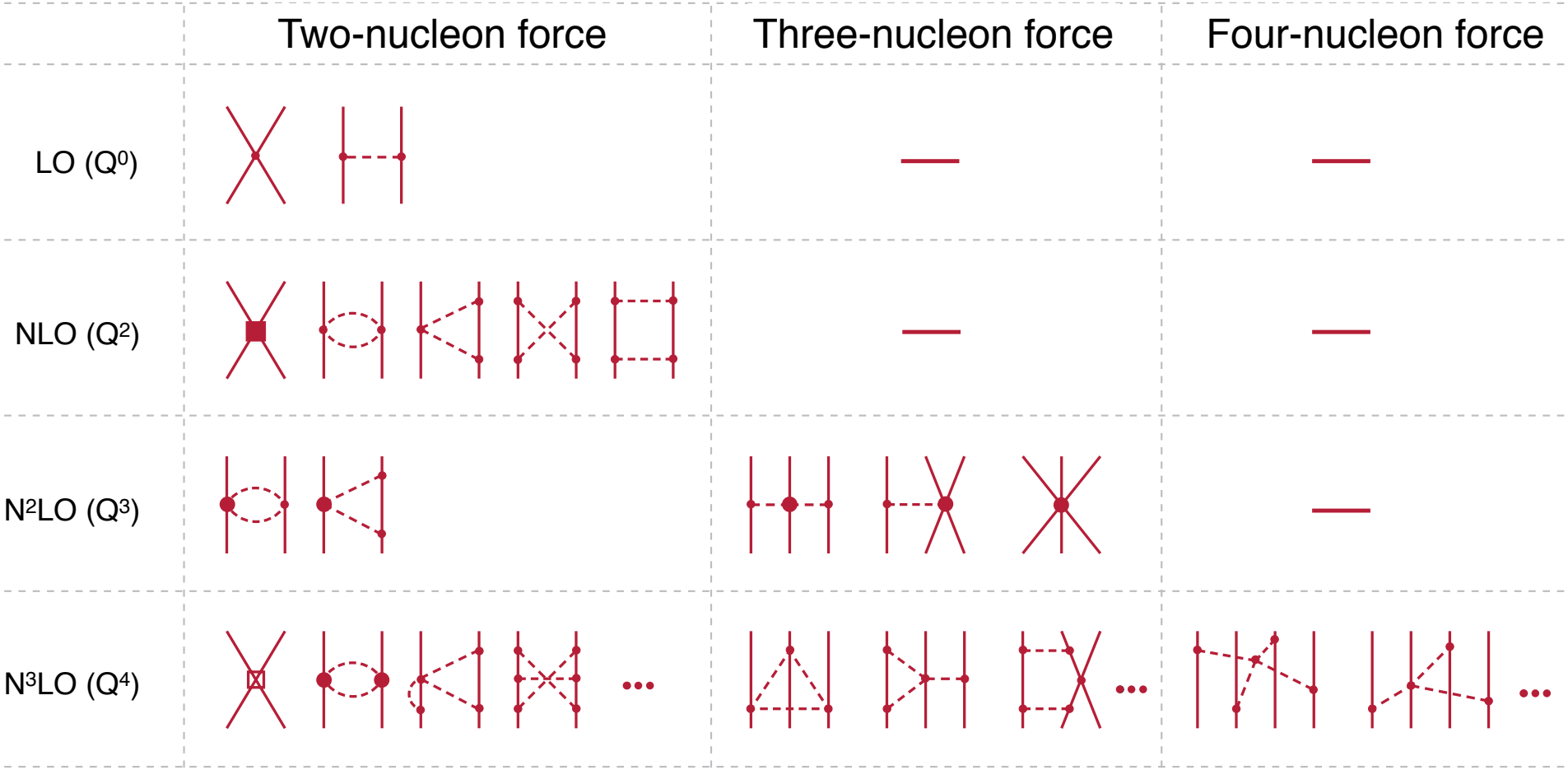
- Theory error bands from varying cutoff over “natural” range

Few-body chiral forces

- At what orders? $\nu = -4 + 2N + 2L + \sum_i (d_i + n_i/2 - 2)$, so adding a nucleon suppresses by Q^2/Λ^2.
- Power counting confirms $2NF \gg 3NF > 4NF$
- NLO diagrams cancel
- 3NF vertices may appear in NN and other processes
- Fits to the c_i's have sizable error bars



Status of chiral EFT forces [H. Krebs, TRIUMF Workshop (2014)]



- converged
- accurate description of NN at least up to $E_{lab} \sim 200$ MeV

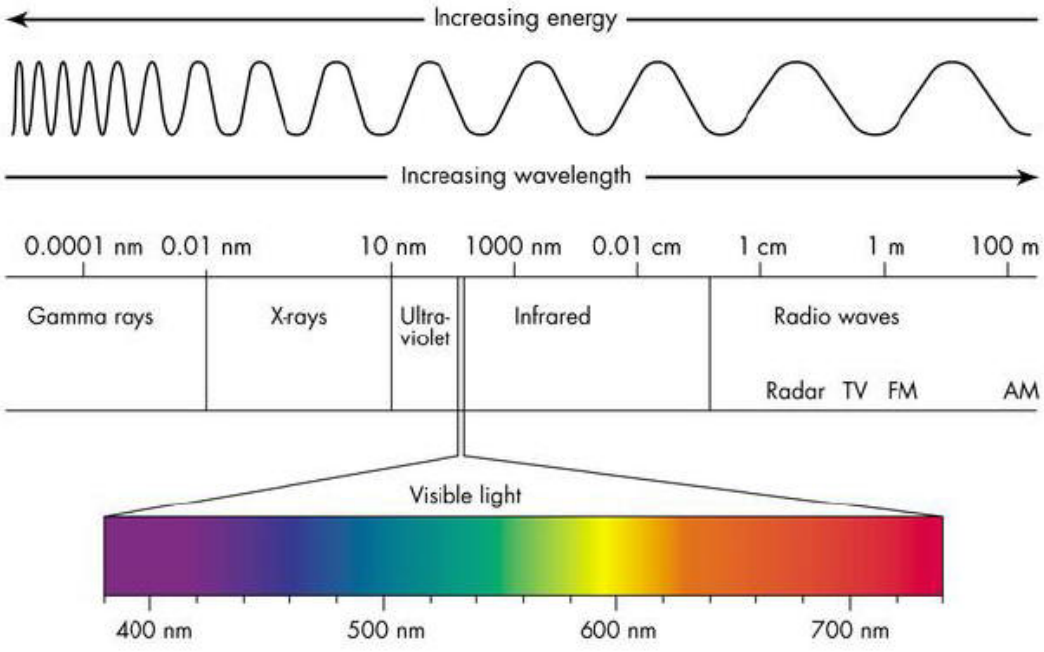
- not yet converged
- higher orders in progress
- impact on few- & many-N systems?

- converged ??

Also in progress: versions with Δ included $\Longrightarrow$ better expansion?

Summary: Conceptual basis of (chiral) effective field theory

- Separate the short-distance (UV) from long-distance (IR) physics $\Longrightarrow$ defines a scale
- Exploit chiral symmetry $\Longrightarrow$ hierarchical treatment of long-distance physics
- Use complete basis for short-distance physics $\Longrightarrow$ hierarchy à la multipoles



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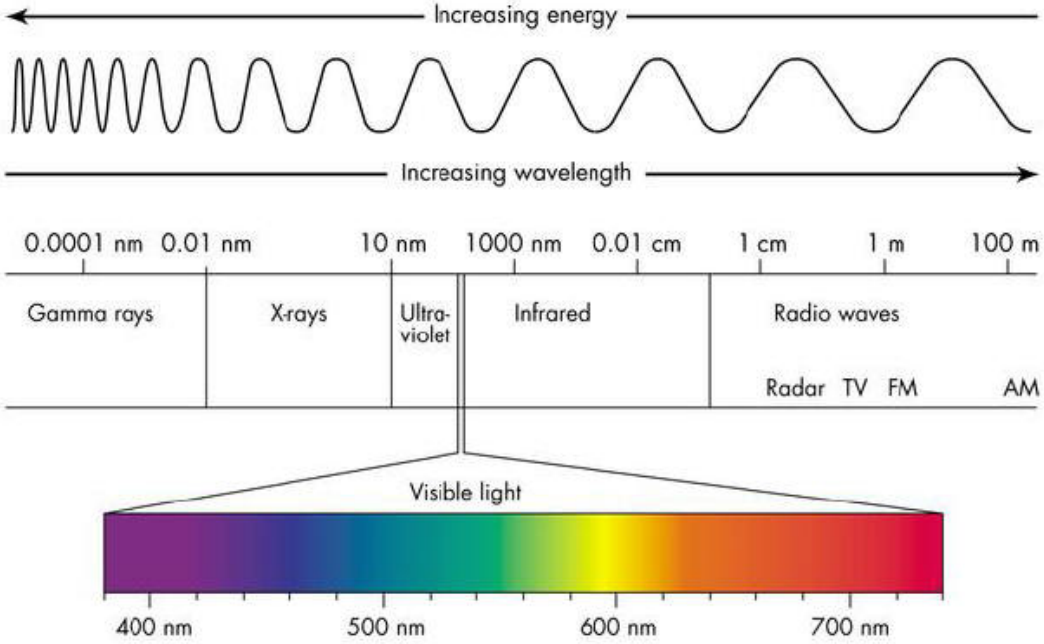
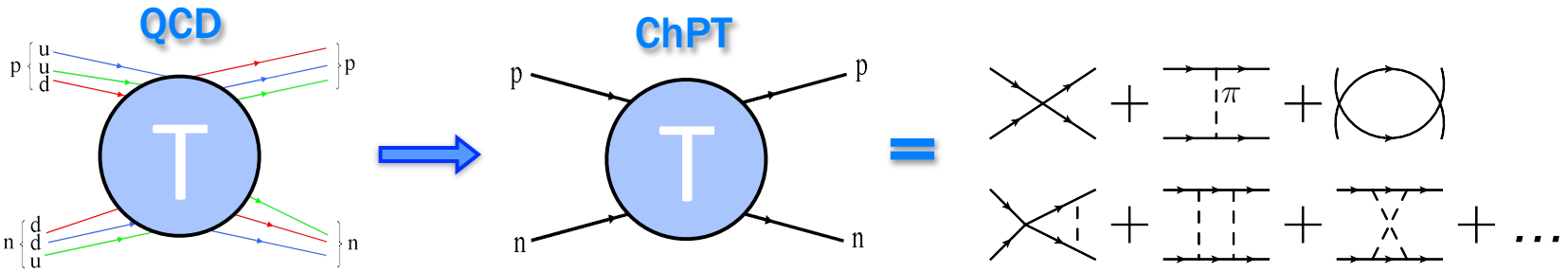




[H. Krebs]

Generate a nonrelativistic potential for many-body methods (controversies!)

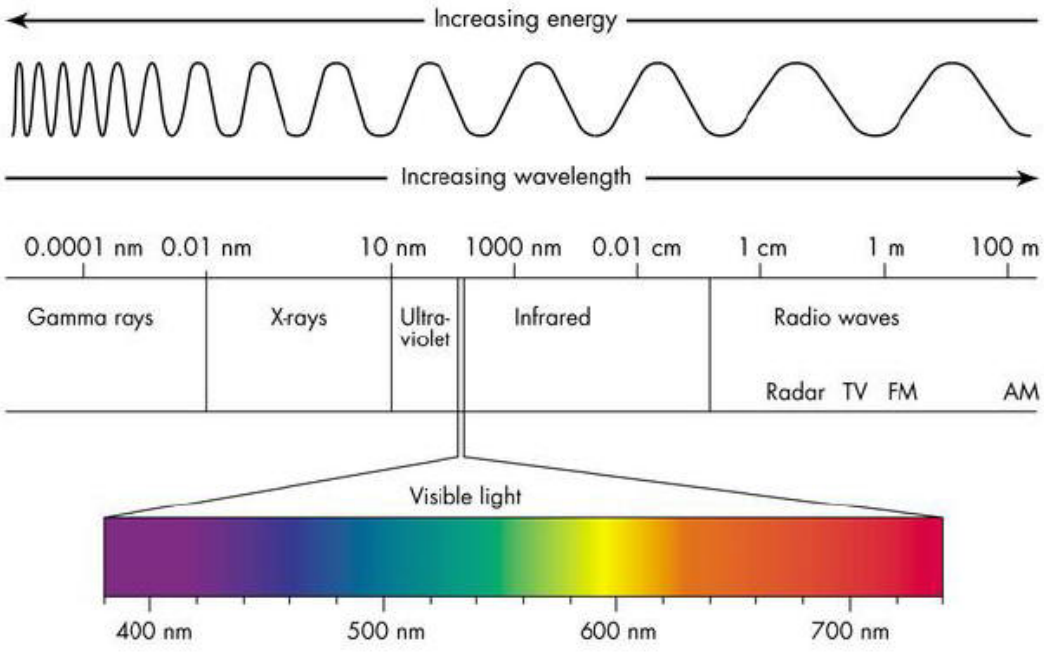
$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle=E|\Psi\rangle$$

Weinberg '91

Where/how do we draw the line? What if we draw it in different places?

How do we draw the line in an EFT? Regulators!

- In coordinate space, define R_0 to separate short and long distance
- In momentum space, use Λ to separate high and low momenta
- Much freedom *how* this is done $\Longrightarrow$ different scales / schemes



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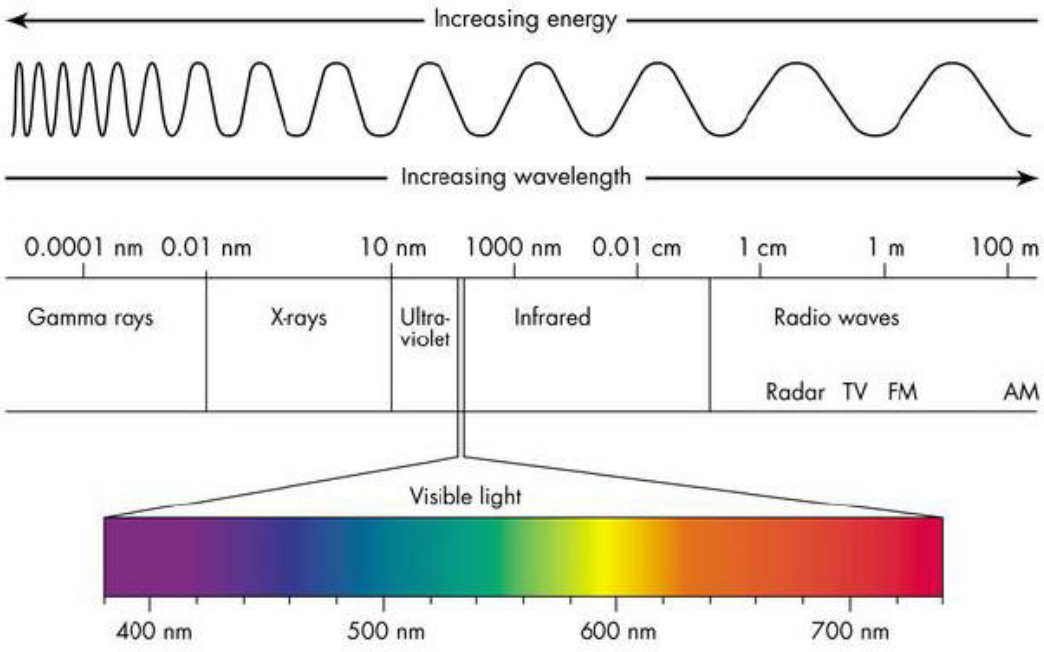


Non-local regulator in momentum (e.g., with $n = 3$ for N^{3}LO):

$$V_{\rm CHPT}(\mathbf{p},\mathbf{p}') \longrightarrow e^{-(\mathbf{p}^2/\Lambda^2)^n} V_{\rm CHPT}(\mathbf{p},\mathbf{p}') e^{-(\mathbf{p}'^2/\Lambda^2)^n}$$

Local regulator in coordinate space for long-range and delta function:

$$V_{\rm long}(\mathbf{r}) \longrightarrow V_{\rm long}(\mathbf{r})(1 - e^{-(r/R_0)^4}) \quad \text{and} \quad \delta(\mathbf{r}) \longrightarrow C e^{-(r/R_0)^4}$$

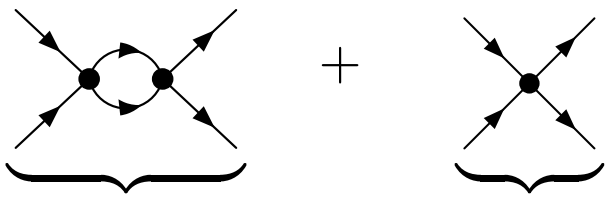
Or local in momentum space [Gazit, Quaglioni, Navratil (2009)]

Rough relation: $\Lambda = 450 \ldots 600\,\text{MeV} \Longleftrightarrow R_0 = 1.0 \ldots 1.2\,\text{fm}$

What does changing a cutoff do in an EFT?

- (Local) field theory version in perturbation theory (diagrams)
  - Loops (sums over intermediate states) $\overset{\Delta\Lambda_c}{\Longleftrightarrow}$ LECs

$$\frac{d}{d\Lambda_c}\Bigg[$$

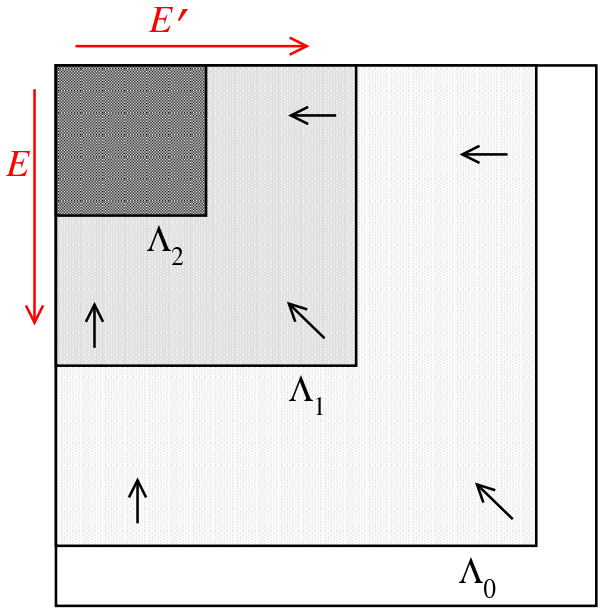


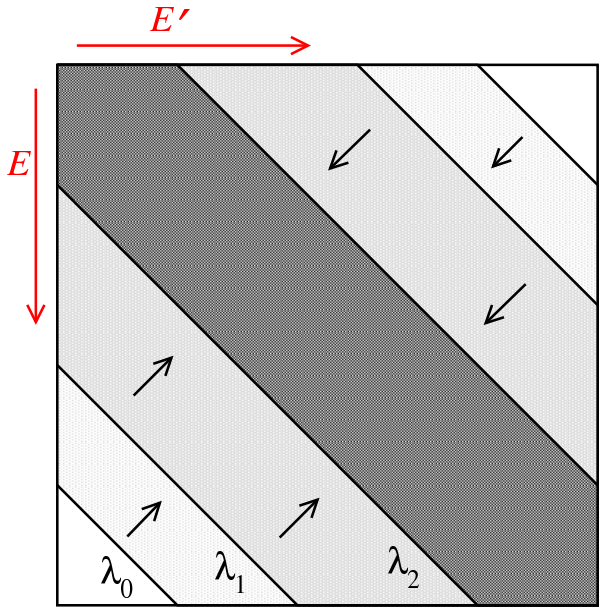
$$\Bigg] = 0$$

$$\int^{\Lambda_c} \frac{d^3q}{(2\pi)^3}\frac{C_0 M C_0}{k^2-q^2+i\epsilon} \qquad C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2}+\cdots$$

  - Momentum-dependent vertices $\Longrightarrow$ Taylor expansion in k^2
- Claim: $V_{\text{low}\,k}$ RG and SRG decoupling work analogously

"$V_{\text{low}\,k}$" SRG ("T" generator)





Outline: Lecture 3

Lecture 3: Effective field theory

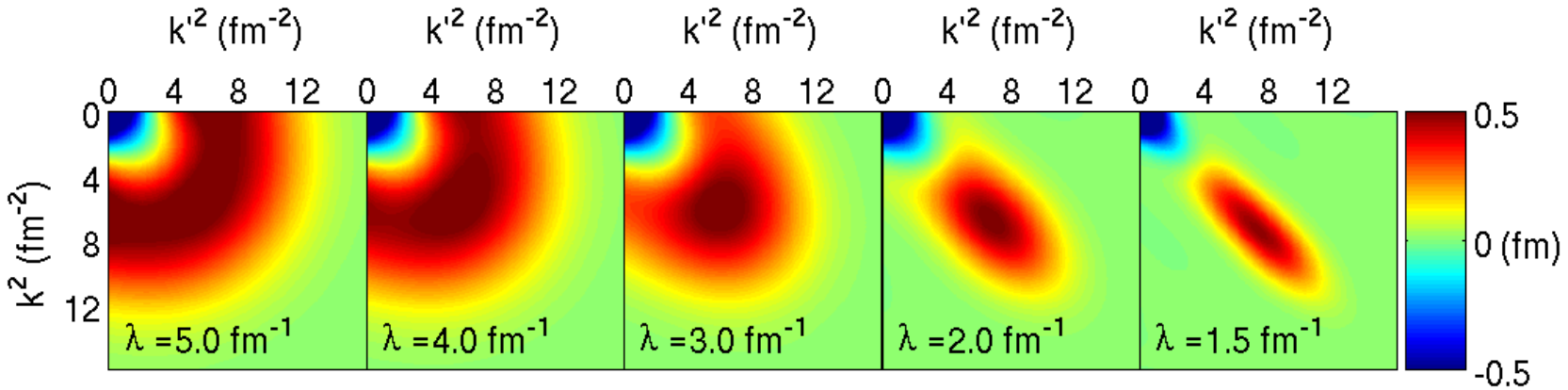
- Recap from lecture 2: How SRG works
- Motivation for nuclear effective field theory
- Chiral effective field theory
- Universal potentials from RG evolution
- Extra: Quantitative measure of perturbativeness

S. Weinberg on the Renormalization Group (RG)

- From "Why the RG is a good thing" [for Francis Low Festschrift]
  "The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - RG: shift between couplings and loop integrals to reduce logs
  - Nuclear: decouple high- and low-momentum modes
- Identifying universality in critical phenomena
  - RG: filter out short-distance degrees of freedom
  - Nuclear: evolve toward universal interactions
- Nuclear: simplifying calculations of structure/reactions
  - Make nuclear physics look more like quantum chemistry!
  - RG gains can violate conservation of difficulty!
  - Use RG scale (resolution) dependence as a *probe* or *tool*

Flow of different N^{3}LO chiral EFT potentials

- $^1S_0$ from N^{3}LO (500 MeV) of Entem/Machleidt



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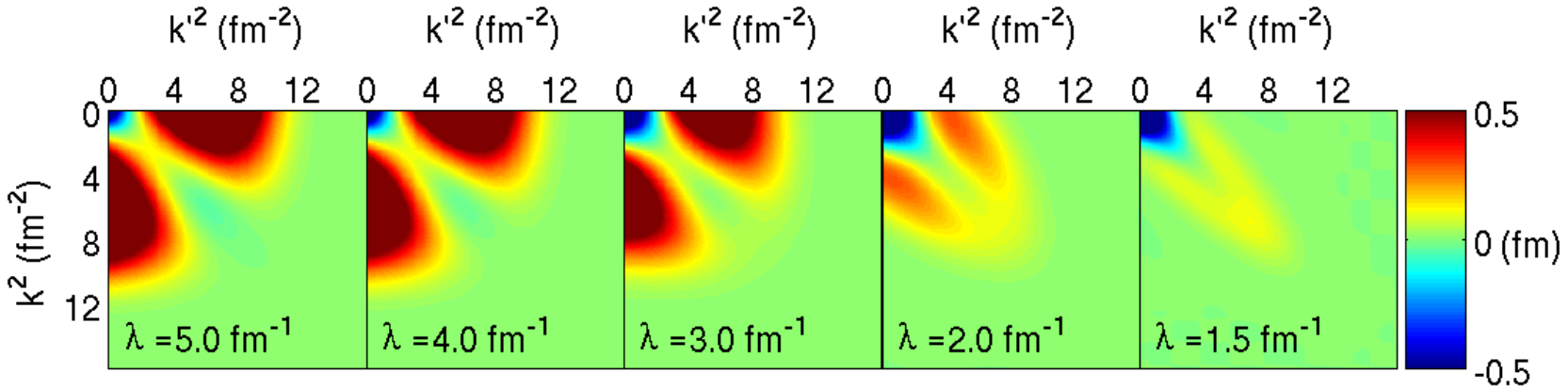

- Decoupling $\Longrightarrow$ perturbation theory is more effective

$$\langle k|V|k\rangle + \sum_{k'} \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2 - k'^2)/m} + \cdots \Longrightarrow V_{ii} + \sum_j V_{ij} V_{ji} \frac{1}{(k_i^2 - k_j^2)/m} + \cdots$$

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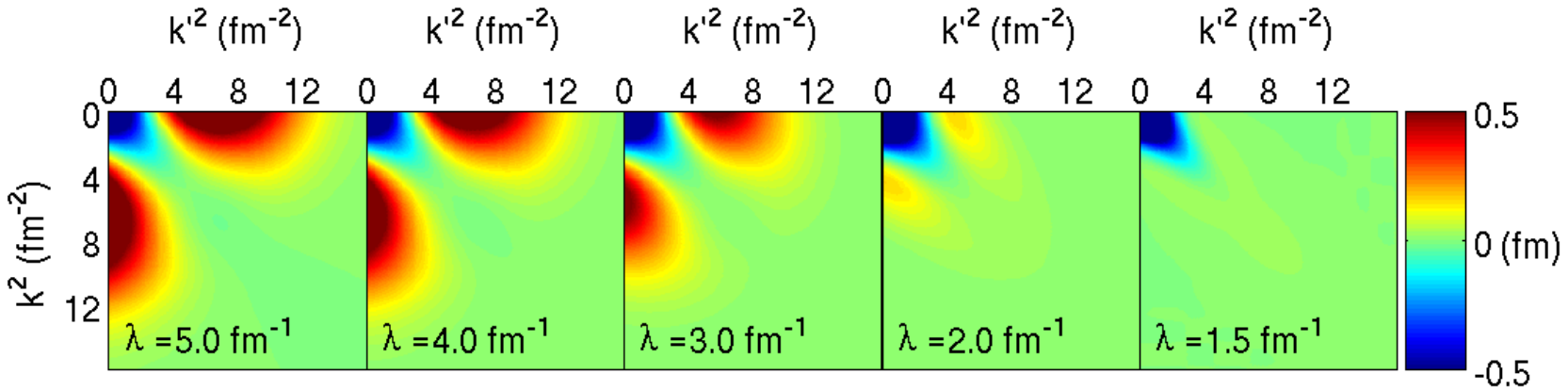

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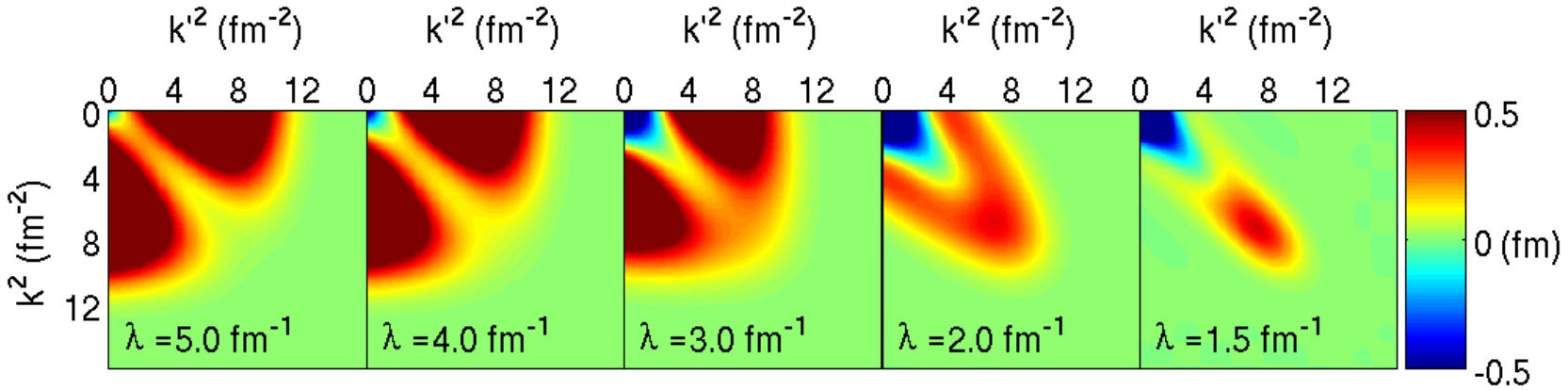

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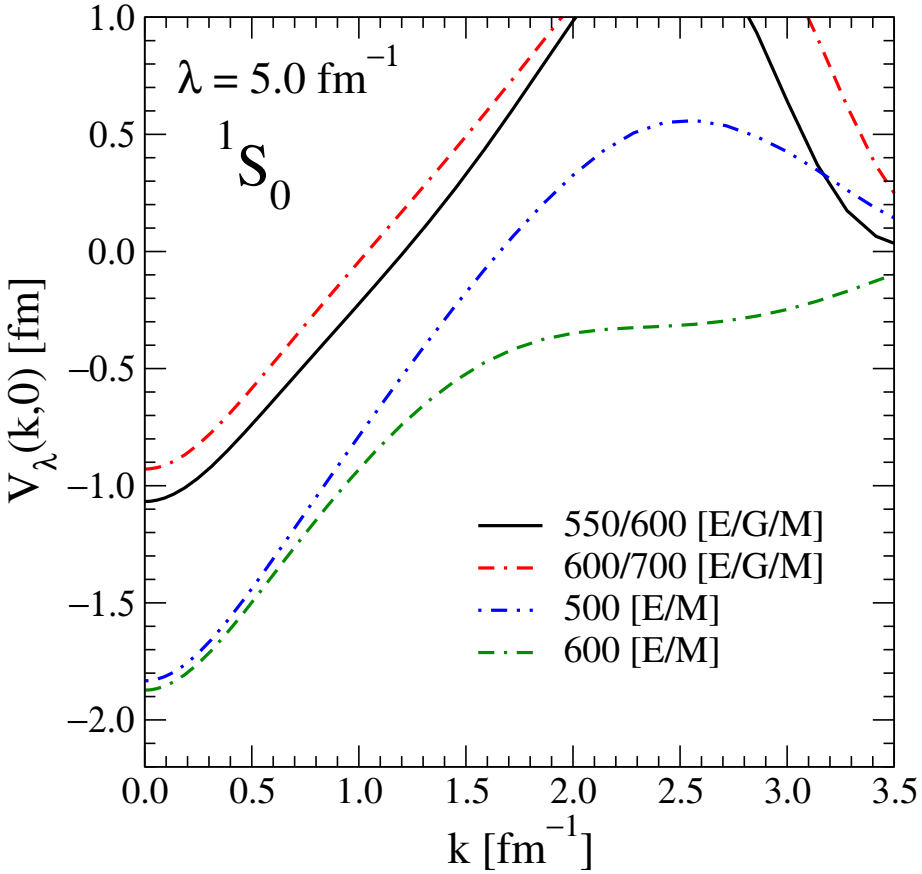
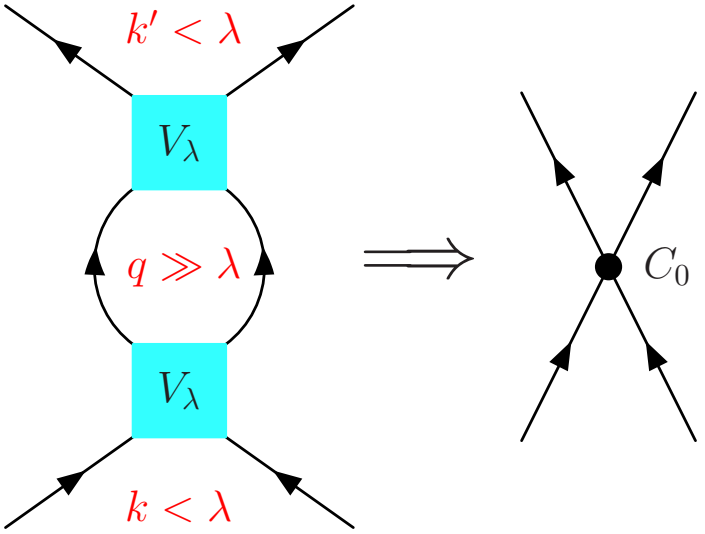
Approach to universality (fate of high-q physics)

Run NN to lower λ via SRG $\Longrightarrow$ $\approx$Universal low-k V_{NN}

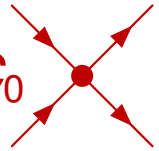
Off-Diagonal $V_\lambda(k,0)$





$q \gg \lambda$ (or λ) intermediate states
$\Longrightarrow$ replace with contact term:
$C_0 \delta^3(\mathbf{x} - \mathbf{x}')$

[cf. $\mathcal{L}_{\rm eff} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots$]

As resolution changes, shift high-k details to contacts, e.g., C_0 

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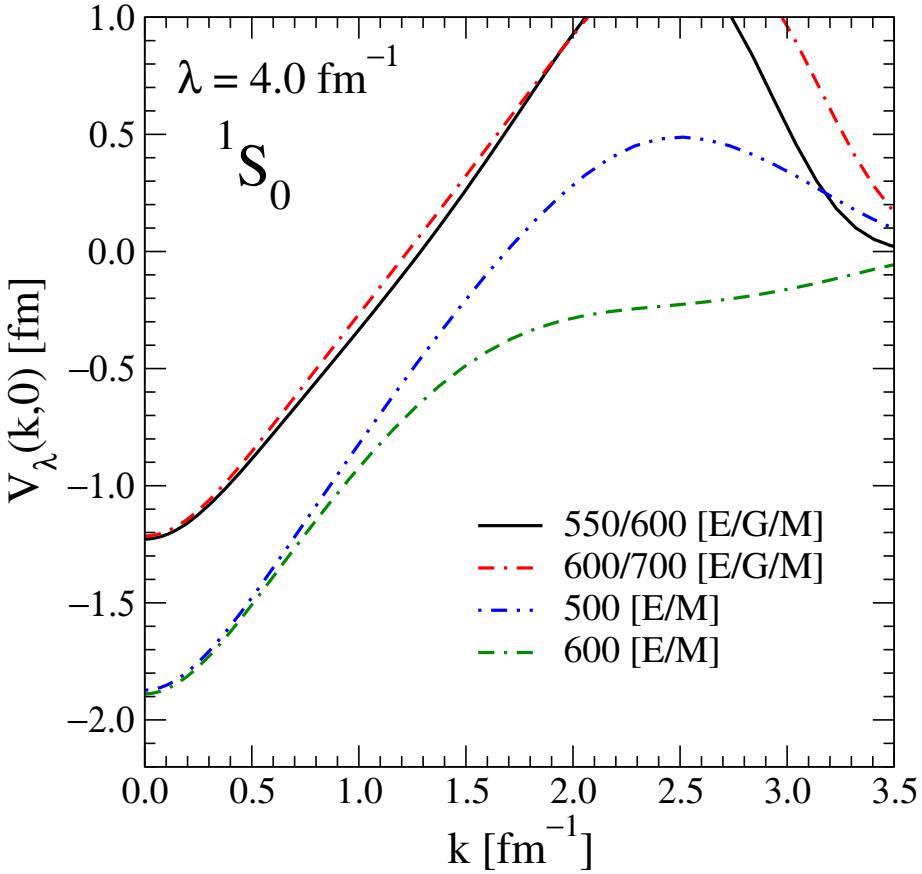

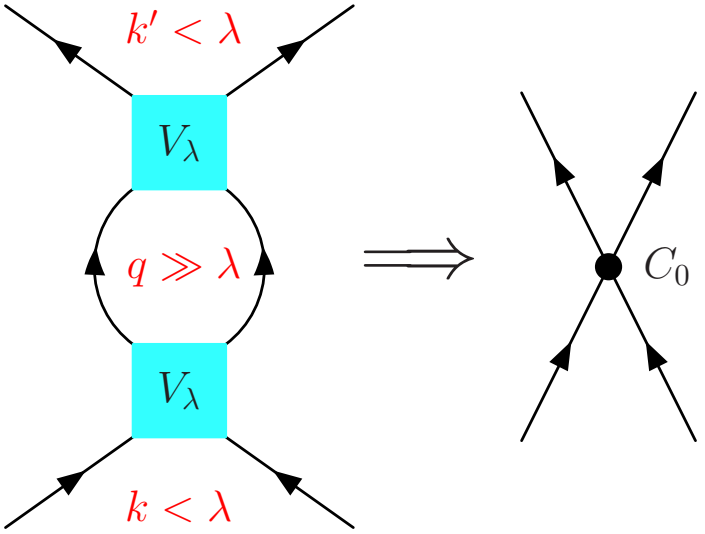

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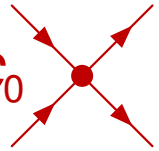
As resolution changes, shift high-k details to contacts, e.g., C_0 

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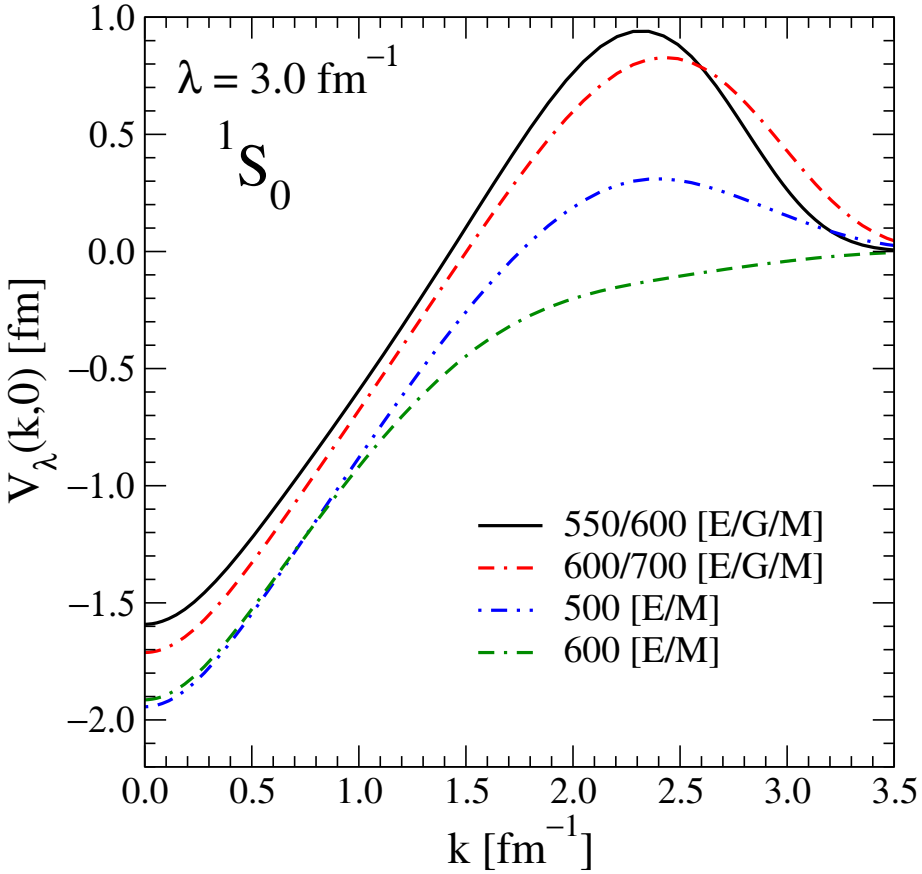

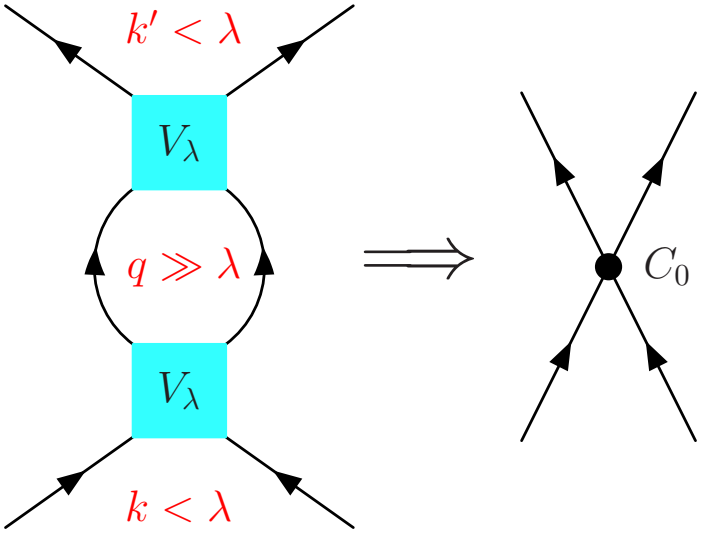

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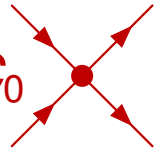
As resolution changes, shift high-k details to contacts, e.g., C_0 

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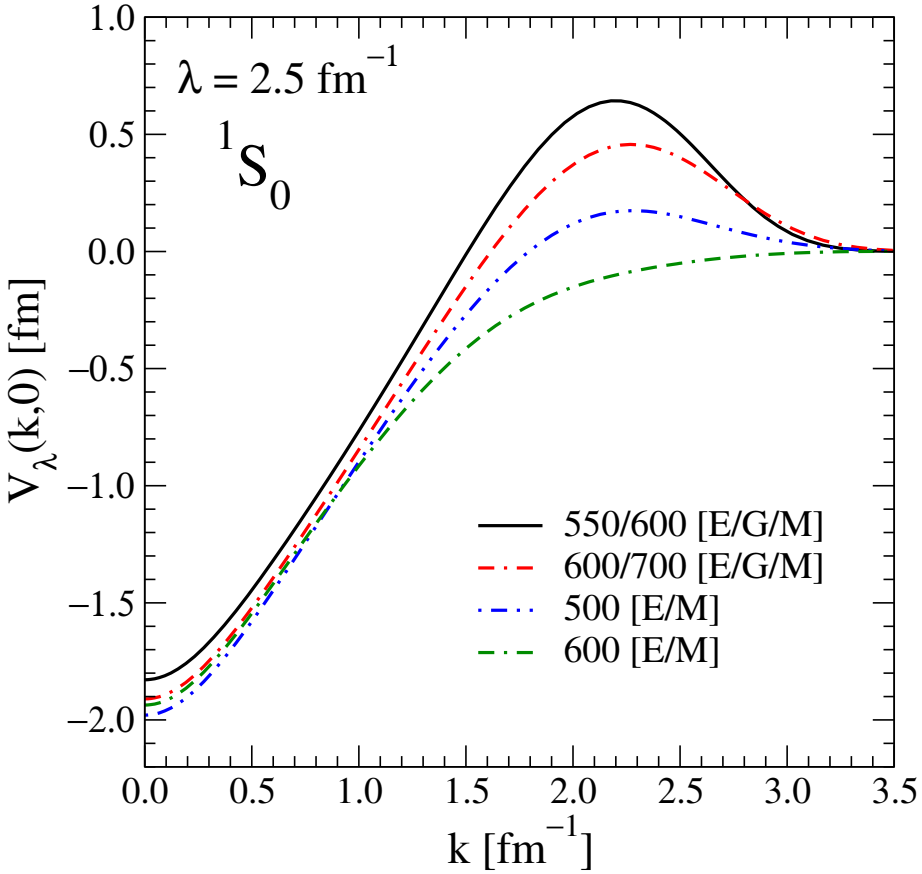

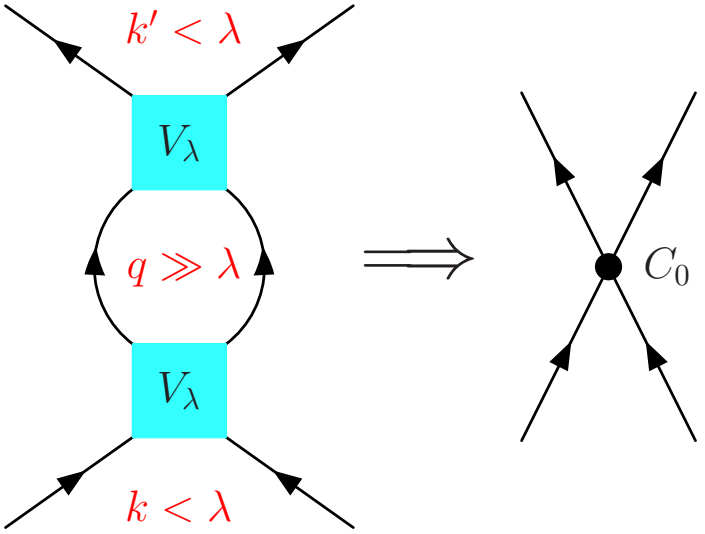

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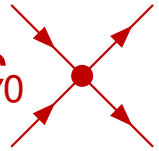
As resolution changes, shift high-k details to contacts, e.g., C_0 

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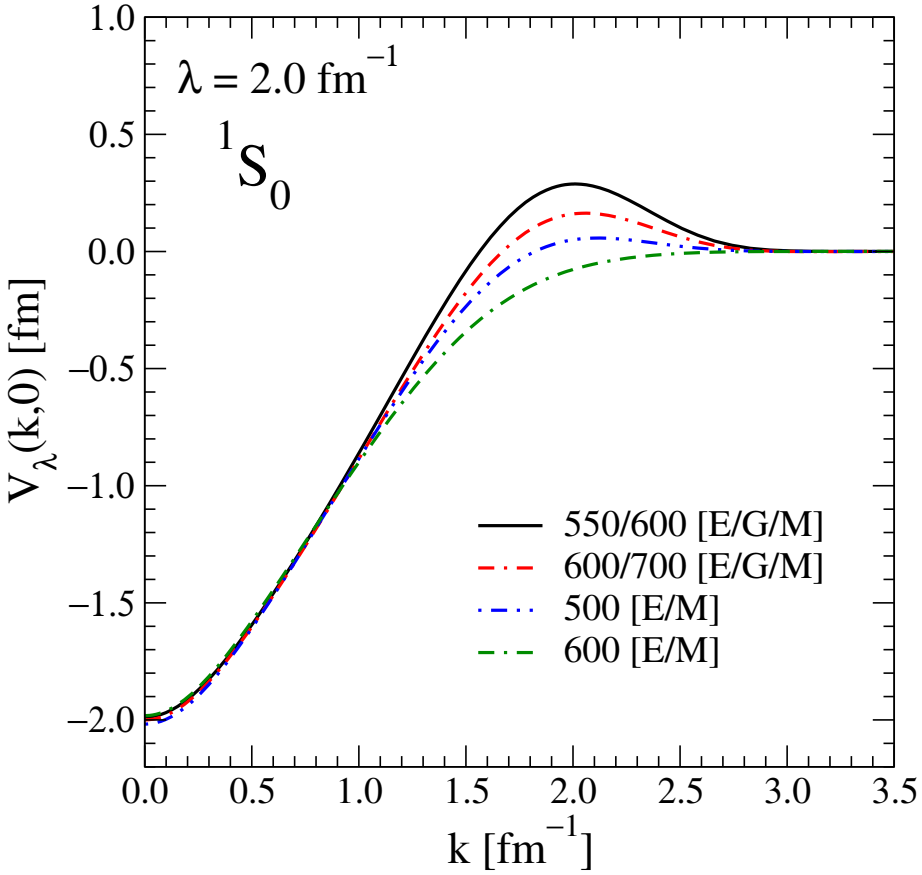

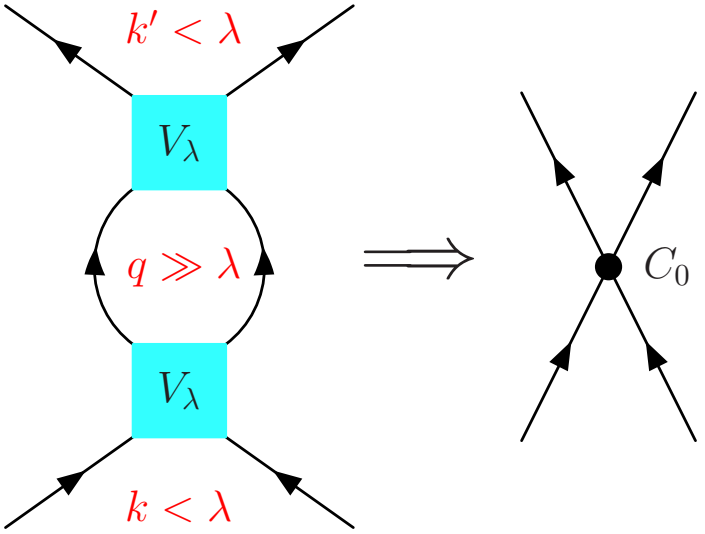

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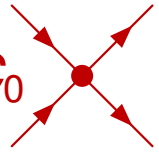
As resolution changes, shift high-k details to contacts, e.g., C_0 

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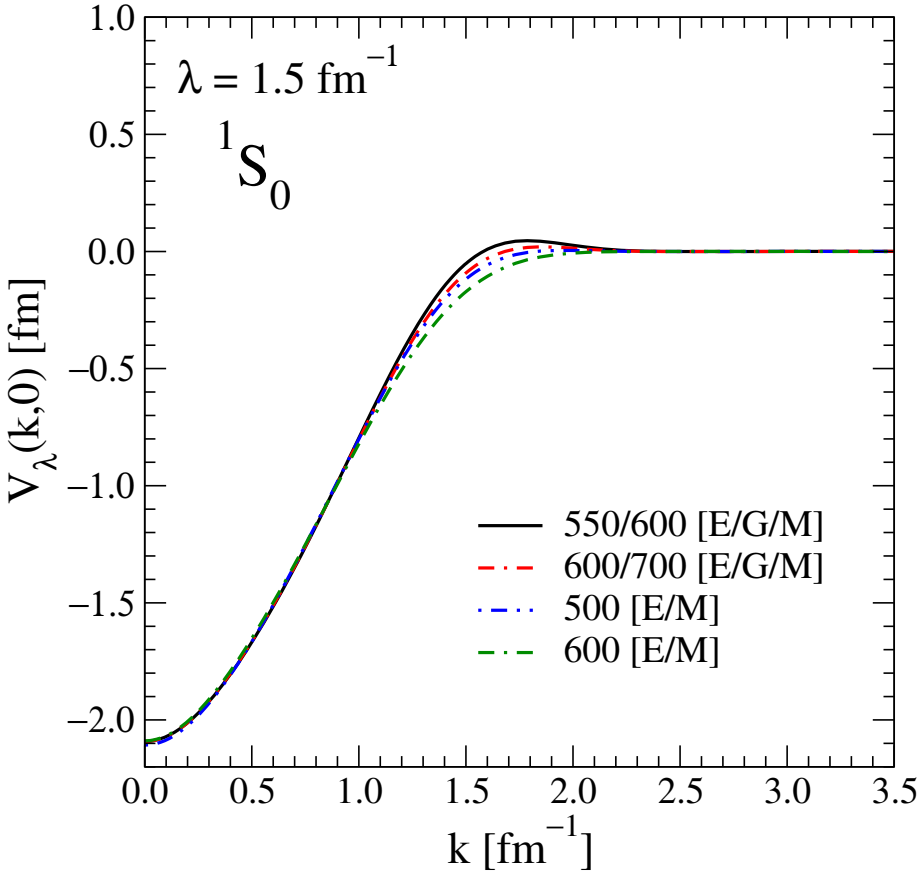

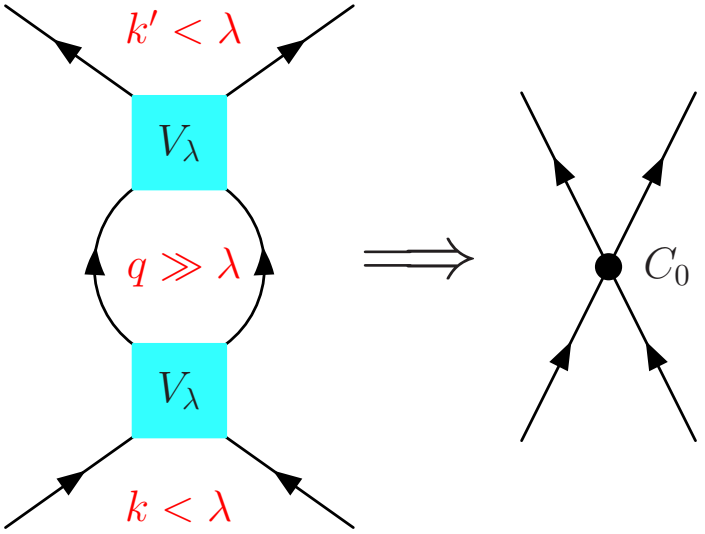

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$\Longrightarrow$ replace with contact term:

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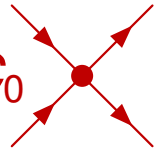
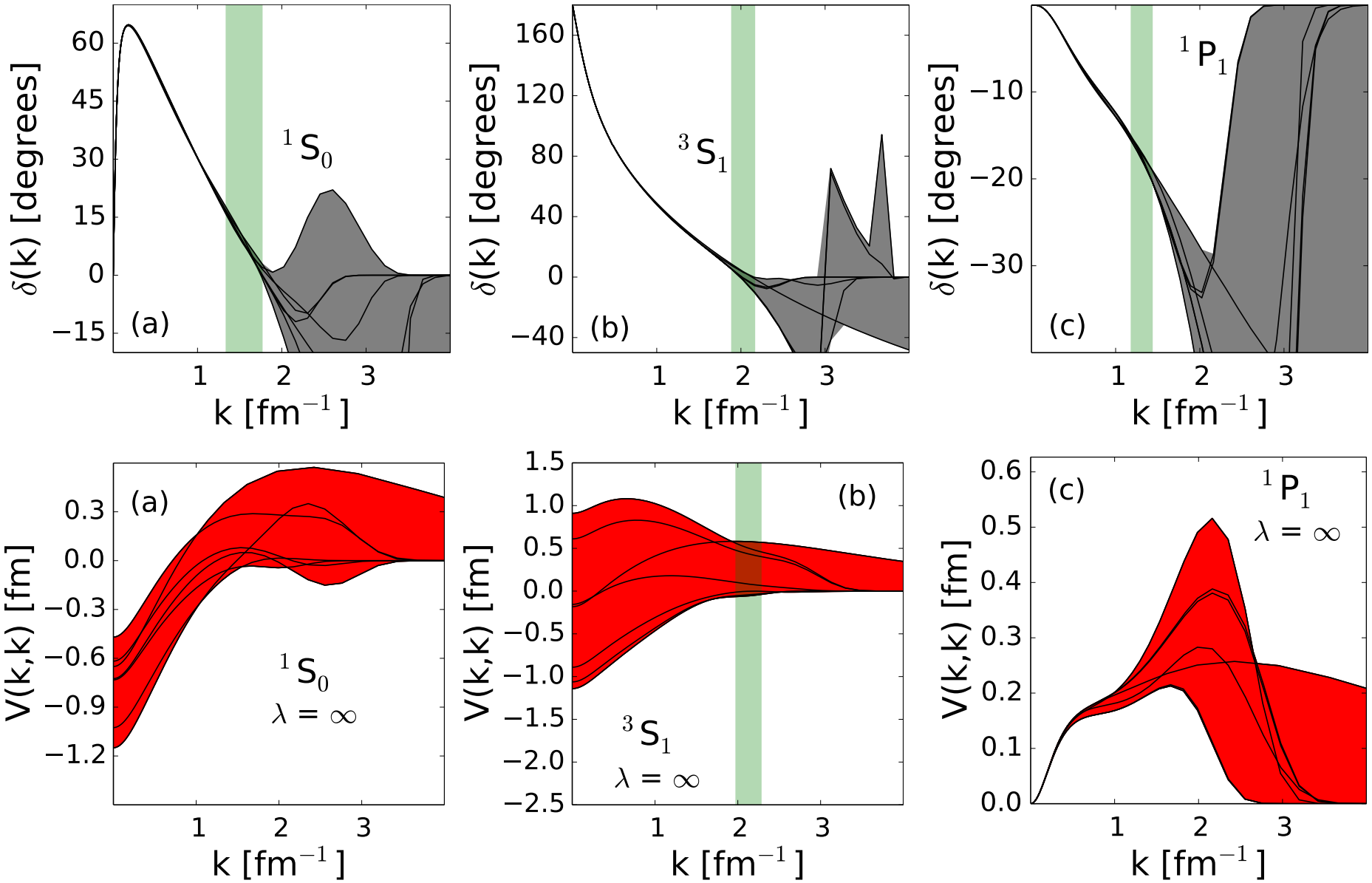
[cf. $\mathcal{L}_{\rm eff} = \cdots + \frac{1}{2} C_0 (\psi^\dagger \psi)^2 + \cdots$]

As resolution changes, shift high-k details to contacts, e.g., C_0 

NN V_{SRG} universality from phase equivalent potentials

Diagonal elements collapse where phase equivalent [Dainton et al, 2014]



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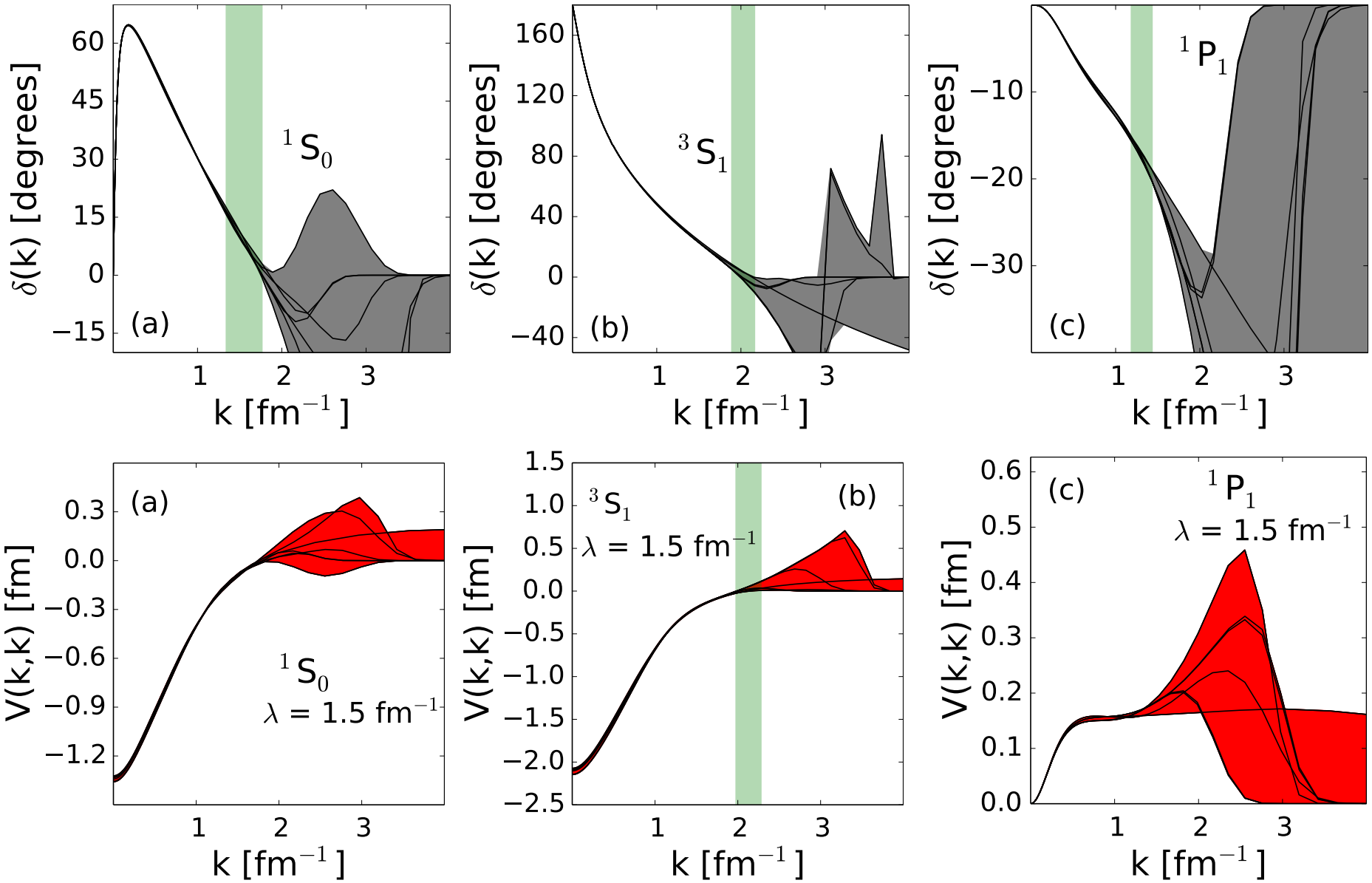
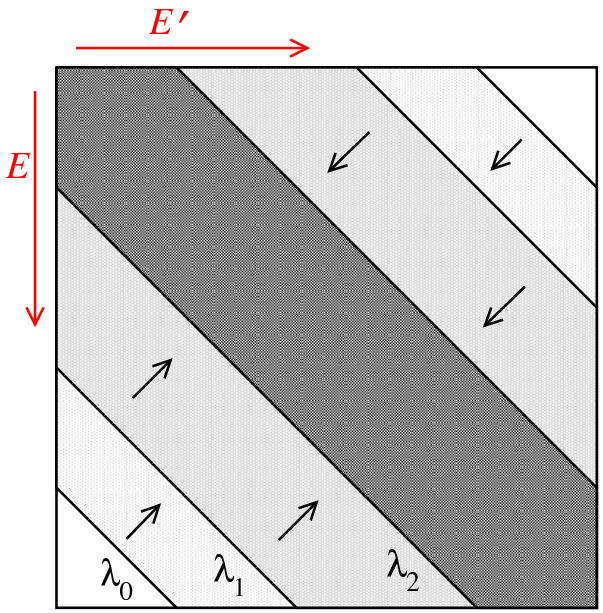
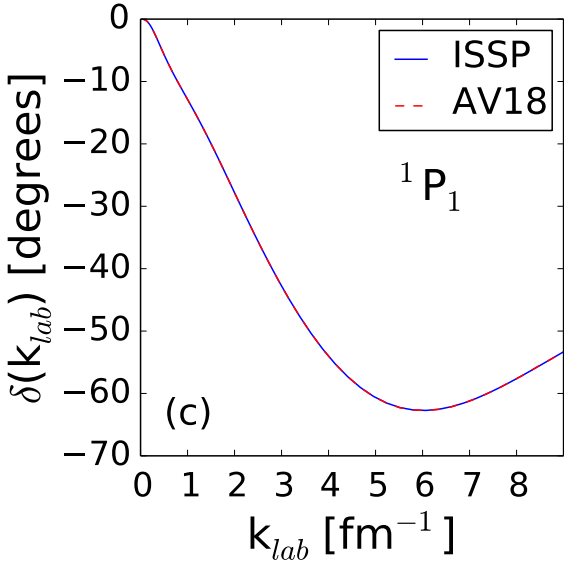
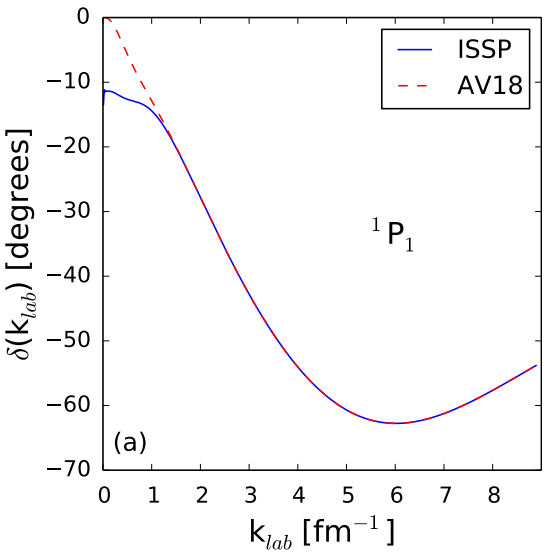


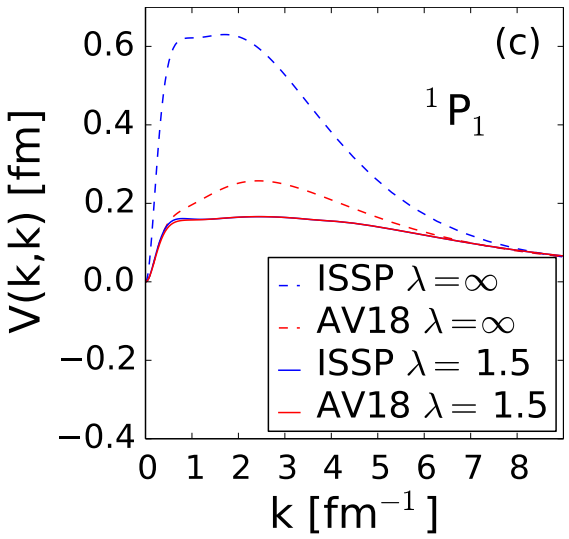
Use universality to probe decoupling

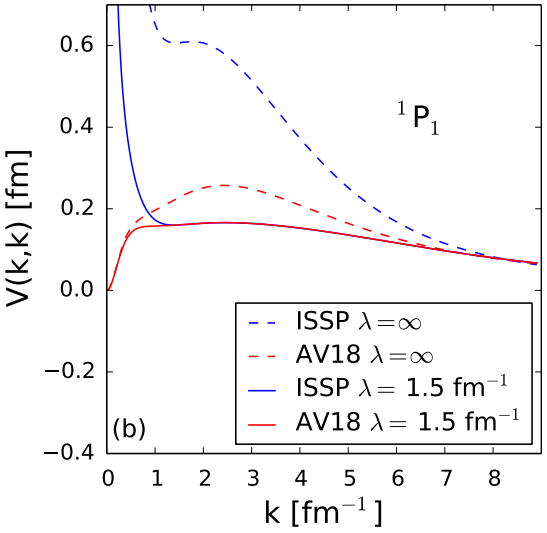
- What if not phase equivalent *everywhere*?
- Use $^1P_1$ as example (for a change :)
- Result: local decoupling!











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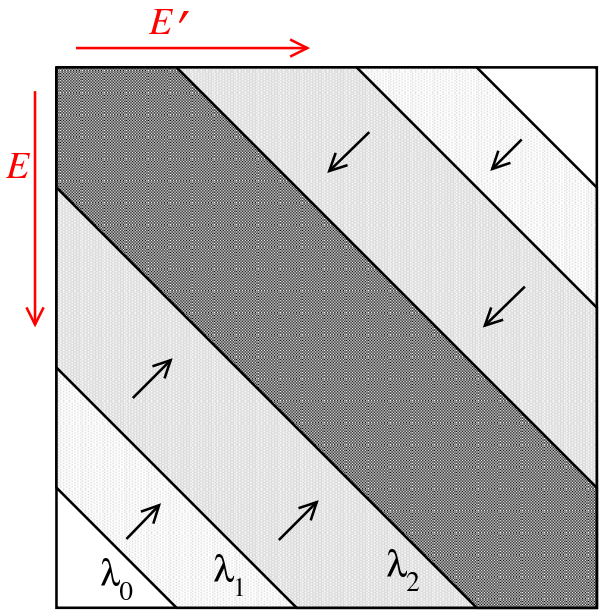
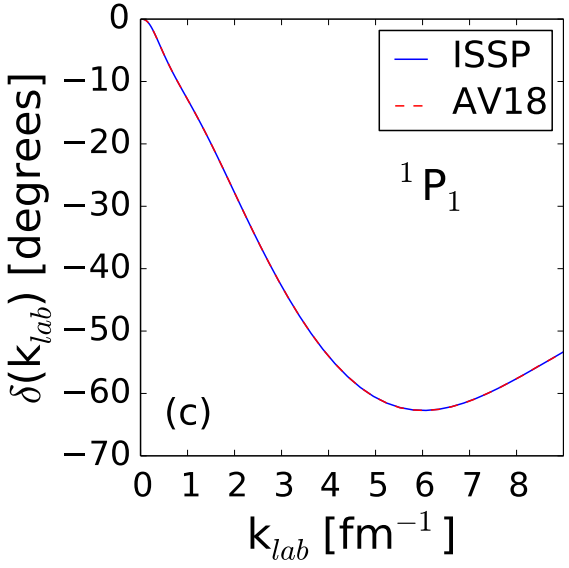
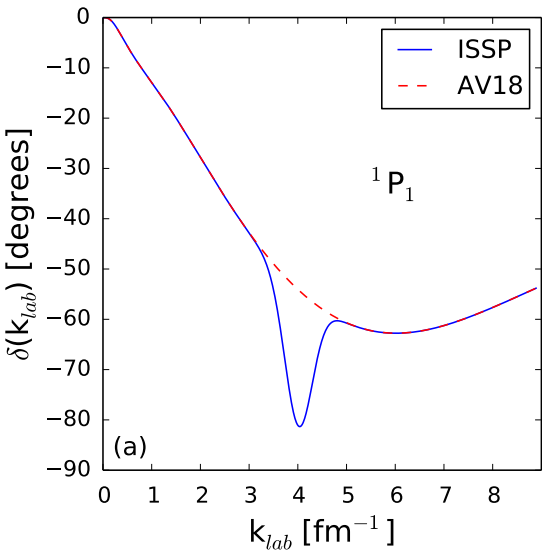
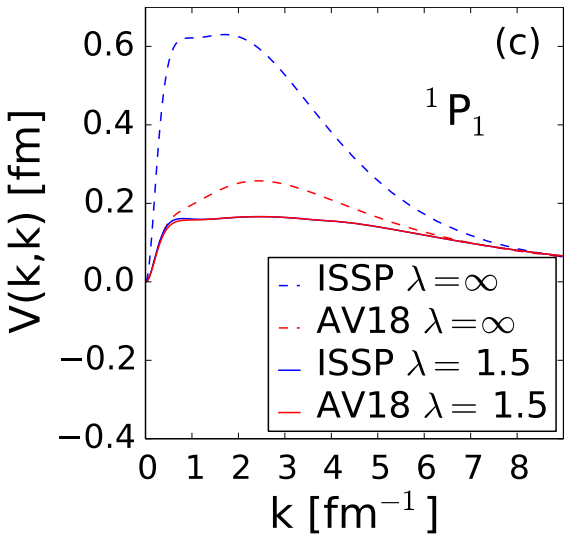
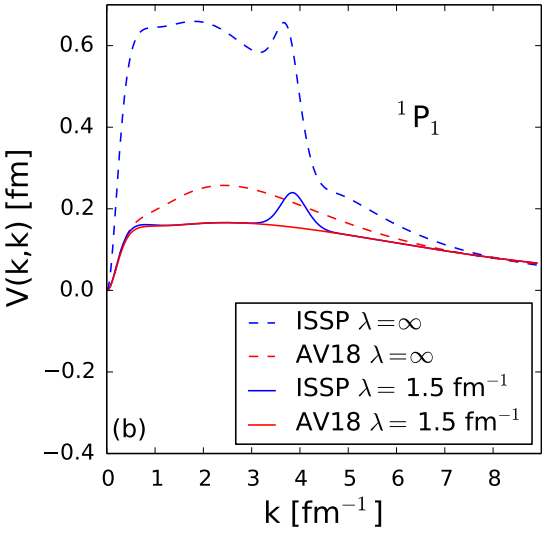










Outline: Lecture 3

Lecture 3: Effective field theory

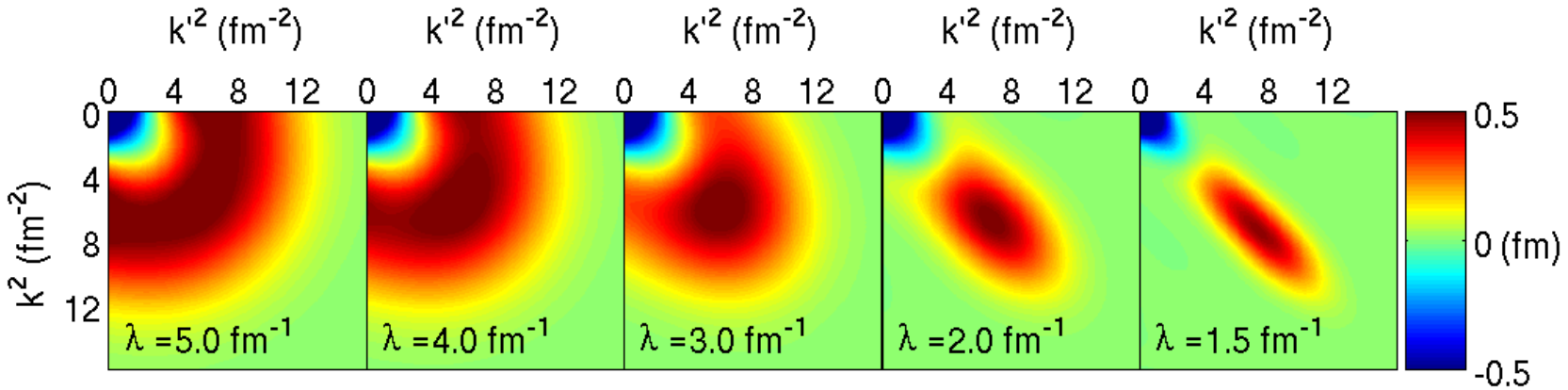
- Recap from lecture 2: How SRG works
- Motivation for nuclear effective field theory
- Chiral effective field theory
- Universal potentials from RG evolution
- Extra: Quantitative measure of perturbativeness

S. Weinberg on the Renormalization Group (RG)

- From "Why the RG is a good thing" [for Francis Low Festschrift]
  "The method in its most general form can I think be understood as a way to arrange in various theories that the degrees of freedom that you're talking about are the relevant degrees of freedom for the problem at hand."
- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - RG: shift between couplings and loop integrals to reduce logs
  - Nuclear: decouple high- and low-momentum modes
- Identifying universality in critical phenomena
  - RG: filter out short-distance degrees of freedom
  - Nuclear: evolve toward universal interactions
- Nuclear: simplifying calculations of structure/reactions
  - Make nuclear physics look more like quantum chemistry!
  - RG gains can violate conservation of difficulty!
  - Use RG scale (resolution) dependence as a *probe* or *tool*

Flow of different N^{3}LO chiral EFT potentials

- $^1S_0$ from N^{3}LO (500 MeV) of Entem/Machleidt



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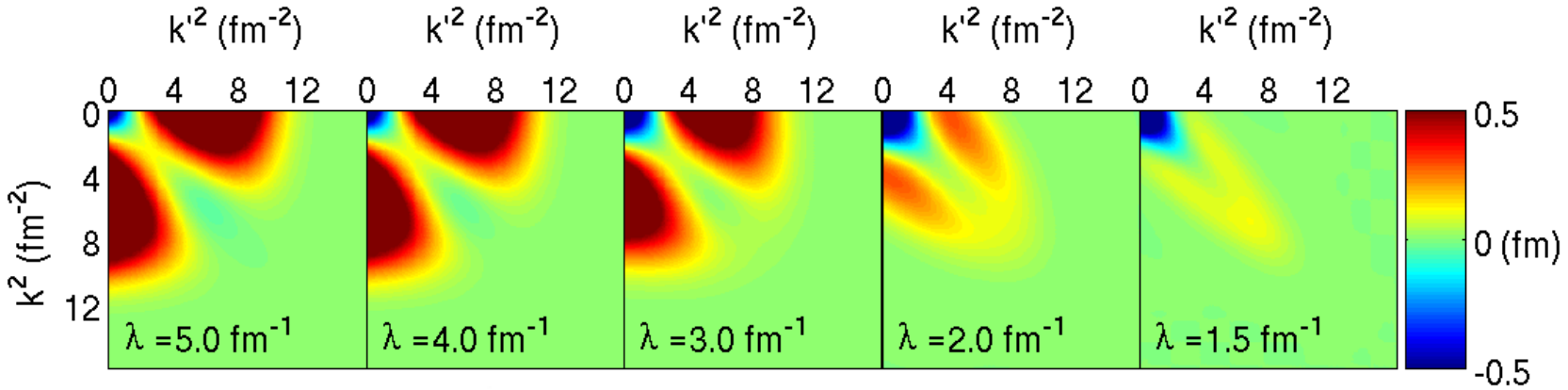

- Decoupling $\Longrightarrow$ perturbation theory is more effective

$$\langle k|V|k\rangle + \sum_{k'} \frac{\langle k|V|k'\rangle\langle k'|V|k\rangle}{(k^2 - k'^2)/m} + \cdots \Longrightarrow V_{ii} + \sum_j V_{ij} V_{ji} \frac{1}{(k_i^2 - k_j^2)/m} + \cdots$$

Flow of different N^{3}LO chiral EFT potentials

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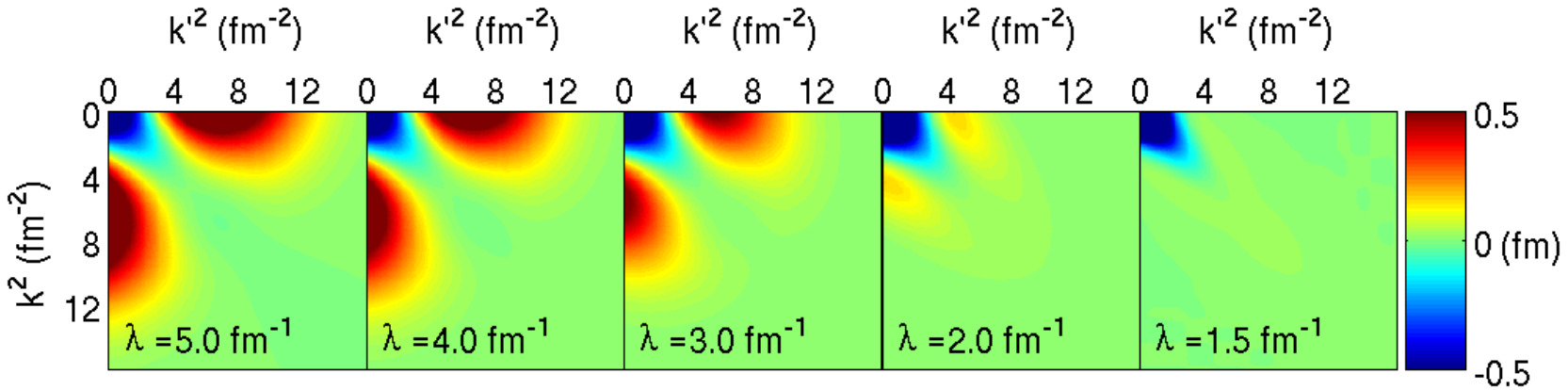

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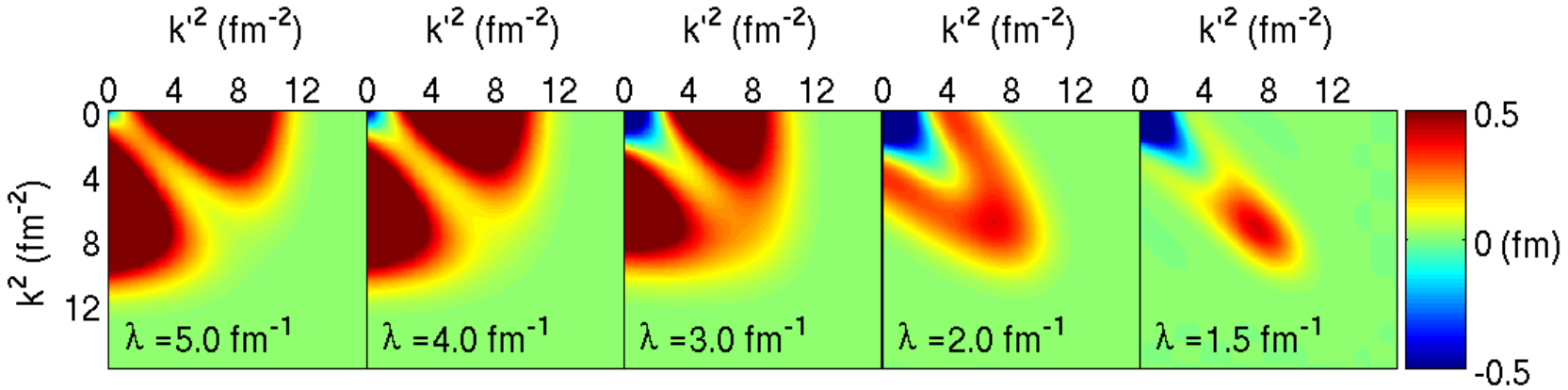

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Convergence of the Born series for scattering

- Consider whether the Born series converges for given z

$$T(z) = V + V\frac{1}{z - H_0}V + V\frac{1}{z - H_0}V\frac{1}{z - H_0}V + \cdots$$

- If bound state $|b\rangle$, series must diverge at $z = E_b$, where

$$(H_0 + V)|b\rangle = E_b|b\rangle \quad \Longrightarrow \quad V|b\rangle = (E_b - H_0)|b\rangle$$

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- For fixed E, generalize to find eigenvalue η_ν [Weinberg]

$$\frac{1}{E_b - H_0}V|b\rangle = |b\rangle \qquad \Longrightarrow \qquad \frac{1}{E - H_0}V|\Gamma_\nu\rangle = \eta_\nu|\Gamma_\nu\rangle$$

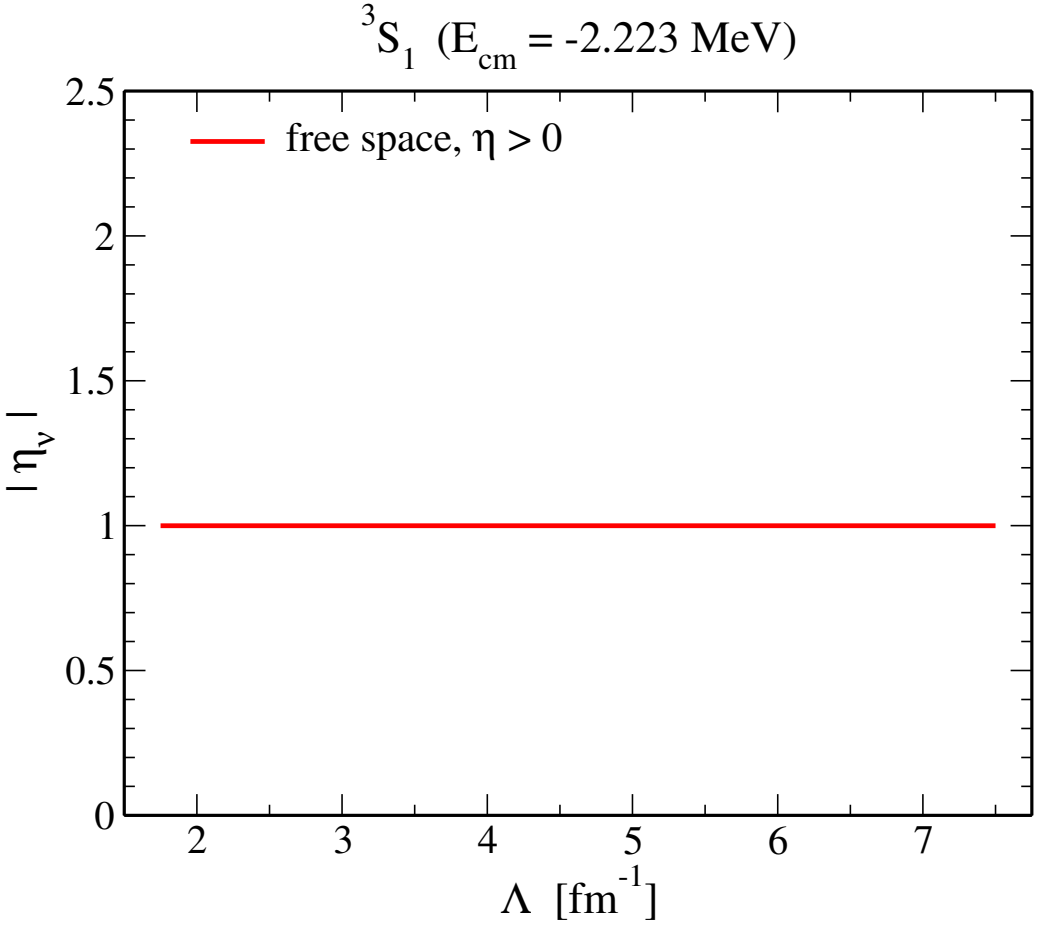
- From T applied to eigenstate, divergence for $|\eta_\nu(E)| \geq 1$:

$$T(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle(1 + \eta_\nu + \eta_\nu^2 + \cdots)$$

$\Longrightarrow$ $T(E)$ diverges if bound state at E for V/η_ν with $|\eta_\nu| \geq 1$

Weinberg eigenvalues as function of cutoff Λ/λ

- Consider $\eta_\nu(E = -2.22\,\text{MeV})$
- Deuteron $\Longrightarrow$ attractive eigenvalue $\eta_\nu = 1$
  - $\Lambda \downarrow \Longrightarrow$ unchanged



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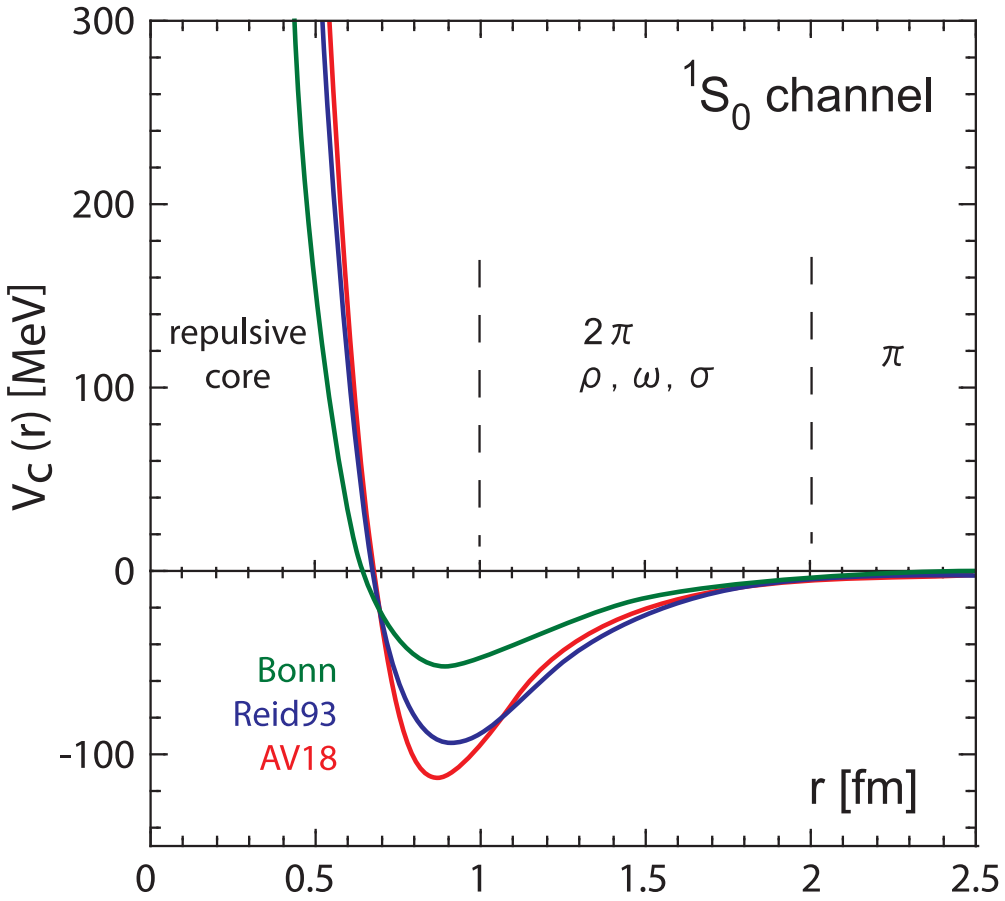

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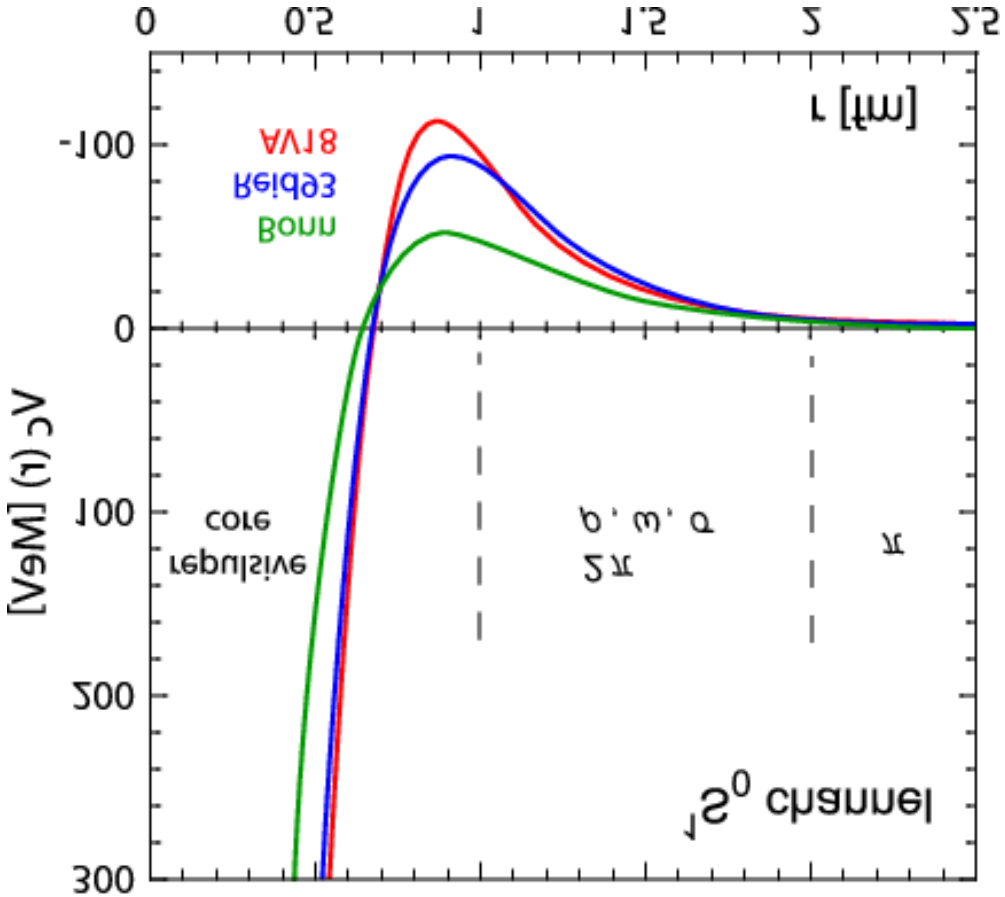

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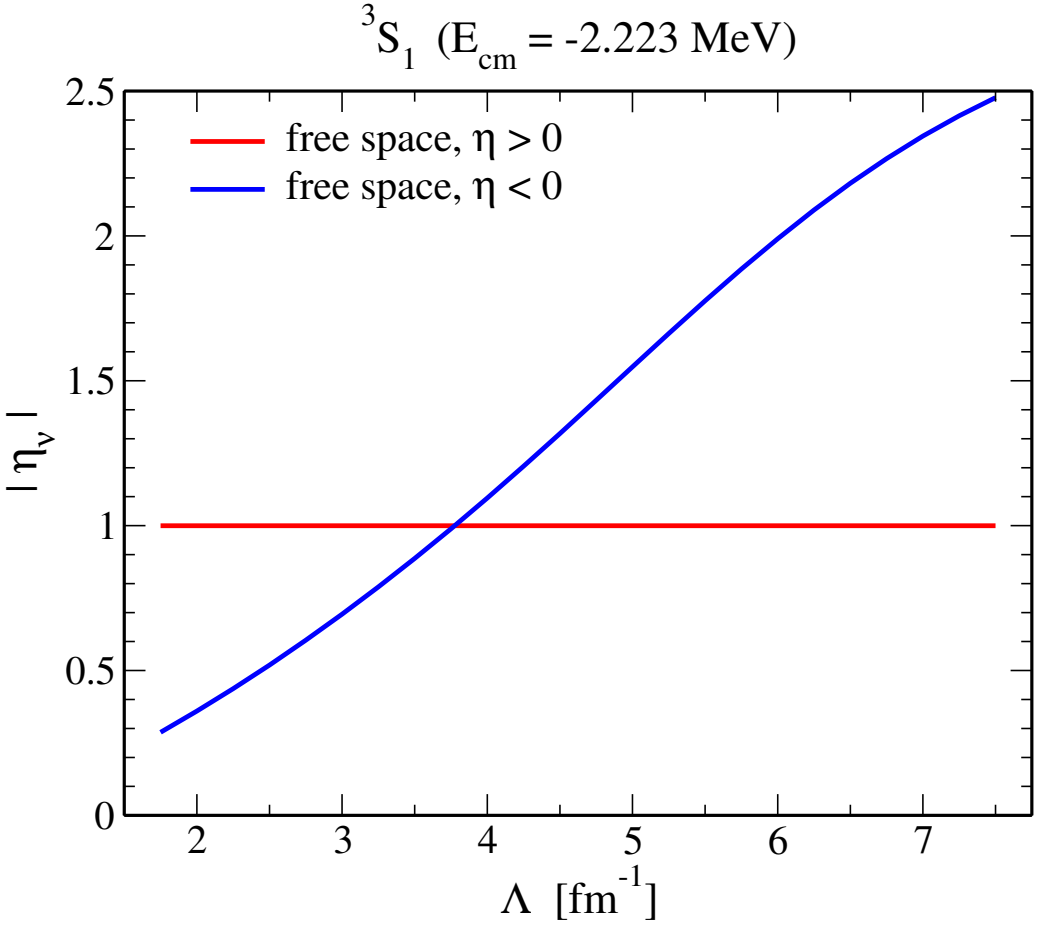
- Hard core $\Longrightarrow$ repulsive eigenvalue η_ν
  - $\Lambda \downarrow \Longrightarrow$ reduced



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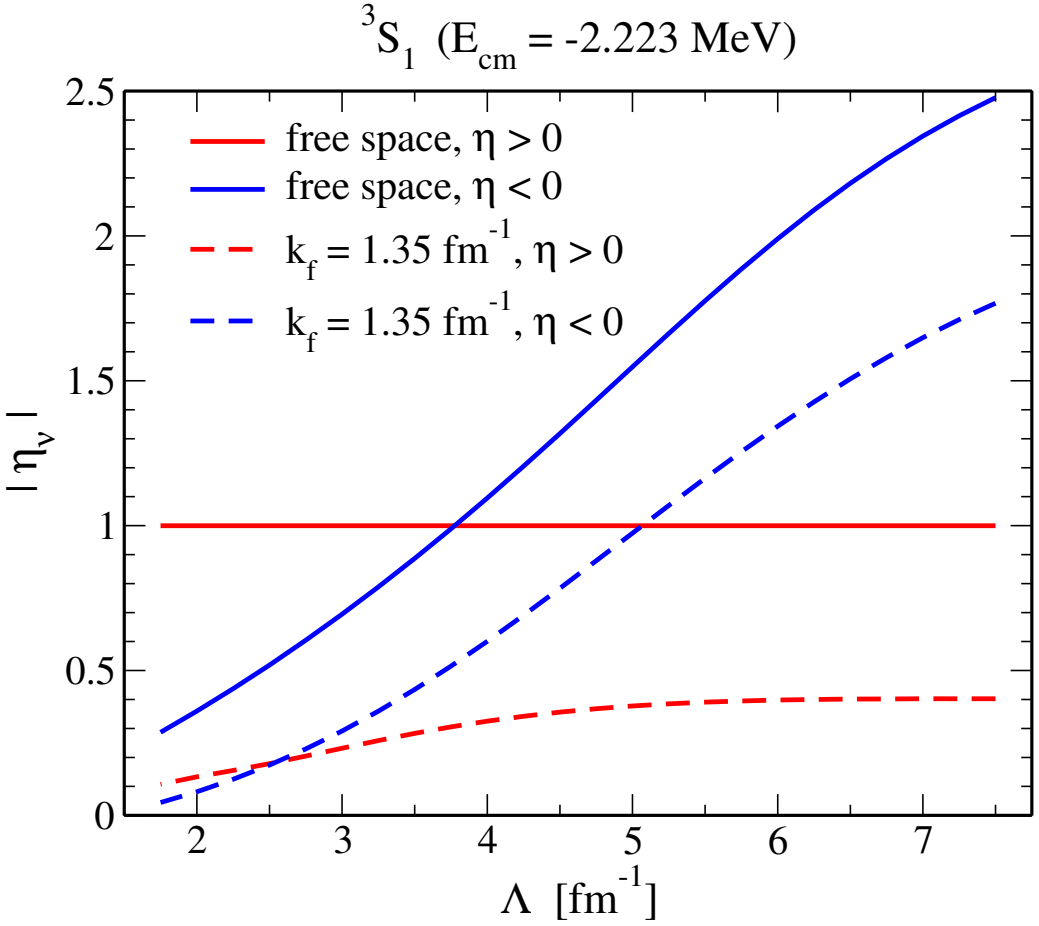
- Deuteron $\Longrightarrow$ attractive eigenvalue $\eta_\nu = 1$
  - $\Lambda \downarrow \Longrightarrow$ unchanged
- But η_ν can be negative, so $V/\eta_\nu \Longrightarrow$ flip potential
- Hard core $\Longrightarrow$ repulsive eigenvalue η_ν
  - $\Lambda \downarrow \Longrightarrow$ reduced
- In medium: both reduced
  - $\eta_\nu \ll 1$ for $\Lambda \approx 2\,\text{fm}^{-1}$

$\Longrightarrow$ perturbative (at least for particle-particle channel)



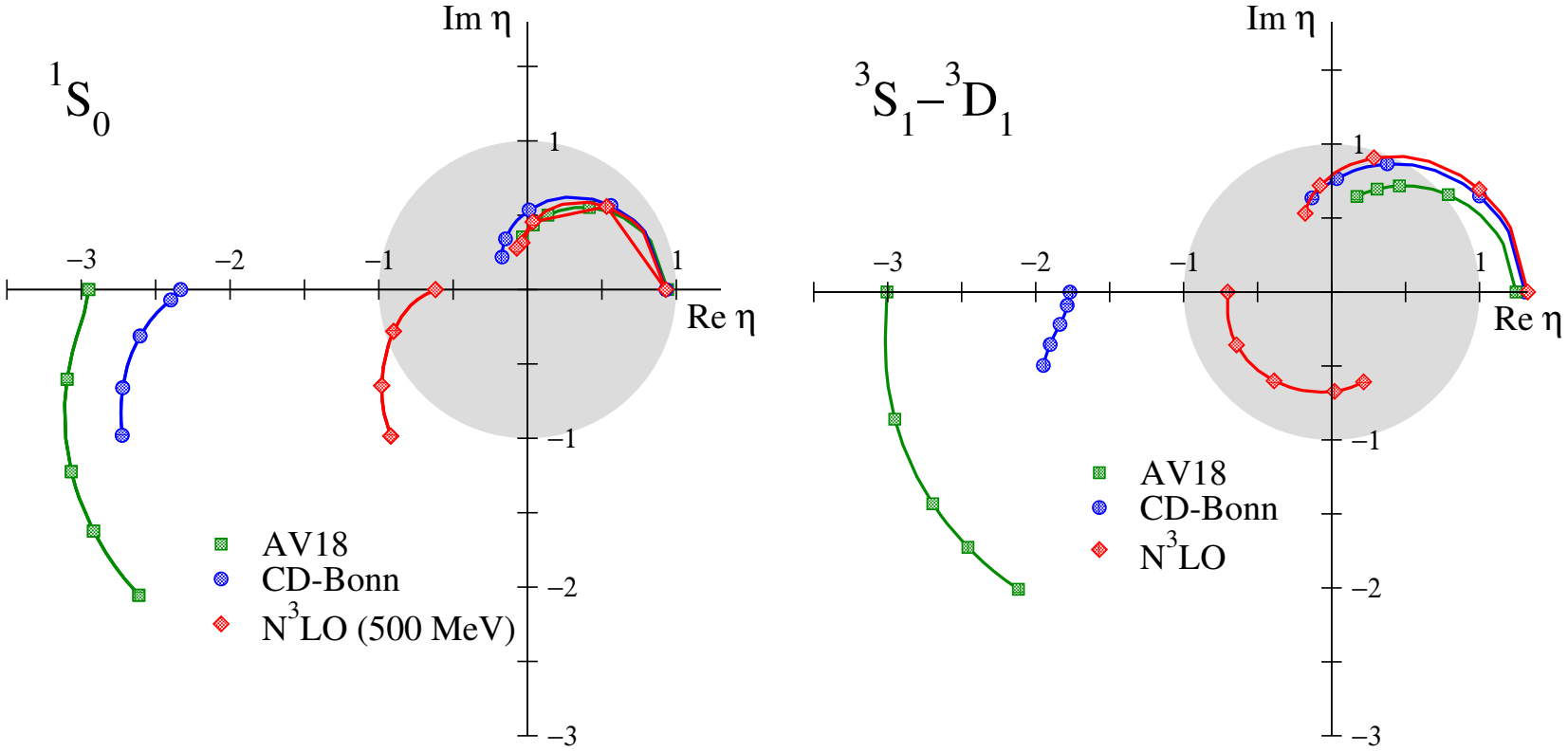
Weinberg eigenvalue analysis of convergence

Born Series: $T(E) = V + V\frac{1}{E-H_0}V + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V + \cdots$

- For fixed E, find (complex) eigenvalues $\eta_\nu(E)$ [Weinberg]

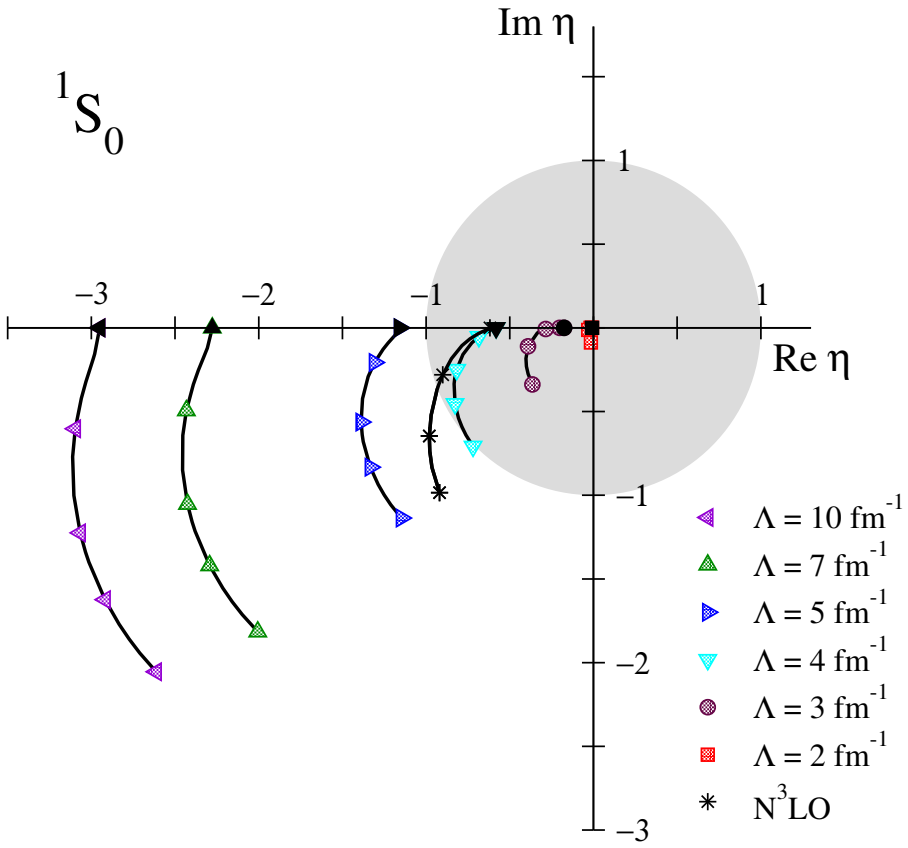
$$\frac{1}{E-H_0}V|\Gamma_\nu\rangle = \eta_\nu|\Gamma_\nu\rangle \quad \Longrightarrow \quad T(E)|\Gamma_\nu\rangle = V|\Gamma_\nu\rangle(1+\eta_\nu+\eta_\nu^2+\cdots)$$

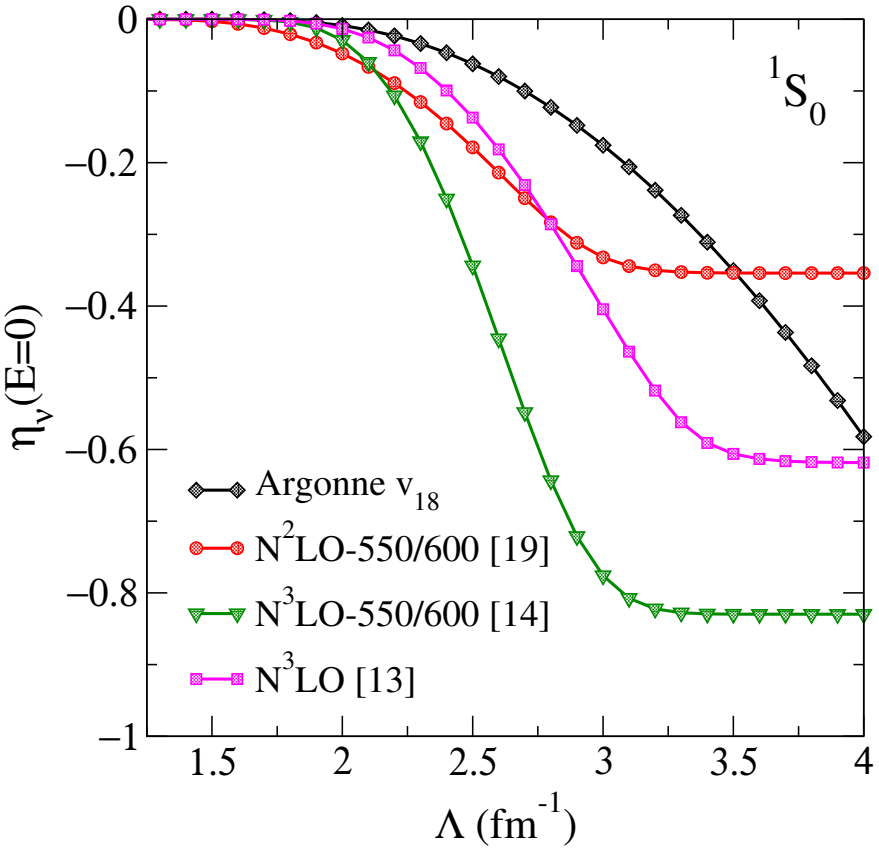
$\Longrightarrow$ T diverges if *any* $|\eta_\nu(E)| \geq 1$ [nucl-th/0602060]



Lowering the cutoff increases “perturbativeness”

- Weinberg eigenvalue analysis (repulsive) [nucl-th/0602060]





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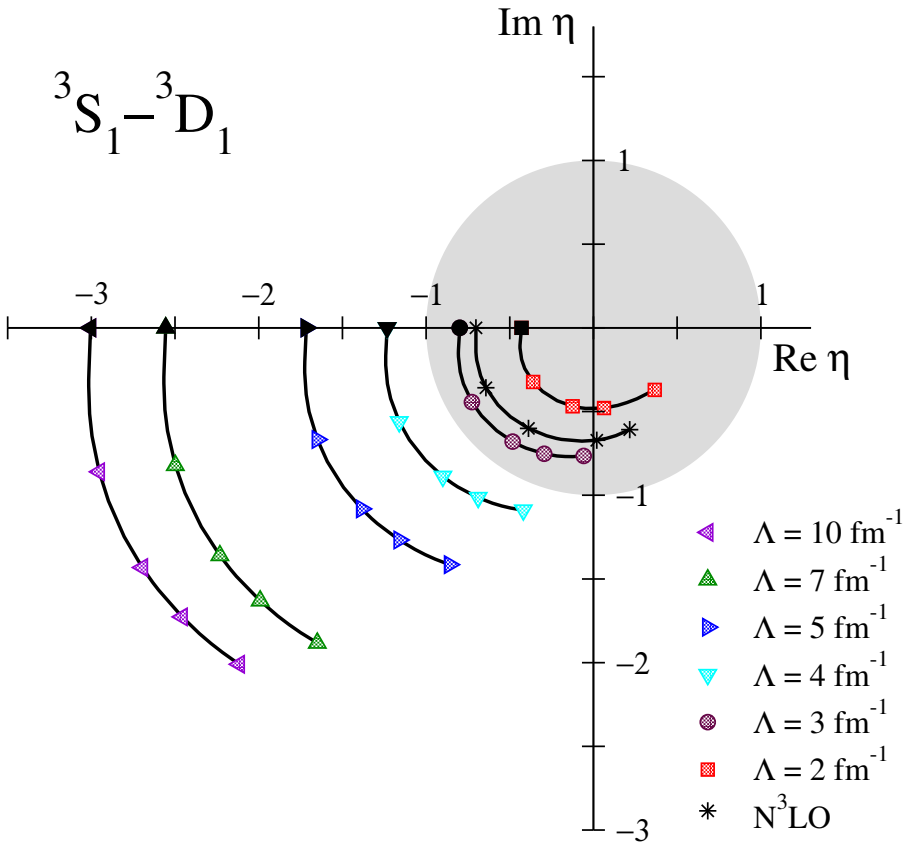
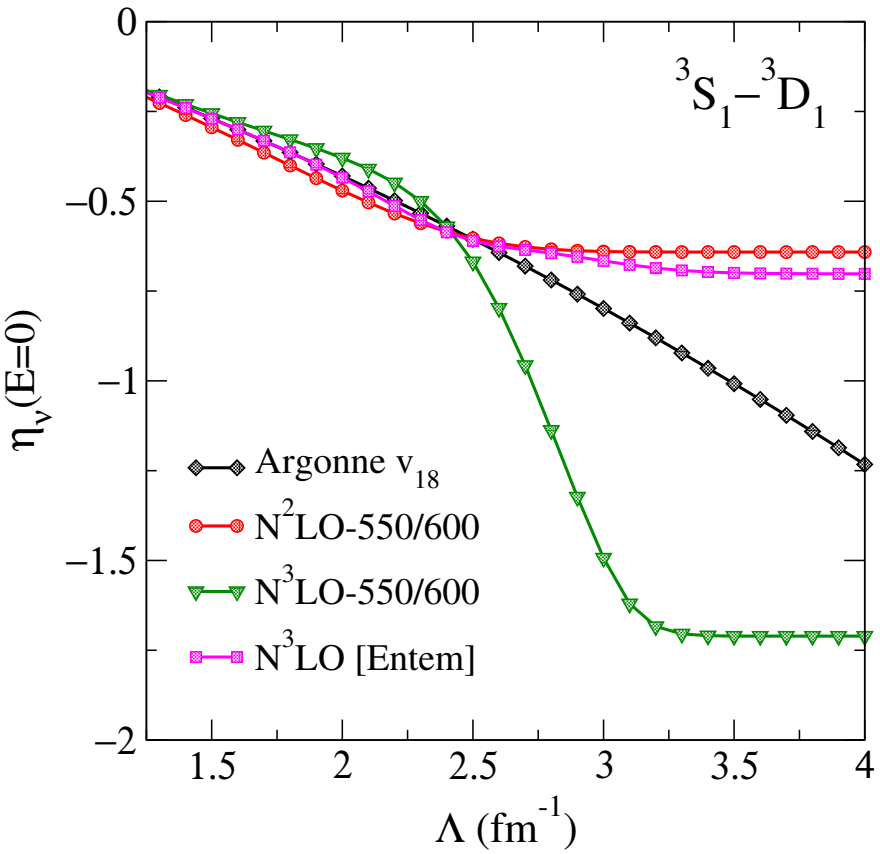
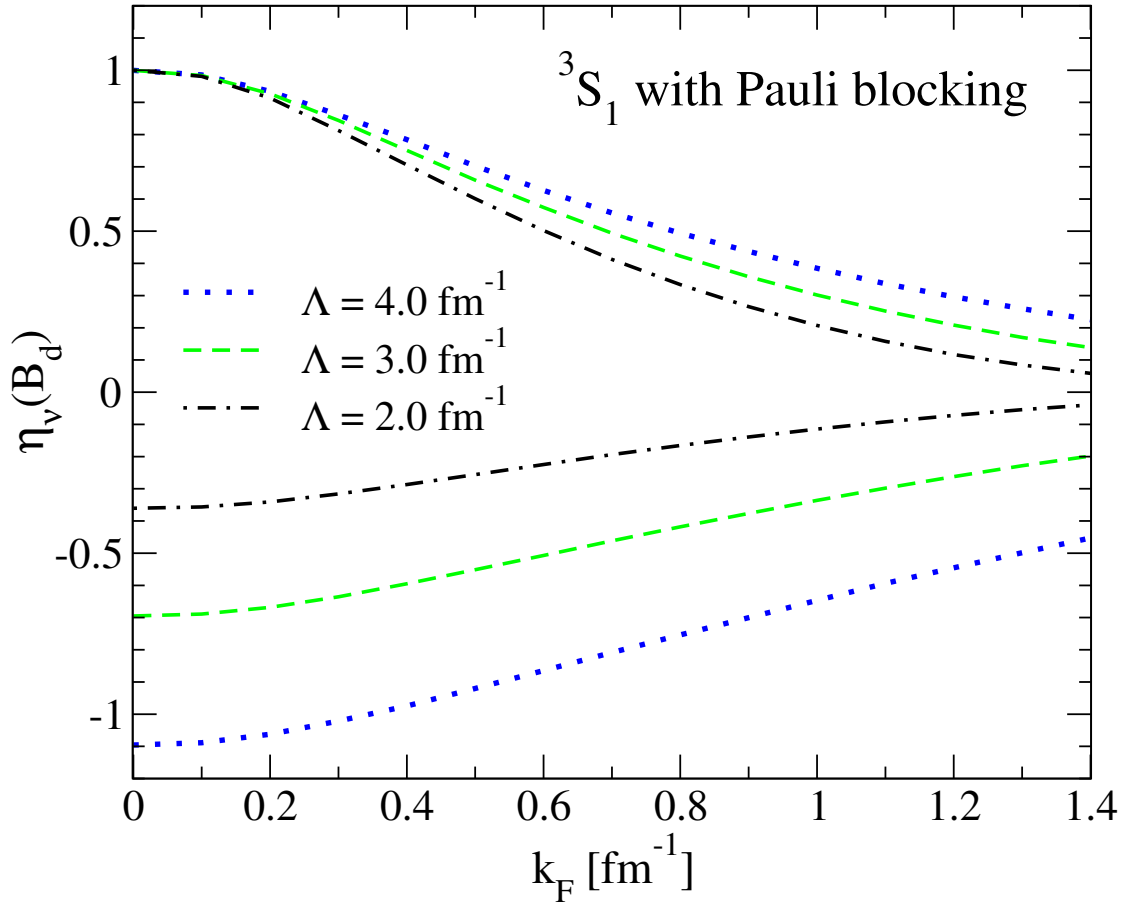




Lowering the cutoff increases "perturbativeness"

- Weinberg eigenvalue analysis
  (η_ν at -2.22 MeV vs. density)



- Pauli blocking in nuclear matter increases it even more!
  - at Fermi surface, pairing revealed by $|\eta_\nu| > 1$